Where do we stand with fuzzy project scheduling?

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Pierre Bonnal[1]; Didier Gourc[2]; and Germain Lacoste[3]

Abstract: Fuzzy project scheduling has interested several researchers in the past two decades; about 20 articles have been written on this issue. Contrary to stochastic project-scheduling approaches that are used by many project schedulers, and even if the axiomatic associated to the theory of probabilities is not always compatible with decision-making situations, fuzzy project-scheduling approaches that are most suited to these situations have been kept in the academic sphere. This paper starts by recalling the differences one can observe between uncertainty and imprecision. Then most of the published research works that have been done in this field are summarized. Finally, a framework for addressing the resource-constrained fuzzy project-scheduling problem is proposed. This framework uses temporal linguistic descriptors, which might become very interesting features to the project-scheduling practitioners.

CE Database subject headings: Scheduling; Project management; Fuzzy sets.

Introduction

"Simple things are wrong, while complex ones are unusable!" French poet Paul Valery said one century ago. Project schedulers are facing a similar dilemma when trying to find the most appropriate tool for scheduling their projects. Project-scheduling models that are simple can sometimes be misleading. Models that are too sophisticated, because of their low reactivity, can be useless.

Critical-path method (CPM) is the simplest method available for modeling the execution of a project. If the complexity of the project is low, one can believe that such an approach may be sufficient. For more complex projects, such as large-scale industrial or construction projects, additional features such as a wider variety of precedence constraints including start–start, start–finish, finish–finish, and with or without a lag; temporal constraints such as milestones; as well as resource constraints shall be considered.

Among these additional features, uncertainty is an issue that is addressed in one of the first publications related to the project-scheduling problem—the Malcolm et al. paper on the program evaluation and review technique (PERT) method (Malcolm 1959). Several interesting papers have been issued following this primary paper, and among them, a paper proposing the use of Monte-Carlo simulations by Van Slyke (1963), also papers extending the stochastic scheduling problem to branching (the so-called generalized project-scheduling problem) (Eisner 1962; Elmaghraby 1964; Elmaghraby 1966; Moore and Clayton 1976; Pritsker 1968), and a few others aiming to criticize the PERT method (Kotiah and Wallace 1973; McCrimmon and Rayvec 1964; Parks and Ramsing 1969).

Additional criticisms came from some writers interested in the implementation of the fuzzy-set and the possibility theories to solve practical decision-making problems. They argued, and still do, that the project-scheduling problem is not a domain that suits the axiomatic associated with the probability theory. Fuzzy or possibilistic approaches are much more appropriate.

The first paper addressing the project-scheduling problem with a fuzzy point of view was by Chanas and Kamburowski (1981) and was published in the early 1980s. In the last 20 years, about 20 papers have been published on that issue. Most of them are very theoretical and cannot be implemented in real-life situations.

We believe that these scheduling approaches are mature enough and that, through some adaptations, project-management practitioners can benefit from them. The aim of this paper is to sum up the knowledge acquired on fuzzy/possibilistic project scheduling over the years and to propose a practical method to add to the project-scheduler toolbox.

Uncertainty versus Imprecision

Some definitions of terms associated with the notions of uncertainty and imprecision are given in this section. From Dubois and Prade (1988) and from Klir and Folger (1988) we can derive the following:

- Uncertainty usually refers to the random nature of a result; this term is of a probabilistic nature, and
- Imprecision refers to the incompletely defined nature of a result; imprecision has a deterministic nature.

A pragmatic approach to the notion of information consists of considering it as a predicate, a truth, to which a balancing qualifier is attached. Several qualifiers come to mind including probable, possible, necessary, plausible, credible, and so on.

Probable

This qualifier gets its meaning when used in the perspective of the probability theory. If we consider that this theory refers to the measurement of the frequency of event occurrences, then the

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probable has a pure arithmetical meaning. In certain cases, the probability theory refers to the feeling of uncertainty, i.e., the price that someone may pay if the proposal he/she believes to be true is really false. In such situations, the probable has a more subjective meaning. In the project-management domain, this is the latter meaning that is usually considered, probable activity duration for instance.

Possible and Necessary

Aristotle already mentioned the duality that exists between these two terms. The possible expresses the difficulty associated with the realization of an event, while the necessary refers to the obligation to have an event realized. If an event is necessary, that means its contrary is impossible. One can locate the probable somewhere in between the possible and the necessary.

Plausible and Credible

Everything that comes from a corpus of knowledge is said to be credible. Everything that does not is said to be plausible.

The probability theory is a proven mathematical theory. Although it has been in existence for several centuries, its axiomatic is quite recent as shown in Kolmogorov's work in the 1930s. This theory has many axioms, but they are clear. Its global consistency is uncontested. Its implementation to a wide range of domains has given valuable results. However, because it is a particularly axiomatic theory, several limitations can be observed when applied to other fields such as project management.

Prior to any calculation, two of its axioms require that all the possible discrete events are identified, and that the sum of all these probabilities equals one. When applied to decision-making processes, these two conditions are rarely satisfied. By definition, the knowledge someone can have of the future is vague. Therefore, it is extremely difficult to identify all the possible events that may occur.

The distribution law of a random variable gives the frequency of occurrence of each of its possible values. Math textbooks are full of distribution laws, but their use supposes that enough data has been gathered to let the analyst check if the envisioned laws fit the data. Unfortunately, the project-management analyst does not have enough information to perform this prerequisite. This is due to the fact that, by definition, a project is a collection of nonrepetitive activities. Even if the closing-out phases are correctly performed, the collected data does not have the quality required for ensuring a good fit of distribution laws.

Project activities are usually not fair dice because project managers obviously have some power to steer the execution of their projects. With this last argument, one has three irrefutable reasons for concluding on the inappropriateness of the probability theory when used in the project-management field, and especially for the resolution of the project-scheduling problem.

Cantor and Dedekind launched the basis of the set theory in the middle of the 19th century. It was quite a controversial mathematical theory at that time. That is not the case anymore. This theory's principle is that all of the elements of the universe can be sorted out in sets, and every set can be considered as a whole. When this theory is applied to real-life problems, it is sometimes difficult to decide whether an element belongs to a set or not. In the 1920s, the Polish logician Łukasiewicz and the Romanian mathematician Moisil, proposed a multivalent logic that handles the notion of doubt to deal with this. If 0 is false and 1 is true, then the notion of uncertainty can be associated with 1/2 for instance. One had to wait until the 1960s and for Zadeh's work to have a complete algebra for treating vagueness and fuzziness. The fuzzy-set and possibility theories have much less axiomatic limitations as compared to the probability theory. For instance, an event can be possible, so does its contrary. Because this theory is compatible with the limitations given above, we are convinced that project schedulers may benefit from using it. This is particularly the case for planning and scheduling large-scale industrial or construction projects.

Deterministic Activity Network Calculations

A project can be broken down into activities that must be performed in a determined sequence. An acyclic-directed graph is usually used to model this sequence of activities. The CPM approach associates activities to vertices and precedence constraints to nodes. Another approach makes the contrary, and is preferred because it is easier to handle. Let us call $A=\{a_1,\ldots,a_n\}$ the project activities and $U=[u_{jk}]$ the precedence matrix made as follows: $u_{jk}=1$ if a_k is an immediate successor of a_j, and $u_{jk}=0$ otherwise. Let us call $\Gamma_j \subset A$ the subset of the activities that are immediate successors of a_j, and $\Gamma_j^{-1} \subset A$ the subset of the activities that are immediate predecessors of a_j. It shall be mentioned that all these subsets derives from U. The resolution of the so-called project-scheduling problem, i.e., of the graph $G=(A,U)$ supposes that all activity duration is known. To simplify the understanding of the calculation procedure, two zero-duration activities are added: a_α and a_ω. a_α precedes all the activities for which $\Gamma_j^{-1}=\varnothing$; a_α is called the initial activity of the network, and its start date is known. The a_ω succeeds all the activities for which $\Gamma_j=\varnothing$ and a_ω is called the terminal activity. The aim of the exercise is to determine the minimum makespan for the project and the earliest start dates t_j and the latest finish dates τ_j of each activity. The t_j are computed throughout the so-called forward calculation at the end of which the minimum makespan is obtained; $t_\omega - t_\alpha$. Then t_ω and τ_ω are made equal in order to compute the τ_j throughout the so-called backward calculation. Total and free floats can be computed when all the dates associated to the activities are known.

$$TF_j=\tau_j-t_j+d_j \quad \text{and} \quad FF_j=\min_{k\in\Gamma_j}\{t_k\}-t_j+d_j$$

Calculations are performed from a_α to a_ω (forward) and from a_ω to a_α (backward). An activity can be scheduled if its immediate predecessors (forward) or successors (backward) are already calculated.

$$t_j=\max_{k\in\Gamma_j^{-1}}\{t_k+d_k\} \quad \text{if} \quad \Gamma_j^{-1}\neq\varnothing \quad \text{and} \quad t_j=t_\alpha \quad \text{otherwise}$$

$$\tau_j=\min_{k\in\Gamma_j}\{\tau_k-d_k\} \quad \text{if} \quad \Gamma_j\neq\varnothing \quad \text{and} \quad \tau_j=\tau_\omega \quad \text{otherwise}$$

In the scheduling of real-life projects, schedulers often need to consider temporal constraints (fixed milestones which correspond to external events) and a wider range of precedence constraints including finish–start, start–start, start–finish, and finish–finish. The introduction of these additional features in the calculation procedure given previously is quite straightforward.

The scheduler may also need to consider resource constraints. The problem becomes more difficult to resolve as no exact solution can be found in a polynomial computational time. This is why several attempts were made, and are still being made, to find heuristics and metaheuristics that give solutions close to the exact one. Several domains of the applied mathematics including beam, light-beam, and taboo search, as well as bound and branch, ge-

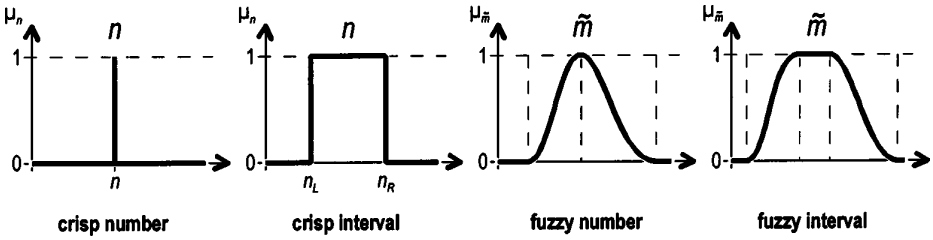

Fig. 1. Fuzzy versus crisp numbers and intervals

netic algorithms, and simulated annealing have been investigated. The use of priority rules is undoubtedly the simpler approach one can implement to resolve the resource-constrained project-scheduling problem (RCPSP). Assuming that the remainder of this paper treats fuzzy project scheduling, this heuristic methodology gives good enough results (Alvares-Valdes and Tamarit 1989; Davis 1973; Davis and Patterson 1973). As a prerequisite to the RCPSP, a calculation of the activity network as per the procedure is given above, i.e., the resource constraints are omitted. They are then added in the following way: $\Sigma_{a_j \in P} r_j^l \leq R^l$, $\forall l$, where P is the set of the activities in progress at a given time, r_j^l is the amount of the lth resource needed for performing activity a_j, and R^l is the availability of the lth resource over the given time period.

The RCPSP calculations are similar to the ones just given. Because of limited resources, it may be possible for just a subset of the eligible activities to be scheduled. In such situations, priority rules are used to make a selection among all the eligible activities. Alvares-Valdes and Tamarit (1989) showed through an activity network benchmarking that the combinations of the greatest-rank positional weight (GRPW) added to the latest start time or GRPW added to the most total successors lead to a satisfactory optimization.

Therefore, we can observe that a limited number of arithmetical operations are required for resolving the project-scheduling problem, and the RCPSP including sum, difference, maximum, minimum, and ranking.

Some Definitions From Fuzzy-Set and Possibility Theories

Most of the results given hereafter can be found in most of the textbooks addressing fuzzy arithmetic or fuzzy logic (Dubois and Prade 1988; Klir and Folger 1988). In the *crisp*-set theory, an element x does or does not belong to a set X. The membership function $\mu_X(x)$ equals 1 if $x \in X$, and zero otherwise, i.e., $\mu_X(x) \in \{0,1\}$. In the fuzzy-set theory, an element may more or less belong to a set: $\mu_X(x) \in [0,1]$. Fuzzy numbers and fuzzy intervals are fuzzy sets over the real line $\mathcal{R}$. Some characteristics of fuzzy vs. crisp numbers and intervals are shown in Fig. 1.

Throughout this paper, and for computational efficiency, the trapezoidal notation is used. A fuzzy interval is defined from four crisp numbers x_{L0}, x_{L1}, x_{R1}, and x_{R0} as follows: $\tilde{x} = \langle x_{L0}, x_{L1}, x_{R1}, x_{R0} \rangle$ (Fig. 2). Additional definitions such as α-cuts are given in Fig. 2.

Addition and Subtraction

The most often used formulas for adding or subtracting fuzzy intervals $\tilde{x} = \langle x_{L0}, x_{L1}, x_{R1}, x_{R0} \rangle$ and $\tilde{y} = \langle y_{L0}, y_{L1}, y_{R1}, y_{R0} \rangle$ are the following:

$$\tilde{x} + \tilde{y} = \langle x_{L0} + y_{L0}, x_{L1} + y_{L1}, x_{R1} + y_{R1}, x_{R0} + y_{R0} \rangle$$

$$\tilde{x} - \tilde{y} = \langle x_{L0} - y_{R0}, x_{L1} - y_{R1}, x_{R1} - y_{L1}, x_{R0} - y_{L0} \rangle$$

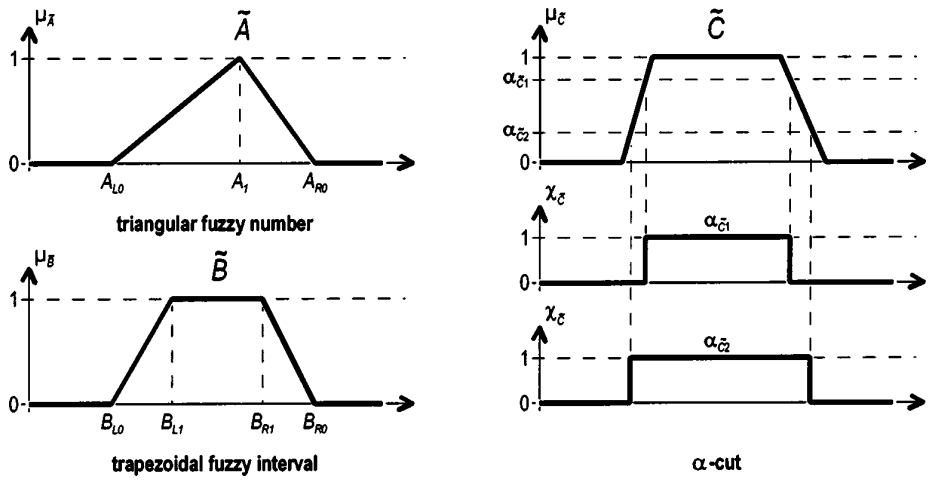

Fig. 2. Trapezoidal notation; α-cuts

These two formulas are known to be pessimistic. To avoid the inflation of the imprecision, the following operators were proposed:

$$\tilde{x}+\tilde{y}=\langle x_{L1}+y_{L1}-[(x_{L1}-x_{L0})^q+(y_{L1}-y_{L0})^q]^{1/q}, x_{L1}+y_{L1},$$
$$x_{R1}+y_{R1}, x_{R1}+y_{R1}+[(x_{R0}-x_{R1})^q+(y_{R0}-y_{R1})^q]^{1/q}\rangle$$
$$\tilde{x}-\tilde{y}=\langle x_{L1}-y_{R1}-[(x_{L1}-x_{L0})^q+(y_{R0}-y_{R1})^q]^{1/q}, x_{L1}-y_{R1},$$
$$x_{R1}-y_{L1}, x_{R1}-y_{L1}+[(x_{R0}-x_{R1})^q+(y_{L1}-y_{L0})^q]^{1/q}\rangle$$

For $q>1$, the two operators give more optimistic results. In some ways, they average the imprecision. We are convinced that in a decision-making context, the imprecision of a result (a sum or a difference for instance) shall have at least the imprecision of the most imprecise of its operands. This is why we are proposing the following alternative operators. Let $z=\langle z_{L0}, z_{L1}, z_{R1}, z_{R0}\rangle$ be the result of $\tilde{x}+\tilde{y}$ and $\tilde{w}=\langle w_{L0}, w_{L1}, w_{R1}, w_{R0}\rangle$ the result of $\tilde{x}-\tilde{y}$.

$$z_{L0}=z_{L1}-\max\{x_{L1}-x_{L0}, y_{L1}-y_{L0}\}\times c_2$$
$$z_{L1}=\frac{1}{2}(x_{L1}+x_{R1}+y_{L1}+y_{R1})-\max\{x_{R1}-x_{L1}, y_{R1}-y_{L1}\}\times c_1$$
$$Z_{R1}=\frac{1}{2}(x_{L1}+x_{R1}+y_{L1}+y_{R1})+\max\{x_{R1}-x_{L1}, y_{R1}-y_{L1}\}\times c_1$$
$$z_{R0}=z_{R1}+\max\{x_{R0}-x_{R1}, y_{R0}-y_{R1}\}\times c_2$$
$$w_{L0}=w_{L1}-\max\{x_{L1}-x_{L0}, y_{R0}-y_{R1}\}\times c_2$$
$$w_{L1}=\frac{1}{2}(x_{L1}+x_{R1}-y_{L1}-y_{R1})-\max\{x_{R1}-x_{L1}, y_{R1}-y_{L1}\}\times c_1$$
$$w_{R1}=\frac{1}{2}(x_{L1}+x_{R1}-y_{L1}-y_{R1})+\max\{x_{R1}-x_{L1}, y_{R1}-y_{L1}\}\times c_1$$
$$w_{R0}=w_{R1}+\max\{x_{R0}-x_{R1}, y_{L1}-y_{L0}\}\times c_2$$

Interesting results can be obtained with $c_1=1/2$ and $c_2=1$.

Maximum and Minimum

The most often used formulas for fuzzy maximum and minimum operations are the following:

$$\max\{\tilde{x},\tilde{y}\}=[\max(x_{L0}, y_{L0}); \max(x_{L1}, y_{L1}); \max(x_{R1}, y_{R1})$$
$$\max(x_{R0}, y_{R0})]$$
$$\min\{\tilde{x},\tilde{y}\}=[\min(x_{L0}, y_{L0}); \min(x_{L1}, y_{L1}); \min(x_{R1}, y_{R1})$$
$$\min(x_{R0}, y_{R0})]$$

These two operators also give quite pessimistic results, but if the operands are fuzzy intervals with similar imprecision ratios, imprecision can be kept within acceptable limits.

Ranking

Let $\tilde{x}=\langle x_{L0}, x_{L1}, x_{R1}, x_{R0}\rangle$ and $\tilde{y}=\langle y_{L0}, y_{L1}, y_{R1}, y_{R0}\rangle$ be two fuzzy intervals. If $x_{L0}\leqslant y_{L0}$, $x_{L1}\leqslant y_{L1}$, $x_{R1}\leqslant y_{R1}$ and $x_{R0}\leqslant y_{R0}$, the ranking of $\tilde{x}$ and $\tilde{y}$ is straightforward $\tilde{y}$ is said to be strongly greater than $\tilde{x}$. If one or two of these four inequalities is/are not true, the strong comparison rule must be abandoned to the advantage of the so-called weak comparison rule (WCR). Several proposals have been made to address this issue including WCR using area compensation, barycentre comparison, measures of possibilities, and of necessities. The WCR that uses the middle of α-cuts, proposed by Dubois, Fargier, and Fortemps (2000) seems to be the most efficient one. It also has the advantage of the linearity. This method associates to each of the fuzzy intervals to be compared, a crisp quantity that is obtained from the middle of the segments generated by α-cuts (Fig. 3).

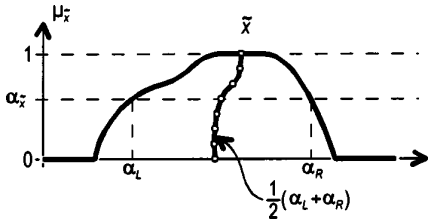

Fig. 3. Comparison rule that uses middle of α-cuts

$$\tilde{x}\mapsto F(\tilde{x})=\frac{1}{2}\int_0^1(\alpha_L+\alpha_R)\mathrm{d}\alpha$$

Also, when the fuzzy intervals to compare are trapezoidal intervals

$$F(\tilde{x})=\frac{1}{4}(x_{L0}+x_{L1}+x_{R1}+x_{R0})$$

Fuzzy/Possibilistic Project-Scheduling Problem

Over the last 20 years, two dozen articles or so tried to address the fuzzy project-scheduling problem. Three different axes have been investigated so far (Galvagnon 2000).

- The membership level of an arc to a graph has been considered. In an activity-on-arrow formulation, it is the existence of an activity which is considered, while in a activity-on-node formulation, it is the existence of a precedence constraint. Very few papers address this issue. There is only one by Dubois and Prade (1978) and a second by Klein (1988).
- The project-scheduling problem can also be seen as a constraint satisfaction problem. In that case, the aim is to calculate a satisfaction level of an activity network (Fargier 1994).
- The imprecision associated with the quantitative information which characterizes an activity network can also be looked at; for instance, the activity duration, the work load required, the resource availability, fixed dates and soon.

The latter axis is by far the one that has been investigated the most. The most significant contributions are numerous; they are briefly described hereafter. The chronological order of publication is used.

Chanas and Kamburowski

The two writers are the very first who processed the project-scheduling problem in a fuzzy environment (Chanas and Kamburowski 1981). The criticisms made by several writers (Kotiah and Wallace 1973; McCrimmon and Rayvec 1964; Parks and Ramsing 1969) toward the PERT probabilistic approach motivated the publication of this paper. Chanas and Kamburowski featured a methodology for addressing the fuzzy project-scheduling problem that slices the activity duration given as fuzzy intervals, in α-cuts. The project-scheduling problem is then solved for each α-cut

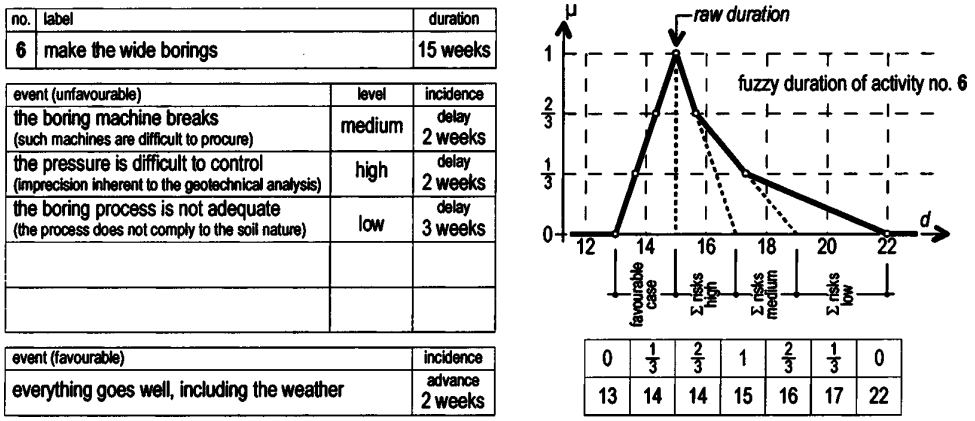

Fig. 4. Estimating fuzzy duration (Lesmann et al.)

level. Hence, it is crisp quantities that are handled. A recombination of all these calculation results gives a fuzzy solution to the project-scheduling problem.

Dubois and Prade

The proposal made by these two writers (Dubois and Prade 1988) is certainly the most significant. It is also the most straightforward. They have also considered that, by definition, activity duration is imprecise, and that this information must be estimated using fuzzy quantities. Further on, they used trapezoidal intervals for modeling fuzzy duration. The methodologies used for solving the project-scheduling problem and the RCPSP are very similar to the ones recalled above. They differ by the fact that addition, subtraction, maximum, minimum, and ranking have been replaced by their fuzzy equivalents. Gazdik's (1983) and Wang's (1999) papers feature methodologies similar to that of Dubois and Prade.

Lootsma

After comparing the benefits of possibilistic scheduling approaches with respect to probabilistic ones, and after recalling the inconsistency of building a probabilistic activity network, Lootsma (1989) proposed a comparative analysis of the two methodologies through an example. From it, he observed that the total project makespan is shorter when the possibilistic approach is used. The dispersion over the project termination date is also greater.

Nasution

In their approaches, none of the writers cited previously mentioned the phenomenon that may occur at the backward calculation of the activity network. Nasution (1994) addresses it through an example that is recalled hereafter. Let G be a project made of two activities a_{12} and a_{23}, which can be performed in parallel. Because the writer prefers an activity-on-arrow formulation, a dummy activity a_{13} is introduced. Let $\tilde{t}_j$ and $\tilde{\tau}_j$ be the early and late date associated with the event e_j. Except $\tilde{t}_1$, which is a crisp date $\tilde{t}_1 = t_1 = \langle 0,0,0 \rangle$, all dates and duration of this microproject are expressed with triangular fuzzy numbers. Let us say that $\tilde{d}_{13} > \tilde{d}_{23}$, then $\tilde{t}_3 = \max\{t_1 + \tilde{d}_{13}, t_1 + \tilde{d}_{23}\} = \tilde{d}_{23}$. e_3 is the terminal event, then prior to the backward calculation of the activity network: $\tilde{\tau}_3 = \tilde{t}_3$ and $\tilde{\tau}_2 = \tilde{\tau}_3 - \tilde{d}_{13} = \tilde{d}_{13} - \tilde{d}_{23}$. If $\tilde{d}_{13} = \langle x_{L0}, x_1, x_{R0} \rangle$ and $\tilde{d}_{23} = \langle y_{L0}, y_1, y_{R0} \rangle$: $\tilde{\tau}_2 = \langle x_{L0} - y_{R0}, x_1 - y_1, x_{R0} - y_{L0} \rangle$. Also, because a_{13} is a dummy activity $\tilde{\tau}_1 = \tilde{\tau}_2$. The expression shows that, even if x_1 and y_1 are positive numbers, $\tilde{\tau}_1$ can be a negative fuzzy number. This phenomenon, also mentioned by Galvagnon et al. (2000), comes from the fact that $\tilde{x} + \tilde{y} - \tilde{y} \neq \tilde{x}$, $\tilde{x}$ and $\tilde{y}$ being two fuzzy numbers. Two proposals have been made to avoid this difficulty. Nasution proposes to replace (in the backward calculation) the ordinary fuzzy subtraction operator by another one he called interactive fuzzy subtractor that makes $\tilde{x} + \tilde{y} - \tilde{y} = \tilde{x}$. Hapke and Słowiński (1996a) propose to use the optimistic fuzzy operator (see previous section).

It should also be mentioned that the core of Nasution paper consists of symbolic calculations, with perhaps some interests from the academic point of view, but due to their complexity, with very few interests to the practitioner.

Geidel

To respect the chronological order, we have to mention Geidel's works (Geidel 1988, 1989). This writer proposes the use of measures of possibility and necessity for a better ranking of activities.

Lesmann et al.

This approach (Lessmann 1994) is similar to the one of Dubois and Prade. They also noted the phenomenon depicted previously. To address the latter, they made two proposals. They argued that in most construction projects, the project due date is specified in the contract. Hence, backward calculations can be initiated from a crisp date. Otherwise, because they preferred to use polynomial fuzzy numbers, they suggested transferring the earliest date of the terminal milestone to the latest date as follows: $\tilde{\tau}_\omega = \tau_\omega = x_1$ assuming $t_\omega = \langle x_{L0}, \ldots, x_1, \ldots, x_{R0} \rangle$. A methodology for estimating fuzzy numbers is also given (Fig. 4).

Hapke and Słowiński

Most of the papers addressing the fuzzy project-scheduling problem are from Hapke and Słowiński together with few others (Hapke and Słowiński 1993, 1996a,b; Hapke et al. 1994, 1997a,b, 1998; Hapke 1995). Their research involves the RCPSP with fuzzy data. To solve this problem, they investigated the use of

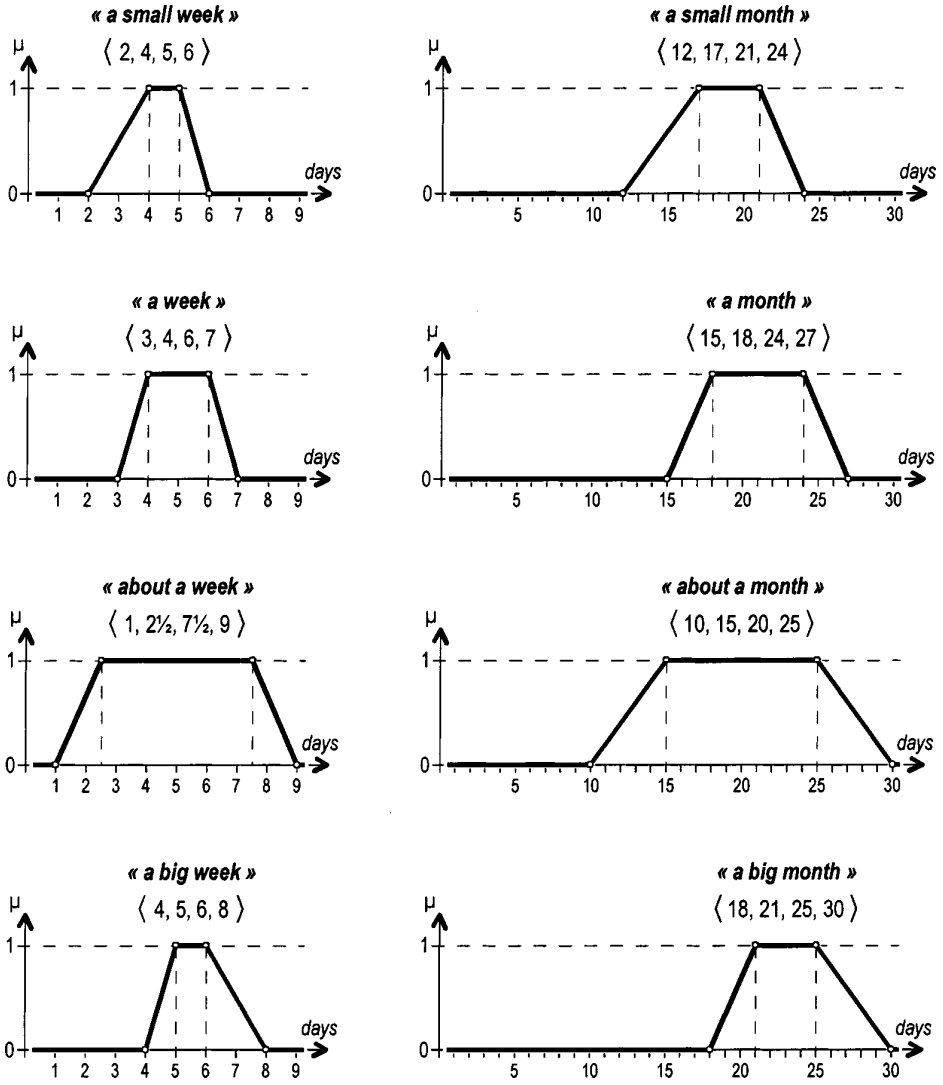

Fig. 5. Linguistic descriptors of duration

several heuristics and metaheuristics such as the Pareto-simulated annealing, the light beam search and so on.

Galvagnon et al.

Their works were motivated by the fact that no satisfactory proposal had been made to tackle the backward calculation problem in a fuzzy project-scheduling environment. Even though their framework addresses this more accurately, no solution can be found in a polynomial computational time unless the activity network is modeled as a series-parallel graph, which is quite a restrictive simplification (Galvagnon et al. 2000; Galvagnon 2000). Despite the analytical rigor and interest of this proposal, it is of low interest to the project practitioner.

Chanas and Zieliński

The two most recent papers by these writers are of great interest especially because no one took care of the problem associated with the determination of critical paths in a fuzzy environment before. In their first paper on this issue (Chanas and Zieliński 2002), they demonstrated that

- The problem that consists in finding a critical path (and demonstrating its criticality) is an easy problem that can be solved in a polynomial computational time.
- The problem of searching for all of the critical paths of an activity network can only be solved in an exponential computational time.

In their second paper, Chanas and Zieliński (2001) propose two algorithms for determining the so-called *f*-criticality index of a path.

Temporal Linguistic Descriptors

Linguistic descriptors are certainly among the very interesting features provided by the fuzzy-set theory. The relationship one can find between everyday speaking for describing things and

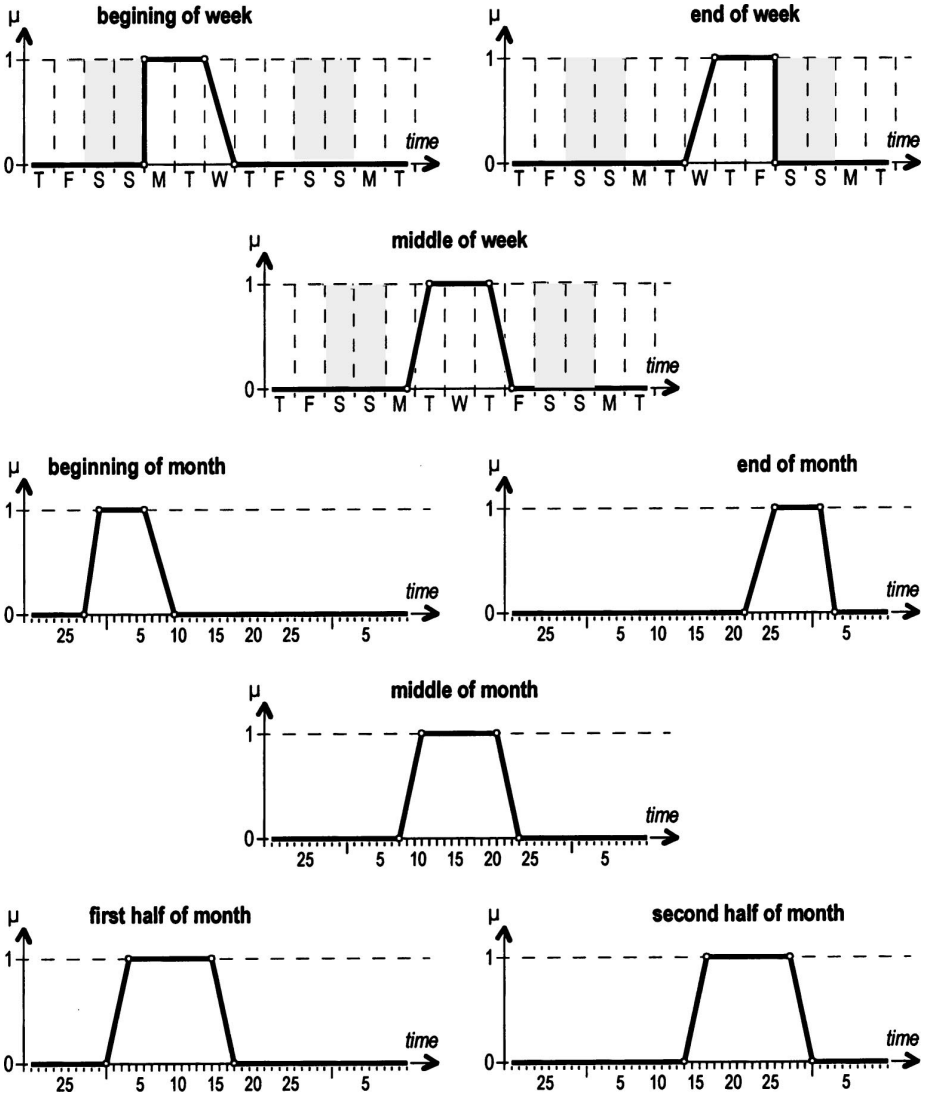

Fig. 6. Linguistic descriptors of dates

situations and their mathematical equivalent is through the so-called linguistic descriptors (LD). We are making some proposals to associate trapezoidal fuzzy intervals to temporal LDs, i.e., LDs of dates and LDs of duration (Figs. 5 and 6).

The benefits of gathering information in a fuzzy project-scheduling environment are straightforward. For instance, when a project planner asks an engineer for the time he needs to complete the writing of a technical specification, an answer of *about 1 week* does certainly not mean exactly 5 working days or 40 working hours. If nothing happens, the specification can be issued within 4 days. On the contrary, if the completion of the specification becomes slightly more difficult than foreseen, then its duration may equal 6 or 7 days. This is just a matter of precision (or of imprecision) of the estimation. In hindsight, the engineer considers that in both cases the precision he has given is accurate enough for the purpose.

One can also apply modifiers to this imprecise duration. *A big week* is slightly longer than *1 week, and approximately 1 month* is more imprecise than *1 month* (Figs. 5 and 6).

Unfortunately, the multiplication of two trapezoidal fuzzy numbers or intervals is a problem that is not as trivial as the ones that consist of adding or subtracting such numbers. The product of trapezoidal fuzzy interval is not a trapezoidal interval! For the remainder of this paper, the multiplication of crisp numbers to trapezoidal fuzzy numbers is sufficient. For instance

$$2 \text{ weeks} = 2 \times 1 \text{ week} = 2 \times \langle 3,4,6,7 \rangle = \langle 6,8,12,14 \rangle$$

The multiplication is also required when resources are involved. We limit ourselves to crisp resources allocation and availability. It should be noted that Galvagnon (2000) has made proposals for using fuzzy intervals for describing workloads in a resource-constrained project-scheduling environment.

Fuzzy Activity-Network Calculations

The methodology proposed here for resolving the resource-constrained fuzzy project-scheduling problem is made of several phases.

Table 1. Input Data to Numerical Example

Activity ID	Temporal LD	Γ^{-1}	Γ	Resources	d_{L0}	d_{L1}	d_{R1}	d_{R0}
α	—	—	A, I	—	0.0	0.0	0.0	0.0
A	3 weeks	α	B, E	*R*1	9.0	12.0	18.0	21.0
B	About 1 month	A	C	*R*2	10.0	15.0	20.0	25.0
C	About 1 month	B	D	*R*3	10.0	15.0	20.0	25.0
D	2 weeks	C, F	L	*R*3	6.0	8.0	12.0	14.0
E	3 big weeks	A	F	*R*2	12.0	15.0	18.0	24.0
F	About 3 weeks	E	D, G	*R*1,*R*3	3.0	7.5	22.5	37.0
G	About 3 weeks	F	H	*R*2	3.0	7.5	22.5	37.0
H	About 2 months	G	ω	*R*2	20.0	30.0	10.0	50.0
I	About 3 weeks	α	J	*R*1	3.0	7.5	22.5	37.0
J	2 small weeks	I	K	*R*1	4.0	8.0	10.0	12.0
K	2 big weeks	J	L	*R*1	8.0	10.0	12.0	16.0
L	About 6 weeks	D, K	ω	*R*1	6.0	15.0	45.0	54.0
ω	—	L, H	—	—	0.0	0.0	0.0	0.0

Phase 1

The transformation of the activity-network information gathered with temporal linguistic descriptors into trapezoidal fuzzy intervals is a prerequisite. This first phase straightforwardly assumes what has been said previously.

Phase 2

The second phase aims to identify critical activities. Because the search of critical paths may sometimes be a problem that cannot be solved in a polynomial computational time, the following procedure is proposed to approximate criticality indices of activities.

The fuzzy activity network is transformed in a crisp network by considering the most optimistic duration of each activity, i.e., $d_j = d_{L0\,j}$. This optimistic network is then calculated (forward and backward processes). At the end of this first treatment, critical and noncritical activities can be distinguished. Let us call *C* the set of the possible critical activities. These calculations on crisp activity networks are reiterated, considering the most optimistic duration ($d_j = d_{L0\,j}$) for the activities that belong to the set *C* and the most pessimistic duration ($d_j = d_{R0\,j}$) for those that are not in this set. At the end of each iteration, newly critical activities are appended to *C*. The process is stopped when the cardinality of *C* has become constant.

This algorithm is bound to be processed in a polynomial computational time because the number of iterations is obviously less than the number of the activities of the network. Let us call *n* the total number of iterations, and p_j the number of iterations at the end of which the activity a_j is found critical. The criticality index CI_j can be appraised as follows: $CI_j = p_i/n$. If $CI_j = 1$, this means that activity a_j is always critical, as per the global imprecision associated with the activity network. If $0 < CI_j < 1$, the activity a_j can be critical in some configurations, and if $CI_j = 0$, one can conclude that there is no way to have the activity a_j critical. One of the interesting features of these criticality indices is that they can be ranked.

Phase 3

Up to that phase, resources have not been taken into account in schedule calculations. The procedure for calculating a resource-constrained fuzzy activity network is similar to the RCPSP using priority rules with the following differences:

- Crisp arithmetic operations are replaced by their fuzzy equivalents,
- The ranking of eligible activities is made with respect to the late start time ($\tilde{\tau}_j - \tilde{d}_j$). The CIs are then used for discriminating equalities. Other ranking combinations can be envisaged, and
- To avoid imprecision propagation, we propose that $\tilde{t}_\omega$ is transferred to $\tilde{\tau}_\omega$ as follows:

$$\tilde{\tau}_\omega = \left\langle \frac{t_{L1}+t_{R1}}{2}, \frac{t_{L1}+t_{R1}}{2}, \frac{t_{L1}+t_{R1}}{2}, \frac{t_{L1}+t_{R1}}{2} \right\rangle$$

Phase 4

It is of interest to any project practitioner to know the critical activities of a project. The critical activities of *C* that have been identified in Phase 2 of the present scheduling methodology may not reflect the results of Phase 3 calculations. Hence, it is wise to relaunch a process similar to the one of Phase 2. In order to use an identical algorithm, pseudo-precedence constraints can be added between activities that share identical resources.

Phase 5

Despite the fact that attenuated arithmetical operators are being used all along this process, the transformation of the calculated dates ($\tilde{t}_j$ and $\tilde{\tau}_j$) into temporal LDs is not straightforward. We propose to use the means ($\bar{t}_j$), associated with an imprecision range (Δt_j) to make the following transformations:

$$\bar{t}_j = \frac{1}{4}(t_{L0}+t_{L1}+t_{R1}+t_{R0})_j \quad \text{and} \quad \bar{\tau}_j = \frac{1}{4}(\tau_{L0}+\tau_{L1}+\tau_{R1}+\tau_{R0})_j$$

$$\Delta t_j = \frac{1}{6}(t_{L0}+2t_{L1}-2t_{R1}-t_{R0})_j \quad \text{and}$$

$$\Delta \tau_j = \frac{1}{6}(\tau_{L0}+2\tau_{L1}-2\tau_{R1}-\tau_{R0})_j$$

If $\Delta t_j < 5$ days ($\Delta \tau_j < 5$ days), the dates $\tilde{t}_j$ ($\tilde{\tau}_j$) can be expressed in weeks. One can easily find the week number associated to $\bar{t}_j$ ($\bar{\tau}_j$), and these figures serve as a basis for the corresponding temporal LDs. For instance, if $\bar{t}_j$ = November *6, 2005*, then, *week No. 45* can be used to express $\tilde{t}_j$.

Table 2. Results of Computations

	Criticality index		Earliest dates			Latest dates		
Activity ID	Without resources	Resources constrained	t_D	t_F	Δt	τ_D	τ_F	$\Delta\tau$
α	1	1	Week 0	Week 0	0	Beginning January	Beginning January	±5 weeks
A	0.7	1	Week 0	Week 3	±1 week	Beginning January	2nd half January	±5 weeks
B	0.3	0.3	Week 9	Week 10	±1 week	Mid-February	1st half March	±5 weeks
C	0.3	0.3	Week 13	Week 15	±1 week	1st half March	Beginning April	±5 weeks
D	0.3	0.3	Week 18	Week 19	±1 week	Beginning April	Mid-April	±5 weeks
E	0.3	0.7	Week 4	Week 6	±1 week	2nd half January	Mid-February	±5 weeks
F	0.3	0.3	1st half February	1st half March	±3 weeks	End February	2nd half March	±3 weeks
G	0.3	0.3	1st half March	End March	±3 weeks	2nd half March	1st half April	±3 weeks
H	0.3	0.3	End March	Mid-May	±3 weeks	1st half April	End may	±2 weeks
I	0.3	0.3	2nd half January	Mid-February	±3 weeks	Beginning March	2nd half March	±5 weeks
J	0.3	0.3	Mid-February	End February	±3 weeks	2nd half March	Beginning April	±5 weeks
K	0.3	0.3	1st half March	End March	±3 weeks	Beginning April	Mid-April	±5 weeks
L	0.7	0.7	Mid-April	End May	±5 weeks	Mid-April	End May	±5 weeks
ω	1	1	End May	End May	±5 weeks	End May	End May	±5 weeks

If $\Delta t_j \geqslant 5$ days ($\Delta\tau_j \geqslant 5$ days), the dates $\tilde{t}_j$ ($\tilde{\tau}_j$) should be expressed in months. Similarly, the day of the month, q_j, can easily be found for each date $\bar{t}_j$ ($\bar{\tau}_j$). In the present methodology, five cases may be distinguished.

if $q_j \in [1,6]$ then t_j = beginning of month

if $q_j \in [7,12]$ then t_j = first half of month

if $q_j \in [13,19]$ then t_j = middle of month

if $q_j \in [20,25]$ then t_j = second half of month

if $q_j \in [26,31]$ then t_j = end of month

For dates that have an imprecision range lower than 5 days, i.e., expressed in weeks, it appears useless to state this information. If $\Delta t_j \geqslant 5$ days or $\Delta\tau_j \geqslant 5$ days, this information is of interest. We propose these imprecision ranges in weeks, and round them to the closest integer. Let us use again $\bar{t}_j$ = November *6, 2005*. If $\Delta\tau_j = 21.6$ days for instance, then $\tilde{t}_j$ = beginning *November 2005, ±4 weeks*.

Numerical Example

A network made of 12 activities has been used as a numerical example. Table 1 gives information on input data, such as activity duration (expressed in the form of temporal LDs), predecessors and successors, and the corresponding trapezoidal interval associated with the activity duration (Fig. 5). The results of the computations are in Table 2.

Conclusions

After about 20 years of maturation in the academic sphere, we believe that fuzzy project scheduling is mature enough to be used in real-world projects. If classical deterministic approaches such as CPM remain the most valuable for small projects, large-scale industrial or construction ones may benefit from fuzzy project-scheduling approaches. The methodologies one can find in the literature on this issue are quite theoretical and difficult to implement. The approach we propose in this paper is pragmatic enough to be understood by project-scheduling practitioners, and thus to be implemented in real-world projects. Among the difficulties one could face, the gathering of imprecise data in the form of fuzzy numbers or intervals could constitute a restraint to the implementation of fuzzy project-scheduling approaches. We are now convinced that this obstacle is removed thanks to the use of temporal LDs. This paper aims at making the link between researchers and practitioners. Feedbacks from field implementations are now required for making research on this issue advance. One of the issues pending is the tuning of temporal LD definitions.

Acknowledgments

The writers' thanks go out to all those who helped us collect all the literature that has been published on the fuzzy project-scheduling problem especially Didier Dubois, Vincent Galvagnon, Joachim Geidel, and Professor Michael Oberguggenberger.

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