Periodic components estimation in chronobiological time series via a Bayesian approach

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PERIODIC COMPONENTS ESTIMATION IN CHRONOBIOLOGICAL TIME SERIES VIA A BAYESIAN APPROACH

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ABSTRACT

In chronobiology a periodic components variation analysis for the signals expressing the biological rhythms is needed. Therefore precise estimation of the periodic components is required. The classical approaches, based on FFT methods, are inefficient considering the particularities of the data (non-stationary, short length and noisy). In this paper we propose a new method using inverse problem and Bayesian approach with sparsity enforcing prior. The considered prior law is the Student-t distribution, viewed as a marginal distribution of an Infinite Gaussian Scale Mixture (IGSM) defined via the inverse variances. For modelling the non stationarity of the observed signal and the noise we use a Gaussian model with unknown variances. To infer those variances as well as the variances of the periodic components we use conjugate priors. From the joint posterior law the unknowns are estimated via Posterior Mean (PM) using the Variational Bayesian Approximation (VBA). Finally, we validate the proposed method on synthetic data and present some preliminary results for real chronobiological data.

Index Terms— Periodic components estimation, Inverse Problem, Variational Bayesian Approximation (VBA), Kullback-Leibler divergence (KL), Infinite Gaussian Scale Mixture (IGSM), Posterior Mean (PM).

1. INTRODUCTION

Chronobiology examines periodic phenomena in living organisms. Those cycles are known as biological rhythms. One particular cycle of main interest is the circadian rhythm [1], The mammalian circadian timing system consists of a master pacemaker in the suprachiasmatic nucleus of hypothalamus and subsidiary molecular clock in most peripheral cell types, being synchronized by the day-night cycle, generating circadian ($\sim$ 24h) oscillations. The development of in vivo bioluminescence recording technologies enables to monitor the circadian biomarkers in peripheral tissues during a certain number of consecutive days, providing time series data [2]. In cancer treatment experiments, such signals presents some particularities: as the cancer tumor grows every day until the death of mice used in the experiments, the result is a non-stationary signal, with an increasing trend and a very short length. The objective of an accurate description of the periodic components variation during the evolution of cancer tumor phase can be formulated as the need for a method that can give an accurate estimation of the periodic components from a limited number of data.

2. CLASSICAL FT BASED METHODS

Spectral analysis of time series is a well known subject for a very long time. The most common methods are Fourier Transform (FT) based methods, which are widely used in many applications due to several obvious advantages: well known, well understood and fast, via FFT. The periodical phenomena was studied with different approaches in different particular conditions, [3], [4], [5], [6], [7], using in general the FFT based methods. Nevertheless, the particularities of the biomedical signals considered in chronobiology experiments show that the classical methods have certain limitations. In particular, for short time series relative to the searched periodic components the precision given by the FFT methods is insufficient to determine the underlying periodic components (in the experiment considered in this article, a 96 hours recorded signal relative to a 24 hours periodic component, linked with the circadian clock). An important point with biological signals is that biologist use more often period than frequency. In FFT based methods the results are presented on an axis which is uniform in frequency. In particular, for a four day (96h) recorded signal, beside the 24h corresponding periodic components, the nearest amplitudes in the periodic components vector correspond to the 32h and 19h. More general, if the prior knowledge sets the principal period around a value

P, in order to obtain a period vector that contains the period P and also the periods $P - 1$ and $P + 1$, via FFT approach the signal must be observed for $(P - 1)(P + 1)$ periods, i.e. $(P - 1)P(P + 1)$, which is beyond the real experiments.

3. INVERSE PROBLEM APPROACH AND GENERAL BAYESIAN INFERENCE

The first step of the proposed method consists in formulating the problem as an inverse problem:

$$g(t_n) = \sum_{m=1}^{M} f(p_m) e^{2\pi j \frac{1}{p_m} t_n} + \epsilon_n, \quad n \in \{1, ..., N\} \quad (1)$$

where $g(t_n)$ represents the observed value at time t_n, p_m represents the mth period, $f(p_m)$ its amplitude and ϵ_n accounts for errors and uncertainties. With the notation $g(t_n) = g_n$ and $f(p_m) = f_m$, defining the vectors $\boldsymbol{f} = [f_1, f_2, \ldots, f_M]^T$, $\boldsymbol{g} = [g_1, g_2, \ldots, g_N]^T$ and $\boldsymbol{\epsilon} = [\epsilon_1, \epsilon_2, \ldots, \epsilon_N]^T$ we obtain the following model:

$$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{\epsilon} \quad (2)$$

In this paper we adopt a Bayesian approach, which consists in infering $\boldsymbol{f}$ via:

$$p(\boldsymbol{f}|\boldsymbol{g}, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2) \propto p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{\theta}_1)\, p(\boldsymbol{f}|\boldsymbol{\theta}_2). \quad (3)$$

A necessary extension for real world application is the case where the hyperparameters $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)$ involved are not known and have to be estimated from the joint posterior law:

$$p(\boldsymbol{f}, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2|\boldsymbol{g}) \propto p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{\theta}_1)\, p(\boldsymbol{f}|\boldsymbol{\theta}_2)\, p(\boldsymbol{\theta}_1) p(\boldsymbol{\theta}_2) \quad (4)$$

In this way we obtain an unsupervised method. The main steps are then the assignment of $p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{\theta}_1)$, $p(\boldsymbol{f}|\boldsymbol{\theta}_2)$, $p(\boldsymbol{\theta}_1)$, $p(\boldsymbol{\theta}_2)$ and then the computation of the posterior $p(\boldsymbol{f}, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2|\boldsymbol{g})$ to infer on $\boldsymbol{f}$ and the hyperparameters $\boldsymbol{\theta}$.

4. A HIERARCHICAL MODEL FOR SPARSITY ENFORCING PRIOR MODEL

We propose to use a non-stationary Gaussian model where v_{ϵ_i} are considered to be unknowns. For having the possibility to estimate them we assign Inverse Gamma distributions:

$$g_i = \sum_{j=1}^{M} H_{ij} f_j + \epsilon_i,$$

$$p(\epsilon_i|v_{\epsilon_i}) = \mathcal{N}(\epsilon_i|0, v_{\epsilon_i}), \ i \in \{1, 2, \ldots, N\}$$

$$p(v_{\epsilon_i}|\alpha_{\epsilon_i 0}, \beta_{\epsilon_i 0}) = \mathcal{IG}(v_{\epsilon_i}|\alpha_{\epsilon_i 0}, \beta_{\epsilon_i 0}), \ i \in \{1, 2, \ldots, N\} \quad (5)$$

which results to:

$$\begin{cases} p(\boldsymbol{\epsilon}|\boldsymbol{v}_\epsilon) = \mathcal{N}(\boldsymbol{\epsilon}|\boldsymbol{0}, \boldsymbol{V}_\epsilon) \\ p(\boldsymbol{v}_\epsilon|\boldsymbol{\alpha}_{\epsilon 0}, \boldsymbol{\beta}_{\epsilon 0}) = \prod_{i=1}^{N} \mathcal{IG}(v_{\epsilon_i}|\alpha_{\epsilon_i 0}, \beta_{\epsilon_i 0}) \end{cases} \quad (6)$$

where we introduced the vector $\boldsymbol{v}_\epsilon$ and the corresponding diagonal matrix $\boldsymbol{V}_\epsilon$:

$$\boldsymbol{v}_\epsilon = \begin{bmatrix} v_{\epsilon_1} \ldots v_{\epsilon_i} \ldots v_{\epsilon_N} \end{bmatrix}^T \ ; \ \boldsymbol{V}_\epsilon = \mathrm{diag}[\boldsymbol{v}_\epsilon] \quad (7)$$

The likelihood $p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{v}_\epsilon)$ is obtained using the considered linear model (2) and the assigned distribution for the error vector $\boldsymbol{\epsilon}$:

$$p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{v}_\epsilon) = \mathcal{N}(\boldsymbol{g}|\boldsymbol{H}\boldsymbol{f}, \boldsymbol{V}_\epsilon) \quad (8)$$

The proposed prior distribution is a Student-t distribution, in order to enforce the sparsity [8] and use the prior knowledge of reduced number of clocks in the periodic component vector. While a direct assignment of a Student-t distribution for the prior law $p(\boldsymbol{f})$ leads to a non-quadratic criterion when estimating $\boldsymbol{f}$, the Student-t distribution corresponding to the prior law can be expressed as an Infinite Gaussian Mixture [9], modelling the inverse variance as a Gamma distribution or the variance as an Inverse Gamma distribution. For the variance of $\boldsymbol{f}$ we assume a general model:

$$\boldsymbol{v}_f = \begin{bmatrix} v_{f_1} \ldots v_{f_j} \ldots v_{f_M} \end{bmatrix}^T \ ; \ \boldsymbol{V}_f = \mathrm{diag}[\boldsymbol{v}_f] \quad (9)$$

This gives us the possibility to propose the following prior for the hierarchical model:

$$\begin{cases} p(\boldsymbol{f}|\boldsymbol{v}_f) = \mathcal{N}(\boldsymbol{f}|\boldsymbol{0}, \boldsymbol{V}_f) \\ p(\boldsymbol{v}_f|\boldsymbol{\alpha}_{f 0}, \boldsymbol{\beta}_{f 0}) = \prod_{j=1}^{M} \mathcal{IG}(v_{f_j}|\alpha_{f_j 0}, \beta_{f_j 0}) \end{cases} \quad (10)$$

where we used the notations:

$$\boldsymbol{\alpha}_{\epsilon 0} = \begin{bmatrix} \alpha_{\epsilon_1 0} \ldots \alpha_{\epsilon_N 0} \end{bmatrix}^T, \quad \boldsymbol{\beta}_{\epsilon 0} = \begin{bmatrix} \beta_{\epsilon_1 0} \ldots \beta_{\epsilon_N 0} \end{bmatrix}^T$$
$$\boldsymbol{\alpha}_{f 0} = \begin{bmatrix} \alpha_{f_1 0} \ldots \alpha_{f_M 0} \end{bmatrix}^T, \quad \boldsymbol{\beta}_{f 0} = \begin{bmatrix} \beta_{f_1 0} \ldots \beta_{f_M 0} \end{bmatrix}^T \quad (11)$$

The error variance prior proposed, the likelihood (8) and the prior (10) represents the proposed IGSM Hierarchical Model, which can be summarized as follows:

$$\begin{cases} p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{v}_\epsilon) \propto |\boldsymbol{V}_\epsilon|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2}\|\boldsymbol{V}_\epsilon^{-\frac{1}{2}}(\boldsymbol{g} - \boldsymbol{H}\boldsymbol{f})\|^2 \right\} \\ p(\boldsymbol{f}|\boldsymbol{v}_f) \propto |\boldsymbol{V}_f|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2}\|(\boldsymbol{V}_f)^{-\frac{1}{2}}\boldsymbol{f}\|^2 \right\} \\ p(\boldsymbol{v}_\epsilon|\boldsymbol{\alpha}_{\epsilon 0}, \boldsymbol{\beta}_{\epsilon 0}) \propto \prod_{i=1}^{N} v_{\epsilon_i}^{-(\alpha_{\epsilon_i 0}+1)} \exp\left\{ -\sum_{i=1}^{N} \beta_{\epsilon_i 0} v_{\epsilon_i}^{-1} \right\} \\ p(\boldsymbol{v}_f|\boldsymbol{\alpha}_{f 0}, \boldsymbol{\beta}_{f 0}) \propto \prod_{j=1}^{M} v_{f_j}^{-(\alpha_{f_j 0}+1)} \exp\left\{ -\sum_{j=1}^{M} \beta_{f_j 0} v_{f_j}^{-1} \right\} \end{cases} \quad (12)$$

From this hierarchical model the posterior distribution can be obtained via the proportionality relation considered in (4):

$$p(\boldsymbol{f}, \boldsymbol{v}_\epsilon, \boldsymbol{v}_f|\boldsymbol{g}) \propto p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{v}_\epsilon)\, p(\boldsymbol{f}|\boldsymbol{v}_f) \\ p(\boldsymbol{v}_\epsilon|\boldsymbol{\alpha}_{\epsilon 0}, \boldsymbol{\beta}_{\epsilon 0})\, p(\boldsymbol{v}_f|\boldsymbol{\alpha}_{f 0}, \boldsymbol{\beta}_{f 0}) \quad (13)$$

5. BAYESIAN COMPUTATION AND PROPOSED ALGORITHM

For estimation of the unknowns, we consider the Posterior Mean (PM). This estimator is used because it minimize the Mean Square Error (MSE). One way to compute the PM in this case is to first approximate the posterior law $p(\boldsymbol{f}, \boldsymbol{v}_\epsilon, \boldsymbol{v}_f | \boldsymbol{g})$ with a separable law $q(\boldsymbol{f}, \boldsymbol{v}_\epsilon, \boldsymbol{v}_f | \boldsymbol{g})$:

$$p(\boldsymbol{f}, \boldsymbol{v}_\epsilon, \boldsymbol{v}_f | \boldsymbol{g}) \approx q(\boldsymbol{f}, \boldsymbol{v}_\epsilon, \boldsymbol{v}_f | \boldsymbol{g}) = q_1(\boldsymbol{f}) \, q_2(\boldsymbol{v}_\epsilon) \, q_3(\boldsymbol{v}_f), \tag{14}$$

in such a way that the approximate $q(\boldsymbol{f}, \boldsymbol{z}, v_\epsilon, v_f)$ is obtained by minimizing of the Kullback Leibler divergence $\mathrm{KL}\,(q:p) = \int q \, \ln \frac{q}{p}$ via alternate optimization. Thanks to the choice of the exponential families for the priors and the conjugate priors for the hyperparameters, we obtain proportionality relations between all distributions $q_1(\boldsymbol{f})$, $q_2(\boldsymbol{v}_\epsilon)$, $q_3(\boldsymbol{v}_f)$ and certain exponential expressions. The argument of the exponential proportional to $q_1(\boldsymbol{f})$ can be written as a quadratic criterion

$$J(\boldsymbol{f}) = \| \left(\widetilde{\boldsymbol{V}_\epsilon^{-1}} \right)^{1/2} (\boldsymbol{g} - \boldsymbol{H}\boldsymbol{f}) \|^2 + \| \left(\widetilde{\boldsymbol{V}_f^{-1}} \right)^{\frac{1}{2}} \boldsymbol{f} \|^2, \tag{15}$$

leading to the conclusion that $q_1(\boldsymbol{f})$ is a Normal distribution. The mean is given by the solution that minimize the criterion $J(\boldsymbol{f})$, then the covariance matrix can be found by identification. It is then easy to show that $q_1(\boldsymbol{f}) = \mathcal{N}\left(\boldsymbol{f} | \widehat{\boldsymbol{f}}_{PM}, \widehat{\boldsymbol{\Sigma}} \right)$, with:

$$\begin{cases} \widehat{\boldsymbol{f}}_{PM} = \left(\boldsymbol{H}^T \widetilde{\boldsymbol{V}_\epsilon^{-1}} \boldsymbol{H} + \widetilde{\boldsymbol{V}_f^{-1}} \right)^{-1} \boldsymbol{H}^T \widetilde{\boldsymbol{V}_\epsilon^{-1}} \boldsymbol{g} \\ \widehat{\boldsymbol{\Sigma}} = \left(\boldsymbol{H}^T \widetilde{\boldsymbol{V}_\epsilon^{-1}} \boldsymbol{H} + \widetilde{\boldsymbol{V}_f^{-1}} \right)^{-1} \end{cases}, \tag{16}$$

that $q_{2i}(v_{\epsilon_i})$ are Inverse Gamma distributions, $q_{2i}(v_{\epsilon_i}) = \mathcal{IG}\left(v_{\epsilon_i} | \alpha_{\epsilon_i}, \beta_{\epsilon_i} \right)$ with

$$\begin{cases} \alpha_{\epsilon_i} = \alpha_{\epsilon_i 0} + \frac{1}{2} \\ \beta_{\epsilon_i} = \beta_{\epsilon_i 0} + \frac{1}{2} \left[\boldsymbol{H}_i \widehat{\boldsymbol{\Sigma}} \boldsymbol{H}_i^T + \left(g_i - \boldsymbol{H}_i \widehat{\boldsymbol{f}}_{PM} \right)^2 \right] \end{cases} \tag{17}$$

where $\boldsymbol{H}_i$ is the line i of the matrix $\boldsymbol{H}$ and finally, that $q_{3j}(v_{f_j}) = \mathcal{IG}\left(v_{f_j} | \alpha_{f_j}, \beta_{f_j} \right)$ with:

$$\begin{cases} \alpha_{f_j} = \alpha_{f_j 0} + \frac{1}{2} \\ \beta_{f_j} = \beta_{f_j 0} + \frac{1}{2} \left(\widehat{f}_{jPM}^2 + \widehat{\boldsymbol{\Sigma}}_{jj} \right) \end{cases} \tag{18}$$

Knowing that $q_{2i}(v_{\epsilon_i})$ and $q_{3j}(v_{f_j})$ are Inverse Gamma distributions and using $\left\langle x^{-1} \right\rangle_{\mathcal{IG}(x|\alpha,\beta)} = \frac{\alpha}{\beta}$, then we obtain the following forms for $\widetilde{\boldsymbol{V}_\epsilon^{-1}}$ and $\widetilde{\boldsymbol{V}_f^{-1}}$ involved in the expression

of the two parameters of the Normal distribution $q_1(\boldsymbol{f})$:

$$\widetilde{\boldsymbol{V}_\epsilon^{-1}} = \begin{bmatrix} \frac{\alpha_{\epsilon_1}}{\beta_{\epsilon_1}} & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & \frac{\alpha_{\epsilon_i}}{\beta_{\epsilon_i}} & \dots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & \frac{\alpha_{\epsilon_N}}{\beta_{\epsilon_N}} \end{bmatrix} \; ; \; \widetilde{\boldsymbol{V}_f^{-1}} = \begin{bmatrix} \frac{\alpha_{f_1}}{\beta_{f_1}} & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & \frac{\alpha_{f_j}}{\beta_{f_j}} & \dots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & \frac{\alpha_{f_M}}{\beta_{f_M}} \end{bmatrix} \tag{19}$$

Finally, these steps lead to an iterative algorithm described as it follows: **(a)** Initialization; **(b)** Use equation (16) to compute $\widehat{\boldsymbol{f}}_{PM}, \widetilde{\boldsymbol{\Sigma}}$; **(c)** Use equation (17) to compute $\widetilde{\alpha_\epsilon}, \widetilde{\beta_\epsilon}$; **(d)** Use equation (18) to compute $\widetilde{\alpha_f}, \widetilde{\beta_f}$. The following scheme summarizes the proposed algorithm:
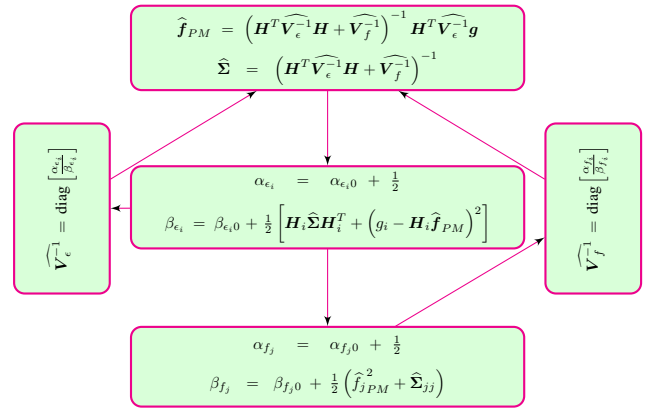
For initializing the algorithm one of the possible choice is assigning values for the following parameters: $\left\{ \alpha_{f_j}^{(0)}, \beta_{f_j}^{(0)} \right\}$, $j \in \{1, 2, \dots, M\}$ representing $\widetilde{\boldsymbol{V}_f^{-1}}^{(0)}$ and $\left\{ \alpha_{\epsilon_i}^{(0)}, \beta_{\epsilon_i}^{(0)} \right\}$, $i \in \{1, 2, \dots, N\}$ representing $\widetilde{\boldsymbol{V}_\epsilon^{-1}}^{(0)}$ This choice for the initialization procedure is sufficient, in the sense that the considered parameters from above represent all the necessary informations for starting the first iteration of the algorithm and computing all other parameters of the algorithm corresponding to step zero, i.e. $\widehat{\boldsymbol{f}}_{PM}^{(0)}$ and $\widehat{\boldsymbol{\Sigma}}^{(0)}$. For the parameters $\alpha_{\epsilon_i}^{(0)}$, $\beta_{\epsilon_i}^{(0)}$ and $\alpha_{f_j}^{(0)}, \beta_{f_j}^{(0)}$, we consider the following initialization:

$$\alpha_{\epsilon_j}^{(0)} = \alpha_{\epsilon_j 0} \;, \;\; \beta_{\epsilon_j}^{(0)} = \beta_{\epsilon_j 0} \;, \;\; \alpha_{f_j}^{(0)} = \alpha_{f_j 0} \;, \;\; \beta_{f_j}^{(0)} = \beta_{f_j 0} \tag{20}$$

A natural choice in this case is Non Informative Prior Law (NIPL). The Inverse Gamma Distribution is weak for parameters $\alpha \to 0$ and $\beta \to 0$, so one possible choice is $\alpha_{\epsilon_j 0} = \beta_{\epsilon_j 0} = 0.001$ and $\alpha_{f_j 0} = \beta_{f_j 0} = 0.001$. In particular, such an approach, is consistent with a non-supervised algorithm

6. SIMULATIONS

For validating the proposed method, first we work with some simulated data. In the real case the theoretical $\boldsymbol{f}$ is unknown,

so the only possible comparison is between the available g (representing the real data) and the estimated $\widehat{g}_{PM}$ (obtained via the reconstruction done with the estimated $\widehat{f}_{PM}$). After validating the method, we present the results corresponding to real data, i.e. signals from chronobiology experiments.

6.1. Synthetic data

An important step for validating the method is to consider signals with known corresponding periodic components, which gives the possibility to compare f and the estimated $\widehat{f}_{PM}$. We consider the following protocol: **(a)** Consider a sparse amplitude periodic components vector f, Figure 1,(a). For the simulations used in this article, we analysed a periodic components for the interval associated with the circadian domain and the possible harmonics, i.e. the interval $[8, 32]$, with one hour precision; **(b)** Compute the corresponding signal g_0
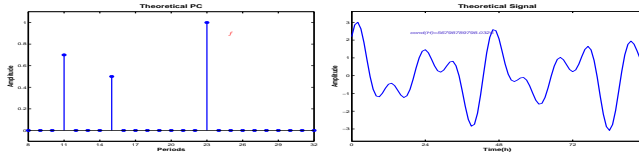

Fig. 1. Periodic Components f and the corresponding theoretical signal $g_0 = Hf$

(4 days length), Figure 1, (b). The matrix operator H used is a real matrix obtained in the same manner as the Fourier Transform Matrix using the considered periods and is defined as a sum of a sine and cosine; **(c)** Generate a noisy signal g (input
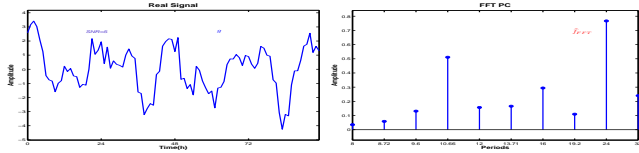

Fig. 2. Noisy Signal $g = Hf + \epsilon$ (a) and the corresponding FFT Spectrum

for the proposed algorithm) by adding some noise (SNR 5dB), Figure 2, (a) and its corresponding spectrum obtained via Fast Fourier Transform, Figure 2, (b). **(d)** Use the noisy signal
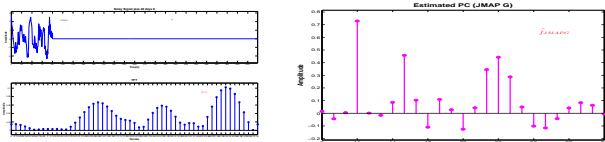

Fig. 3. Periodic components estimation via zero padding and L_2 regularization

g to estimate the periodic components via the zero padding

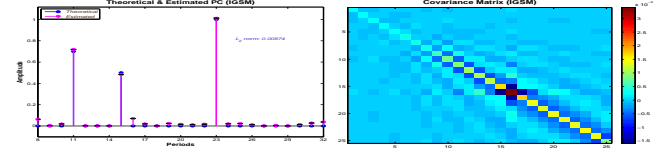method Figure 3, (a) and via L_2 regularization method Figure 3, (b). **(e)** Compare the periodic components vector (f)

Fig. 4. (IP:Student-t) Method: periodic components reconstruction and the covariance matrix

with the estimated $\widehat{f}_{PM}$ one via proposed method, Figure 4, (a). The proposed method also indicates the variances; the covariance matrix is presented in Figure 4, (b). **(f)** Compare
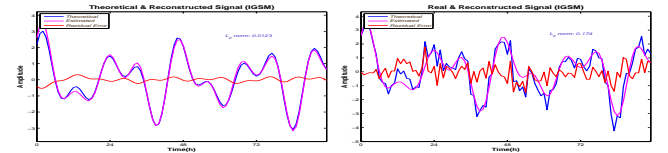

Fig. 5. (IP:Student-t) Method: signal and noisy signal reconstruction

the original signal g_0 and the reconstructed one $\widehat{g}_{PM}$, Figure 5, (a) and the noisy signal g with the reconstructed one $\widehat{g}_{PM}$. As a conclusion of these simulations, we can see that neither FFT based methods (with or without zero padding) nor Least Squares (LS) or even the quadratic regularization methods can give satisfactory results. The proposed method seems to be appropriate for this application.

6.2. Real data

For this section we consider a real signal expressing the photon absorption. The experiment was realized over mice,
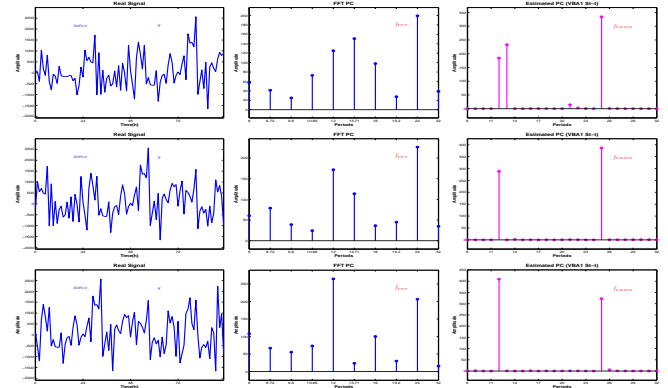

Fig. 6. (IP:Student-t) Method vs. FFT: Real data

investigating the tumor clock gene expression in freely

moving mice along the course of tumor growth using RT-Biolumicorder units (Lesa-technology SA, Switzerland). The hepatocarcinoma cells with bioluminescent clock gene Per2 (Hepa1-6Per2::luc) were inoculated subcutaneously in mice. The tumor photon emission was recorded with a photomultiplier tube in mice. The question addressed by the biologists is the stability of the periodic components so for every four days of the signal we apply the proposed method and we compare the results with the FFT spectrum. From a 10 days length signal we present three consecutive windows (4 days length, 1 day shift) in Figure 6. On the left column are presented the windows, in the center the corresponding periodic components estimated via FFT and on the right column the periodic components estimated by the proposed method. Figure 7 shows another three consecutive windows and the
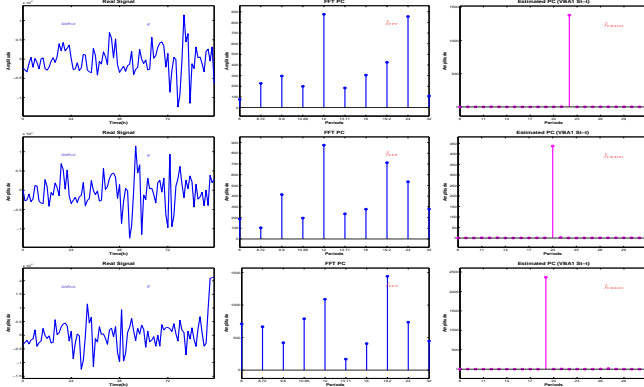

Fig. 7. (IP:Student-t) Method vs. FFT: Real data

corresponding periodic component vector estimation for FFT and the proposed method.

7. DISCUSSIONS AND CONCLUSIONS

We have presented a new method that can estimate the periodic components of short signals. In the synthetic data subsection 6.1 we have showed the drawbacks of the FFT method: for the known sparse periodic component vector, all the picks are wrongly estimated, making the analysis of the stability of the period impossible for such short signals. The zero padding method and the L_2 regularization method are also providing inaccurate estimations. The proposed method is accurately estimating the periodic component vector. For the example presented in Figure 4,(a) the reconstruction error, is $\frac{\|\widehat{\boldsymbol{f}}_{PM}-\boldsymbol{f}\|^2}{\|\boldsymbol{f}\|^2} = 0.00874$. The residual error for the reconstructed signal $\widehat{\boldsymbol{g}}_{PM}$ and $\widehat{\boldsymbol{g}}$, Figure 5,(b) is consistent with the signal to noise ratio of 5 dB. The proposed method is providing also the covariance matrix for $\boldsymbol{f}$, i.e. the variances for the estimated amplitudes. For the real data, Figure 7 shows how the proposed method is able to detect the variability of the period in the signal, by precisely estimating the positions

of the non-zero periodic components for each window. The results obtained via the FFT method can not detect this variability. Our model makes no assumptions concerning the exact number of non-zero picks. This allows the biologist to visualize all the periodic phenomena from the recorded signals, Figure 6.

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