



HAL
open science

Solution of the Implicit Colebrook Equation for Flow Friction Using Excel

Dejan Brkić

► **To cite this version:**

Dejan Brkić. Solution of the Implicit Colebrook Equation for Flow Friction Using Excel. Spreadsheets in Education, 2017, 10 (2). hal-01586190

HAL Id: hal-01586190

<https://hal.science/hal-01586190>

Submitted on 12 Sep 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

8-30-2017

Solution of the Implicit Colebrook Equation for Flow Friction Using Excel

Dejan Brkic

-, dejanbrkic0611@gmail.com

Follow this and additional works at: <http://epublications.bond.edu.au/ejsie>



This work is licensed under a [Creative Commons Attribution-Noncommercial-No Derivative Works 4.0 License](https://creativecommons.org/licenses/by-nc-nd/4.0/).

Recommended Citation

Brkic, Dejan (2017) Solution of the Implicit Colebrook Equation for Flow Friction Using Excel, *Spreadsheets in Education (eJSiE)*: Vol. 10: Iss. 2, Article 2.

Available at: <http://epublications.bond.edu.au/ejsie/vol10/iss2/2>

This In the Classroom Article is brought to you by the Bond Business School at [epublications@bond](mailto:epublications@bond.edu.au). It has been accepted for inclusion in *Spreadsheets in Education (eJSiE)* by an authorized administrator of [epublications@bond](mailto:epublications@bond.edu.au). For more information, please contact [Bond University's Repository Coordinator](#).

Solution of the Implicit Colebrook Equation for Flow Friction Using Excel

Abstract

Empirical Colebrook equation implicit in unknown flow friction factor (λ) is an accepted standard for calculation of hydraulic resistance in hydraulically smooth and rough pipes. The Colebrook equation gives friction factor (λ) implicitly as a function of the Reynolds number (Re) and relative roughness (ϵ/D) of inner pipe surface; i.e. $\lambda_0=f(\lambda_0, Re, \epsilon/D)$. The paper presents a problem that requires iterative methods for the solution. In particular, the implicit method used for calculating the friction factor λ_0 is an application of fixed-point iterations. The type of problem discussed in this "in the classroom paper" is commonly encountered in fluid dynamics, and this paper provides readers with the tools necessary to solve similar problems. Students' task is to solve the equation using Excel where the procedure for that is explained in this "in the classroom" paper. Also, up to date numerous explicit approximations of the Colebrook equation are available where as an additional task for students can be evaluation of the error introduced by these explicit approximations $\lambda \approx f(Re, \epsilon/D)$ compared with the iterative solution of implicit equation which can be treated as accurate.

Keywords

Colebrook equation, Hydraulic friction, Turbulence, Pipes, Flow

Distribution License



This work is licensed under a [Creative Commons Attribution-Noncommercial-No Derivative Works 4.0 License](https://creativecommons.org/licenses/by-nc-nd/4.0/).

Solution of the Implicit Colebrook Equation for Flow Friction Using Excel

Introduction

Hydraulic resistance depends on flow rate. Similar situation is with electrical resistance when a diode is in circuit [1]. To be more complex, widely used empirical Colebrook equation (1) is iterative i.e. implicit in fluid flow friction factor because the unknown friction factor (λ) appears on both sides of the equation [2].

$$\frac{1}{\sqrt{\lambda_0}} = -2 \cdot \log_{10} \left(\frac{2.51}{\text{Re} \cdot \sqrt{\lambda_0}} + \frac{\varepsilon}{3.71 \cdot D} \right) \quad (1)$$

In Colebrook's equation, λ is Darcy flow friction factor (dimensionless), Re is Reynolds number (dimensionless) and ε/D is relative roughness of inner pipe surface (dimensionless). Practical domain of the Reynolds number (Re) is between 4000 and 10^8 while for the relative roughness (ε/D) is up to 0.05. Index 0 here denotes the values of friction factor (λ) calculated using the implicit Colebrook equation through iterative procedure, i.e. it denotes the solution conditionally accepted as accurate, or let's say the most accurate compared with other approaches such as use of explicit approximate formulas.

The Colebrook equation is based on joint experiment which Colebrook as PhD student conducted with his professor White [3]. Later Rouse followed by Moody made flow friction diagram based on these results [4, 5].

The Colebrook equation is valuable for determination of hydraulic resistances for turbulent regime in smooth and rough pipes but it is not valid for laminar regime. It describes a monotonic change in the friction factor in commercial pipes from smooth to fully rough. This equation has become the accepted standard of accuracy for calculation of hydraulic friction factor despite the fact that many new experiments have disputed its accuracy [6].

The empirical and implicit Colebrook equation cannot be rearranged to derive and calculate friction factor (λ) directly in one step [7]. The most accurate procedure to calculate this unknown friction factor (λ) is through iterative procedure [8]; $\lambda_0 = f(\lambda_0, \text{Re}, \varepsilon/D)$. This can be accomplished relative easily in spreadsheet environment and the detailed procedure is explained in this "in the classroom" paper. In addition to the iterative procedure, many explicit approximations are available; $\lambda \approx f(\text{Re}, \varepsilon/D)$, but they introduce certain error [9] which can be predicted in advance and which is not distributed uniformly through the practical domain of the Reynolds number (Re) and the relative roughness (ε/D) [10]. An additional task for students is evaluation of this relative error caused by using of approximations compared with the iterative solution which can be treated as accurate [11].

In summary, the main students' tasks are:

1. To calculate flow friction (λ_0) in Excel using implicit Colebrook's equation, and

2. To calculate flow friction (λ) in Excel using explicit approximations of Colebrook's equation and to evaluate relative error. In addition diagrams that represent distribution of error can be drawn in Excel.
3. Additional tasks: Lambert W-function, networks of pipes with loops, MATLAB (Genetic Algorithms – GA and Artificial Neural Networks - ANN), Excel fitting tool, etc.

This “in the classroom” paper contains Excel file as Electronic Annex.

1. Iterative solution in Excel using implicit Colebrook equation

To solve the implicit Colebrook equation, one must start by somehow estimating the value of the friction factor (λ_0) on the right side of the equation, to calculate the new λ_0 on the left, enter the new value of λ_0 back on the right side, and continue this process until there is a balance on both sides of the equation within an arbitrary small difference without causing endless computations.

The Colebrook equation can be expressed as (2):

$$\frac{1}{\sqrt{\lambda_0}} = -2 \cdot \log_{10} \left(\frac{2.51}{\text{Re} \cdot \sqrt{\lambda_0}} + \frac{\varepsilon}{3.71 \cdot D} \right) = -2 \cdot \log_{10} \left(\frac{a}{\sqrt{\lambda_0}} + B \right) = -2 \cdot \log_{10}(A + B) \quad (2)$$

Under the logarithm, the term A represents partially turbulent flow through hydraulically smooth pipes proposed by Prandtl while the second term, B, represents turbulent flow through hydraulically rough pipes proposed by von Karman. As can be seen from Figure 1, one pipe can be hydraulically smooth or rough not only depending on the state of its inner roughness but also on the state of boundary sub-layer of fluid in motion near the inner wall surface of the pipe [12].

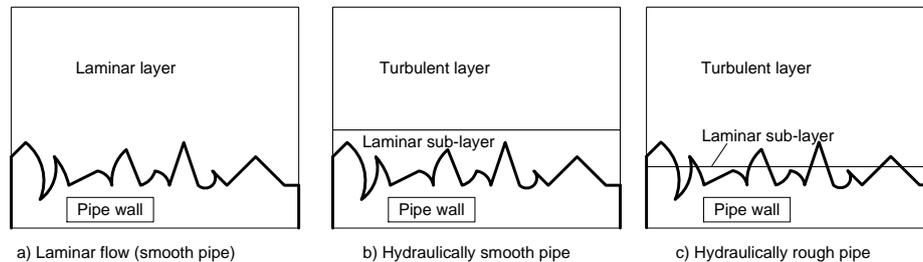


Figure 1: Different hydraulic regimes of flow in one pipe

Using Prandtl's and von Karman's equations separately the sharp change in values of friction factor between smooth and rough regime will occur. On the other hand Colebrook and White during their experiments did not detect this sharp change. According to them the transition from the hydraulically smooth regime of turbulence to the fully rough is smooth as can be seen from Figure 2. Note that $\log(A)+\log(B) \neq \log(A+B)$, where the separate use of $\log(A)$ and $\log(B)$ produce two lines in related diagrams with sharp intersection while $\log(A+B)$ produces one smooth line.

So, Colebrook's equation has virtually two parts, smooth Prandtl (A in eq. 2) and rough von Karman (B in eq. 2) with smooth transition between them. Only smooth Prandtl's part is implicit in unknown flow friction factor (λ_0). Knowing that only the first smooth part is equal to zero in the first iteration ($A=0$; $Re \rightarrow \infty$), while the second part has value different than zero ($B \neq 0$), the arbitrary estimation of the value of the flow friction factor (λ_0) in the first iteration can be avoided where the initial value in the first iteration is from $\frac{1}{\sqrt{\lambda_0}} = -2 \cdot \log_{10}(B)$.

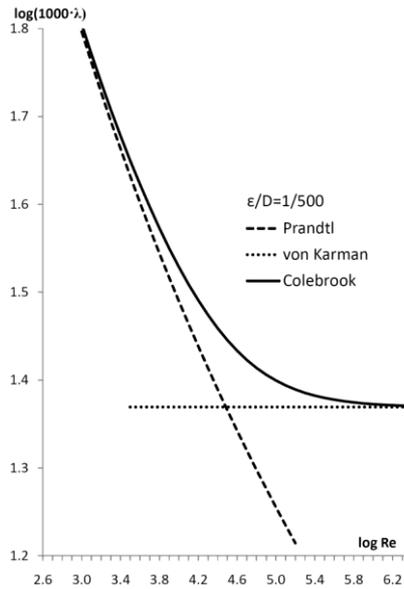


Figure 2: Findings of Colebrook and White shows smooth transitions from smooth to rough turbulent flow

To implement this procedure in Excel in order to solve the implicit Colebrook-White equation the 'Office button' at the upper left corner of the screen need to be pressed (Figure 3) where in 'Excel options', 'Formulas' needs to be selected (this procedure can be slightly different in some version of Excel). As shown in Figure 4, in the window 'Formulas', box 'Enable iterative calculation', need to be ticked and desired number of iteration need to be chosen (max. allowed is 32767).

Excel-code for the implicit Colebrook equation is (result will appear in C1);

$=-2*\text{LOG}10(((1/3.71)*B1)+((2.51/A1)*C1))$ where B1 is cell with the relative roughness (ϵ/D), A1 cell with the Reynolds number (Re) and C1 is iterative reference.

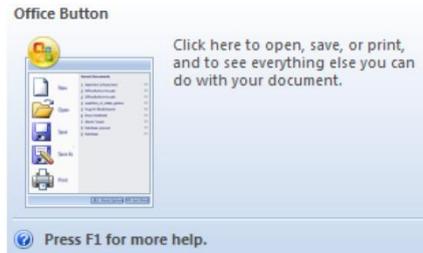


Figure 3: Office button in Excel

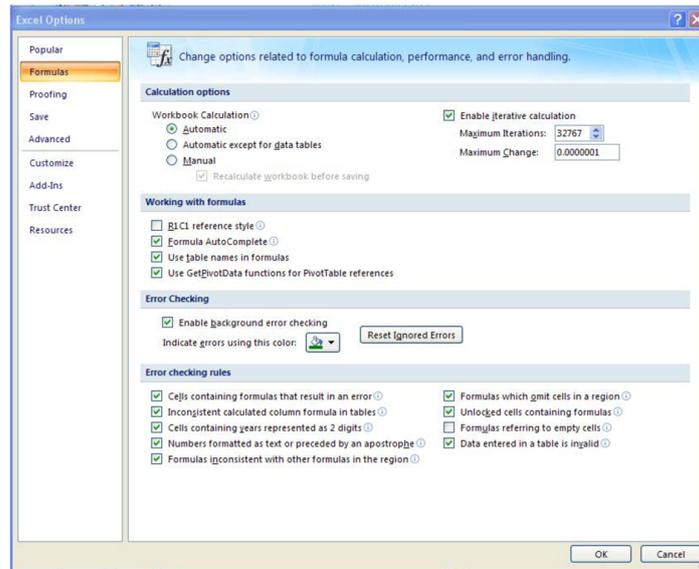


Figure 4: Settings for iterative calculation in Excel

For example:

- for $Re=10^4$ and $\varepsilon/D=10^{-6}$, $\lambda_0=0.0308844939$;
- for $Re=5.8 \cdot 10^6$ and $\varepsilon/D=3 \cdot 10^{-3}$, $\lambda_0=0.0261693581$;
- for $Re=3 \cdot 10^7$ and $\varepsilon/D=4.3 \cdot 10^{-4}$, $\lambda_0=0.0161582229$;
- for $Re=6 \cdot 10^4$ and $\varepsilon/D=2 \cdot 10^{-4}$, $\lambda_0=0.0208369171$;
- for $Re=4 \cdot 10^5$ and $\varepsilon/D=0.03$, $\lambda_0=0.0571868356$; etc.

Results are obtained from Excel file attached to this “in the classroom paper” as Electronic Annex.

2. Explicit approximations of Colebrook’s equation

Numerous of explicit approximations of Colebrook’s equation exist [13-35]. They introduce certain error which can be estimated in advance. The error is not distributed uniformly through the domain of the Reynolds number (Re) and the relative roughness (ε/D).

Students’ task is to find few approximations of the Colebrook equation in available literature [10, 11, 36-41] and to estimate their relative error compared with the accurate iterative solution of the original Colebrook’s equation. For this purpose friction factor calculated using approximations can be noted as λ while from the original implicit equation as λ_0 . In that way relative percentage error can be calculated as $\delta\%=[(\lambda-\lambda_0)/\lambda_0] \cdot 100\%$. Also the whole domain of applicability of the Colebrook equation can be covered with mesh where in nodes the relative error can be calculated. In that way diagram of error can be constructed. Good resolution for that should be achieved with at least 500 mesh nodes over the whole practical domain of the Reynolds number (Re) and the relative roughness (ε/D).

Following calculation of the error performed in the Excel file from Annex of this “in the classroom” paper, students need to prepare an additional Excel file in which error analysis will be performed in order to construct diagram of error distribution. In this file mesh which will allow construction of diagram of error over the domain of the Reynolds number (Re) and the relative roughness (ε/D) is formed.

As an illustrative example, Brkić approximation [14] is examined (3). First part in Eq. 3 is with the original values of coefficients while the second is altered using genetic algorithms [36, 37] in order to decrease maximal relative error ($\delta\%$). Distribution of the relative error of Brkić approximation before and after genetic optimisation [36] can be seen in Figure 5. Maximal relative error before optimisation [14] is about 2.2% and after [36] about 1.29%. Note that the optimisation that is performed to cut maximal error over the domain [36, 37]. Such approach in many cases can cause increase of the relative error in certain points of the domain.

Related this task, students need to produce similar diagrams such as 3D as in Figure 5, but also 2D as well as other appropriate types.

$$\left. \begin{aligned} \frac{1}{\sqrt{\lambda}} &\approx -2 \cdot \log_{10} \left(\frac{2.18 \cdot a_1}{\text{Re}} + \frac{1}{3.71} \cdot \frac{\varepsilon}{D} \right) \\ a_1 &\approx \ln \frac{\text{Re}}{1.816 \cdot \ln \left(\frac{1.1 \cdot \text{Re}}{\ln(1 + 1.1 \cdot \text{Re})} \right)} \end{aligned} \right\} \rightarrow \left. \begin{aligned} \frac{1}{\sqrt{\lambda}} &\approx -2.013 \cdot \log_{10} \left(\frac{2.261 \cdot A_1}{\text{Re}} + \frac{1}{3.71} \cdot \frac{\varepsilon}{D} \right) \\ A_1 &\approx \ln \frac{\text{Re}}{2.479 \cdot \ln \left(\frac{1.1 \cdot \text{Re}}{\ln(1 + 1.1 \cdot \text{Re})} \right)} \end{aligned} \right\} \quad (3)$$

For example for $\text{Re}=7 \cdot 10^4$ and for $\varepsilon/D=10^{-4}$, $\lambda_0=0.019832705$ from iterative procedure, $\lambda=0.019942264$ according to Brkić approximation (3) before with $\delta\%=0.55\%$ and $\lambda=0.019679583$ after genetic approximation with $\delta\%=0.77\%$.

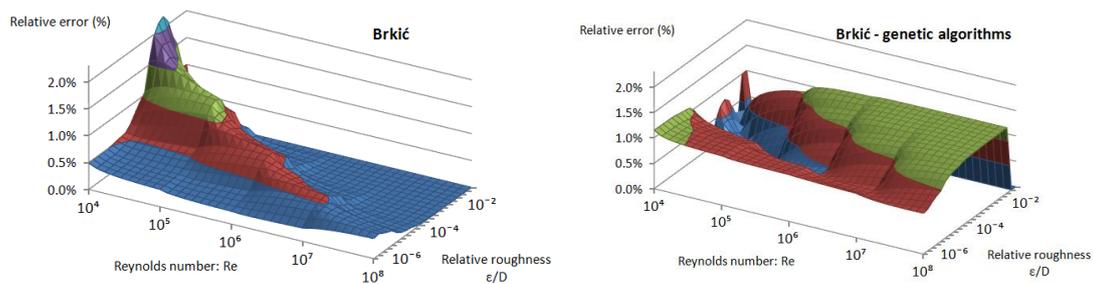


Figure 5: Distribution of error of Brkić approximation before and after genetic optimisation

This “in the classroom” paper is supplied with Excel file which contains certain number of approximations. The file is set also for iterative calculation and hence the error introduced by selected approximation can also be calculated. Using that pattern, students can code in Excel additional approximations found in literature (here as example is shown approximation by Brkić; Eq. 3). Also, as inverse task, already Excel-

coded approximation can be extracted from the file and can be compared with the form from original sources [36]. Excel-code for Brkić approximation before optimisation (3) is $=2*\text{LOG}10((B1/3.71)+(2.18*E1/A1))$ where B1 is cell with the relative roughness (ε/D), A1 cell with the Reynolds number (Re) and E1 auxiliary term $=\text{LN}(A1/(1.816*\text{LN}(1.1*A1/\text{LN}(1+1.1*A1))))$.

3. Additional tasks

Colebrook equation can be expressed in explicit form through Lambert W-function [42-47]. Further to evaluate friction factor (λ) instead of specific approximations developed for Colebrook's equation, general approximations for the Lambert W-function can be used [48-51]. Students need to find in available literature these approximations of the Lambert W-function, to find available forms of Colebrook's equation expressed through the Lambert W-function [42-45, 52-55] and to implement related calculation in Excel. Students should note that some expressions of Colebrook's equation through the Lambert W-function contain exponential form which makes for some combination of the Reynolds number (Re) and the relative roughness (ε/D) calculation impossible due to limited capability of registers of computer to accommodate extremely large or small numbers [54, 55].

As additional task, student can repeat all activities in e.g. MATLAB or similar software packages. Colebrook equation also can be simulated with Artificial Neural Networks – ANN and such task can be also performed in MATLAB [56].

Excel contains fitting tools [57] which can be used for optimisation of approximations similarly as mentioned optimisation through genetic algorithm which is performed in MATLAB [36, 37]. This activity can be used as a task for advance students.

Further, students can use Excel for more capable task such as calculation of water distribution networks (both tree- and loop-like) where multiple simultaneous calculation of friction factor is needed [58-70].

Conclusion

Colebrook's equation suffers from being implicit in unknown flow friction factor (λ), but on the other hand this equation is relatively simple which makes it ideal for students to train implementation of iterative procedures in spreadsheet environment, to increase their capability to make diagrams, to perform error analysis, etc. All activities can be performed in spreadsheet environment but also in MATLAB or similar software packages specialised for calculation.

The tasks described in this "in the classroom" paper are in the first place for students of hydraulics, petroleum engineering and water resources [71] but also for students of all engineering branches where fluid flow can occur [72-74], including fuel cells [75, 76].

References

- [1] Brkić, D. (2014). Discussion of “Method to cope with zero flows in Newton solvers for water distribution systems” by Nikolai B. Gorev et al. *Journal of Hydraulic Engineering ASCE*, 140(4), 456-459.
[https://doi.org/10.1061/\(ASCE\)HY.1943-7900.0000769](https://doi.org/10.1061/(ASCE)HY.1943-7900.0000769)
- [2] Colebrook, C.F. (1939). Turbulent flow in pipes with particular reference to the transition region between the smooth and rough pipe laws. *Journal of the Institution of Civil Engineers (London)*, 11(4), 133-156.
<https://dx.doi.org/10.1680/ijoti.1939.13150>
- [3]. Colebrook, C., and White, C. (1937). Experiments with fluid friction in roughened pipes. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 161(906), 367-381.
<https://dx.doi.org/10.1098/rspa.1937.0150>
- [4]. Moody, L.F. (1944). Friction factors for pipe flow. *Transactions of ASME*, 66(8), 671-684. Available from:
http://user.engineering.uiowa.edu/~me_160/lecture_notes/MoodyLFpaper1944.pdf
- [5]. LaViolette M. (2017). On the history, science, and technology included in the Moody diagram. *Journal of Fluids Engineering ASME*, 139(3), 030801-030801-21. <https://dx.doi.org/10.1115/1.4035116>
- [6]. Allen, J.J., Shockling, M.A., Kunkel, G.J., and Smits, A.J. (2007). Turbulent flow in smooth and rough pipes. *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 365(1852), 699-714. <https://dx.doi.org/10.1098/rsta.2006.1939>
- [7]. Mikata, Y., and Walczak, W.S. (2016). Exact analytical solutions of the Colebrook-White equation. *Journal of Hydraulic Engineering ASCE*, 142(2), 04015050. [https://dx.doi.org/10.1061/\(ASCE\)HY.1943-7900.0001074](https://dx.doi.org/10.1061/(ASCE)HY.1943-7900.0001074)
- [8]. Brkić, D. (2012). Determining friction factors in turbulent pipe flow. *Chemical Engineering (New York)*, 119(3), 34-39. Available from:
<http://www.chemengonline.com/determining-friction-factors-in-turbulent-pipe-flow/?printmode=1>
- [9]. Brkić, D. (2014). Discussion of “Gene expression programming analysis of implicit Colebrook–White equation in turbulent flow friction factor calculation” by Saeed Samadianfard [*J. Pet. Sci. Eng.* 92–93 (2012) 48–55]. *Journal of Petroleum Science and Engineering*, 124, 399-401.
<https://dx.doi.org/10.1016/j.petrol.2014.06.007>
- [10]. Winning, H.K., and Coole, T. (2013). Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. *Flow, Turbulence and Combustion*, 90(1), 1-27. <https://dx.doi.org/10.1007/s10494-012-9419-7>
- [11]. Brkić, D. (2011). Review of explicit approximations to the Colebrook relation for flow friction. *Journal of Petroleum Science and Engineering*, 77(1), 34-48.
<https://dx.doi.org/10.1016/j.petrol.2011.02.006>
- [12]. Brkić, D. (2012). Can pipes be actually really that smooth?. *International Journal of Refrigeration*, 35(1), 209-215. <https://dx.doi.org/10.1016/j.ijrefrig.2011.09.012>

- [13]. Biberg, D. (2017). Fast and accurate approximations for the Colebrook equation. *Journal of Fluids Engineering ASME*, 139(3), 031401-031401-3. <https://dx.doi.org/10.1115/1.4034950>
- [14]. Fang, X., Xu, Y., and Zhou, Z. (2011). New correlations of single-phase friction factor for turbulent pipe flow and evaluation of existing single-phase friction factor correlations. *Nuclear Engineering and Design*, 241(3), 897-902. <https://dx.doi.org/10.1016/j.nucengdes.2010.12.019>
- [15]. Brkić, D. (2011). An explicit approximation of Colebrook's equation for fluid flow friction factor. *Petroleum Science and Technology*, 29(15), 1596-1602. <https://dx.doi.org/10.1080/10916461003620453>
- [16]. Ghanbari, A., Fred, F.F., and Rieke, H.H. (2011). Newly developed friction factor correlation for pipe flow and flow assurance. *Journal of Chemical Engineering and Materials Science*, 2(6), 83-86. Available from: <http://www.academicjournals.org/journal/JCEMS/article-abstract/43BC5B11677>
- [17]. Brkić, D. (2011). New explicit correlations for turbulent flow friction factor. *Nuclear Engineering and Design*, 241(9), 4055-4059. <https://dx.doi.org/10.1016/j.nucengdes.2011.07.042>
- [18]. Papaevangelou, G, Evangelides, C., Tzimopoulos, C. (2010). A new explicit relation for the friction factor coefficient in the Darcy–Weisbach equation. In *Proceedings of the Protection and Restoration of the Environment, Corfu, Greece, 5–9 July 2010*; pp. 166–172. Available from: http://blogs.sch.gr/geopapaevan/files/2010/07/full-paper_pre1128act.pdf
- [19]. Avci, A., and Karagoz, I. (2009). A novel explicit equation for friction factor in smooth and rough pipes. *Journal of Fluids Engineering ASME*, 131(6), 061203-061203-4. <https://dx.doi.org/10.1115/1.3129132>
- [20]. Buzzelli, D. (2008). Calculating friction in one step. *Machine Design*, 80(12), 54-55. Available from: http://images.machinedesign.com/images/archive/72728061908flui_00000051017.pdf
- [21]. Romeo, E., Royo, C., and Monzón, A. (2002). Improved explicit equations for estimation of the friction factor in rough and smooth pipes. *Chemical Engineering Journal*, 86(3), 369-374. [https://dx.doi.org/10.1016/S1385-8947\(01\)00254-6](https://dx.doi.org/10.1016/S1385-8947(01)00254-6)
- [22]. Manadili, G. (1997). Replace implicit equations with signomial functions. *Chemical Engineering (New York)*, 104(8), 129-130.
- [23]. Chen, J.J.J. (1984). A simple explicit formula for the estimation of pipe friction factor. *Proceedings of the Institution of Civil Engineers (London)*, 77(1), 49-55. <https://dx.doi.org/10.1680/iicep.1984.1272>
- [24]. Serghides, T. K. (1984). Estimate friction factor accurately. *Chemical Engineering (New York)*, 91(5), 63-64.
- [25]. Haaland, S.E. (1983). Simple and explicit formulas for the friction factor in turbulent pipe flow. *Journal of Fluids Engineering ASME*, 105(1), 89-90. <https://dx.doi.org/10.1115/1.3240948>

- [26]. Zigrang, D.J., and Sylvester, N.D. (1982). Explicit approximations to the solution of Colebrook's friction factor equation. *AIChE Journal*, 28(3), 514-515. <https://dx.doi.org/10.1002/aic.690280323>
- [27]. Barr, D.I.H. (1981). Solutions of the Colebrook-White function for resistance to uniform turbulent flow. *Proceedings of the Institution of Civil Engineers (London)*, 71(2), 529-535. <https://dx.doi.org/10.1680/icep.1981.1895>
- [28]. Round, G.F. (1980). An explicit approximation for the friction factor-Reynolds number relation for rough and smooth pipes. *The Canadian Journal of Chemical Engineering*, 58(1), 122-123. <https://dx.doi.org/10.1002/cjce.5450580119>
- [29]. Schorle, B.J., Churchill, S.W., and Shacham, M. (1980). Comments on "An explicit equation for friction factor in pipe". *Industrial & Engineering Chemistry Fundamentals*, 19(2), 228-230. <https://dx.doi.org/10.1021/i160074a019>
- [30]. Chen, N.H. (1979). An explicit equation for friction factor in pipe. *Industrial & Engineering Chemistry Fundamentals*, 18(3), 296-297. <https://dx.doi.org/10.1021/i160071a019>
- [31]. Swanee, P.K., and Jain, A.K. (1976). Explicit equations for pipeflow problems. *Journal of the Hydraulics Division ASCE*, 102(5), 657-664.
- [32]. Eck, B. (1973). *Technische Stromungslehre*; Springer: New York, NY, USA
- [33]. Wood, D.J. (1966). An explicit friction factor relationship. *Civil Engineering*, 36(12), 60-61.
- [34]. Moody, L.F. (1947). An approximate formula for pipe friction factors. *Transactions of ASME*, 69(12), 1005-1011.
- [35]. Brkić, D. (2016). A note on explicit approximations to Colebrook's friction factor in rough pipes under highly turbulent cases. *International Journal of Heat and Mass Transfer*, 93, 513-515. <https://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.08.109>
- [36]. Brkić, D., and Čojbašić, Ž. (2017). Evolutionary optimization of Colebrook's turbulent flow friction approximations. *Fluids*, 2(2), 15. <https://dx.doi.org/10.3390/fluids2020015>
- [37]. Čojbašić, Ž., and Brkić, D. (2013). Very accurate explicit approximations for calculation of the Colebrook friction factor. *International Journal of Mechanical Sciences*, 67, 10-13. <https://dx.doi.org/10.1016/j.ijmecsci.2012.11.017>
- [38]. Gregory, G.A., and Fogarasi, M. (1985). Alternate to standard friction factor equation. *Oil & Gas Journal*, 83(13), 120 and 125-127.
- [39]. Genić, S., Arandžević, I., Kolendić, P., Jarić, M., Budimir, N., and Genić, V. (2011). A review of explicit approximations of Colebrook's equation. *FME transactions*, 39(2), 67-71. Available from: http://www.mas.bg.ac.rs/_media/istrazivanje/fme/vol39/2/04_mjaric.pdf
- [40]. Zigrang, D.J., and Sylvester, N.D. (1985). A review of explicit friction factor equations. *Journal of Energy Resources Technology ASME* 107(2), 280-283. <https://dx.doi.org/10.1115/1.3231190>

- [41]. Giustolisi, O., Berardi, L., and Walski, T. M. (2011). Some explicit formulations of Colebrook–White friction factor considering accuracy vs. computational speed. *Journal of Hydroinformatics*, 13(3), 401-418.
<https://dx.doi.org/10.2166/hydro.2010.098>
- [42]. Brkić, D. (2011). W solutions of the CW equation for flow friction. *Applied Mathematics Letters*, 24(8), 1379-1383.
<https://dx.doi.org/10.1016/j.aml.2011.03.014>
- [43]. Brkić, D. (2017). Discussion of “Exact analytical solutions of the Colebrook-White equation” by Yozo Mikata and Walter S. Walczak. *Journal of Hydraulic Engineering ASCE*, 143(9), 0701700, [https://dx.doi.org/10.1061/\(ASCE\)HY.1943-7900.0001341](https://dx.doi.org/10.1061/(ASCE)HY.1943-7900.0001341)
- [44]. Keady, G. (1998). Colebrook-White formula for pipe flows. *Journal of Hydraulic Engineering ASCE*, 124(1), 96-97.
[https://dx.doi.org/10.1061/\(ASCE\)0733-9429\(1998\)124:1\(96\)](https://dx.doi.org/10.1061/(ASCE)0733-9429(1998)124:1(96))
- [45]. Rollmann, P., and Spindler, K. (2015). Explicit representation of the implicit Colebrook–White equation. *Case Studies in Thermal Engineering*, 5, 41-47.
<https://dx.doi.org/10.1016/j.csite.2014.12.001>
- [46]. Brkić, D. (2010). Efficiency of Distribution and Use of Natural Gas in Households (Ефикасност дистрибуције и коришћења природног гаса у домаћинствима, In Serbian). Ph.D. Thesis, University of Belgrade, Belgrade, Serbia, Available from: <http://nardus.mpn.gov.rs/123456789/2654>,
<http://eteze.bg.ac.rs/application/showtheses?thesesId=1127>,
<https://fedorabg.bg.ac.rs/fedora/get/o:7888/bdef:Content/download>, and
<http://vbs.rs/scripts/cobiss?command=DISPLAY&base=70036&RID=36621839>
- [47]. Sonnad, J.R., and Goudar, C.T. (2007). Explicit reformulation of the Colebrook–White equation for turbulent flow friction factor calculation. *Industrial & Engineering Chemistry Research*, 46(8), 2593-2600.
<https://dx.doi.org/10.1021/ie0340241>
- [48]. Barry, D.A., Parlange, J.Y., Li, L., Prommer, H., Cunningham, C.J., and Stagnitti, F. (2000). Analytical approximations for real values of the Lambert W-function. *Mathematics and Computers in Simulation*, 53(1), 95-103.
[https://dx.doi.org/10.1016/S0378-4754\(00\)00172-5](https://dx.doi.org/10.1016/S0378-4754(00)00172-5)
- [49]. Boyd, J.P. (1998). Global approximations to the principal real-valued branch of the Lambert W-function. *Applied Mathematics Letters*, 11(6), 27-31.
[https://dx.doi.org/10.1016/S0893-9659\(98\)00097-4](https://dx.doi.org/10.1016/S0893-9659(98)00097-4)
- [50]. Corless, R.M., Gonnet, G.H., Hare, D.E., Jeffrey, D.J., and Knuth, D.E. (1996). On the Lambert W function. *Advances in Computational Mathematics*, 5(1), 329-359. <https://dx.doi.org/10.1007/BF02124750>
- [51]. Hayes, B. (2005). Why W? *American Scientist*, 93(2), 104–108.
<https://dx.doi.org/10.1511/2005.2.104>
- [52]. Brkić, D. (2012). Lambert W function in hydraulic problems. *Mathematica Balkanica*, 26(3-4), 285-292. Available from:
<http://www.math.bas.bg/infres/MathBalk/MB-26/MB-26-285-292.pdf>

- [53]. Goudar, C. T., and Sonnad, J. R. (2003). Explicit friction factor correlation for turbulent flow in smooth pipes. *Industrial & Engineering Chemistry Research*, 42(12), 2878-2880. <https://dx.doi.org/10.1021/ie0300676>
- [54]. Sonnad, J.R., and Goudar, C.T. (2004). Constraints for using Lambert W function-based explicit Colebrook–White equation. *Journal of Hydraulic Engineering ASCE*, 130(9), 929-931. [https://dx.doi.org/10.1061/\(ASCE\)0733-9429\(2004\)130:9\(929\)](https://dx.doi.org/10.1061/(ASCE)0733-9429(2004)130:9(929))
- [55]. Brkić, D. (2012). Comparison of the Lambert W-function based solutions to the Colebrook equation. *Engineering Computations*, 29(6), 617-630. <https://dx.doi.org/10.1108/02644401211246337>
- [56]. Brkić, D., and Čojbašić, Ž. (2016). Intelligent flow friction estimation. *Computational Intelligence and Neuroscience*, 5242596. <https://dx.doi.org/10.1155/2016/5242596>
- [57]. Vatankhah, A.R. (2014). Comment on “Gene expression programming analysis of implicit Colebrook–White equation in turbulent flow friction factor calculation”. *Journal of Petroleum Science and Engineering*, 124, 402-405. <https://dx.doi.org/10.1016/j.petrol.2013.12.001>
- [58]. Cross, H. (1936). Analysis of flow in networks of conduits or conductors. University of Illinois at Urbana Champaign, College of Engineering. Engineering Experiment Station. 34, 3-29. Available from: <http://hdl.handle.net/2142/4433>
- [59]. Brkić, D. (2016). Spreadsheet-based pipe networks analysis for teaching and learning purpose. *Spreadsheets in Education (eJSiE)*, 9(2), Available from: <http://epublications.bond.edu.au/ejsie/vol9/iss2/4/>
- [60]. Brkić, D. (2011). Iterative methods for looped network pipeline calculation. *Water Resources Management*, 25(12), 2951-2987. <https://dx.doi.org/10.1007/s11269-011-9784-3>
- [61]. Simpson, A., and Elhay, S. (2010). Jacobian matrix for solving water distribution system equations with the Darcy-Weisbach head-loss model. *Journal of Hydraulic Engineering ASCE*, 137(6), 696-700. [https://dx.doi.org/10.1061/\(ASCE\)HY.1943-7900.0000341](https://dx.doi.org/10.1061/(ASCE)HY.1943-7900.0000341)
- [62]. Brkić, D. (2012). Discussion of “Jacobian matrix for solving water distribution system equations with the Darcy-Weisbach head-loss model” by Angus Simpson and Sylvan Elhay. *Journal of Hydraulic Engineering ASCE*, 138(11), 1000-1001. [https://dx.doi.org/10.1061/\(ASCE\)HY.1943-7900.0000529](https://dx.doi.org/10.1061/(ASCE)HY.1943-7900.0000529)
- [63]. Spiliotis, M., and Tsakiris, G. (2010). Water distribution system analysis: Newton-Raphson method revisited. *Journal of Hydraulic Engineering ASCE*, 137(8), 852-855. [https://dx.doi.org/10.1061/\(ASCE\)HY.1943-7900.0000364](https://dx.doi.org/10.1061/(ASCE)HY.1943-7900.0000364)
- [64]. Brkić, D. (2012). Discussion of “Water distribution system analysis: Newton-Raphson method revisited” by M. Spiliotis0 and G. Tsakiris. *Journal of Hydraulic Engineering ASCE*, 138(9), 822-824. [https://dx.doi.org/10.1061/\(ASCE\)HY.1943-7900.0000555](https://dx.doi.org/10.1061/(ASCE)HY.1943-7900.0000555)

- [65]. Brkić, D. (2009). An improvement of Hardy Cross method applied on looped spatial natural gas distribution networks. *Applied Energy*, 86(7), 1290-1300. <https://dx.doi.org/10.1016/j.apenergy.2008.10.005>
- [66]. Niazkar, M., and Afzali, S.H. (2017). Analysis of water distribution networks using MATLAB and Excel spreadsheet: h-based methods. *Computer Applications in Engineering Education*, 25(1), 129-141. <https://dx.doi.org/10.1002/cae.21786>
- [67]. Niazkar, M., and Afzali, S.H. (2017). Analysis of water distribution networks using MATLAB and Excel spreadsheet: Q-based methods. *Computer Applications in Engineering Education*, 25(2), 277-289. <https://dx.doi.org/10.1002/cae.21796>
- [68]. Brkić, D. (2011). A gas distribution network hydraulic problem from practice. *Petroleum Science and Technology*, 29(4), 366-377. <https://dx.doi.org/10.1080/10916460903394003>
- [69]. Brkić, D. (201x). Discussion of "Economics and statistical evaluations of using Microsoft Excel solver in pipe network analysis" by I.A. Oke; A. Ismail; S. Lukman; S.O. Ojo; O.O. Adeosun; and M.O. Nwude, *Journal of Pipeline Systems Engineering and Practice ASCE*, /in press/
- [70]. Demir, S., Manav Demir, N., and Karadeniz, A. (201x). An MS Excel tool for water distribution network design in environmental engineering education. *Computer Applications in Engineering Education*, /in press/. <https://dx.doi.org/10.1002/cae.21870>
- [71]. Pandit, A. (2016). *Water Engineering with the Spreadsheet: A Workbook for Water Resources Calculations Using Excel*. American Society of Civil Engineers – ASCE, <https://dx.doi.org/10.1061/9780784414040>
- [72]. Brkić, D., and Tanasković, T.I. (2008). Systematic approach to natural gas usage for domestic heating in urban areas. *Energy*, 33(12), 1738-1753. <https://dx.doi.org/10.1016/j.energy.2008.08.009>
- [73]. Pambour, K.A., Bolado-Lavin, R., and Dijkema, G.P. (2016). An integrated transient model for simulating the operation of natural gas transport systems. *Journal of Natural Gas Science and Engineering*, 28, 672-690. <https://dx.doi.org/10.1016/j.jngse.2015.11.036>
- [74]. Praks, P., Kopustinskas, V., and Masera, M. (2015). Probabilistic modelling of security of supply in gas networks and evaluation of new infrastructure. *Reliability Engineering & System Safety*, 144, 254-264. <https://dx.doi.org/10.1016/j.ress.2015.08.005>
- [75]. Barreras, F., López, A. M., Lozano, A., and Barranco, J.E. (2011). Experimental study of the pressure drop in the cathode side of air-forced open-cathode proton exchange membrane fuel cells. *International Journal of Hydrogen Energy*, 36(13), 7612-7620. <https://doi.org/10.1016/j.ijhydene.2011.03.149>
- [76]. Brkić, D. (2012). Comments on "Experimental study of the pressure drop in the cathode side of air-forced open-cathode proton exchange membrane fuel cells" by Barreras et al. *International Journal of Hydrogen Energy*, 37(14), 10963-10964. <https://doi.org/10.1016/j.ijhydene.2012.04.074>