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CONSISTENT ANISOTROPIC WIENER FILTERING FOR AUDIO SOURCE SEPARATION

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ABSTRACT

For audio source separation applications, it is common to apply a Wiener-like filtering to a time-frequency (TF) representation of the data, such as the short-time Fourier transform (STFT). This approach, in which the phase of the original mixture is assigned to each component, is limited when sources overlap in the TF domain. In this paper, we propose to improve this technique by accounting for two properties of the phase. First, we model the sources by anisotropic Gaussian variables: this model accounts for the non-uniformity of the phase, and permits us to incorporate some prior information about the phase that originates from a sinusoidal model. Second, we exploit the STFT consistency, which is the relationship between STFT coefficients that is due to the redundancy of the STFT. We derive a conjugate gradient algorithm for estimating the corresponding filter, which we refer to as the consistent anisotropic Wiener filter. Experiments conducted on music pieces show that the proposed approach yields results similar to or better than the state-of-the-art with a dramatic reduction of the computation time.

Index Terms— Wiener filtering, phase recovery, sinusoidal modeling, STFT consistency, audio source separation.

1. INTRODUCTION

Audio source separation consists in extracting underlying components called sources that add up to form an observable audio signal called mixture. Many separation techniques act on a time-frequency (TF) representation of the data, such as the short-term Fourier transform (STFT), because the structure of sound is more prominent in that domain. Most methods, whether based on graphical models [1], non-negative matrix factorization [2], or deep neural networks [3, 4], only process some function of the STFT modulus (e.g., magnitude, power, or log-magnitude spectrogram), discarding the phase information. However, when it comes to resynthesizing time-domain signals, an estimate for the phase of the corresponding complex-valued STFT is necessary [5, 6].

In the single-channel source separation framework, a common practice consists in applying a Wiener-like filtering [7], which assigns the phase of the mixture to each extracted component. Such a filter, which is optimal in a minimum mean square error (MMSE) sense under a Gaussian [7] or stable [8] assumption, originates from the observation that the phase appears as uniformly distributed [9]. However, even if this filter leads to quite satisfactory results in practice [10, 7], it has been pointed out [11] that when sources overlap in the TF domain, it is responsible for residual interference and artifacts in the separated signals.

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One approach to obtaining better phase estimates is to promote consistency [12]: indeed, a complex-valued matrix (for instance the output of a Wiener filter) may in general not be consistent, that is, it may not correspond to the STFT of an actual time-domain signal. Such methods [13, 12] iteratively compute a complex-valued matrix in order to maximize its consistency. Some recent works [14, 15, 16] attempted to combine Wiener filtering and consistency-based techniques in a unified framework for audio source separation. Consistent Wiener filtering [16] has so far been shown to be the most promising candidate for this task.

Alternatively, phase recovery can be performed by using phase models based on signal analysis. For instance, the widely used model of mixtures of sinusoids [17, 18] leads to explicit constraints for phase reconstruction that are based on the relationships between adjacent TF bins [19]. Such an approach has been exploited for time-stretching in the phase vocoder algorithm [20], as well as for speech enhancement [21, 22], audio restoration [19], and source separation [23]. In [24], we introduced an anisotropic Gaussian (AG) model in which the phase is no longer uniform, which allows us to incorporate some prior information about the phase that arises from a sinusoidal model. We derived an MMSE estimator which generalizes Wiener filtering to AG variables.

In this paper, we propose to combine these two approaches by exploiting both a consistency constraint and some phase information based on a signal model. We propose to address this issue by extending the consistent Wiener filtering to the AG case. Our approach consists in minimizing an objective cost function which penalizes the reconstruction error in the AG model, to which is added a regularization term which promotes consistency. This function is minimized by means of the preconditioned conjugate gradient algorithm. Experiments conducted on realistic music signals for a vocals/accompaniment separation task show that exploiting those two phase constraints within a unified framework outperforms both approaches taken separately.

This paper is organized as follows. Section 2 presents the generalized anisotropic Wiener filtering and details the estimation of the sources under a consistency constraint. Section 3 experimentally validates the potential of this method for an audio source separation task. Finally, Section 4 draws some concluding remarks.

2. CONSISTENT ANISOTROPIC WIENER FILTERING

2.1. Anisotropic Gaussian model

Let \( X \in \mathbb{C}^{F \times T} \) be the STFT of a single-channel audio signal. \( X \) is the linear and instantaneous mixture of \( J \) sources \( S_j \), such that for all TF bin \( ft \), \( X_{ft} = \sum_j S_{jft} \). Since all TF bins are treated similarly, we remove the indices \( ft \) when appropriate for more clarity. We assume that each source \( S_j \) follows a complex normal distri-
bution: \( S_j \sim \mathcal{N}(m_j, \gamma_j, c_j) \), where \( m_j, \gamma_j \) and \( c_j \) are the mean, variance and relation term of \( S_j \). The covariance matrix is:

\[
\Gamma_j = \begin{pmatrix} \gamma_j & c_j \\ c_j & \gamma_j \end{pmatrix},
\]

where \( \bar{z} \) denotes the complex conjugate of \( z \). Many previous studies [7, 16, 25] model the sources as circular-symmetric (or isotropic) variables [7] (i.e., such that \( m_j = c_j = 0 \)), which is equivalent to assuming that the phase of each source is uniformly distributed.

In this paper, we adopt a different standpoint, originally developed in [24]: we model the source signals by mixtures of sinusoids, which leads to explicit relationships between the phases of adjacent TF bins [19] and therefore to some prior phase estimate \( \phi_j \). Assuming that the phase of each source is uniformly distributed, it can be shown [19] that the phase follows the unwrapping equation:

\[
\phi_{j,t} = \phi_{j,t-1} + 2 \pi l \nu_{j,t},
\]

where \( l \) is the hop size (in samples) of the STFT and \( \nu_{j,t} \) is the normalized frequency in channel \( f \). We then consider that the phases should be distributed around the values \( \phi_j \) with a concentration parameter \( \kappa \in [0, +\infty] \). Thus, we propose to structure the moments of the distribution as follows:

\[
m_j = \lambda \sqrt{\bar{\tau}} e^{i\phi_j}, \quad \gamma_j = (1 - \lambda^2) v_j, \quad \text{and} \quad c_j = \rho v_j e^{i2\phi_j},
\]

where \( \lambda \) and \( \rho \) are defined as in [24], and \( v_j \) is an estimate of the source power \( |S_j|^2 \). The relation terms \( c_j \) are non-zero in general, which conveys the property of anisotropy of the distribution, hence the name of anisotropic Gaussian (AG) model. Finally, \( X \sim \mathcal{N}(m_X, \gamma_X, c_X) = \mathcal{N}(\sum_j m_j, \sum_j \gamma_j, \sum_j c_j) \), and \( \Gamma_X = \sum_j \Gamma_j \).

2.2. MMSE estimation without constraint

We seek to obtain an estimator of the sources for performing the separation task. We consider the posterior distribution of the sources given the mixture. Due to the mixing constraint, the conditional distribution lies on a subspace of dimension \( J' = J - 1 \), so we focus on a subset of free variables. Without loss of generality, we consider the first \( J' \) sources as free variables given the mixture and denote them as \( S = [S_1, ..., S_{J'}]^T \) in each TF bin \( f,t \), where \( J' \) denotes the transpose. It can be shown [26] that \( S|X \) follows a multivariate complex normal distribution with mean vector \( \mu = [\mu_1, ..., \mu_{J'}]^T \) such that:

\[
\bar{\mu}_j = m_j + \Gamma_j \Gamma_X^{-1} (X - \bar{X}),
\]

where \( \bar{x} = (x - \bar{x})^T \). The posterior covariance matrix is:

\[
\Xi = \begin{pmatrix} \Gamma_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \Gamma_{J'} \end{pmatrix}^{-1} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \vdots \\ \Gamma_{J'} \end{pmatrix}^T.
\]

In particular, the posterior covariance matrix of each source is \( \Gamma_j = \Gamma_j - \Gamma_j \Gamma_X^{-1} \Gamma_j \). Using the Woodbury identity, we obtain the precision matrix \( \Lambda \) defined as the inverse of the covariance matrix:

\[
\Lambda = \Xi^{-1} = \begin{pmatrix} \Gamma_1^{-1} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \Gamma_{J'}^{-1} \end{pmatrix} + \begin{pmatrix} 0 & \cdots & 0 & \Gamma_{J'}^{-1} \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \Gamma_1^{-1} & \cdots & \cdots & \Gamma_{J'}^{-1} \end{pmatrix}.
\]

Therefore, the negative log-likelihood of the posterior distribution \( -\log p(S|X) \) is equal, up to an additive constant and to a positive scaling factor, to the following quadratic loss function:

\[
\Psi(S) = \sum_{f,t} (S_{f,t} - \mu_{f,t})^H \Lambda_{f,t} (S_{f,t} - \mu_{f,t}),
\]

where \( S = [S_1, S_2, ..., S_J]^T \) and \( \mu \) denotes the conjugate transpose. We focus on the case \( J = 2 \) (i.e., \( J' = 1 \)). This corresponds to many source separation applications where only 2 sources interact, such as speech/noise or singing voice/musical accompaniment. Moreover, the general case can be reduced to this special case by considering in turn each source against all others. Since in this case \( S_{f,t} \) reduces to \( S_{f,t} \), we shall remove the index \( j \) = 1 for clarity.

The cost function (7) then rewrites:

\[
\Psi(S) = \sum_{f,t} (S_{f,t} - \mu_{f,t})^H \Lambda_{f,t} (S_{f,t} - \mu_{f,t}),
\]

where \( \Lambda_{f,t} = \Gamma_{1,f,t}^{-1} + \Gamma_{J',f,t}^{-1} = \Gamma_{f,t}^{-1} \). Setting the gradient of \( \Psi \) in (8) w.r.t. \( S_{f,t} \) to 0 leads to the MMSE solution: \( S_{f,t} = \mu_{f,t} \forall f,t \).

2.3. Consistency constraint

When the STFT is computed using overlapping analysis windows (which is usual in practice), it is a redundant TF representation which implies that certain relationships must hold between its TF coefficients. This results in the fact that not all matrices in \( \mathbb{C}^{F \times T} \) are the STFT of a time-domain signal. We will then say that a matrix \( S \) is consistent [12] if it is equal to the STFT of its inverse STFT, or, equivalently, if \( \mathcal{F}(S) = 0 \), where:

\[
\forall S \in \mathbb{C}^{F \times T}, \mathcal{F}(S) = S - \text{STFT} \circ \text{iSTFT}(S).
\]

The Wiener filter output does not generally satisfy this constraint, so that \( \text{STFT} \circ \text{iSTFT}(\mu) \) no longer minimizes the loss function (7).

As in [16], we propose to promote consistency in the form of a soft penalty added to the cost (8), which results in:

\[
\Psi_S(S) = \Psi(S) + 2\delta \| \mathcal{F}(S) \|^2,
\]

where \( \| . \| \) denotes the Frobenius norm for matrices. The greater \( \delta \), the more consistent the resulting source estimate will be.

We can find the complex spectrogram\(^2\) \( S \) minimizing \( \Psi_S \) by setting the gradient of \( \Psi_S(S) \) to 0 and then solving. The consistency term is identical to that in [16], but the gradient of \( \Psi(S) \) is slightly more involved. To make its derivation easier to understand, it helps to consider the whole complex spectrogram \( S \) as the equivalent vector \( \bar{X} \) obtained by concatenating the real and imaginary parts of all the frames of \( S \). The gradient of \( \Psi(S) \) can be derived with respect to the elements of \( \bar{X} \), leading to an \( \mathbb{R} \)-linear operator on \( \bar{S} - \bar{\mu} \), which can be reformulated as an \( \mathbb{R} \)-linear operator on \( S - \mu \). We

\(^2\)For convenience, we call “complex spectrogram” any complex-valued matrix, even if it is not the STFT of an actual signal.
eventually obtain the gradient of $\Psi(S)$ w.r.t. $S_{ft}$ as:

$$\nabla_{S_{ft}} \Psi(S) = 4\Omega_{ft}(S_{ft} - \mu_{ft}),$$

(11)

where

$$\Omega_{ft}(y) = \frac{1}{\det(\Gamma_{ft})} (\gamma_{ft} y - c_{ft} y)_{\forall y \in C},$$

(12)

and $|\Gamma'_{ft}| = \gamma_{ft}^2 - |c_{ft}|^2$. Altogether, setting the gradient of $\Psi(S)$ to 0 leads to

$$(\Omega + \delta \mathcal{F}^* \circ \mathcal{F}) S = \Omega \mu,$$

(13)

where $^*$ denotes the Hermitian adjoint and $\Omega$ is defined as the real-linear operator that consists in independently applying $\Omega_{ft}$ to each TF bin $Y_{ft}$ of a complex spectrogram $Y$:

$$(\Omega Y)_{ft} = \Omega_{ft}(Y_{ft}), \quad \forall Y \in \mathbb{C}^{F \times T}.$$ 

Since $\mathcal{F}$ is a projector, then $\mathcal{F} \circ \mathcal{F} = \mathcal{F}$. Furthermore, if the analysis and synthesis windows are equal up to a scaling factor (which is generally the case in practice), then it can be shown [16] that $\mathcal{F}$ is Hermitian, i.e., $\mathcal{F}^* = \mathcal{F}$, and the global minimum verifies:

$$(\Omega + \delta \mathcal{F}^* \circ \mathcal{F}) S = \Omega \mu.$$ 

(14)

Drawing on [16], we propose to solve (14) with the preconditioned conjugate gradient method [27], since the operator $\Omega + \delta \mathcal{F}$ is ill-conditioned. The preconditioner $M$ is derived similarly to [16], leading here at each TF bin to

$$(MY)_{ft} = \Omega_{ft}(Y_{ft}) + \delta \frac{FT - L}{FT} Y_{ft},$$

(15)

where $L$ is the time-signal length. Inverting $M$ is slightly more involved than in [16], where it amounted to a simple scalar multiplication, because $\Omega_{ft}(Y_{ft})$ here involves both $Y_{ft}$ and $Y_{ft}$ as can be seen in Eq. (12). A short calculation leads to

$$(M^{-1}(Y))_{ft} = \frac{1}{\eta_{ft}} \left\{ \left( \frac{\gamma_{ft}}{|\Gamma_{ft}|} + \delta \frac{FT - L}{FT} \right) Y_{ft} + \frac{c_{ft}'}{|\Gamma_{ft}|} Y_{ft} \right\},$$

(16)

where $\eta_{ft} = \left( \frac{\gamma_{ft}}{|\Gamma_{ft}|} + \delta \frac{FT - L}{FT} \right)^2 - \frac{|c_{ft}'|^2}{|\Gamma_{ft}|^2}$.

The full procedure is summarized in Algorithm 1, and a MATLAB implementation is available at [28].

### 3. EXPERIMENTAL EVALUATION

#### 3.1. Dataset and protocol

We propose to experimentally assess the potential of the consistent anisotropic Wiener filtering procedure described in Algorithm 1. We consider 100 music songs from the Demixing Secrets Database (DSD100), a remastered version of the database used for the SiSEC 2015 campaign [29]. The database is split into two sets of 50 songs, a training set and a test set. Each song is made up of $J = 2$ sources: the vocal track and the musical accompaniment track (which may contain various instruments such as guitar, bass, drums, piano...). The signals are sampled at $F_s = 44100$ Hz and the STFT is computed with a 46 ms long Hann window and 75% overlap.

Two scenarios are considered. First, an Oracle scenario, in which the powers $v$ are assumed to be known (i.e., equal to the ground truth). Second, an Informed scenario, as in an informed source separation framework [25]: an NMF with Kullback-Leibler divergence [10] is applied to the spectrogram of each isolated source, which provides an estimate of the powers $v_{ft}$. Each NMF uses 100 iterations of multiplicative update rules and a rank of factorization $K = 50$. This scenario will inform us about the performance of the methods when the power estimates differ from the ground truth, while still remaining of relatively good quality.

The following approaches are compared: first, we consider two non-iterative techniques, namely Wiener filtering [7] and anisotropic Wiener (AW) filtering [24], which correspond to applying (4) with $\kappa = 0$ and $\kappa \neq 0$, respectively. These estimates are then used to initialize Algorithm 1, respectively leading to the consistent Wiener filtering (CW) [16] and to the proposed consistent anisotropic Wiener filtering (CAW). As in [16], the stopping criterion is chosen as $\epsilon = 10^{-6}$.

Source separation quality is measured with the signal-to-distortion, signal-to-interference, and signal-to-artifact ratios (SDR, SIR, and SAR) [30] expressed in dB, where only a rescaling (not a refiltering) of the reference is allowed.

A demonstration on an audio excerpt is available at [28].

#### 3.2. Influence of the consistency weight

First, similarly as in [24], we study the impact of the anisotropy parameter $\kappa$ on the separation quality on the training set: the best

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**Algorithm 1 Consistent anisotropic Wiener filtering.** Note: matrix operations are element-wise.

**Inputs:**
- Prior power $v \in \mathbb{R}^{2 \times F \times T}$ and phase $\phi \in [0, 2\pi]^{2 \times F \times T}$.
- Stopping criterion $\epsilon > 0$.

**Posterior moments**
- $\gamma_1 = (1 - \lambda^2)v_1$, $\gamma_2 = (1 - \lambda^2)v_2$, $\gamma_X = \gamma_1 + \gamma_2$.
- $c_1 = \rho v_1 e^{i2\phi_1}$, $c_2 = \rho v_2 e^{i2\phi_2}$, $c_X = c_1 + c_2$.
- $\gamma' = \gamma_1 - (\gamma_X c_1^* c_1 - |c_1|^2) / (\gamma_X^2 - |c_X|^2)$.
- $c' = c_1 - (2\gamma_X c_1 c_1^* c_X - c_1^* c_X) / (\gamma_X^2 - |c_X|^2)$.
- $|\Gamma'| = |\gamma' - |c'||^2$.

**Preconditioned conjugate gradient**
- $\Omega$ as defined in (12) and $M^{-1}$ as defined in (16),
- $S_0 = \mu$, $R_0 = -\delta \mathcal{F}(S_0)$, $P_0 = M^{-1}(R_0)$, $\xi_{new} = \langle R_0, P_0 \rangle$, $k = 0$.
- **repeat**
  - $Q_k = \Omega(P_k) + \delta \mathcal{F}(P_k)$.
  - $a_k = \xi_{new} / \langle P_k, Q_k \rangle$.
  - $S_{k+1} = S_k + a_k P_k$, $R_{k+1} = R_k - a_k Q_k$.
  - $Z_{k+1} = M^{-1}(R_{k+1})$, $\xi_{old} = \xi_{new}$.
  - $\xi_{new} = \langle R_{k+1}, Z_{k+1} \rangle$.
  - $P_{k+1} = Z_{k+1} + \beta_k P_k$.
  - $k = k + 1$.
- **until** $a_k^2 ||P_{k-1}||^2 < \epsilon ||S_k||^2$

**Output:**
- $S_k \in \mathbb{C}^{F \times T}$. 
results in terms of SDR, SIR, and SAR are obtained for $\kappa = 1$ in the Oracle scenario, and $\kappa = 0.8$ in the Informed scenario.

We then investigate here the influence of the consistency parameter $\delta$ on the separation quality. The results in terms of SDR averaged over the 50 songs composing the training set are presented in Fig. 1 (similar trends are observed for the SIR and SAR). We observe that promoting consistency leads to improving the separation quality over other approaches that do not account for this property (i.e., when $\delta \to 0$), whether the magnitude values are known or estimated beforehand. The optimal value of $\delta$ is dependent on the data, with a peak in the SDR here at 10 in the Oracle scenario and 1 in the Informed scenario. This corresponds to a trade-off between excessively promoting the consistency and only accounting for the MMSE estimates.

### 3.3. Separation results

We now consider the 50 songs that form the test set, and set $\delta$ to its learned optimal value. The results averaged over the dataset are presented in Table. 1.

In the Oracle scenario, the proposed method outperforms all the other approaches. While the AW technique improves the separation quality over the Wiener estimates, it performs slightly worse than the CW filtering. The proposed CAW method overcomes this limit, since it combines the potential of both AW and CW approaches, and improves the criteria by approximately 0.5 dB over the CW technique. In the Informed scenario, the improvement is less important (about 0.2 dB), which suggests that even if the proposed phase retrieval method can improve the separation quality over the other techniques, its full potential is reached when the power estimates are close to the ground truth.

### 4. CONCLUSION

The consistent anisotropic Wiener filtering procedure introduced in this paper is a promising approach for recovering the phase of the components in a source separation framework, since it combines a phase property that originates from signal modeling, and a consistency constraint which accounts for the redundancy of the STFT. Future work will focus on extending this procedure to the case of more than two sources and to multichannel mixtures. In addition, such a technique can be implemented in an online fashion through a frame-by-frame processing, similarly as in some real-time implementations of the Griffin and Lim algorithm [31].
5. REFERENCES


