Determination of the Complete Penetration Magnetic Field of a HTS Pellet from the Measurements of the Magnetic Field at its Top-Center Surface

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***Abstract*— In previous papers a new method to characterize HTS pellets has been presented This method is based on the measurement of their complete penetration magnetic flux density B_P. It allows determining the critical current density J_C and the n-value of the classical power law model. In this paper, a method of determination of B_P of a unique HTS pellet from the measurements of the magnetic field at its top-center surface is presented. This is a simpler and cheaper method than previous methods. It uses analytical calculation based on Biot and Savard law and numerical calculation based on power law. The maximum magnetic flux density produced by the SC pellet B_{ZSCM} is deduced from the measurement. B_P is deduced from B_{ZSCM} multiplying it by a coefficient analytically calculated. The influence of $J_C(B)$ on this method is presented and analyzed.**

***Index Terms*—Superconductors, magnetic field diffusion, critical current density.**

I. Introduction

A LOT of projects of superconducting motors, generators or magnetic levitation systems have been studying on the base of high temperature superconductor (HTS) bulks because of increasing of critical current density and improvement of cryogenics [1]-[7]. The critical state model (CSM) is commonly used [8] to calculate magnetization, strength and other physical parameters. In the CSM, the current density J can only be equals to 0 or the critical current density J_C independent of the rate of variation of the externally applied magnetic field. Thanks to CSM, J_C can be simply determined by experiments [9]-[10].

If a uniform axial magnetic field $B_a(t)$ is applied to a cylindrical HTS pellet, Fig. 1, the magnetic flux density at the center of the pellet $B_0(t)$ starts to rise after some time delay T_P, related to the moment at which B_a reaches B_P, Fig. 2. Biot–Savart law, assuming CSM with a magnetic field independent J_C, allows calculating an analytic expression for the complete penetration field, named B_{PB}, for cylinders, i.e. [10]-[12]:

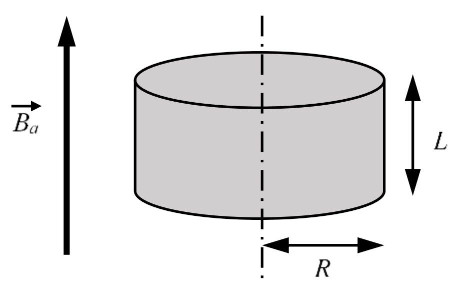

Fig. 1. Cylindrical bulk superconducting pellet submitted to a uniform axial magnetic field B_a(t).

$$B_{PB} = \frac{\mu_0 J_C . L}{4} . \ln\left(\frac{\sqrt{R^2 + \left(\frac{L}{2}\right)^2} + R}{\sqrt{R^2 + \left(\frac{L}{2}\right)^2} - R} \right) \quad (1)$$

where L is the length of cylinder and R is its radius.

The power law model (PLM) (2) is commonly used for HTS for temperatures above 50 K, instead of CSM [11], [12].

$$E = E_C \left(\frac{J}{J_C}\right)^n = \frac{E_C}{J_C} \cdot \left(\frac{J}{J_C}\right)^{n-1} J \quad (2)$$

The calculation of the magnetization of superconducting samples assuming PLM generally requires numerical simulations. The method of determination of the J_C and n parameters using PLM is given in [11]. It has been shown that the complete penetration magnetic field B_P depends on the n-value and the applied magnetic field rising rate V_b, Fig.2:

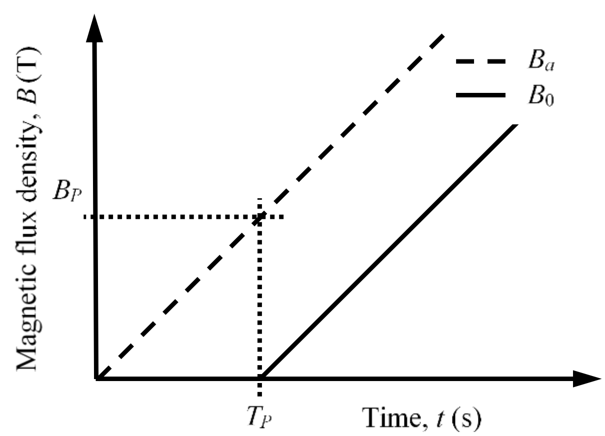

Fig. 2. Linearly growing applied magnetic field (B_a) and theoretical magnetic field (B_0) at the center of the pellet versus time (t).

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$$B_P = B_{PB}\left(1 + \frac{\alpha \ln V_b + \beta}{n}\right) \tag{3}$$

with $\alpha = 1.2$ and $\beta = 3.4$ [11].

In [12], it has been shown that there is no influence of geometrical parameters R and L on these values of α and β for commonly used HTS pellets.

Based on (3) PLM, J_C and n-value of a cylindrical sample are calculated from experimental measurements using two identical pellets, the magnetic field being measured by axial Hall probe between the two HTS pellets [11]. In this paper a method of determination of B_P of a unique HTS pellet from the measurements of the magnetic field at its top-center surface is presented. This could be a simpler and cheaper method than the previous method using two pellets. This method uses one Hall probe placed at a distance d from the top-center surface of the pellet (Fig. 3). In this case the issue is to know the relationship between the measured magnetic flux density by Hall probe and B_P. From B_{P1} and B_{P2} measured at two rise rates of applied magnetic field V_{b1} and V_{b2}, n-value, B_{PB} and finally J_C can be deduced [11], Fig. 4.

II. Calculations of the Relationship Between the Measured Magnetic Flux Density and B_P

A combination of numerical simulations and analytical calculations has been used to determine the relationship between the measured magnetic flux density by Hall probe and B_P.

A. Numerical simulations

Firstly, numerical simulations of a HTS pellet (1 cm radius R and 1 cm length L) submitted to linearly rising applied magnetic field as in [11]. This allows us to determine the magnetic flux density inside and outside the HTS pellet and the current density inside the HTS pellet.

Fig. 5 represents the distributions of J/J_C in a HTS pellet for three different V_b (0.1T/s, 10T/s and 1000T/s) for the same value of B_a. In these simulations the n-value chosen is 15, that is a weak n-value for HTS at 77K [11-15], J_C is constant and equals to 100 A/mm². As shown in Fig. 5 and confirmed by other simulations, whatever V_b, geometrical parameters and sufficiently high n-value ($n > 10$), J/J_C is almost constant all over J exists. However J/J_C depends on n-value and V_b that are dynamic parameters. So in complete penetration case when B_a equals B_P, J/J_C can be considered constant all over the HTS pellet as in the case of Bean model.

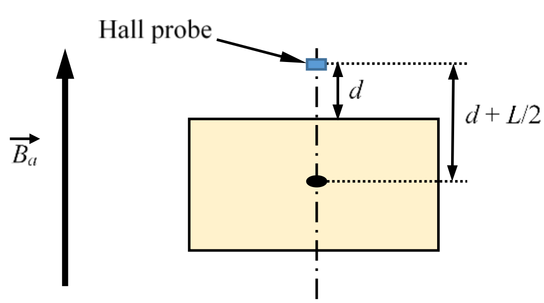

Fig. 3. Hall probe location above the HTS pellet.

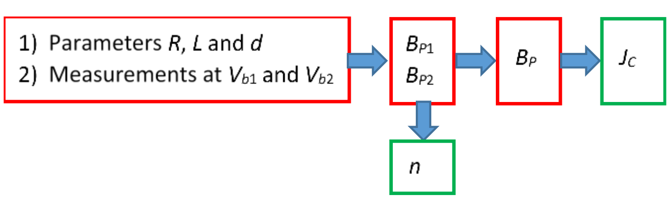

Fig. 4. Determination of J_C and *n-value*.

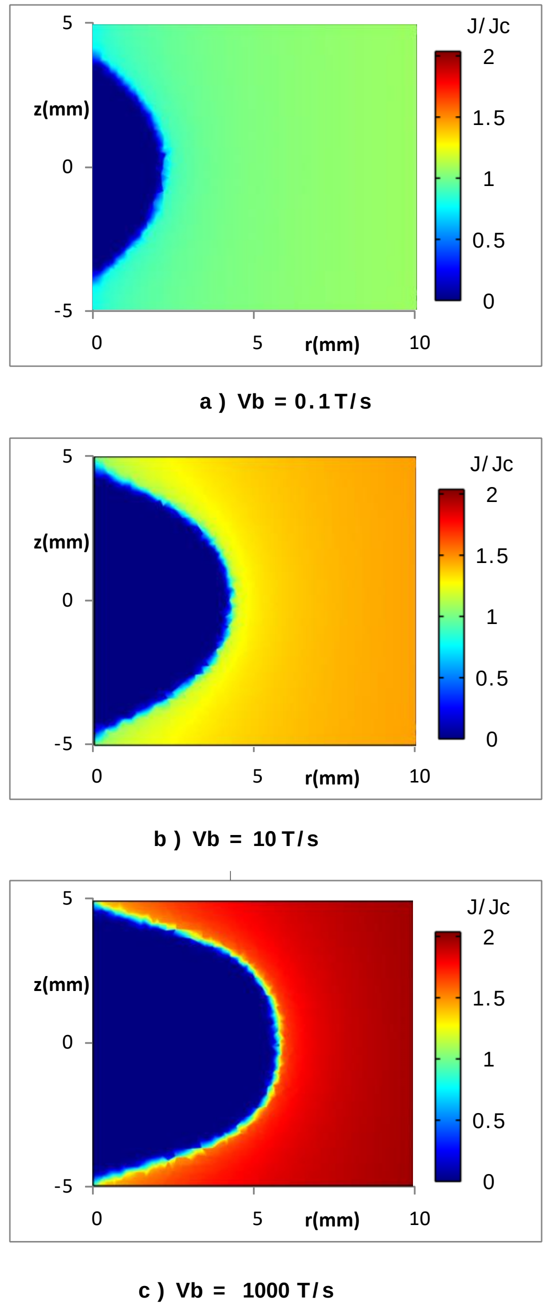

Fig. 5. J/J_C distributions in the pellet for different rise rates for the same applied magnetic field.

The magnetic field at the center pellet is named $B_0(t)$ and the magnetic field at the position of the Hall probe is named $B_z(d,t)$. For $n = 15$ $B_a(t)$ and $B_z(d{=}0.5\text{mm}, t)$ are represented in Fig. 6

for constant J_C and equals to 100 A/mm². The difference $B_a(t)–B_z(d=0.5\text{mm},t)$ represented in Fig. 6 is the magnetic flux density produced by the induced current inside the superconducting pellet named $B_{ZSC}(d, t)$. Fig.6 shows that $B_{ZSC}(d=0.5\text{mm}, t)$ reaches a maximum value B_{ZSCM} before $B_a(t)–B_0(t)$ reaches B_P. B_{ZSCM} is directly linked to B_P because in complete penetration case, J/J_C is almost constant all over the HTS pellet and the HTS pellet can produce B_P and not more B_P. The relationshipe between B_{ZSCM} and B_P is linked to geometrical parameters R and like with CSM as it will be shown in part II-*B*.

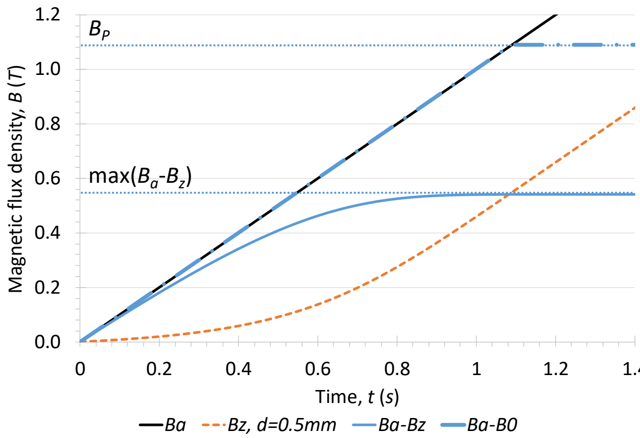

Fig. 6. Numerical calculation of $B_a(t)–B_z(d=0.5\text{mm}, t)$ and $B_a(t)–B_0$, with $n = 15$, $V_b = 1$T/s.

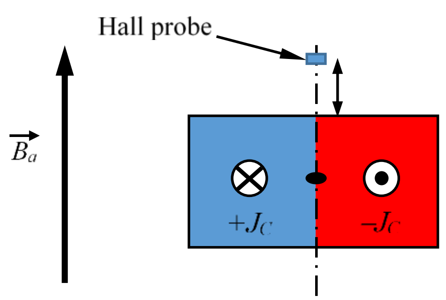

Fig. 7. Complete penetration of the current density with CSM.

B. Analytical calculation of the magnetic flux density along the pellet axis

In the case of the magnetic field complete penetration and CSM, the magnetic flux density along the axis of the cylindrical pellet B_{ZSCB} can be calculated using Biot–Savart law in the same way as the B_{PB} (1). B_{ZSCB} is produced by homogenous and constant critical current J_C in the volume of the pellet (Fig. 7).

The formula of B_{ZSCB} is obviously similar to (1):

$$B_{ZSCB}(d,R,L)=$$

$$\frac{\mu_0.J_C}{4}\left[(d+L).\ln\left(\frac{1+\dfrac{R}{\sqrt{(d+L)^2+R^2}}}{1-\dfrac{R}{\sqrt{(d+L)^2+R^2}}}\right)\right] \tag{4}$$

$$-\frac{\mu_0.J_C}{4}\left[d.\ln\left(\frac{1+\dfrac{R}{\sqrt{d^2+R^2}}}{1-\dfrac{R}{\sqrt{d^2+R^2}}}\right)\right]$$

Fig. 8 shows B_{ZSCB} versus d for realistic values of d due the thickness of Hall probes.

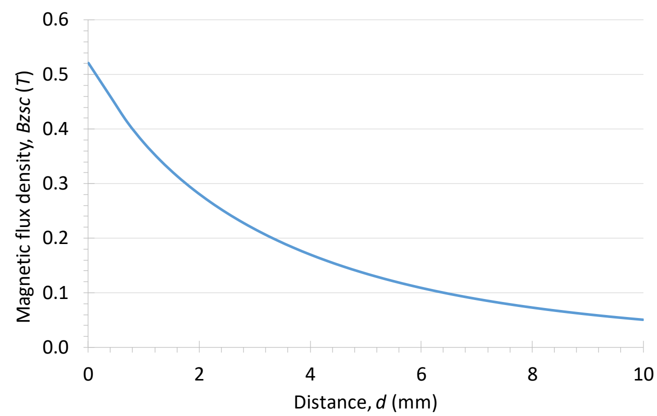

Fig. 8. Analytical calculation of B_{ZSC} along the axis of the pellet in complete penetration case.

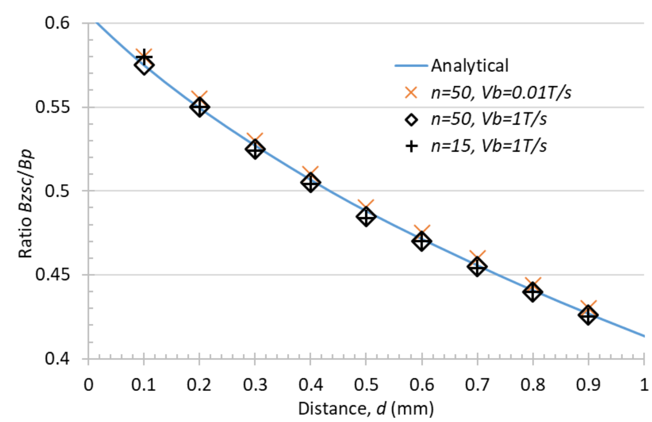

Fig. 9. Comparison of numerical calculation of B_{ZSCM}/B_P and analytical calculation of B_{ZSCB}/B_{PB}.

C. Comparison of analytical and numerical calculation of B_{ZSCM}/B_P for complete penetration case

The last question is to know if the numerically calculated ratio $B_{ZSCM}(d)/B_P$ and the analytically calculated ratio $B_{ZSCB}(d)/B_{PB}$ are equivalent or different. $B_{ZSCM}(d)/B_{PB}$ has been numerically calculated with various *n*-value from 15 to 50 and various magnetic field rise rate from 1 T/s to 1000 T/s. Fig. 9

clearly shows that these two ratios are equivalent for these numerous values of both these parameters. It means that $B_{ZSCB}(d)/B_{PB}$ is relevant to deduce B_P from $B_{ZSCM}(d)$ drawn from measurement whatever n-value and V_b:

$$B_P = \frac{B_{ZSCM}(d)_{measured}}{\left(B_{ZSCB}(d)/B_{PB}\right)_{analytical}} \quad (5)$$

It should be added that the ratio $B_{ZSCB}(d)/B_{PB}$ is J_C independent and depends only on geometrical parameters R and L.

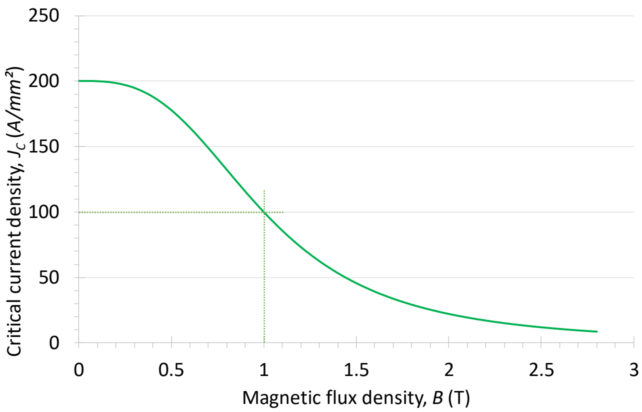

Fig. 10. $J_C(B)$ used in numerical simulations.

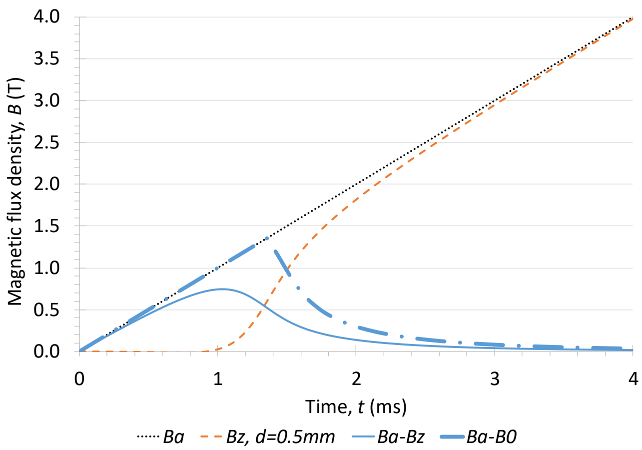

Fig. 11. Numerical calculation of $B_a(t)$–$B_z(d$=0.5mm, $t)$ and $B_a(t)$–B_0, with n = 15, V_b = 1T/s and $J_C(B)$.

D. Influence of Jc(B) on results of this determination method

The most important assumption made in this method is constant J_C. In this section the influence of $J_C(B)$ is analyzed. Numerical simulations of a HTS pellet submitted to linearly rising applied magnetic field has been made using $J_C(B)$ as:

$$Jc(B) = \frac{J_{C0}}{1 + \left(|B|/B_{C0}\right)^3} \quad (6)$$

with J_{C0} = 200 A/mm² and B_{C0} = 1 T.

J_{C0} and B_{C0} are arbitrarily chosen to have huge variation of $J_C(B)$, represented in Fig. 10, around B_P = 1.1 T which has been previously calculated with constant J_C. This numerical calculation has been made with the same dynamics parameters as those in Fig. 6, namely n = 15 and V_b = 1 T/s.

The results of this simulations shown in Fig. 11 that the magnetic flux density produced by HTS pellet $B_{ZSC}(t)$ equals to $B_a(t)$–$B_z(t)$ presents a unique maximum value B_{ZSCM} instead of a saturation plateau as in the case of constant J_C, in Fig. 6. B_P is the maximum value of $B_a(t)$–$B_0(t)$ and equals 1.35 T, in Fig. 11. For d = 0.5 mm, the calculated B_{ZSCM} is 0.745 T and it corresponds to $B_{ZSCM}(d)/B_P$ = 0.55 instead of 0.49 for constant J_C, Fig. 9. This difference is due to the non-homogeneity of the magnetic flux density and current density inside the HTS pellet. In this case (4) has to be recalculated which is not analytically possible with Biot-Savart law and can only be made by numerical simulations. This remaining work has to be done in the next step of our work.

III. Conclusion

The main idea of this paper is to determine B_P of a unique pellet HTS pellet from the measurement of the maximum magnetic flux density at the top-center surface B_{ZSCM} of this pellet. Analytical calculated ratio $B_{ZSCB}(d)/B_{PB}$ similar as numerically calculated ratio $B_{ZSCM}(d)/B_{PB}$ allows us to simply determine B_P. The influence of $J_C(B)$ of this B_P, n-value and J_C determination method has to be studied further in next studies to improve the relevance of our method. The comparison of this method with previous ones [13]-[15] has to be done too.

Acknowledgment

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