Iterative Methods for Looped Network Pipeline Calculation

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Dear editor,
All your suggestions are now accepted.
Sincerely yours,
Dejan Brkić, PhD

Iterative methods for looped network pipeline calculation

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Abstract:

Since the value of the hydraulic resistance depends on flow rate, problem of flow distribution per pipes in a gas or water distributive looped pipelines has to be solved using iterative procedure. A number of iterative methods for determining of hydraulic solution of pipeline networks, such as, Hardy Cross, Modified Hardy Cross, Node-Loop method, Modified Node method and M.M. Andrijašev method are shown in this paper. Convergence properties are compared and discussed using a simple network with three loops. In a municipal gas pipeline, natural gas can be treated as incompressible fluid. Even under this circumstance, calculation of water pipelines cannot be literary copied and applied for calculation of gas pipelines. Some differences in calculations of networks for distribution of these two fluids, i.e. water apropos natural gas are also noted.

Keywords: Pipeline networks, Water distribution system, Natural gas distribution system, Calculation methods, Flow, Hydraulic pipeline systems

1. Introduction

A number of iterative methods for determining the hydraulic solution of water and natural gas pipeline networks which take ring-like form, such as, Hardy Cross, Modified Hardy Cross (including Andrijašev method), Node-Loop method and Modified Node method are compared in this paper. All presented methods assume equilibrium among pressure and friction forces in steady and incompressible flow. As a result, they cannot be successfully used in unsteady and compressible flow calculations with large pressure drops where inertia force is important. Problem of flow in pipes and open conduits was one which had been of considerable interest to engineers for nearly 250 years. Even today, this problem is not solved definitively. The difficulty to solve the turbulent flow problems lies in the fact that the friction factor is a complex function of relative surface roughness and Reynolds number. Precisely, hydraulic resistance depends on flow rate and hence flow problem in hydraulic networks has to be solved iteratively. Similar situation is with electric current when diode is in circuit. With common resistors in electrical circuits where the electric resistances do not depend on the value of electric current in the conduit, problem is linear and no iterative procedure has to be used. So problem of flow through a single tube is already complex. Despite of it, very efficient procedures are available for solution of flow problem in a complex pipeline such as looped pipeline like waterworks or natural gas distribution network is. Most of the shown methods are based on solution of the loop equations while the node equations are used only as control of accuracy. Node-Loop method is also based on solution of the loop equations but the node equations are also used in calculation and not only for control purposes. Node method and here shown Modified Node method is based on solution of the node equations while the loop equations is used for control purposes.

In this paper, speed of convergence of presented methods are compared and discussed. This is done for one simple network with three loops, both, for water and gas distribution networks. Similar example can be done for the air ventilation systems in the buildings or mines. For ventilation problem readers can consult paper of Aynsley (1997), Mathews and Köhler (1995), Wang and Hartman (1967), etc. While the disposal of water or gas distribution system is in single plane with certain elevation of pipes, ventilation network is almost always spatial (Brkić 2009).

2. Hydraulics frictions and flow rates in pipes

Each pipe is connected to two nodes at its ends. In a pipe network system, pipes are the channels used to convey fluid from one location to another. Pipes are sometimes referred to as tubes or conduits, and as a part of network to as lines, edges, arcs or branches (in the graph theory, special kinds of branches are referred to as trees). The physical characteristics of a pipe include the length, inside diameter, roughness, and minor loss coefficient. When fluid is conveyed through the pipe, hydraulic energy is lost due to the friction between the moving fluid and the stationary pipe surface. This friction loss is a major energy loss in pipe flow (Farshad et al. 2001, Kumar 2010).

Various equations were proposed to determinate the head losses due to friction, including the Darcy-Weisbach, Fanning, Chezy, Manning, Hazen-Williams and Scobey formulas. These equations relate the friction losses to physical characteristics of the pipe and various flow parameters. The Darcy-Weisbach formula for calculating of friction loss is more accurate than the Hazen-Williams. Beside this, the Hazen-Williams relation is only for water flow.

2.1 Gas flow rates and pressure drops in pipes

When a gas is forced to flow through pipes it expands to a lower pressure and changes its density. Flow-rate, i.e. pressure drop equations for condition in gas distribution networks assumes a constant density of a fluid within the pipes. This assumption applies only to incompressible, i.e. for liquids flows such as in water distribution systems for municipalities (or any other liquid, like crude oil, etc.). For the small pressure drops in typical gas distribution networks, gas density can be treated as constant, which means that gas can be treated as incompressible fluid (Pretorius et al. 2008), but not as liquid flow. Assumption of gas incompressibility means that it is compressed and forced to convey through pipes, but inside the pipeline system pressure drop of already compressed gas is small and hence further changes in gas density can be neglected. Fact is that gas is actually compressed and hence that volume of gas is decreased and then such compressed volume of gas is conveying with constant density through gas distribution pipeline. So, mass of gas is constant, but volume is decreased while gas density is according to this, increased. Operate pressure for distribution gas network is $4 \cdot 10^5$ Pa abs i.e. $3 \cdot 10^5$ Pa gauge and accordingly volume of gas is decreased four times compared to volume of gas at normal conditions.

Inner surface of polyethylene pipes which are almost always used in gas distribution networks are practically smooth and hence flow regime in the typical network is hydraulically ‘smooth’ (Sukharev et al 2005). For this regime is suitable Renouard’s equation adjusted for natural gas flow (1):

$$F_g = C = p_1^2 - p_2^2 = \frac{4810 \cdot Q_n^{1.82} \cdot L \cdot \rho_r}{D_{in}^{4.82}} \tag{1}$$

Regarding to Renouard’s formula has to be careful since it does not relate pressure drop but actually difference of the quadratic pressure at the input and the output of pipe. This means that $\sqrt{C}$ is not actually pressure drop in spite of the same unit of measurement, i.e. same unit is used for the pressure (Pa). For gas pipeline calculation is very useful fact that when $\sqrt{C} \to 0$ this consecutive means that also $C \to 0$. Parameter $\sqrt{C}$ can be noted as pseudo-pressure drop.

2.2. Liquid flow in pipes

In Renoard's equation adjusted for gas pipelines (1) friction factor is rearranged in the way to be expressed using other flow parameters and also using some thermodynamic properties of natural gas. Using formulation for Darcy friction factor in hydraulically smooth region Renouard suggests his equation for liquid flow (2):

$$\lambda = \frac{0.172}{\mathrm{Re}^{0.18}} \tag{2}$$

Then pressure drop (Ekinci and Konak 2009) can be found very easily using Darcy-Weisbach equation (3):

$$F_w = \Delta p = p_1 - p_2 = \lambda \cdot \frac{\mathrm{L}}{\mathrm{D}_{\mathrm{in}}^5} \cdot \frac{8 \cdot Q^2}{\pi^2} \cdot \rho \tag{3}$$

Renouard's equation (2) is based on power law. Liquid flow is better fit using some kind of logarithmic formula like Colebrook's (4):

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \log\left(\frac{2.51}{\mathrm{Re} \cdot \sqrt{\lambda}} + \frac{\varepsilon}{3.71 \cdot D_{in}} \right) \tag{4}$$

Colebrook's equation is suitable especially for flow through steel pipes (Colebrook 1939). Some researchers adopt a modification of the Colebrook equation, using the 2.825 constant instead of 2.51 especially for gas flow calculation (Haaland 1983, Coelho and Pinho 2007).

In the case of waterworks, pressures will also be expressed in Pa, not in meter (m) equivalents. In that way, clear comparison with gas distribution network can be done. Pressures expressed in Pa can be very easily recalculated in heads expressed in meters (m) for water networks knowing water density. Example network is located in a single-flat area with no variation in elevation (otherwise the correction term must be used which is significant for water network).

3. Looped pipeline networks for distribution of fluids

A pipeline network is a collection of elements such as pipes, compressors, pumps, valves, regulators, tanks, and reservoirs interconnected in a specific way. In this article focus is on pipes. The behavior of the network is governed by the specific characteristics of the elements and how the elements are connected together. Our assumption is that pipes are connected in a smooth way, i.e. so called minor hydraulic loses are neglected. Including other elements different than pipes is a subject of sufficient diversity and complexity to merit a separate review.

The analysis of looped pipeline systems by formal algebraic procedures is very difficult if the systems are complex. Electrical models had been used in study of this problem in the time before advanced computer became available as background to support demandable numerical procedures (Mah and Shacham 1978). Here presented methods use successive corrections or better to say these methods require iterative procedure. The convergence properties of presented methods are the main subject of this article. All of such methods can be divided into two groups (1) Methods based on solution of the loop equations, and (2) Methods based on solution of the node equations.

Most of the methods used commonly in engineering practice belong to the group based on solution of the loop equations. Such method presented in this paper is Hardy Cross method (Cross 1936). Method of Andrijašev is variation of Hardy Cross method (Andrijašev 1964)[1].

[1] There are some difficulties with citation of Russian methods. Both methods, Lobačev and Andrijašev, are actually from 1930's but original papers from Soviet time are problem to be found. Many books in Russian language explain methods of Lobačev and Andrijašev but in these books reference list is not available.

Contemporary with Hardy Cross, soviet author V.G. Lobačev (Latišenkov and Lobačev 1956) developed very similar method compared to original Hardy Cross method. Modified Hardy Cross method proposed Epp and Fowler (1970) which considers entire system simultaneously is also sort of loop method. Node-Loop method proposed by Wood and Charles (1972) and later improved by Wood and Rayes (1981) is combination of the loop and node oriented methods, but despite of its name is essentially belong to the group of loop methods. Node-Loop method is based on solution of the loop equations while the node equations are also involved in calculation. Only Node method proposed by Shamir and Howard (1968) is real representative of node oriented methods. Node method uses idea of Hardy Cross but to solve node equations instead of loops ones. Improved Node method uses similar idea as Epp and Fowler (1970) suggested for the improvement of original Hardy Cross method, but of course here applied to the Node method.

Example network with three loops is shown in the figure 1. Before calculation, maximal consumption for each node including one of more inlet nodes has to be determined. Pipe diameters and node inputs and outputs cannot be changed during the iterative procedure. Goal is to find final flow distribution for this pipeline system (simulation problem).

Figure 1. Example of pipeline network

Pipe lengths and pipes diameters are listed in figure 1. Final flows do not depend on first assumed fluid flows per pipes in the case of loop oriented methods or on first assumed pressure drops or pseudo-pressure drops in the case of node oriented methods. To have a better appreciation of the utility of these representations, first will be considered the laws that govern flow rates and pressure drops in a pipeline network. These are the counterparts to

Kirchhoff's laws for electrical circuits and accordingly for hydraulic networks, namely, (i) the algebraic sum of flows at each node must be zero; and (ii) the algebraic sum of pressure drops for water network (i.e. pseudo-pressure drop for gas network) around any cyclic path (loop) must be approximately zero. For a hydraulic network, fact that both laws are satisfied simultaneously means that calculation for steady state for simulation problem is finished.

3.1 Methods based on solution of the loop equations

Loops are sometimes referred also to as contours or paths. Note that contours and loops are not synonyms in the M.M. Andrijašev method.

For the loop oriented methods first Kirchhoff's law must be satisfied for all nodes in all iterations. Second Kirchhoff's law for each loop must be satisfied with acceptable tolerance at the end of the calculation. Or, in other words, final flows are these ones which values are not changed between two successive iterations (must be satisfied for flow in each pipe).

In a loop oriented methods, first initial flow pattern must be chosen to satisfied first Kirchhoff's law. Endless number of flow combinations can satisfy this condition. Someone can conclude that it is crucial to start with good initial guesses. But, how does one obtain good initial guesses? And how sensitive are the methods to the initial guesses? Answer is that initial flow pattern does not have significant influence on convergence properties of observed method. These indicate that the choice of initial flows is not critical and need to be chosen only to satisfied first Kirchhoff's law for all nodes (Gay and Middleton 1971). It would be equally satisfactory to generate the initial distribution within the computer program. Initial flow pattern for our example network is shown in the figure 2. Loops are also defined in the figure 2.

Figure 2. Example network initial parameters prepared for loop oriented calculation

Results of calculation using original Hardy Cross, Modified Hardy Cross, M.M. Andrijašev method are not actually flows Q but correction of flows ΔQ calculated for each loop. These corrections have to be added algebraically to flows from previous iteration for each pipe according to specific rules. A pipe common to two loops receives two corrections. The upper plus or minus sign (shown in tables 1-8) indicates direction of flow in that conduit in these two loops and is obtained from Q for previous iteration. The upper sign is the same as the sign in front of Q if the flow direction in each loop coincides with the assumed flow direction in the particular loop under consideration, and opposite if it does not. The lower sign is copied from the primary loop for this correction (sign from the loop where this correction is first, sign preceding the first iteration from adjacent contour for the conduit taken into consideration). The rules for sign of corrections ΔQ_2 are: (1). the algebraic operation for correction ΔQ_1 should be the opposite of its sign; i.e. add when the sign is minus. (2). the algebraic operation for corrections ΔQ_2 should be the opposite of their lower signs when their upper signs are the same as the sign in front of Q, and as indicated by their lower signs when their upper signs are opposite to the sign in front of Q. For details of sing of corrections consult paper of Brkić (2009) and article Corfield et al (1974) from Gas Engineers Handbook.

3.1.1 Hardy Cross method for the pipeline calculation

Hardy Cross, American engineer, developed method in 1936, later named after him (Cross 1936). Hardy Cross calculation for gas pipeline network is shown in table 1 and for water network in table 2. Only two iterations will be shown in details. Flow Q_1 calculated in first iteration become initial flow Q for second iteration. The plus or minus preceding the flow Q, indicates the direction of the conduit flow for the particular loop (Figure 2). A plus sign denotes clockwise flow in the conduit within the loop; a minus sign, counter-clockwise. A flow correction ΔQ_1 as shown in table 1 and 2 is computed for each loop. This correction must be subtracted algebraically from the assumed gas flow according to specific rules explained previously.

Correction for each loop ΔQ_1 is calculated using first derivative of pressure function C for pipes defined by Renouard's equation for gas networks (5) and derivative of pressure drop Δp for pipes defined by Darcy-Weisbach for water networks (6) where flow Q is treated as variable:

$$F' = \frac{\partial(C)}{\partial Q} = \frac{\partial\left(\frac{4810 \cdot Q_n^{1.82} \cdot L \cdot \rho_r}{D_{in}^{4.82}}\right)}{\partial Q} = \frac{\partial\left(R \cdot Q^{1.82}\right)}{\partial Q} = \frac{1.82 \cdot 4810 \cdot Q_n^{0.82} \cdot L \cdot \rho_r}{D_{in}^{4.82}} = 1.82 \cdot R \cdot Q_n^{0.82}$$

(5)

$$F' = \frac{\partial(\Delta p)}{\partial Q} = \frac{\partial\left(\frac{8 \cdot \rho \cdot \lambda \cdot \mathrm{L} \cdot Q^2}{\pi^2 \cdot \mathrm{D}^5}\right)}{\partial Q} = \frac{\partial\left(R \cdot Q^2\right)}{\partial Q} = \frac{16 \cdot \rho \cdot \lambda \cdot \mathrm{L} \cdot Q}{\pi^2 \cdot \mathrm{D}^5} = 2 \cdot R \cdot Q$$

(6)

Derivative for loops is calculated using assumed loop shown in the figure 2 with no reference to direction. For the loop I this derivative for gas network (7) and for water network (8) can be written:

$$F_I^{'} = \frac{\partial C_I(Q)}{\partial Q_I} = \frac{\partial\left(\left|-C_3(Q_3)\right| + \left|C_4(Q_4)\right| + \left|-C_7(Q_7)\right|\right)_I}{\partial Q_I} = 1.82 \cdot 4810 \cdot \rho_r \cdot \left(\frac{Q_3^{0.82} \cdot L_3}{D_3^{4.82}} + \frac{Q_4^{0.82} \cdot L_4}{D_4^{4.82}} + \frac{Q_7^{0.82} \cdot L_7}{D_7^{4.82}}\right)_I$$

(7)

$$F_I^{'} = \frac{\partial \Delta p_I(Q)}{\partial Q_I} = \frac{\partial(|-\Delta p_3(Q_3)| + |\Delta p_4(Q_4)| + |-\Delta p_7(Q_7)|)_I}{\partial Q_I} = \frac{16 \cdot \rho}{\pi^2} \cdot \left(\frac{\lambda_3 \cdot L_3 \cdot Q_3}{D_3^5} + \frac{\lambda_4 \cdot L_4 \cdot Q_4}{D_4^5} + \frac{\lambda_7 \cdot L_7 \cdot Q_7}{D_7^5} \right)_I$$

(8)

In presented example loop I begins and ends in node II via pipes 3, 4, and 7.

Correction of flow (ΔQ_1) for each loop can be calculated (9):

$$\Delta Q_1 = -\frac{F}{|F^{'}|} \quad (9)$$

Table 1. Hardy Cross calculation for example gas network

Table 2. Hardy Cross calculation for example water network

In the original Hardy Cross method, each loop correction is determined independently of other loops. Original Hardy Cross method is a sort of Newton–Raphson method but used to solve each single loop equation solely, one by one. Hence, Hardy Cross method is also known as the single contour adjustment method. In matrix form, original Hardy Cross method for the example network for water distribution from figure 1 can be noted as (10):

$$\begin{bmatrix} \frac{\partial F_I(|-Q_3|,|Q_4|,|-Q_7|)}{\partial(\Delta Q_I)} & 0 & 0 \\ 0 & \frac{\partial F_{II}(|Q_1|,|-Q|_2,|-Q_4|,|Q_5|)}{\partial(\Delta Q_{II})} & 0 \\ 0 & 0 & \frac{\partial F_{III}(|-Q_5|,|Q_6|,|Q_7|,|-Q_8|)}{\partial(\Delta Q_{III})} \end{bmatrix} x \begin{bmatrix} \Delta Q_I \\ \Delta Q_{II} \\ \Delta Q_{III} \end{bmatrix} = \begin{bmatrix} F_I \\ F_{II} \\ F_{III} \end{bmatrix} \quad (10)$$

Using numerical values from table 2 for the first iteration of calculation of water network matrix equation can be written (11):

$$\begin{bmatrix} 129435625 & 0 & 0 \\ 0 & 53910587 & 0 \\ 0 & 0 & 193795963 \end{bmatrix} x \begin{bmatrix} \Delta Q_I \\ \Delta Q_{II} \\ \Delta Q_{III} \end{bmatrix} = \begin{bmatrix} -15928498 \\ 5040086 \\ 2070510 \end{bmatrix} \quad (11)$$

Then set of correction for water network calculation in the first iteration is $[\Delta Q_I, \Delta Q_{II}, \Delta Q_{III}]^T=[-12306\cdot 10^{-5}, 9349\cdot 10^{-5}, 1068\cdot 10^{-5}]^T$ which is identical as in table 2.

3.1.2 Modified Hardy Cross method for the pipeline calculation

Modified Hardy Cross method (somewhere called improved) is also known as the simultaneous loop adjustment method. As seen in figure 1, several loops have common pipes, so corrections to these loops will cause energy losses around more than one loop. In figure 1, pipe 4 belongs to two loops (loop I and II), pipe 7 to loop I and III, and finally pipe 5 to II and III. Modified Hardy Cross method is a sort of Newton–Raphson method used to solve unknown flow corrections taking into consideration whole system simultaneously. Epp and Fowler (1970) gave idea for this approach. To increase efficiency of Hardy Cross method zeros from non-diagonal term in matrix equation (10) will be replaced to include influence of pipes mutual with adjacent loop (12).

$$\begin{bmatrix} \frac{\partial F_I(|-Q_3|,|Q_4|,|-Q_7|)}{\partial(\Delta Q_I)} & -\left|\frac{\partial F_I(|-Q_4|)}{\partial(\Delta Q_{II})}\right| & -\left|\frac{\partial F_I(|Q_7|)}{\partial(\Delta Q_{III})}\right| \\ -\left|\frac{\partial F_{II}(|-Q_4|)}{\partial(\Delta Q_I)}\right| & \frac{\partial F_{II}(|Q_1|,|-Q_2|,|-Q_4|,|Q_5|)}{\partial(\Delta Q_{II})} & -\left|\frac{\partial F_{II}(|-Q_5|)}{\partial(\Delta Q_{III})}\right| \\ -\left|\frac{\partial F_{III}(|Q_7|)}{\partial(\Delta Q_I)}\right| & -\left|\frac{\partial F_{III}(|-Q_5|)}{\partial(\Delta Q_{II})}\right| & \frac{\partial F_{III}(|-Q_5|,|Q_6|,|Q_7|,|-Q_8|)}{\partial(\Delta Q_{III})} \end{bmatrix} x \begin{bmatrix} \Delta Q_I \\ \Delta Q_{II} \\ \Delta Q_{III} \end{bmatrix} = \begin{bmatrix} F_I \\ F_{II} \\ F_{III} \end{bmatrix}$$

(12)

Presented matrix is symmetrical; for example (13):

$$\frac{\partial F_I(Q_7)}{\partial(\Delta Q_{III})} = \frac{\partial F_{III}(Q_7)}{\partial(\Delta Q_I)} \quad (13)$$

This is because pipe 7 is mutual for two adjacent loops (loop I and loop III). Non-diagonal terms have always opposite sign than diagonal. Spatial networks common for ventilation systems in buildings or mines are exceptions (Brkić 2009).

Using numerical values from table 2 for the first iteration of calculation of water network matrix equation can be written (14):

$$\begin{bmatrix} 12943562525 & -4978573 & -70157780 \\ -4978573 & 53910587 & -429677 \\ -70157780 & -429677 & 193795963 \end{bmatrix} x \begin{bmatrix} \Delta Q_{I} \\ \Delta Q_{II} \\ \Delta Q_{III} \end{bmatrix} = \begin{bmatrix} -159284985 \\ 50400861 \\ 20705102 \end{bmatrix}$$

(14)

First two iterations from our example using the modified Hardy Cross method is shown in table 3 for gas network and in table 4 for water network.

Table 3. Calculation after the modified Hardy Cross method for example gas network

Table 4. Calculation after the modified Hardy Cross method for example water network

3.1.2 Modified Andrijašev method for the pipeline calculation

Andrijašev method can be used in the formulation as in the original Hardy Cross method and as in Modified Hardy Cross method. Here it will be given in notation as improved method because this approach shows better convergence performance (example for two iteration of gas network calculation is shown in table 5 and for water network in table 6). It can be noted that some pipes in table 1-4 received only one correction per iteration (for example pipe 3 in contour I). This means that pipe 3 belongs only to one loop. In method of M.M. Andrijašev contours can be defined to include few loops. Thus, contours can be defined in other way and then each pipe in the network belongs to two contours (Figure 3). Loop is not synonym with contour in M.M. Andrijašev method as in Hardy Cross approach. Andrijašev's contour will be marked with special sign (^{O}).

Figure 3. Contours for method of M.M. Andrijašev calculation

Now contour I^{O} (red circuit in figure 3) starting and ending in node I via pipes 4, 5. 6, 8, 3, contour II^{O} (green circuit in figure 3) starting and ending in referent node via pipes 1, 6, 8, 7, 4, 2, and finally contour III^{O} (blue circuit in figure 3) starting and ending in referent node via pipes 1, 5, 7, 3, 2.

Matrix formulation of this method for example gas network can be written as (15):

$$\begin{bmatrix} \frac{\partial F_I^o(|-Q_3|,|Q_4|,|-Q_5|,|Q_6|,|-Q_8|)}{\partial(\Delta Q_I^o)} & \frac{\partial F_I^o(|-Q_4|,|Q_6|,|Q_8|)}{\partial(\Delta Q_{II}^o)} & \frac{\partial F_I^o(|Q_3|,|-Q_5|)}{\partial(\Delta Q_{III}^o)} \\ \frac{\partial F_{II}^o(|-Q_4|,|Q_6|,|Q_8|)}{\partial(\Delta Q_I^o)} & \frac{\partial F_{II}^o(|-Q_1|,|-Q_2|,|-Q_4|,|Q_6|,|Q_7|,|Q_{-8}|)}{\partial(\Delta Q_{II}^o)} & \frac{\partial F_{II}^o(|Q_1|,|Q_2|,|-Q_7|)}{\partial(\Delta Q_{III}^o)} \\ \frac{\partial F_{III}^o(|Q_3|,|-Q_5|)}{\partial(\Delta Q_I^o)} & \frac{\partial F_{III}^o(|Q_1|,|Q_2|,|-Q_7|)}{\partial(\Delta Q_{II}^o)} & \frac{\partial F_{III}^o(|Q_1|,|-Q_2|,|-Q_3|,|Q_5|,|-Q_7|)}{\partial(\Delta Q_{III}^o)} \end{bmatrix} x \begin{bmatrix} \Delta Q_I^o \\ \Delta Q_{II}^o \\ \Delta Q_{III}^o \end{bmatrix} = \begin{bmatrix} F_I^o \\ F_{II}^o \\ F_{III}^o \end{bmatrix} \quad (15)$$

Here has to be very careful because non-diagonal terms are not always negative as in modified Hardy Cross method (16):

$$\begin{bmatrix} 40489565894 & 25862392143 & 11719977603 \\ 25862392143 & 52949765748 & -4166255733 \\ 11719977603 & -4166255733 & 36379348868 \end{bmatrix} x \begin{bmatrix} \Delta Q_I^o \\ \Delta Q_{II}^o \\ \Delta Q_{III}^o \end{bmatrix} = \begin{bmatrix} -3277663792 \\ 1492371894 \\ -2518358643 \end{bmatrix} \quad (16)$$

For example term in first row and second column from (16) is 25862392143=4298435730+22897756035-1333799622 according to table 5. Same value has term in second row, first column, etc. Presented matrix is symmetrical. Similar can be done for water network (table 6).

Table 5. Calculation after Modified M.M. Andrijašev method for example gas network

Table 6. Calculation after Modified M.M. Andrijašev method for example water network

M.M. Andrijašev method does not improve convergence properties compared with Modified Hardy Cross method. It only complicates calculation of non-diagonal terms in the matrix which is used for calculation. Furthermore, calculation of single-flat pipeline networks using M.M. Andrijašev method is equally complex as calculation of spatial, multidimensional network, using Modified Hardy Cross method (Brkić 2009).

3.2 Methods based on solution of the node equations

Nodes are sometimes also referred to as junctions, points or vertices.

Shamir and Howard (1968) introduced first node oriented method. For the node oriented methods second Kirchhoff's law must be satisfied for each loop in all iterations. First Kirchhoff's law for each node must be satisfied with acceptable tolerance at the end of the calculation. Here will be presented only one method. Improvement of presented method is done according to the same idea as used to improve original Hardy Cross method but here applied to the matrix solution for the node equations.

3.2.1 Node method

Pipe equations in previous text were expressed as Δp=F(Q) for waterworks, or C= $p_1^2 - p_2^2$=F(Q) for gas networks. These relations can be rewritten in form as Q=f(Δp) for waterworks or Q=f(C) for gas networks. After that, Renouard's equation (1) can be rearranged (17):

$$f_g = Q(C) = \left(\frac{\left(p_1^2 - p_2^2\right) \cdot D_{in}^{4.82}}{4810 \cdot L \cdot \rho_r}\right)^{\frac{1}{1.82}} = \left(\frac{C \cdot D_{in}^{4.82}}{4810 \cdot L \cdot \rho_r}\right)^{\frac{1}{1.82}} \quad (17)$$

Similar, can be done for Darcy-Weisbach equation (18):

$$f_w = Q(\Delta p) = \sqrt{\frac{\left(p_1 - p_2\right) \cdot \mathrm{D}_{\mathrm{in}}^5 \cdot \pi^2}{8 \cdot \lambda \cdot \mathrm{L} \cdot \rho}} = \sqrt{\frac{\Delta p \cdot \mathrm{D}_{\mathrm{in}}^5 \cdot \pi^2}{8 \cdot \lambda \cdot \mathrm{L} \cdot \rho}} \quad (18)$$

In Node method, (C) for each pipe in a gas network and (Δp) for each pipe in a water network has to be assumed, not flows. These assumed functions of pressure must be chosen to satisfy second Kirchhoff's law (Figure 4). First Kirchhoff's law will be fulfilled with demanded tolerance at the end of calculation. For gas network, correction of C, noted as ΔC must be calculated, and for water network, correction of Δp, here noted as $\Delta_{\Delta p}$. Same algebraic rules as for loop oriented methods are valid. Correction ΔC or $\Delta_{\Delta p}$ will be calculated using first derivative of Q=f(Δp) for waterworks and Q=f(C) for gas networks, where Δp, i.e. C is treated as variable (19):

$$\Delta x = -\frac{f}{\left|f'\right|} \quad (19)$$

In previous equation x is Δp or C. Example network adjusted for node oriented calculation is shown in figure 4 (red letters for gas network and blue for water network).

Figure 4. Example of pipeline gas network with three loops adjusted for node oriented methods

Calculation of the network using non-improved Node method is not shown here.

3.2.2 Modified Node method

Modified Node method is also referred to as the simultaneous node adjustment method. For the improvement of Node method, same idea as for the improvement of the Hardy Cross method is used. One node must be chosen to be omitted from the calculation shown in table 7 and 8, because linear independency among node equations is preserved in that way (Mathews and Köhler 1995, Mah and Shacham 1978). More details about graph theory of networks will be explained in further text (section 3.4). Gas network calculation after Modified Node method is shown in table 7:

Table 7. Calculation after Modified Node method for example gas network with three loops

Corrections in the table 7 and 8 are calculated using matrix equation (20):

$$\begin{bmatrix} f_1'(|+x_2|,|+x_3|,|-x_4|) & -|f_1'(x_3)| & -|f_1'(x_4)| & 0 & 0 \\ -|f_2'(x_3)| & f_2'(|-x_3|,|-x_7|,|-x_8|) & -|f_2'(x_7)| & 0 & -|f_2'(x_8)| \\ -|f_3'(x_4)| & -|f_3'(x_7)| & f_3'(|+x_4|,|+x_5|,|+x_7|) & -|f_3'(x_5)| & 0 \\ 0 & 0 & -|f_4'(x_5)| & f_4'(|+x_1|,|-x_5|,|-x_6|) & -|f_4'(x_6)| \\ 0 & -|f_5'(x_8)| & 0 & -|f_5'(x_6)| & f_5'(|+x_6|,|+x_8|) \end{bmatrix} x \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \\ \Delta x_5 \end{bmatrix} = \begin{bmatrix} \sum Q_2 + Q_3 - Q_4 - Q_I \\ \sum - Q_3 - Q_7 - Q_8 + Q_{II} \\ \sum Q_4 + Q_5 + Q_7 - Q_{III} \\ \sum Q_1 - Q_5 - Q_6 - Q_{IV} \\ \sum Q_6 + Q_8 - Q_V \end{bmatrix} \quad (20)$$

When non-diagonal terms in the first matrix (21) are equalized with zero, results are equal as for Node method in basic form (19). For the first iteration, numerical values for the matrix equation are extracted from table 6 (21):

$$\begin{bmatrix} 3.69\cdot 10^{-10} & -6.21\cdot 10^{-11} & -6.28\cdot 10^{-11} & 0 & 0 \\ -6.21\cdot 10^{-11} & 1.28\cdot 10^{-10} & -4.19\cdot 10^{-11} & 0 & -2.46\cdot 10^{-11} \\ -6.28\cdot 10^{-11} & -4.18\cdot 10^{-11} & 3.40\cdot 10^{-10} & -2.36\cdot 10^{-10} & 0 \\ 0 & 0 & -2.36\cdot 10^{-10} & 3.54\cdot 10^{-10} & -3.30\cdot 10^{-11} \\ 0 & -2.46\cdot 10^{-11} & 0 & -3.30\cdot 10^{11} & 5.76\cdot 10^{-11} \end{bmatrix} x \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \\ \Delta x_5 \end{bmatrix} = \begin{bmatrix} +0.7665 \\ -0.9121 \\ +2.9302 \\ -2.2805 \\ +0.4417 \end{bmatrix} \quad (21)$$

For the node oriented calculation applied for water networks, pressure function (C) must be replaced with pressure drop (Δp). Further analogy is clear (Table 8).

Table 8. Calculation after Modified Node method for example water network with three loops

3.4 Node-Loop method

Conditionally, Node-Loop method can be sorted in the group of methods based on solution of the loop equations according to the previous discussion, but better solution is to be treated as combination of the loop and node oriented methods as its name unambiguously suggest. Node-Loop method is also known as the flow adjustment method.

Wood and Charles (1972) developed the flow adjustment method by coupling the loop equations with the node equations. Wood and Rayes (1981) improved this method. Rather than solve for loop corrections, in this method, conservation of energy around a loop is written directly in the terms of the pipe flow rates. Final result after this method is not flow correction, but even better flow itself.

Pipeline network with three loops and six nodes is used as example in this article. Graph has X branches (pipes) and Y nodes where in our example, X=8 and Y=6. Graph with Y nodes (in our case 6) has Y-1 independent nodes (in our case 5) and X-Y+1 independent loops (in our case 3). Tree is a set of connected branches chosen to connect all nodes, but not to make any closed path (not to form a loop). Branches, which do not belong to a tree, are links (number of links are X-Y+1). Loops in the network are formed using pipes from tree and one more chosen among the link pipes. Number of the loops is determined by number of links. Network from example has six nodes and five independent nodes. One node can be omitted from calculation while no information on the topology in that way will be lost. Rows in the node matrix with all node included are not linearly independent. To obtain linear independence any row of the node matrix has to be omitted. For example, pipe 6 is between node IV and V, and reasonable assumption is that if node IV is output node for flow through pipe 6, then node V must be input node for flow through this pipe. In our example, node VI will be omitted. First

Kirchhoff's law for the initial flow pattern shown in figure 2 can be written using set of equations (22):

$$\begin{aligned}
&node_{I} \sim -Q_{I} + Q_{2} - Q_{3} - Q_{4} = 0\\
&node_{II} \sim Q_{II} + Q_{3} - Q_{7} - Q_{8} = 0\\
&node_{III} \sim -Q_{III} + Q_{4} + Q_{5} + Q_{7} = 0\\
&node_{IV} \sim -Q_{IV} + Q_{1} - Q_{5} - Q_{6} = 0\\
&node_{V} \sim -Q_{V} + Q_{6} + Q_{8} = 0\\
&node_{VI} \sim Q_{VI} - Q_{1} - Q_{2} = 0
\end{aligned} \qquad (22)$$

Node VI will be omitted from the node matrix to assure linear independency of the rows as shown in figure 4 (23):

$$[N] = \begin{bmatrix} 0 & +1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & +1 & +1 & 0 & +1 & 0 \\ +1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & +1 \end{bmatrix} \qquad (23)$$

First Kirchhoff's law must be fulfilled in all iterations for all nodes. Second Kirchhoff's law for the initial water flow pattern shown in the figure 1 can be written using set of equations (24):

$$\begin{aligned}
&loop_{I} \sim -F_{3} + F_{4} - F_{7} = F_{I}\\
&loop_{II} \sim F_{1} - F_{2} - F_{4} + F_{5} = F_{II}\\
&loop_{III} \sim -F_{5} + F_{6} + F_{7} - F_{8} = F_{III}
\end{aligned} \qquad (24)$$

Second Kirchhoff's law for the initial flow pattern shown in the figure 1 also can be noted in matrix form using loop matrix (25):

$$[L]=\begin{bmatrix} 0 & 0 & -1 & +1 & 0 & 0 & -1 & 0 \\ +1 & -1 & 0 & -1 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & +1 & +1 & -1 \end{bmatrix} \qquad (25)$$

Second Kirchhoff's law must be fulfilled for all loops at the end of calculation with demanded accuracy (i.e. $F_I \rightarrow 0$, $F_{II} \rightarrow 0$ and $F_{III} \rightarrow 0$).

In Node-Loop method these two matrices become one with some modifications. The nodes and the loops equations shown in previous text here will be united in one coherent system by coupling these two set of equations. To introduce matrix calculation, the node-loop matrix [NL], matrix of calculated flow in observed iteration [Q], and [V] matrix will be defined (26):

$$[V]=\begin{bmatrix} |Q_I| \\ -|Q_{II}| \\ |Q_{III}| \\ |Q_{IV}| \\ |Q_V| \\ -F_I + \left(-|F_3|\cdot|F_3'| + |F_4|\cdot|F_4'| - |F_7|\cdot|F_7'|\right) \\ -F_{II} + \left(|F_1|\cdot|F_1'| - |F_2|\cdot|F_2'| - |F_4|\cdot|F_4'| + |F_5|\cdot|F_5'|\right) \\ -F_{III} + \left(-|F_5|\cdot|F_5'| + |F_6|\cdot|F_6'| + |F_7|\cdot|F_7'| - |F_8|\cdot|F_8'|\right) \end{bmatrix} \qquad (26)$$

Sign minus preceding some of the flows Q in matrix [V] means that this particular Q is not consumption (sing minus represent inlet of fluid). Node-Loop matrix [NL] can be defined using node matrix, loop matrix and first derivative of Renouard’s function for gas pipes or of Darcy-Weisbach for water networks, as follows (27):

$$[NL]=\begin{bmatrix} 0 & +1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & +1 & +1 & 0 & +1 & 0 \\ +1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & +1 \\ 0 & 0 & -1\cdot\left|F_3^{'}\right| & +1\cdot\left|F_4^{'}\right| & 0 & 0 & -1\cdot\left|F_7^{'}\right| & 0 \\ +1\cdot\left|F_1^{'}\right| & -1\cdot\left|F_2^{'}\right| & 0 & -1\cdot\left|F_4^{'}\right| & +1\cdot\left|F_5^{'}\right| & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1\cdot\left|F_5^{'}\right| & +1\cdot\left|F_6^{'}\right| & +1\cdot\left|F_7^{'}\right| & -1\cdot\left|F_8^{'}\right| \end{bmatrix}$$

(27)

Further, vector [Q] of the unknown flows can be calculated in the first iteration (28).

$$[Q]=\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix}=inv[NL]x[V] \qquad (28)$$

Possible sign minus in a front of flow Q in the matrix [Q] means that calculated flow direction is opposite compared to shown one in the previous iteration (or in the figure 1 in our case for the first calculated values of flows compared with initials flow pattern). If all values of pressure drops sums calculated after (24) are not approximate zero with reasonable accuracy, calculation has to be repeated using values calculated in previous iteration. At the end of calculation calculated set of flows [Q] stays unchanged.

Here will be used values from table 1 for gas network (29) and from table 2 for water network (30) as example. These values are valid for the first iteration. First five rows (first matrix) are from node equation, and next three is from loop equation but multiplied with first derivate marked in tables as F':

$$\begin{bmatrix} 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -11839776055 & 1333799622 & 0 & 0 & -14293015047 & 0 \\ 8812326713 & -1314432601 & 0 & -1333799622 & 119798452.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -119798452.1 & 4298435730 & 14293015047 & -22897756035 \end{bmatrix} x \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = \begin{bmatrix} 0.055 \\ -0.277 \\ 0.361 \\ 0.222 \\ 0.194 \\ -2988241676 \\ 923187587 \\ 300557365 \end{bmatrix}$$

(29)

$$\begin{bmatrix} 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -54299272 & 4978573 & 0 & 0 & -70157780 & 0 \\ 426229242 & -5879434 & 0 & -4978573 & 429677 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -429677 & 16717415 & 70157780 & -106491089 \end{bmatrix} x \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = \begin{bmatrix} 0.055 \\ -0.277 \\ 0.361 \\ 0.222 \\ 0.194 \\ -15928498 \\ 5040086 \\ 2070510 \end{bmatrix}$$

(30)

For gas network last three rows are calculated as follows using values from table 1,

for loop I; -2988241676=

=-3644197165+(-0.194·11839776055+0.027·1333799622+(-0.305)·14293015047)

for loop II; 923187587=1125838521+

+(0.277·8812326713+(-0.277·1314432601)+(-0.027)·1333799622)+0.027·119798452)

for loop III; 300557365=366533372+

+(-0.027·119798452+0.027·4298435730+0.305·14293015047+(-0.166·22897756035))

After first iteration for gas network vector of flows is [0.198409265, 0.357146291, 0.043307855, 0.25828288, -0.094469817, 0.07065686, 0.197298048, 0.123787585]^T. Minus in front of flow in pipe 5 means: change assumed flow direction from previous iteration. After first iteration for water network vector of flows is [0.197719798, 0.357835758, 0.052496097, 0.249784106, -0.092806697, 0.068304272, 0.204133702, 0.126140172]^T. Flows are expressed in m^3/s.

Excellent book for waterworks calculation by Boulos et al. (2006) can be recommended for further reading. In this book, authors instead to omit one node in the node matrix to preserve linear independency of rows in this matrix introduce one pseudo-loop in loop matrix. This procedure is not practical because at least two nodes with equal pressure must be found in the network. This is not always possible. Further in that way the node-loop matrix has two additional rows which could be avoided. Mathews and Köhler (1995) in his discussion use simplest way, i.e. they omit one row.

4. Comparison of solution techniques for looped piping networks

Final flows are unique after all presented methods, and will be listed in table 9, both for water and for gas network.

Table 9. Final flows for network presented in this paper

Each method has advantages and shortcomings. Convergence performances will be compared for all presented methods in figure 5. Note that Modified Node method cannot be compared literary because initial values cannot be equalized. In all other methods initial patterns are given in the form of flows and equalized, while in Node method initial pattern is in the form of pressures (better to say function of pressure).

Figure 5. Comparisons of convergence for presented methods (gas network)

Best way to compare water and gas distribution network is to compare velocity of gas and water through pipes. For water, this can be done using (31):

$$v_w = \frac{4 \cdot Q}{D_{in}^2 \cdot \pi} \qquad (31)$$

Velocity of gaseous fluids depends on the pressure in pipe since they are compressible (32):

$$v_g = \frac{4 \cdot p_n \cdot Q_n}{p \cdot D_{in}^2 \cdot \pi} \quad (32)$$

Velocities for water and gas for calculated flows through the pipes in our example is listed in table 10:

Table 10: Velocities for water and gas for calculated flows from example network

5. Conclusion

Comparison between analyzed methods was carried out, taking as a criterion the number of iteration to achieve final results. Modified Hardy Cross method, Modified M.M. Andrijahshev method and Node-Loop method have equal performances according to above adopted criterion. For more complex networks, using Node-Loop method, number of required iterations is smaller even compared with Modified Hardy Cross method. Among these three methods, Node-Loop method is superior because it does not require complex numerical scheme for algebraic addition of corrections in each of iterations. In Node-Loop method, final results of each of the iterations are flows directly and not correction of flows. Modified Andrijašev method is more complex compared with Modified Hardy Cross method but without any improvement in the properties of the convergence. Node method has the worst performance of convergence, but this method is different in its approach compared with all other methods shown in this paper. Node method cannot be rejected easily based only on calculation shown in this paper. Hardy Cross method in original form has historical value and should be replaced with Modified Hardy Cross method, or even better with Node-Loop method.

Node-Loop method, presented among others in the text, is powerful numerical procedure for calculation of flows in looped fluid distribution networks. Main advantage is that flow in each pipe can be calculated directly, which is not possible after other available methods. In other methods, results of calculation are flow corrections which have to be added to flows calculated in previous iteration using complex algebraic rules. Node-Loop method is recommended to be used.

In the real network, consumers are not concentrated in the nodes. This can cause two-way flow in some pipes (Brkić 2009). This can cause disturbance in convergence properties of certain method. In such case, method should be changed. Some details on convergence properties can be found in the paper of Mah (1974), Mah and Lin (1980) and Altman and Boulos (1995). Simulation problem today can be solved using different software (Huddleston et al 2004, Lopes 2004) in which can be implemented shown methods. Method for solution of pipe equations proposed by Hamam and Brameller (1971) for gas networks and Todini and Pilati (1988) for water networks is available, but not shown here.

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Notations:

p-pressure (Pa)

Δp-pressure drop (Pa)

C-pressure function in gas pipelines (Pa^2)

F-pressure function C or Δp (Pa^2 or Pa)

f- flow function (m^3/s)

λ-Darcy friction factor (-)

L-pipe length (m)

ρ-water density (kg/m^3)

ρ_r-relative gas density (-)

Q-flow (m^3/s)

D-pipe diameter (m)

Re-Reynolds number (-)

ε-pipe roughness (m)

ΔQ-correction of flow (m^3/s)

Δx-correction of pressure function (Pa^2 or Pa)

π~3.1415

subscripts

n-normal (~101325 Pa, ~274 K)

in-inner

r-relative

w-water

g-gas

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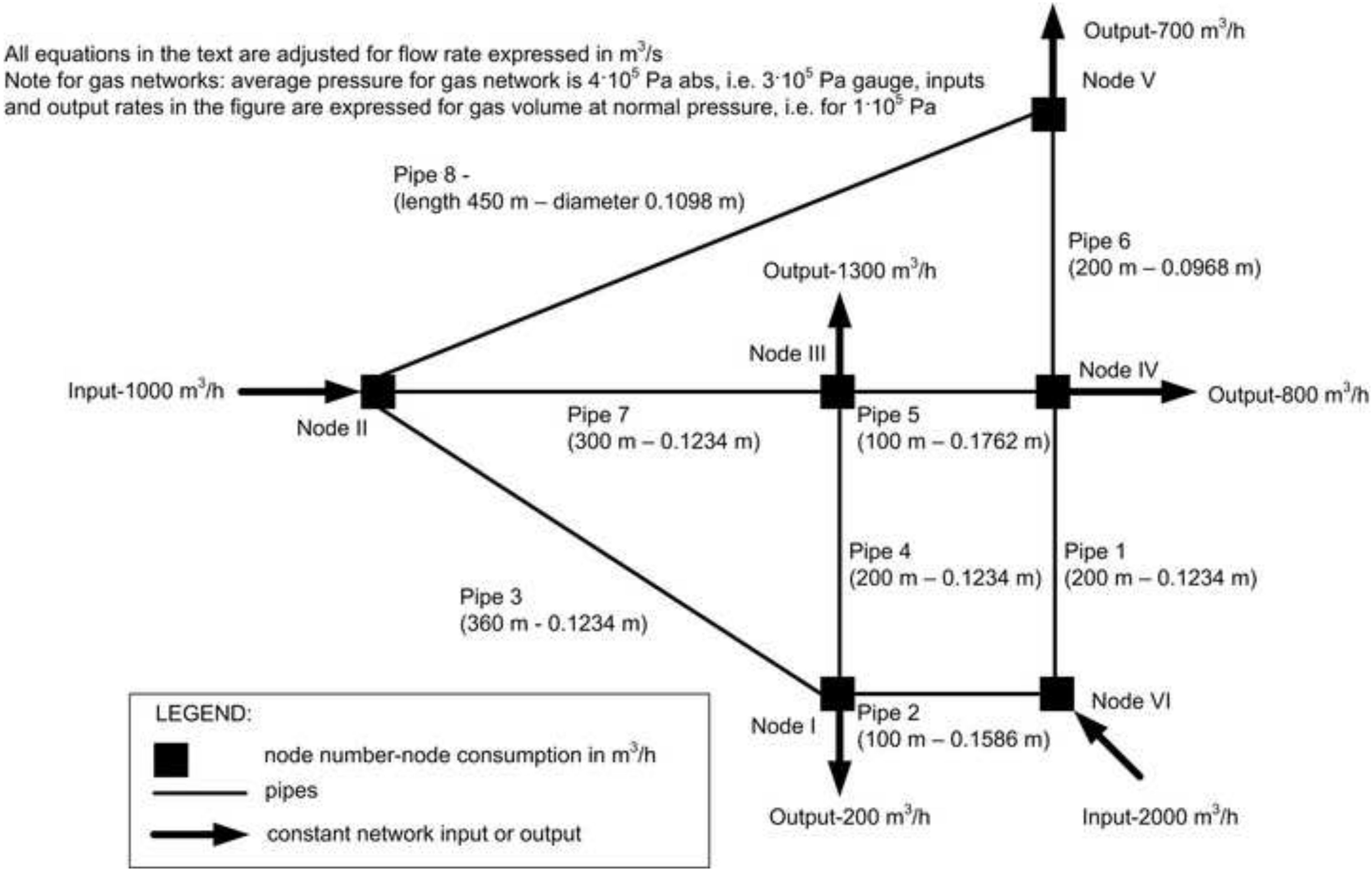

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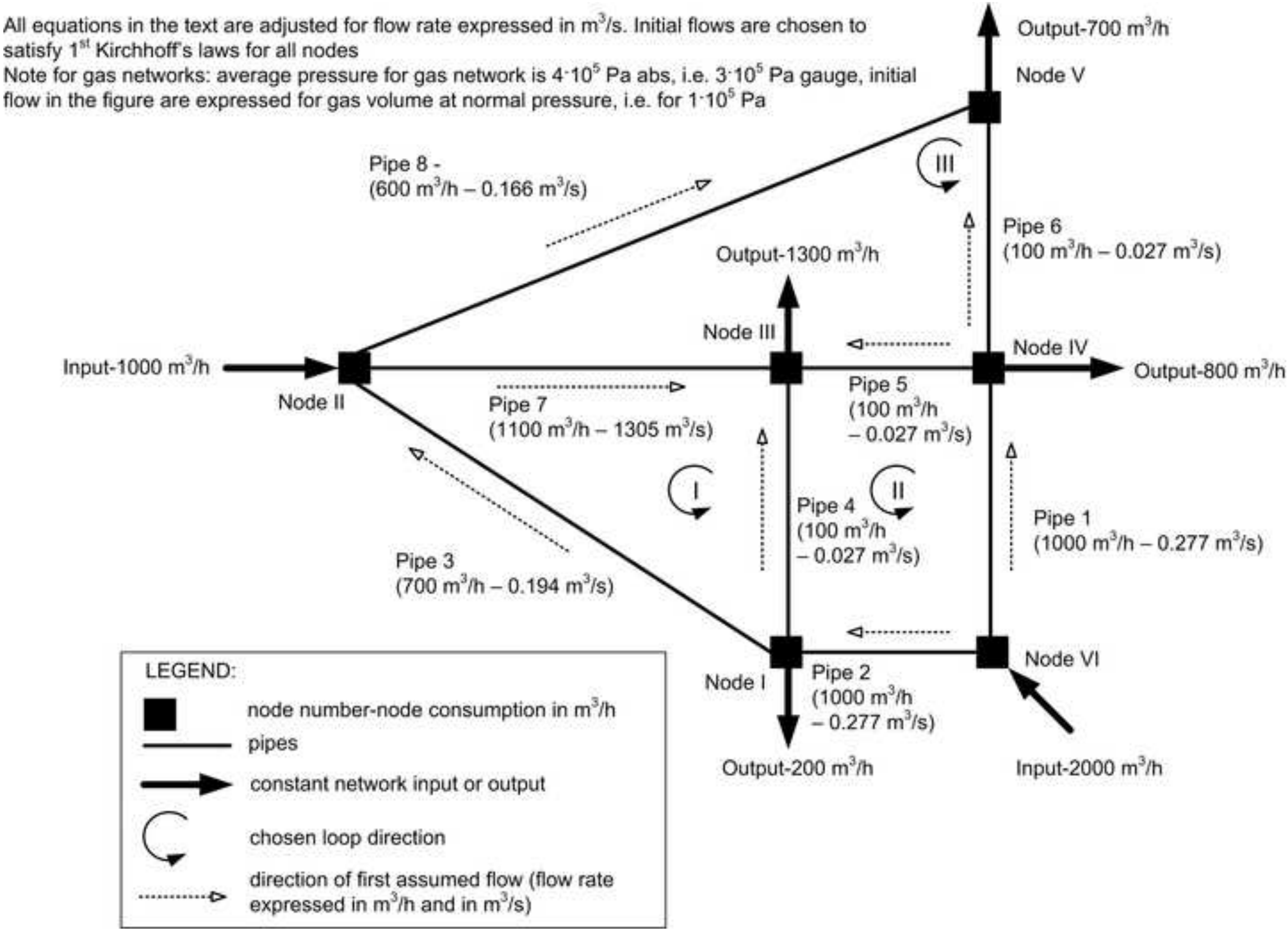

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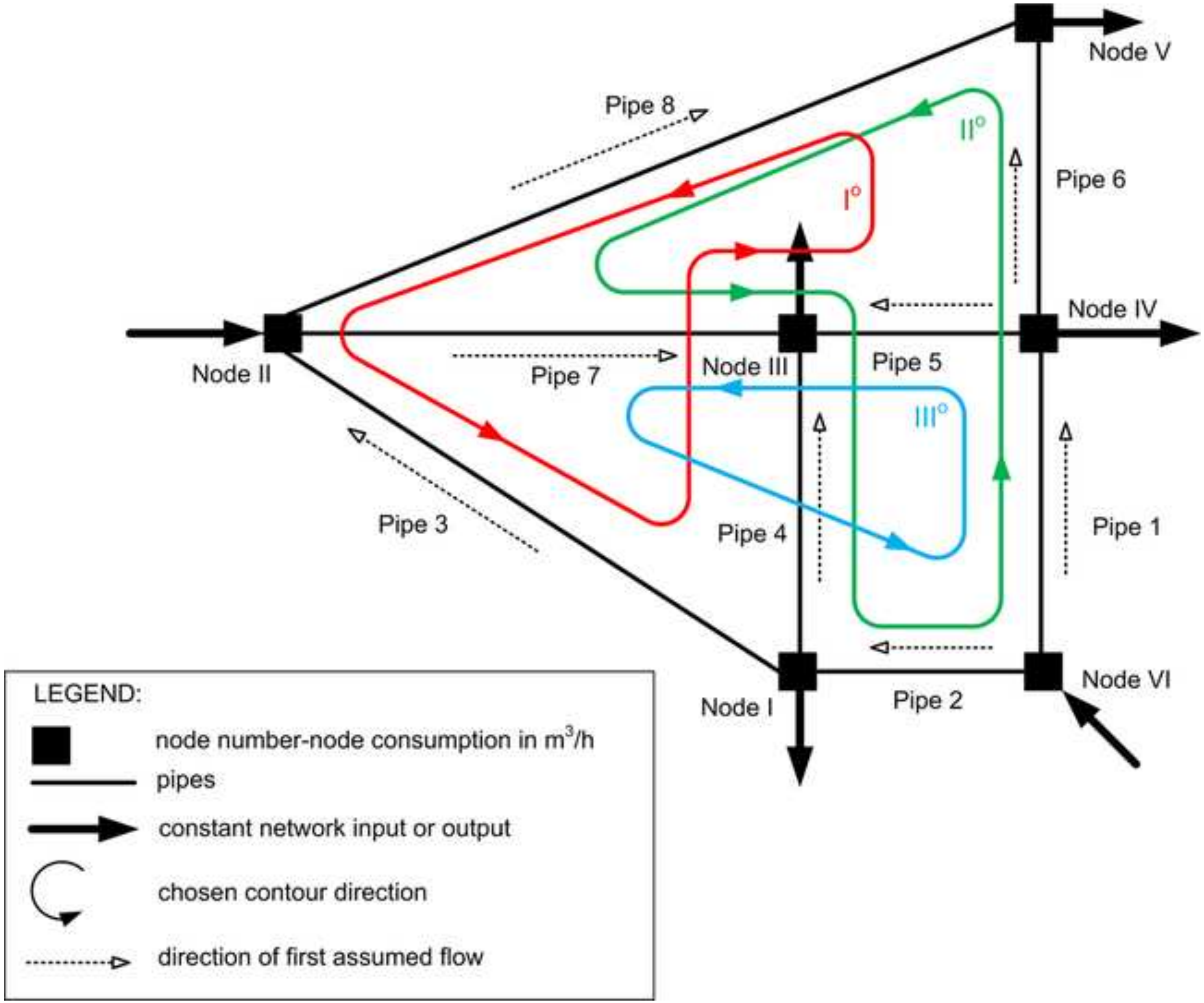

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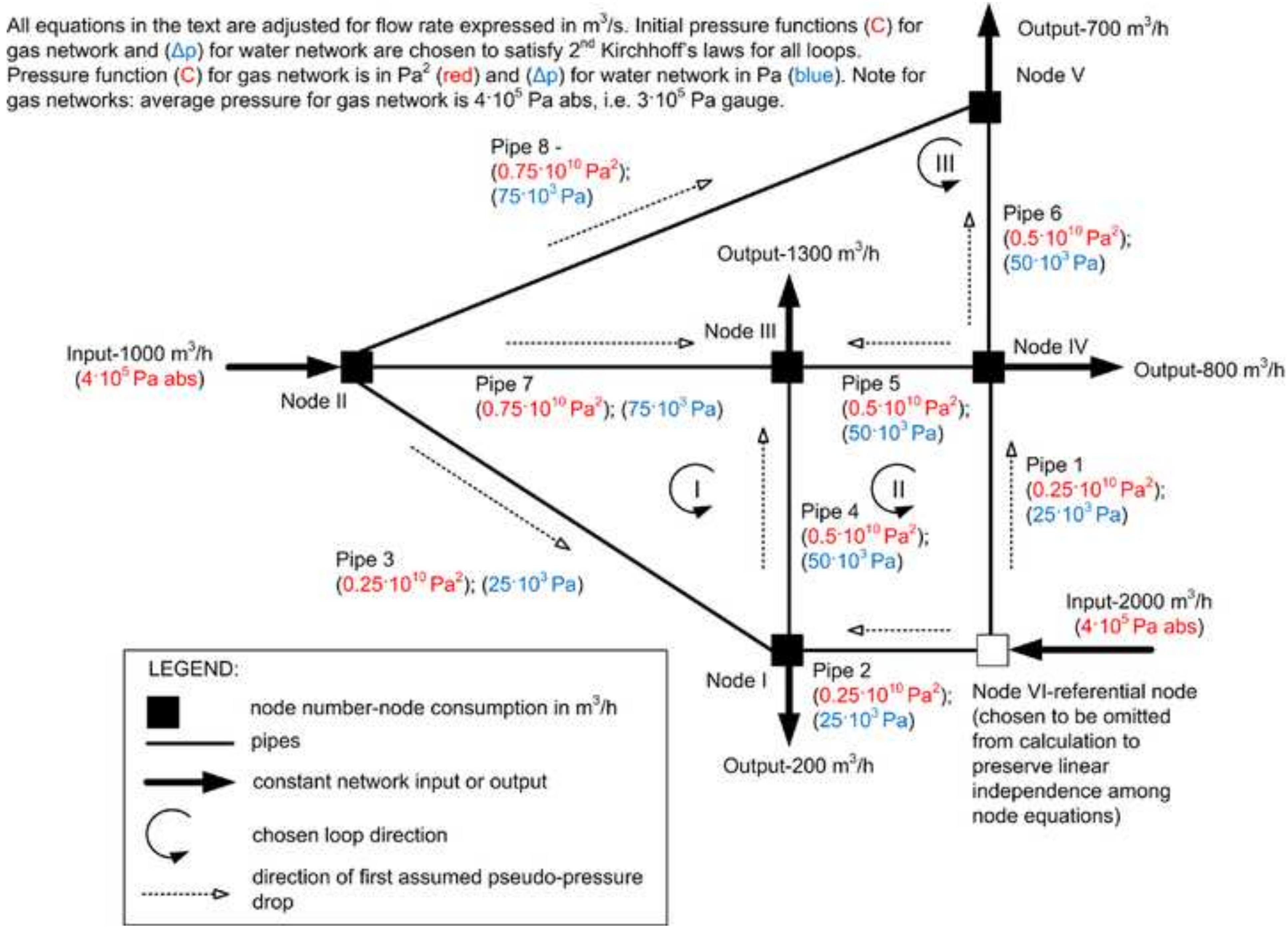

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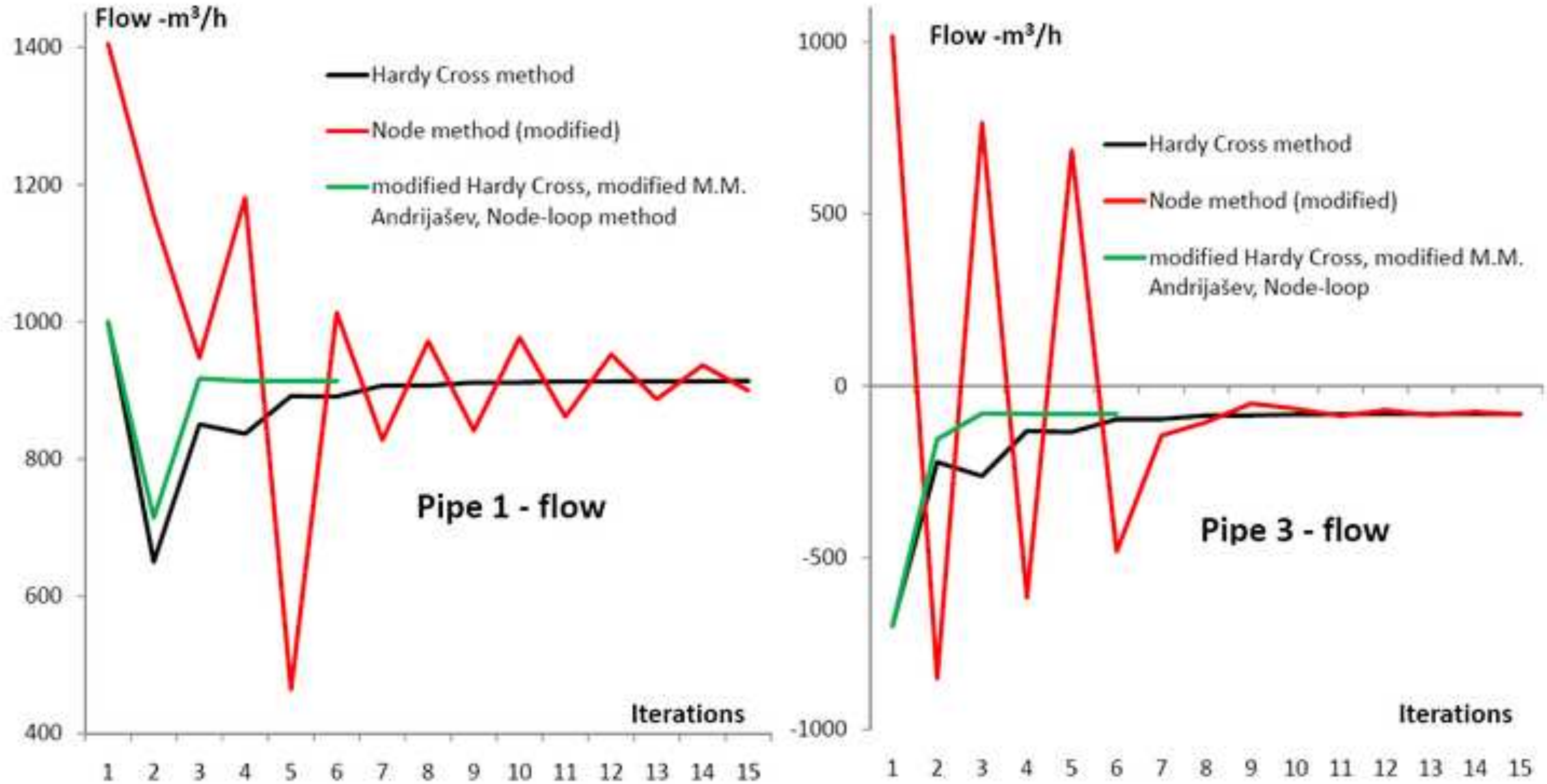

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Table 1. The Hardy Cross calculation for example gas network

		Iteration 1						Iteration 2				
Loop	Pipe	[a]Q	[b]C= $p_1^2 - p_2^2$	F'= $\left\lvert \frac{\partial C(Q)}{\partial Q} \right\rvert$	[c]$\Delta Q_1 = \frac{F}{F'}$	[d]ΔQ_2	[e]Q_1=Q	[b]C= $p_1^2 - p_2^2$	F'= $\left\lvert \frac{\partial C(Q)}{\partial Q} \right\rvert$	[c]$\Delta Q_1 = \frac{F}{F'}$	[d]ΔQ_2	Q_2=Q
I	3	$-19444\cdot10^{-5}$	-1264933339	11839776055	$+13268\cdot10^{-5}$	-	$-6177\cdot10^{-5}$	-156904917	4623293467	$-1128\cdot10^{-5}$	-	$-7305\cdot10^{-5}$
	4	$+2778\cdot10^{-5}$	+20357137	1333799622	$+13268\cdot10^{-5}$	$+9722\cdot10^{-5}$ ∓	$+25767\cdot10^{-5}$	+1173109335	8285863414	$-1128\cdot10^{-5}$	$-5572\cdot10^{-5}$=	$+19067\cdot10^{-5}$
	7	$-30556\cdot10^{-5}$	-2399620963	14293015047	$+13268\cdot10^{-5}$	$+881\cdot10^{-5}$‡	$-16407\cdot10^{-5}$	-773797561	8583648143	$-1128\cdot10^{-5}$	$-4154\cdot10^{-5}$ ±	$-21689\cdot10^{-5}$
		Σ	F_I=-3644197165	27466590725				F_I=+242406855	21492805024			
II	1	$+27778\cdot10^{-5}$	+1344982709	8812326713	$-9722\cdot10^{-5}$	-	$+18056\cdot10^{-5}$	+614087396	6189913452	$+5572\cdot10^{-5}$	-	$+23628\cdot10^{-5}$
	2	$-27778\cdot10^{-5}$	-200615476	1314432601	$-9722\cdot10^{-5}$	-	$-37500\cdot10^{-5}$	-346390930	1681162091	$+5572\cdot10^{-5}$	-	$-31927\cdot10^{-5}$
	4	$-2778\cdot10^{-5}$	-20357137	1333799622	$-9722\cdot10^{-5}$	$-13268\cdot10^{-5}$ ±	$-25767\cdot10^{-5}$	-1173109335	8285863414	$+5572\cdot10^{-5}$	$+1128\cdot10^{-5}$‡	$-19067\cdot10^{-5}$
	5	$+2778\cdot10^{-5}$	+1828425	119798452	$-9722\cdot10^{-5}$	$+881\cdot10^{-5}$ ∓	$-6063\cdot10^{-5}$[f]	-7569671	227216622.4	$+5572\cdot10^{-5}$	$-4154\cdot10^{-5}$ ±	$-4645\cdot10^{-5}$
		Σ	F_{II}=+1125838521	11580357388				F_{II}=-912982541	16384155579			
III	5	$-2778\cdot10^{-5}$	-1828425	119798452	$-881\cdot10^{-5}$	$+9722\cdot10^{-5}$‡	$+6063\cdot10^{-5}$[f]	+7569671	227216622	$+4154\cdot10^{-5}$	$-5572\cdot10^{-5}$=	$+4645\cdot10^{-5}$
	6	$+2778\cdot10^{-5}$	+65604940	4298435730	$-881\cdot10^{-5}$	-	$+1897\cdot10^{-5}$	+32767180	3143915857	$+4154\cdot10^{-5}$	-	$+6051\cdot10^{-5}$
	7	$+30556\cdot10^{-5}$	+2399620963	14293015047	$-881\cdot10^{-5}$	$-13268\cdot10^{-5}$=	$+16407\cdot10^{-5}$	+773797561	8583648143	$+4154\cdot10^{-5}$	$+1128\cdot10^{-5}$ ∓	$+21689\cdot10^{-5}$
	8	$-16667\cdot10^{-5}$	-2096864105	22897756035	$-881\cdot10^{-5}$	-	$-17548\cdot10^{-5}$	-2302927589	23885524961	$+4154\cdot10^{-5}$	-	$-13394\cdot10^{-5}$
		Σ	F_{III}=+366533372	41609005264				F_{III}=-1488793175	35840305583			

Pipe lengths and diameters are shown in figure 1 and initial flow pattern[a] in figure 2; 6 iterations are enough to achieve results shown in table 9 (this is for $\Sigma F \rightarrow 0$)
[b]this is F calculated using (1), [c]also using (9), [d]ΔQ_2 is ΔQ_1 from adjacent loop, [e]final calculated flow in the first iteration is used for the calculation in the second iteration, [f]opposite flow direction than in previous iteration

Table 2. The Hardy Cross calculation for example water network

		Iteration 1						Iteration 2				
Loop	Pipe	[a]Q	[b]$\Delta p=p_1-p_2$	$F'=\left\lvert\frac{\partial \Delta p(Q)}{\partial Q}\right\rvert$	[c]$\Delta Q_1=\frac{F}{F'}$	[d]ΔQ_2	[e]$Q_1=Q$	[b]$\Delta p=p_1-p_2$	$F'=\left\lvert\frac{\partial \Delta p(Q)}{\partial Q}\right\rvert$	[c]$\Delta Q_1=\frac{F}{F'}$	[d]ΔQ_2	$Q_2=Q$
I	3	$-19444\cdot10^{-5}$	-5279095	54299272	$+12306\cdot10^{-5}$	-	$-7138\cdot10^{-5}$	-750378	21023936	$-400\cdot10^{-5}$	-	$-7538\cdot10^{-5}$
	4	$+2778\cdot10^{-5}$	+69146	4978573	$+12306\cdot10^{-5}$	$+9349\cdot10^{-5}\mp$	$+24433\cdot10^{-5}$	+4596449	37625131	$-400\cdot10^{-5}$	$-4573\cdot10^{-5}=$	$+19459\cdot10^{-5}$
	7	$-30556\cdot10^{-5}$	-10718549	70157780	$+12306\cdot10^{-5}$	$+1068\cdot10^{-5}$‡	$-17181\cdot10^{-5}$	-3450640	40167998	$-400\cdot10^{-5}$	$-3920\cdot10^{-5}\pm$	$-21501\cdot10^{-5}$
	Σ		$F_I=-15928498$	129435625				$F_I=+395430$	98817066			
II	1	$+27778\cdot10^{-5}$	+5919850	42622924	$-9349\cdot10^{-5}$	-	$+18429\cdot10^{-5}$	+2639603	28646495	$+4573\cdot10^{-5}$	-	$+23002\cdot10^{-5}$
	2	$-27778\cdot10^{-5}$	-816585	5879413	$-9349\cdot10^{-5}$	-	$-37127\cdot10^{-5}$	-1443784	7777597	$+4573\cdot10^{-5}$	-	$-32553\cdot10^{-5}$
	4	$-2778\cdot10^{-5}$	-69146	4978573	$-9349\cdot10^{-5}$	$-12306\cdot10^{-5}\pm$	$-24433\cdot10^{-5}$	-4596449	37625131	$+4573\cdot10^{-5}$	$+400\cdot10^{-5}$‡	$-19459\cdot10^{-5}$
	5	$+2778\cdot10^{-5}$	+5967	429677	$-9349\cdot10^{-5}$	$+1068\cdot10^{-5}\mp$	$-5503\cdot10^{-5}$[f]	-21346	775842	$+4573\cdot10^{-5}$	$-0.03920\pm$	$-4850\cdot10^{-5}$
	Σ		$F_{II}=+5040086$	53910587				$F_{II}=-3421977$	74825067			
III	5	$-2778\cdot10^{-5}$	-5967	429677	$-1068\cdot10^{-5}$	$+9349\cdot10^{-5}$‡	$+5503\cdot10^{-5}$[f]	+21346	775842	$+3920\cdot10^{-5}$	$-4573\cdot10^{-5}=$	$+4850\cdot10^{-5}$
	6	$+2778\cdot10^{-5}$	+232186	16717415	$-1068\cdot10^{-5}$	-	$+1709\cdot10^{-5}$	+92718	10848205	$+3920\cdot10^{-5}$	-	$+5629\cdot10^{-5}$
	7	$+30556\cdot10^{-5}$	+10718549	70157780	$-1068\cdot10^{-5}$	$-12306\cdot10^{-5}=$	$+17181\cdot10^{-5}$	+3450640	40167998	$+3920\cdot10^{-5}$	$+400\cdot10^{-5}\mp$	$+21501\cdot10^{-5}$
	8	$-16667\cdot10^{-5}$	-8874257	106491089	$-1068\cdot10^{-5}$	-	$-17735\cdot10^{-5}$	-10028129	113088168	$+3920\cdot10^{-5}$	-	$-13815\cdot10^{-5}$
	Σ		$F_{III}=+2070510$	193795963				$F_{III}=-6463424$	164880214			

Pipe lengths and diameters are shown in figure 1 and initial flow pattern[a] in figure 2; 6 iterations are enough to achieve results shown in table 9 (this is for $\Sigma F\rightarrow 0$)
[b]this is F calculated using (3), [c]also using (9) or (10), [d]ΔQ_2 is ΔQ_1 from adjacent loop, [e]final calculated flow in the first iteration is used for the calculation in the second iteration, [f]opposite flow direction than in previous iteration

table 3 DB

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Table 3. Calculation after the modified Hardy Cross method for example gas network

		Iteration 1						Iteration 2				
Loop	Pipe	[a]Q	[b]C= $p_1^2 - p_2^2$	F'= $\left\lvert \frac{\partial C(Q)}{\partial Q} \right\rvert$	[c]ΔQ_1	[d]ΔQ_2	[e]Q_1=Q	[b]C= $p_1^2 - p_2^2$	F'= $\left\lvert \frac{\partial C(Q)}{\partial Q} \right\rvert$	[c]ΔQ_1	[d]ΔQ_2	Q_2=Q
I	3	$-19444\cdot10^{-5}$	-1264933339	11839776055	$+15114\cdot10^{-5}$	-	$-4331\cdot10^{-5}$	-82226369	3455539181	$+2078\cdot10^{-5}$	-	$-2253\cdot10^{-5}$
	4	$+2778\cdot10^{-5}$	+20357137	1333799622	$+15114\cdot10^{-5}$	$+7937\cdot10^{-5}$ ∓	$+25828\cdot10^{-5}$	+1178151884	8301891430	$+2078\cdot10^{-5}$	$-5572\cdot10^{-5}$=	$+22263\cdot10^{-5}$
	7	$-30556\cdot10^{-5}$	-2399620963	14293015047	$+15114\cdot10^{-5}$	$-4288\cdot10^{-5}$ ±	$-19730\cdot10^{-5}$	-1082435403	9985057887	$+2078\cdot10^{-5}$	$-4154\cdot10^{-5}$‡	$-17620\cdot10^{-5}$
		Σ	F_I=-3644197166	27466590725				F_I=+13490112	21742488499			
II	1	$+27778\cdot10^{-5}$	+1344982710	8812326713	$-7937\cdot10^{-5}$	-	$+19841\cdot10^{-5}$	+729037716	6687432899	$+5643\cdot10^{-5}$	-	$+25484\cdot10^{-5}$
	2	$-27778\cdot10^{-5}$	-200615476	1314432601	$-7937\cdot10^{-5}$	-	$-35715\cdot10^{-5}$	-316967670	1615251716	$+5643\cdot10^{-5}$	-	$-30072\cdot10^{-5}$
	4	$-2778\cdot10^{-5}$	-20357137	1333799622	$-7937\cdot10^{-5}$	$-15114\cdot10^{-5}$ ±	$-25828\cdot10^{-5}$	-1178151884	8301891430	$+5643\cdot10^{-5}$	$+1128\cdot10^{-5}$ ±	$-22263\cdot10^{-5}$
	5	$+2778\cdot10^{-5}$	+1828426	119798452	$-7937\cdot10^{-5}$	$-4288\cdot10^{-5}$=	$-9447\cdot10^{-5}$[f]	-16966070	326858346.3	$+5643\cdot10^{-5}$	$-4154\cdot10^{-5}$‡	$-3772\cdot10^{-5}$
		Σ	F_{II}=+1125838522	11580357388				F_{II}=-783047908	16931434392			
III	5	$-2778\cdot10^{-5}$	-1828425	119798452	$+4288\cdot10^{-5}$	$+7937\cdot10^{-5}$‡	$+9447\cdot10^{-5}$[f]	+16966070	326858346.3	$-32\cdot10^{-5}$	$-5572\cdot10^{-5}$=	$+3772\cdot10^{-5}$
	6	$+2778\cdot10^{-5}$	+65604940	4298435730	$+4288\cdot10^{-5}$	-	$+7066\cdot10^{-5}$	+358812846	9242405949	$-32\cdot10^{-5}$	-	$+7034\cdot10^{-5}$
	7	$+30556\cdot10^{-5}$	+2399620963	14293015047	$+4288\cdot10^{-5}$	$-15114\cdot10^{-5}$=	$+19730\cdot10^{-5}$	+1082435403	9985057887	$-32\cdot10^{-5}$	$+1128\cdot10^{-5}$=	$+17620\cdot10^{-5}$
	8	$-16667\cdot10^{-5}$	-2096864105	22897756035	$+4288\cdot10^{-5}$	-	$-12379\cdot10^{-5}$	-1220331619	17942054130	$-32\cdot10^{-5}$	-	$-12411\cdot10^{-5}$
		Σ	F_{III}=+366533372	41609005264				F_{III}=+237882701	37496376312			

Pipe lengths and diameters are shown in figure 1 and initial flow pattern[a] in figure 2; 3 iterations are enough to achieve results shown in table 9 (this is for ΣF→0)
[b]this is F calculated using (1), [c]using (12) where (Δp) is replaced with (C), [d]ΔQ_2 is ΔQ_1 from adjacent loop, [e]final calculated flow in the first iteration is used for the calculation in the second iteration, [f]opposite flow direction than in previous iteration

table 4 DB

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Table 4. Calculation after the modified Hardy Cross method for example water network

Loop	Pipe	Iteration 1						Iteration 2				
Loop	Pipe	[a]Q	[b]$\Delta p=p_1-p_2$	$F'=\left\lvert\frac{\partial\Delta p(Q)}{\partial Q}\right\rvert$	[c]ΔQ_1	[d]ΔQ_2	[e]$Q_1=Q$	[b]$\Delta p=p_1-p_2$	$F'=\left\lvert\frac{\partial\Delta p(Q)}{\partial Q}\right\rvert$	[c]ΔQ_1	[d]ΔQ_2	$Q_2=Q$
I	3	$-19444\cdot10^{-5}$	-5279095	54299272	$+14195\cdot10^{-5}$	-	$-5250\cdot10^{-5}$	-416024	15849700	$+2505\cdot10^{-5}$	-	$-2744\cdot10^{-5}$
	4	$+2778\cdot10^{-5}$	+69146	4978573	$+14195\cdot10^{-5}$	$+8006\cdot10^{-5}\mp$	$+24978\cdot10^{-5}$	+4800902	38440414	$+2505\cdot10^{-5}$	$-5312\cdot10^{-5}=$	$+22172\cdot10^{-5}$
	7	$-30556\cdot10^{-5}$	-10718549	70157780	$+14195\cdot10^{-5}$	$-4053\cdot10^{-5}\pm$	$-20413\cdot10^{-5}$	-4840373	47423556	$+2505\cdot10^{-5}$	$-107\cdot10^{-5}\pm$	$-18015\cdot10^{-5}$
		Σ	$F_I=-15928498$	129435625				$F_I=-455494$	101713670			
II	1	$+27778\cdot10^{-5}$	+5919850	42622924	$-8006\cdot10^{-5}$	-	$+19772\cdot10^{-5}$	+3030676	30656275	$+5312\cdot10^{-5}$	-	$+25084\cdot10^{-5}$
	2	$-27778\cdot10^{-5}$	-816585	5879413	$-8006\cdot10^{-5}$	-	$-35784\cdot10^{-5}$	-1342790	7505063	$+5312\cdot10^{-5}$	-	$-30472\cdot10^{-5}$
	4	$-2778\cdot10^{-5}$	-69146	4978573	$-8006\cdot10^{-5}$	$-14195\cdot10^{-5}\pm$	$-24978\cdot10^{-5}$	-4800902	38440414	$+5312\cdot10^{-5}$	$-2505\cdot10^{-5}\pm$	$-22172\cdot10^{-5}$
	5	$+2778\cdot10^{-5}$	+5967	429677	$-8006\cdot10^{-5}$	$-4053\cdot10^{-5}=$	$-9281\cdot10^{-5}$	-57430	1237632	$+5312\cdot10^{-5}$	$-107\cdot10^{-5}\pm$	$-4076\cdot10^{-5}$
		Σ	$F_{II}=+5040086$	53910587				$F_{II}=-3170446$	77839384			
III	5	$-2778\cdot10^{-5}$	-5967	429677	$+4053\cdot10^{-5}$	$+8006\cdot10^{-5}$‡	$+9281\cdot10^{-5}$	+57430	1237632	$+107\cdot10^{-5}$	$-5312\cdot10^{-5}=$	$+4076\cdot10^{-5}$
	6	$+2778\cdot10^{-5}$	+232186	16717415	$+4053\cdot10^{-5}$	-	$+6830\cdot10^{-5}$	+1312311	38425447	$+107\cdot10^{-5}$	-	$+6937\cdot10^{-5}$
	7	$+30556\cdot10^{-5}$	+10718549	70157780	$+4053\cdot10^{-5}$	$-14195\cdot10^{-5}=$	$+20413\cdot10^{-5}$	+4840373	47423556	$+107\cdot10^{-5}$	$-2505\cdot10^{-5}=$	$+18015\cdot10^{-5}$
	8	$-16667\cdot10^{-5}$	-8874257	106491089	$+4053\cdot10^{-5}$	-	$-12614\cdot10^{-5}$	-5136681	81444010	$+107\cdot10^{-5}$	-	$-12507\cdot10^{-5}$
		Σ	$F_{III}=+2070510$	193795963				$F_{III}=+1073434$	168530645			

Pipe lengths and diameters are shown in figure 1 and initial flow pattern[a] in figure 2; 3 iterations are enough to achieve results shown in table 9 (this is for $\Sigma F\rightarrow 0$)

[b]this is F calculated using (3), [c]using (12), [d]ΔQ_2 is ΔQ_1 from adjacent loop, [e]final calculated flow in the first iteration is used for the calculation in the second iteration, [f]opposite flow direction than in previous iteration

table 5 DB

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Table 5. Calculation after Modified M.M. Andrijašev method for example gas network

		Iteration 1						Iteration 2				
Contour[O]	Pipe	[a]Q	[b]C= $p_1^2 - p_2^2$	F'= $\left\lvert\frac{\partial C(Q)}{\partial Q}\right\rvert$	[c] ΔQ_1^o	[d] ΔQ_2^o	[e]Q_1=Q	[b]C= $p_1^2 - p_2^2$	F'= $\left\lvert\frac{\partial C(Q)}{\partial Q}\right\rvert$	[c] ΔQ_1^o	[d] ΔQ_2^o	Q_2=Q
	6	$+2778\cdot10^{-5}$	65604941	4298435730	$+13669\cdot10^{-5}$	$-9381\cdot10^{-5}$‡	$+7066\cdot10^{-5}$	358812846	9242405949	$-1798\cdot10^{-5}$	$+1767\cdot10^{-5}$±	$+7034\cdot10^{-5}$
	8	$-16667\cdot10^{-5}$	-2096864106	22897756035	$+13669\cdot10^{-5}$	$-9381\cdot10^{-5}$∓	$-12379\cdot10^{-5}$	-1220331619	17942054130	$-1798\cdot10^{-5}$	$+1767\cdot10^{-5}$=	$-12411\cdot10^{-5}$
I[O]	3	$-19444\cdot10^{-5}$	-1264933339	11839776055	$+13669\cdot10^{-5}$	$+1444\cdot10^{-5}$=	$-4331\cdot10^{-5}$	-82226369	3455539181	$-1798\cdot10^{-5}$	$+3876\cdot10^{-5}$=	$-2253\cdot10^{-5}$
	4	$+2778\cdot10^{-5}$	20357137	1333799622	$+13669\cdot10^{-5}$	$+9381\cdot10^{-5}$∓	$+25828\cdot10^{-5}$	1178151884	8301891430	$-1798\cdot10^{-5}$	$-1767\cdot10^{-5}$=	$+22263\cdot10^{-5}$
	5	$-2778\cdot10^{-5}$	-1828426	119798452	$+13669\cdot10^{-5}$	$-1444\cdot10^{-5}$±	$+9447\cdot10^{-5}$[f]	16966070	326858346	$-1798\cdot10^{-5}$	$-3876\cdot10^{-5}$=	$+3772\cdot10^{-5}$
	Σ		F°=-3277663793	40489565894				F°=251372813	39268749037			
	6	$+2778\cdot10^{-5}$	65604941	4298435730	$-9381\cdot10^{-5}$	$+13669\cdot10^{-5}$±	$+7066\cdot10^{-5}$	358812846	9242405949	$+1767\cdot10^{-5}$	$-1798\cdot10^{-5}$‡	$+7034\cdot10^{-5}$
	8	$-16667\cdot10^{-5}$	-2096864106	22897756035	$-9381\cdot10^{-5}$	$+13669\cdot10^{-5}$=	$-12379\cdot10^{-5}$	-1220331619	17942054130	$+1767\cdot10^{-5}$	$-1798\cdot10^{-5}$∓	$-12411\cdot10^{-5}$
II[O]	7	$+30556\cdot10^{-5}$	2399620963	14293015047	$-9381\cdot10^{-5}$	$-1444\cdot10^{-5}$=	$+19730\cdot10^{-5}$	1082435403	9985057887	$+1767\cdot10^{-5}$	$-3876\cdot10^{-5}$=	$+17620\cdot10^{-5}$
	4	$-2778\cdot10^{-5}$	-20357137	1333799622	$-9381\cdot10^{-5}$	$-13669\cdot10^{-5}$±	$-25828\cdot10^{-5}$	-1178151884	8301891430	$+1767\cdot10^{-5}$	$+1798\cdot10^{-5}$‡	$-22263\cdot10^{-5}$
	2	$-27778\cdot10^{-5}$	-200615476	1314432601	$-9381\cdot10^{-5}$	$+1444\cdot10^{-5}$=	$-35715\cdot10^{-5}$	-316967670	1615251716	$+1767\cdot10^{-5}$	$+3876\cdot10^{-5}$=	$-30072\cdot10^{-5}$
	1	$+27778\cdot10^{-5}$	1344982710	8812326713	$-9381\cdot10^{-5}$	$+1444\cdot10^{-5}$±	$+19841\cdot10^{-5}$	729037716	6687432899	$+1767\cdot10^{-5}$	$+3876\cdot10^{-5}$±	$+25484\cdot10^{-5}$
	Σ		F°=1492371895	52949765748				F°=-545165207	53774094012			
	1	$+27778\cdot10^{-5}$	1344982710	8812326713	$+1444\cdot10^{-5}$	$-9381\cdot10^{-5}$‡	$+19841\cdot10^{-5}$	729037716	6687432899	$+3876\cdot10^{-5}$	$+1767\cdot10^{-5}$±	$+25484\cdot10^{-5}$
	5	$+2778\cdot10^{-5}$	1828426	119798452	$+1444\cdot10^{-5}$	$-13669\cdot10^{-5}$=	$-9447\cdot10^{-5}$[f]	-16966070	326858346	$+3876\cdot10^{-5}$	$+1798\cdot10^{-5}$‡	$-3772\cdot10^{-5}$
III[O]	7	$-30556\cdot10^{-5}$	-2399620963	14293015047	$+1444\cdot10^{-5}$	$+9381\cdot10^{-5}$‡	$-19730\cdot10^{-5}$	-1082435403	9985057887	$+3876\cdot10^{-5}$	$-1767\cdot10^{-5}$±	$-17620\cdot10^{-5}$
	3	$-19444\cdot10^{-5}$	-1264933339	11839776055	$+1444\cdot10^{-5}$	$+13669\cdot10^{-5}$=	$-4331\cdot10^{-5}$	-82226369	3455539181	$+3876\cdot10^{-5}$	$-1798\cdot10^{-5}$∓	$-2253\cdot10^{-5}$
	2	$-27778\cdot10^{-5}$	-200615476	1314432601	$+1444\cdot10^{-5}$	$-9381\cdot10^{-5}$∓	$-35715\cdot10^{-5}$	-316967670	1615251716	$+3876\cdot10^{-5}$	$+1767\cdot10^{-5}$=	$-30072\cdot10^{-5}$
	Σ		F°=-2518358644	36379348868				F°=-769557797	22070140031			

Pipe lengths and diameters are shown in figure 1, initial flow pattern in figure 3 and numerical values for initial flows[a] in figure 2; 3 iterations are enough to achieve results shown in table 9 (this is for ΣF→0)
[b]this is F calculated using (1), [c]using (15), [d] ΔQ_2^o is ΔQ_1^o from adjacent contour, [e]final calculated flow in the first iteration is used for the calculation in the second iteration, [f]opposite flow direction than in previous iteration

table 6 DB

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Table 6. Calculation after Modified M.M. Andrijašev method for example water network

Contour^{O}	Pipe	Iteration 1 [a]Q	[b]$\Delta p=p_1-p_2$	F'= $\left\lvert\frac{\partial \Delta p(Q)}{\partial Q}\right\rvert$	[c] ΔQ_1^o	[d] ΔQ_2^o	[e]Q_1=Q	Iteration 2 [b]$\Delta p=p_1-p_2$	F'= $\left\lvert\frac{\partial \Delta p(Q)}{\partial Q}\right\rvert$	[c] ΔQ_1^o	[d] ΔQ_2^o	Q_2=Q
	6	$+2778\cdot10^{-5}$	+232186	16717415	$+13127\cdot10^{-5}$	$-9074\cdot10^{-5}$‡	$+6830\cdot10^{-5}$	+1312311	19212723	$-1350\cdot10^{-5}$	$+1457\cdot10^{-5}$ ±	$+6937\cdot10^{-5}$
	8	$-16667\cdot10^{-5}$	-8874257	106491089	$+13127\cdot10^{-5}$	$-9074\cdot10^{-5}$ ∓	$-12614\cdot10^{-5}$	-5136681	40722005	$-1350\cdot10^{-5}$	$+1457\cdot10^{-5}$=	$-12507\cdot10^{-5}$
I^{O}	3	$-19444\cdot10^{-5}$	-5279095	54299272	$+13127\cdot10^{-5}$	$+1068\cdot10^{-5}$=	$-5250\cdot10^{-5}$	-416024	7924850	$-1350\cdot10^{-5}$	$+3855\cdot10^{-5}$=	$-2744\cdot10^{-5}$
	4	$+2778\cdot10^{-5}$	+69146	4978573	$+13127\cdot10^{-5}$	$+9074\cdot10^{-5}$ ∓	$+24978\cdot10^{-5}$	4800902	19220207	$-1350\cdot10^{-5}$	$-1457\cdot10^{-5}$=	$+22172\cdot10^{-5}$
	5	$-2778\cdot10^{-5}$	-5967	429677	$+13127\cdot10^{-5}$	$-1068\cdot10^{-5}$ ±	$+9281\cdot10^{-5}$[f]	+57430	618816	$-1350\cdot10^{-5}$	$-3855\cdot10^{-5}$=	$+4076\cdot10^{-5}$
		Σ	F°=-13857988	182916028				F°=617939	87698601			
	6	$+2778\cdot10^{-5}$	+232186	16717415	$-9074\cdot10^{-5}$	$+13127\cdot10^{-5}$ ±	$+6830\cdot10^{-5}$	1312311	19212723	$+1457\cdot10^{-5}$	$-1350\cdot10^{-5}$‡	$+6937\cdot10^{-5}$
	8	$-16667\cdot10^{-5}$	-8874257	106491089	$-9074\cdot10^{-5}$	$+13127\cdot10^{-5}$=	$-12614\cdot10^{-5}$	-5136681	40722005	$+1457\cdot10^{-5}$	$-1350\cdot10^{-5}$ ∓	$-12507\cdot10^{-5}$
II^{O}	7	$+30556\cdot10^{-5}$	+10718549	70157780	$-9074\cdot10^{-5}$	$-1068\cdot10^{-5}$=	$+20413\cdot10^{-5}$	4840373	23711778	$+1457\cdot10^{-5}$	$-3855\cdot10^{-5}$=	$+18015\cdot10^{-5}$
	4	$-2778\cdot10^{-5}$	-69146	4978573	$-9074\cdot10^{-5}$	$-13127\cdot10^{-5}$ ±	$-24978\cdot10^{-5}$	-4800902	19220207	$+1457\cdot10^{-5}$	$+1350\cdot10^{-5}$‡	$-22172\cdot10^{-5}$
	2	$-27778\cdot10^{-5}$	-816585	5879413	$-9074\cdot10^{-5}$	$+1068\cdot10^{-5}$=	$-35784\cdot10^{-5}$	-1342790	3752532	$+1457\cdot10^{-5}$	$+3855\cdot10^{-5}$=	$-30472\cdot10^{-5}$
	1	$+27778\cdot10^{-5}$	+5919850	42622924	$-9074\cdot10^{-5}$	$+1068\cdot10^{-5}$ ±	$+19772\cdot10^{-5}$	3030676	15328137	$+1457\cdot10^{-5}$	$+3855\cdot10^{-5}$ ±	$+25084\cdot10^{-5}$
		Σ	F°=+7110597	246847196				F°=-2097013	121947382			
	1	$+27778\cdot10^{-5}$	+5919850	42622924	$+1068\cdot10^{-5}$	$-9074\cdot10^{-5}$‡	$+19772\cdot10^{-5}$	3030676	15328137	$+3855\cdot10^{-5}$	$+1457\cdot10^{-5}$ ±	$+25084\cdot10^{-5}$
	5	$+2778\cdot10^{-5}$	+5967	429677	$+1068\cdot10^{-5}$	$-13127\cdot10^{-5}$=	$-9281\cdot10^{-5}$[f]	-57430	618816	$+3855\cdot10^{-5}$	$+1350\cdot10^{-5}$‡	$-4076\cdot10^{-5}$
III^{O}	7	$-30556\cdot10^{-5}$	-10718549	70157780	$+1068\cdot10^{-5}$	$+9074\cdot10^{-5}$‡	$-20413\cdot10^{-5}$	-4840373	23711778	$+3855\cdot10^{-5}$	$-1457\cdot10^{-5}$ ±	$-18015\cdot10^{-5}$
	3	$-19444\cdot10^{-5}$	-5279095	54299272	$+1068\cdot10^{-5}$	$+13127\cdot10^{-5}$=	$-5250\cdot10^{-5}$	-416024	7924850	$+3855\cdot10^{-5}$	$-1350\cdot10^{-5}$ ∓	$-2744\cdot10^{-5}$
	2	$-27778\cdot10^{-5}$	-816585	5879413	$+1068\cdot10^{-5}$	$-9074\cdot10^{-5}$ ∓	$-35784\cdot10^{-5}$	-1342790	3752532	$+3855\cdot10^{-5}$	$+1457\cdot10^{-5}$=	$-30472\cdot10^{-5}$
		Σ	F°=-10888413	173389067				F°=-3625941	51336113			

Pipe lengths and diameters are shown in figure 1, initial flow pattern in figure 3 and numerical values for initial flows[a] in figure 2; 3 iterations are enough to achieve results shown in table 9 (this is for ΣF→0)
[b]this is F calculated using (3), [c]using (15), [d] ΔQ_2^o is ΔQ_1^o from adjacent contour, [e]final calculated flow in the first iteration is used for the calculation in the second iteration, [f]opposite flow direction than in previous iteration

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Table 7. Calculation after Modified Node method for example gas network with three loops

		Iteration 1						Iteration 2				
Node	Pipe	C (Pa^2)	[a]Q=f(C)	[b]f’	[c]ΔC_1	[d]ΔC_2	[e]C_1=C	[a]Q=f(C)	[b]f’	[c]ΔC_1	[d]ΔC_2	C_2=C
I	2	$+0.25\cdot10^{10}$	+1.1108	$24415\cdot10^{-14}$	$-36.1\cdot10^{8}$	-	$-11.1\cdot10^{8f}$	-0.7109	$35204\cdot10^{-14}$	$+28.4\cdot10^{8}$	-	$+17.3\cdot10^{8f}$
	3	$+0.25\cdot10^{10}$	+0.2827	$6214\cdot10^{-14}$	$-36.1\cdot10^{8}$	$-6.99\cdot10^{8}$=	$-18.1\cdot10^{8f}$	-0.2367	$7189\cdot10^{-14}$	$+28.4\cdot10^{8}$	$+4.57\cdot10^{8}$‡	$+14.9\cdot10^{8f}$
	4	$-0.50\cdot10^{10}$	-0.5715	$6280\cdot10^{-14}$	$-36.1\cdot10^{8}$	$+97.0\cdot10^{8}$‡	$+10.9\cdot10^{8f}$	+0.2475	$12474\cdot10^{-14}$	$+28.4\cdot10^{8}$	$-66.9\cdot10^{8}$‡	$-27.6\cdot10^{8f}$
Constant output flow			-0.0555					-0.0555				
			Σf=+0.7665	$36909\cdot10^{-14}$				Σf=-0.7557	$54868\cdot10^{-14}$			
II	3	$-0.25\cdot10^{10}$	-0.2827	$6214\cdot10^{-14}$	$+6.99\cdot10^{8}$	$+36.1\cdot10^{8}$‡	$+18.1\cdot10^{8f}$	+0.2367	$7189\cdot10^{-14}$	$-4.57\cdot10^{8}$	$-28.4\cdot10^{8}$=	$-14.9\cdot10^{8f}$
	7	$-0.75\cdot10^{10}$	-0.5715	$4187\cdot10^{-14}$	$+6.99\cdot10^{8}$	$+97.0\cdot10^{8}$‡	$+29.0\cdot10^{8f}$	+0.3390	$6425\cdot10^{-14}$	$-4.57\cdot10^{8}$	$-66.9\cdot10^{8}$=	$-42.5\cdot10^{8f}$
	8	$-0.75\cdot10^{10}$	-0.3357	$2459\cdot10^{-14}$	$+6.99\cdot10^{8}$	$+78.0\cdot10^{8}$‡	$+9.99\cdot10^{8f}$	+0.1109	$6099\cdot10^{-14}$	$-4.57\cdot10^{8}$	$-39.4\cdot10^{8}$=	$-34.0\cdot10^{8f}$
Constant input flow			+0.2777					+0.2777				
			Σf=-0.9121	$12860\cdot10^{-14}$				Σf=+0.9644	$19713\cdot10^{-14}$			
III	4	$+0.50\cdot10^{10}$	+0.5715	$6280\cdot10^{-14}$	$-97.0\cdot10^{8}$	$+36.1\cdot10^{8}$∓	$-10.9\cdot10^{8f}$	-0.2475	$12474\cdot10^{-14}$	$+66.9\cdot10^{8}$	$-28.4\cdot10^{8}$±	$+27.6\cdot10^{8f}$
	5	$+0.50\cdot10^{10}$	+2.1483	$23608\cdot10^{-14}$	$-97.0\cdot10^{8}$	$+7.52\cdot10^{8}$∓	$-39.5\cdot10^{8f}$	-1.8867	$26260\cdot10^{-14}$	$+66.9\cdot10^{8}$	$5.29\cdot10^{8}$‡	$+32.7\cdot10^{8f}$
	7	$+0.75\cdot10^{10}$	+0.5715	$4187\cdot10^{-14}$	$-97.0\cdot10^{8}$	$-6.99\cdot10^{8}$=	$-29.0\cdot10^{8f}$	-0.3390	$6425\cdot10^{-14}$	$+66.9\cdot10^{8}$	$4.57\cdot10^{8}$‡	$+42.5\cdot10^{8f}$
Constant output flow			-0.3611					-0.3611				
			Σf=+2.9302	$34075\cdot10^{-14}$				Σf=-2.8343	$45160\cdot10^{-14}$			
IV	1	$+0.25\cdot10^{10}$	+0.3905	$8582\cdot10^{-14}$	$-7.52\cdot10^{8}$	-	$+17.5\cdot10^{8}$	+0.3208	$10084\cdot10^{-14}$	$-5.29\cdot10^{8}$	-	$+12.2\cdot10^{8}$
	5	$-0.50\cdot10^{10}$	-2.1483	$23608\cdot10^{-14}$	$-7.52\cdot10^{8}$	$+97.0\cdot10^{8}$‡	$+39.5\cdot10^{8f}$	+1.8867	$26260\cdot10^{-14}$	$-5.29\cdot10^{8}$	$-66.9\cdot10^{8}$=	$-32.7\cdot10^{8f}$
	6	$-0.50\cdot10^{10}$	-0.3004	$3302\cdot10^{-14}$	$-7.52\cdot10^{8}$	$+78.0\cdot10^{8}$‡	$+20.5\cdot10^{8f}$	+0.1840	$4937\cdot10^{-14}$	$-5.29\cdot10^{8}$	$-39.4\cdot10^{8}$=	$-24.3\cdot10^{8f}$
Constant output flow			-0.2222					-0.2222				
			Σf=-2.2805	$35492\cdot10^{-14}$				Σf=2.1693	$41281\cdot10^{-14}$			
V	6	$+0.50\cdot10^{10}$	+0.3004	$3302\cdot10^{-14}$	$-78.0\cdot10^{8}$	$+7.52\cdot10^{8}$∓	$-20.5\cdot10^{8f}$	-0.1840	$4937\cdot10^{-14}$	$39.4\cdot10^{8}$	$5.29\cdot10^{8}$‡	$+24.3\cdot10^{8f}$
	8	$+0.75\cdot10^{10}$	+0.3357	$2459\cdot10^{-14}$	$-78.0\cdot10^{8}$	$-6.99\cdot10^{8}$=	$-9.99\cdot10^{8f}$	-0.1109	$6099\cdot10^{-14}$	$39.4\cdot10^{8}$	$4.57\cdot10^{8}$‡	$+34.0\cdot10^{8f}$
Constant output flow			-0.1944					-0.1944				
			Σf=+0.4417	$5761\cdot10^{-14}$				Σf=-0.4893	$11036\cdot10^{-14}$			

Pipe lengths and diameters are shown in figure 1; see figure 4 for initial pattern (red letters); 26 iterations are enough to achieve results shown in table 9 (this is for Σf→0)

[a]using (17), [b]F’=|∂Q(C)/∂C|,[c]ΔC_1 after eq. (20), [d]ΔC_2 is ΔC_1 from adjacent node, [e]final calculated pressure function in the first iteration is used for the calculation in the second iteration, [f]opposite flow direction than in previous iteration

Table 8. Calculation after Modified Node method for example water network with three loops

Node	Pipe	Iteration 1						Iteration 2				
		Δp (Pa)	[a]$Q=f(\Delta p)$	[b]f'	[c]$\Delta_{\Delta p1}$	[d]$\Delta_{\Delta p2}$	[e]$\Delta p_1=\Delta p$	[a]$Q=f(\Delta p)$	[b]f'	[c]$\Delta_{\Delta p1}$	[d]$\Delta_{\Delta p2}$	$\Delta p_2=\Delta p$
I	2	$+25\cdot10^{3}$	$+4953\cdot10^{-5}$	$9907\cdot10^{-10}$	+177553	-	+202553	$+12974\cdot10^{-5}$	$3203\cdot10^{-10}$	+552198	-	+754750
	3	$+25\cdot10^{3}$	$+1360\cdot10^{-5}$	$2720\cdot10^{-10}$	+177553	-167854=	+34699	$+1396\cdot10^{-5}$	$2010\cdot10^{-10}$	+552198	-730668=	-143771[f]
	4	$-50\cdot10^{3}$	$-2580\cdot10^{-5}$	$2500\cdot10^{-10}$	+177553	-798781±	-671229	$-8616\cdot10^{-5}$	$642\cdot10^{-10}$	+552198	-2779111±	-2898142
Constant output flow			$-5556\cdot10^{-5}$					$-5556\cdot10^{-5}$				
		Σf	$-1823\cdot10^{-5}$	$15207\cdot10^{-10}$				$+198\cdot10^{-5}$	$5856\cdot10^{-10}$			
II	3	$-25\cdot10^{3}$	$-1360\cdot10^{-5}$	$2720\cdot10^{-10}$	+167854	-177553±	-34699	$-1396\cdot10^{-5}$	$2010\cdot10^{-10}$	+730668	-552198±	+143771[f]
	7	$-75\cdot10^{3}$	$-2580\cdot10^{-5}$	$1720\cdot10^{-10}$	+167854	-798781±	-705927	$-7214\cdot10^{-5}$	$511\cdot10^{-10}$	+730668	-2779111±	-2754371
	8	$-75\cdot10^{3}$	$-1555\cdot10^{-5}$	$1037\cdot10^{-10}$	+167854	-1244652±	-1151798	$-5453\cdot10^{-5}$	$237\cdot10^{-10}$	+730668	-3619538±	-4040669
Constant input flow			$27778\cdot10^{-5}$					$+27778\cdot10^{-5}$				
		Σf	$22282\cdot10^{-5}$	$5477\cdot10^{-10}$				$+13715\cdot10^{-5}$	$2759\cdot10^{-10}$			
III	4	$+50\cdot10^{3}$	$+2580\cdot10^{-5}$	$2580\cdot10^{-10}$	+798781	-177553=	+671229	$+8616\cdot10^{-5}$	$642\cdot10^{-10}$	+2779111	-552198=	+2898142
	5	$+50\cdot10^{3}$	$+9206\cdot10^{-5}$	$9206\cdot10^{-10}$	+798781	-854621=	-5840[f]	$-2958\cdot10^{-5}$	$25329\cdot10^{-10}$	+2779111	-2785111±	-11840
	7	$+75\cdot10^{3}$	$+2580\cdot10^{-5}$	$1720\cdot10^{-10}$	+798781	-167854=	+705927	$+7214\cdot10^{-5}$	$511\cdot10^{-10}$	+2779111	-730668=	+2754371
Constant output flow			$-36111\cdot10^{-5}$					$-36111\cdot10^{-5}$				
		Σf	$-21744\cdot10^{-5}$	$13507\cdot10^{-10}$				$-23239\cdot10^{-5}$	$26482\cdot10^{-10}$			
IV	1	$+25\cdot10^{3}$	$+1825\cdot10^{-5}$	$3649\cdot10^{-10}$	+854621	-	+879621	$+9639\cdot10^{-5}$	$548\cdot10^{-10}$	+2785111	-	+3664732
	5	$-50\cdot10^{3}$	$-9206\cdot10^{-5}$	$9206\cdot10^{-10}$	+854621	-798781±	+5840[f]	$+2958\cdot10^{-5}$	$25329\cdot10^{-10}$	+2785111	-2779111=	+11840
	6	$-50\cdot10^{3}$	$-1372\cdot10^{-5}$	$1372\cdot10^{-10}$	+854621	-1244652±	-440031	$-3672\cdot10^{-5}$	$417\cdot10^{-10}$	+2785111	-3619538±	-1274458
Constant output flow			$-22222\cdot10^{-5}$					$-22222\cdot10^{-5}$				
		Σf	$-30976\cdot10^{-5}$	$14228\cdot10^{-10}$				$-13297\cdot10^{-5}$	$26294\cdot10^{-10}$			
V	6	$+50\cdot10^{3}$	$+1372\cdot10^{-5}$	$1372\cdot10^{-10}$	+1244652	-854621=	+440031	$+3672\cdot10^{-5}$	$417\cdot10^{-10}$	+3619538	-2785111=	+1274458
	8	$+75\cdot10^{3}$	$+1555\cdot10^{-5}$	$1037\cdot10^{-10}$	+1244652	-167854=	+1151798	$+5453\cdot10^{-5}$	$237\cdot10^{-10}$	+3619538	-730668=	+4040669
Constant output flow			$-19444\cdot10^{-5}$					$-19444\cdot10^{-5}$				
		Σf	$-16517\cdot10^{-5}$	$2409\cdot10^{-10}$				$-10320\cdot10^{-5}$	$654\cdot10^{-10}$			

Pipe lengths and diameters are shown in figure 1; see figure 4 for initial pattern (blue letters); 9 iterations are enough to achieve results shown in table 9 (this is for $\Sigma f \rightarrow 0$)

[a]using (18); to calculate friction factor λ, Reynolds number has to be calculated and for this velocity have to be chosen (this velocity does not have effect on final results, here is chosen extremely large velocity 100 m/s), [b]$F'=|\partial Q(\Delta p)/\partial \Delta p|$,[c]$\Delta_{\Delta p1}$ after eq. (20), [d]$\Delta_{\Delta p2}$ is $\Delta_{\Delta p1}$ from adjacent node, [e]final calculated pressure function in the first iteration is used for the calculation in the second iteration, [f]opposite flow direction than in previous iteration

Table 9. Final flows for network presented in this paper

Flows (m^3/h)								
Pipe	1	2	3	4	5	6	7	8
Water	902.27	1097.73	94.86	802.87	-146.23[a]	248.50	643.36	451.50
Gas	913.72	1086.28	82.01	804.27	-137.86[a]	251.58	633.60	448.42

[a]sing minus means flow direction opposite than first assumed in figure 2

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Table 10: Velocities for water and gas for calculated flows from example network

Velocity (m/s)								
Pipe	1	2	3	4	5	6	7	8
Water	21.0	15.4	2.2	18.6	-1.7[a]	9.4	14.9	13.2
Gas	5.3	3.8	0.5	4.7	-0.4[a]	2.4	3.7	3.3

[a]sing minus means flow direction opposite than first assumed in figure 2