A QUANTIFIED APPROACH TO THE LAWS OF GRAVITATION IN A PLATONIC QUADRIDIMENSIONAL SPACE

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TITLE:

A QUANTIFIED APPROACH TO THE LAWS OF GRAVITATION
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Abstract:

The properties of the platonic quadridimensional space, the modeling of De Broglie waves and the resulting concept of mass (see hal-01165196, v1 ; hal-01207447, v1 ; hal-01213447, v1) seem to allow a quantum approach to the laws of gravitation by a postulate of quantified declination.
The main idea of this article is that the ***guiding angle*** β of the trajectory of a massive body C in gravitational interaction with others is ***modified by a quantum*** $\Delta\beta$ ***, independent of the chosen observation frame***, at each perception of an occurrence of the De Broglie waves generated by one of these other massive bodies. This elementary quantized hypothesis, associated with the conservation of three absolute physical quantities, leads surprisingly rapidly to the results expected by the classical laws of gravitation.
We propose here an introduction, restricted to the relatively simple case where the guiding angle of the observation frame is equal to $\frac{\pi}{2}$.

Résumé :

Les propriétés de l'espace quadridimensionnel platonicien, la modélisation de l'onde de phase et le concept de masse qui en découlent (cf. les documents hal-01081576, v1; hal-01205805, v1; hal-01213062, v1) semblent permettre une approche quantique des lois de la gravitation par un postulat de déclinaison quantifiée.

L'idée directrice de cet article est que ***l'angle directeur*** β de la trajectoire d'un corps massif C en interaction gravitationnelle avec d'autres est ***modifié d'un quantum*** $\Delta\beta$***, indépendant du référentiel d'observation choisi,*** à chaque perception d'une occurrence de l'onde de phase de De Broglie générée par l'un de ces autres corps massifs. Cette hypothèse quantifiée élémentaire, associée à la conservation de trois quantités physiques absolues, conduit rapidement, de façon surprenante, aux résultats attendus par les lois classiques de la gravitation.
Nous en proposons ici une introduction, restreinte au cas relativement simple où l'angle directeur du référentiel d'observation est égal à $\frac{\pi}{2}$.

1. The geometrical framework

This modeling is based on the Platonic space outlined in the following articles:

« UN MODÈLE PLATONICIEN (EUCLIDIEN-PROJECTIF) POUR LA THÉORIE DE LA RELATIVITÉ RESTREINTE » (pré-publication hal-01081576, version 1).

« A PLATONIC (EUCLIDEAN-PROJECTIVE) MODEL FOR THE SPECIAL THEORY OF RELATIVITY » (pre-publication hal-01165196, version 1).

$\left(O,\vec{i},\vec{j},\vec{k},\vec{h}\right)$ is a frame for the four-dimensional Euclidean space whose axes are denoted (OX), (OY), (OZ), (Ow); the direction of the projection is that of the vector $\vec{h}$.
Following the hal-01207447 v1 and hal-01213447 v1 articles, the notion of relativistic mass of a particle is described here as a result of its interaction with a stratification of the four-dimensional Platonic space by a sequence of hyperplanes $H^{(n)}$ which are orthogonal to the direction of the projection $\vec{h}$, regularly spaced by a distance $\Delta w_0 > 0$.

These concepts are detailed in the HAL articles below:

hal-01165196, v1 : A platonic (euclidean-projective) model for the special theory of relativity.

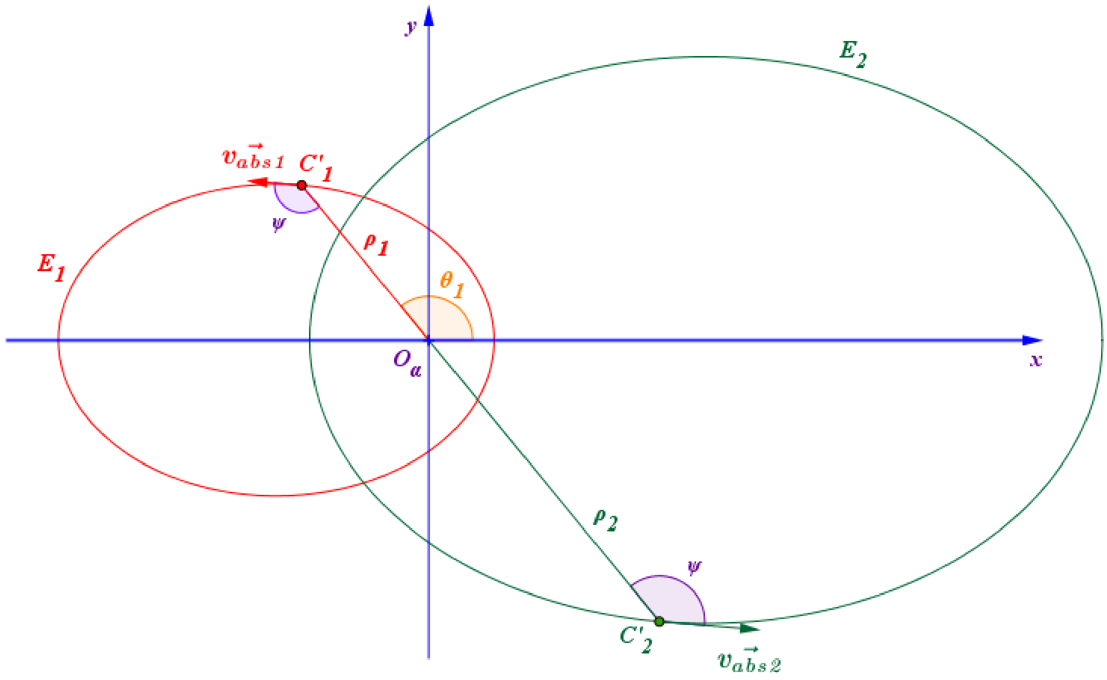
hal-01207447, v1 : Towards a modeling of De Broglie waves in a platonic quadridimensional space.

hal-01213447, v1 : An idea of the mass of a particle in a platonic quadridimensional space.

hal-01340134, v1 : One-dimensional elastic collisions in a platonic quadridimensional space.

hal-01378215, v1 : About time measurement in a platonic quadridimensional space.

2. Presentation of the object of this study



Observers of a reference frame R_α study the movement of two bodies C'_1 and C'_2 in gravitational interaction.

These two objects are in fact the projections of two bodies C_1 and C_2 moving in the Platonic space (see also the diagram in paragraph 3).

Let us denote by β_1 and β_2 the measures of the angles formed by the velocities $\vec{v_1}$ and $\vec{v_2}$ of these two bodies with the hyperplanes of respective equations $W = W(C_1)$ and $W = W(C_2)$.

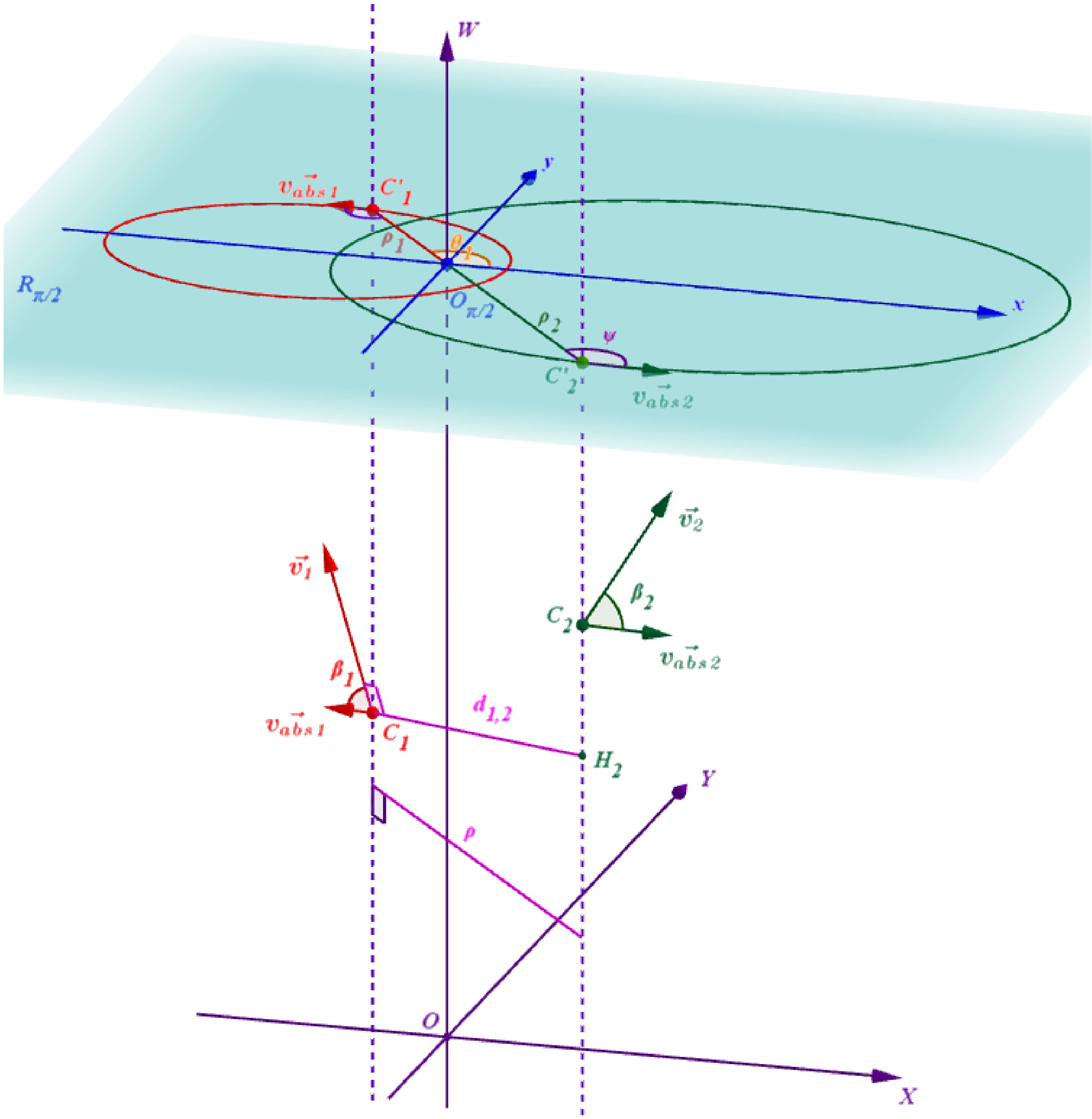
We shall introduce a postulate of ***quantified declination of these angles (generated by the De Broglie's waves linked to their masses)***, which, by generating the mutual accelerations of these two bodies in the Platonic space, will show us that the elliptic trajectories of C'_1 and C'_2 here observed in the reference frame R_α are the ***projections of the quantified quadridimensional trajectories*** thus obtained from the bodies C_1 and C_2.

To simplify the calculations and the presentation of this quantum gravitational approach, the observation frame chosen is $R_{\pi/2}$, the orbits of C'_1 and C'_2 in this reference frame are in a plane whose equation is $z = z_0$, the axes of $R_{\pi/2}$ are positioned so as to have their origin $O_{\pi/2}$ on the axis (OW) and their guiding vectors $\overrightarrow{i_{\pi/2}}$, $\overrightarrow{j_{\pi/2}}$, $\overrightarrow{k_{\pi/2}}$ coincide with the guinding vectors $\vec{i}$, $\vec{j}$, $\vec{k}$ of the axes (OX), (OY), (OZ).

The orbits chosen for C'_1 and C'_2 in this reference frame are ellipses (the absolute velocities considered being small with respect to the speed of light).

See diagram above and the following diagrams (axes $(O_{\pi/2}z)$ and (OZ) are not shown).

3. Quantified declination postulate



Referring to the diagram above, consider the De Broglie mass wave generated by the body C_i and perceived by the body C_j. Let us note $d_{i,j}$ the distance C_iH_j, where H_j designates the projection of C_j on the hyperplane H_{Pi} associated with C_i (that is to say the hyperplane orthogonal to $\vec{v_i}$ and passing through C_i).

We shall adopt the following assumption: in the Platonic space, the ***angular variation*** $\Delta\beta_{i,j}$ corresponding to the change of the trajectory undergone by the body C_j ***perceiving an occurrence of the mass wave*** emitted by the body C_i is ***independent of the reference frame*** R_α (see also paragraph 8) and has the following value:

$$\Delta\beta_{i,j} = \frac{Gh}{c^3\left(d_{i,j}\right)^2}\sin\beta_j\cos\psi\,. \qquad [1]$$

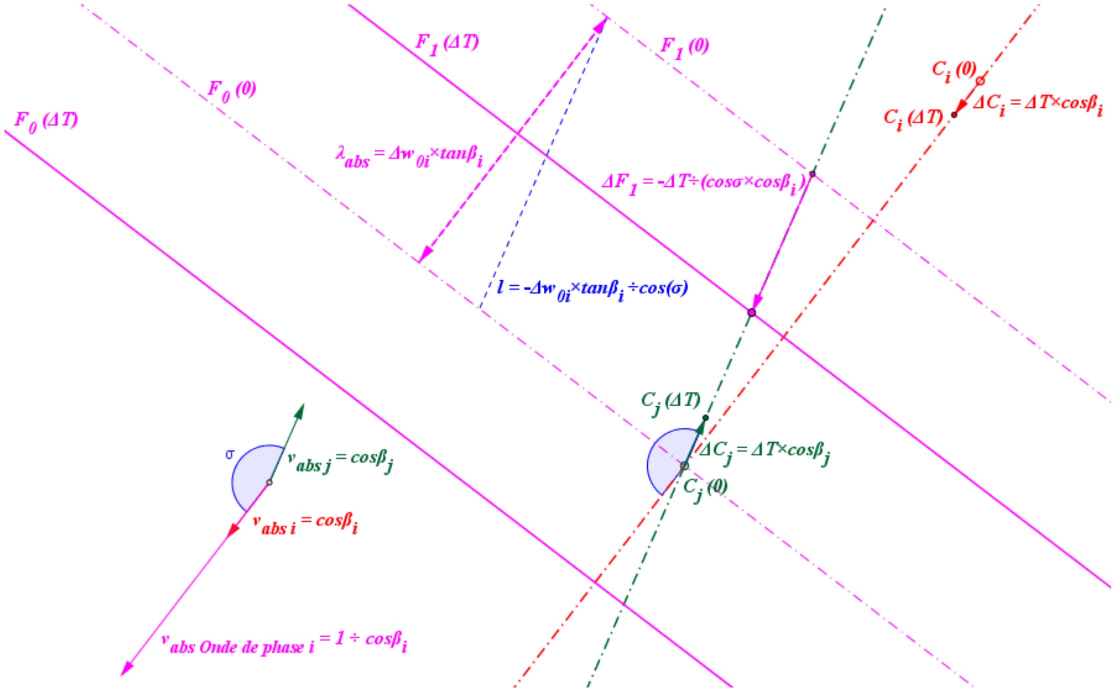
Note that if one sets $d_{i,j} = n_{i,j} \times l_P$, where $l_P = \sqrt{\dfrac{Gh}{2\pi c^3}}$ denotes the Planck length, then the relation [1] becomes: $\Delta\beta_{i,j} = \dfrac{2\pi}{\left(n_{i,j}\right)^2} \sin\beta_j \cos\psi$ (with, in fact, $n_{i,j} = E\left(\dfrac{d_{i,j}}{l_P}\right)$).

Moreover, we have $\left(d_{i,j}\right)^2 = \rho^2 \dfrac{\cos^2\beta_i \cos^2\psi + \sin^2\beta_i}{\sin^2\beta_i}$, (see details in paragraph 9)

it comes therefore:

$$\Delta\beta_{i,j} = \frac{Gh}{c^3\rho^2} \frac{\sin^2\beta_i \sin\beta_j \cos\psi}{\sin^2\beta_i + \cos^2\beta_i \cos^2\psi}. \qquad [1^{bis}]$$

4. Frequency of the mass wave perceived by the body C_j and variation of the angle β_j of its trajectory



Referring to the diagram above, consider the mass wave generated by the body C_i and perceived by the body C_j.

Let us denote by σ a measure of the angle $\left(\overrightarrow{v_{abs\ i}}, \overrightarrow{v_{abs\ j}}\right)$.

A first wavefront $F_0(0)$ is perceived by C_j when it is at $C_j(0)$.

The next wavefront F_1 is then located at $F_1(0)$, separated by the absolute wavelength $\lambda_{abs} = \Delta w_{0i} \tan\beta_i$ (in addition, see the diagram in paragraph 8).

After an absolute time ΔT, the bodies C_i and C_j are at $C_i(\Delta T)$ and $C_j(\Delta T)$ and the wavefront F_1 is at $F_1(\Delta T)$, with the relations:

$$\Delta F_1 = \frac{-\Delta T}{\cos\beta_i \cos\sigma} \quad \text{and} \quad \Delta C_j = \Delta T \cos\beta_j .$$

Thus, the body C_j perceives the wavefront F_1 when the absolute duration ΔT verifies the equation: $l = \Delta C_j + \Delta F_1$, i.e. $-\dfrac{\Delta w_{0i} \tan \beta_i}{\cos \sigma} = \dfrac{-\Delta T}{\cos \beta_i \cos \sigma} + \Delta T \cos \beta_j$,

i.e. : $\Delta T = \dfrac{\Delta w_{0i} \sin \beta_i}{1 - \cos \beta_i \cos \beta_j \cos \sigma}$ (***absolute period*** of the mass wave).

It follows that the ***absolute frequency*** $f_{i,j}$ of this mass wave is given by:

$$f_{i,j} = \frac{1}{\Delta T} = \frac{1 - \cos \beta_i \cos \beta_j \cos \sigma}{\Delta w_{0i} \sin \beta_i}. \qquad [1^{\text{ter}}]$$

With the equality $\sigma = \pi$ in the particular case studied in these first paragraphs, the absolute frequency $f_{i,j}$ of this phase wave becomes:

$$f_{i,j} = \frac{1 + \cos \beta_i \cos \beta_j}{\Delta w_{0i} \sin \beta_i}.$$

This result allows us to estimate the derivative with respect to the absolute time T of the direction angle β_j of the trajectory followed by the body C_j:

$$\frac{d\beta_j}{dT} = f_{i,j} \times \Delta\beta_{i,j} = \frac{Gh}{c^3 \rho^2 \Delta w_{0i}} \frac{1 + \cos \beta_i \cos \beta_j}{\sin^2 \beta_i + \cos^2 \beta_i \cos^2 \psi} \sin \beta_i \sin \beta_j \cos \psi. \qquad \textbf{[2]}$$

5. Absolute energy, absolute momentum and absolute angular momentum

Taking up and completing the definitions adopted in the articles cited in reference, we shall use here the following quantities:

absolute velocity of a particle (without unit): $v_{abs} = \cos\beta$, (cf. **note** in § 9)

absolute mass of a particle (in kg): $m_{abs} = \dfrac{h}{c\Delta w_0 \sin\beta}$,

absolute energy of a particle (in J): $E_{abs} = m_{abs}c^2 = \dfrac{hc}{\Delta w_0 \sin\beta}$.

For the choice of the following definitions, we shall also draw upon the well-known results of the two-body problem.
Thus, by choosing for ***absolute potential energy of the system*** the quantity:

$$E_{Pabs} = -\frac{Gm_{abs1}m_{abs2}}{\rho} = -\frac{G}{\rho}\frac{h^2}{c^2\Delta w_{01}\Delta w_{02}\sin\beta_1\sin\beta_2},$$

the ***conservation of energy*** in the two-body problem becomes:

$$\frac{hc}{\Delta w_{01}\sin\beta_1} + \frac{hc}{\Delta w_{02}\sin\beta_2} - \frac{G}{\rho}\frac{h^2}{c^2\Delta w_{01}\Delta w_{02}\sin\beta_1\sin\beta_2} = k_1 \qquad [3]\ *.$$

Similarly, by setting $\rho = C_1'C_2'$, $\rho_1 = \Omega C_1'$ and $\rho_2 = \Omega C_2'$, where Ω denotes the barycenter of the points C_1' and C_2' respectively assigned to the coefficients m_{abs1} and m_{abs2}, the ***conservation of the absolute angular momentum of the system*** leads to:

$$\rho_1 \frac{h}{c\Delta w_{01}\tan\beta_1}\sin\psi + \rho_2 \frac{h}{c\Delta w_{02}\tan\beta_2}\sin\psi = k_2 \qquad [4]\ *$$

with the relations $\rho = \rho_1 + \rho_2$; $\rho_1 = \rho\dfrac{m_{abs2}}{m_{abs1}+m_{abs2}} = \rho\dfrac{\Delta w_{01}\sin\beta_1}{\Delta w_{01}\sin\beta_1 + \Delta w_{02}\sin\beta_2}$;

$$\rho_2 = \rho\frac{m_{abs1}}{m_{abs1}+m_{abs2}} = \rho\frac{\Delta w_{02}\sin\beta_2}{\Delta w_{01}\sin\beta_1 + \Delta w_{02}\sin\beta_2} \quad \text{and} \quad \rho_2 = \frac{m_{abs1}}{m_{abs2}}\rho_1 .$$

And the consideration of ***absolute momentum*** leads to:

$$\frac{h}{c\Delta w_{01}\tan\beta_1} - \frac{h}{c\Delta w_{02}\tan\beta_2} = 0 \qquad [5].$$

(**N.B.** : to avoid factors $\tan\beta_i$, prefer **[5bis]** : $\dfrac{h\cos\beta_1}{c\Delta w_{01}\sin\beta_1} - \dfrac{h\cos\beta_2}{c\Delta w_{02}\sin\beta_2} = 0$.)

*(k_1 and k_2 are two constants).

6. Study of the movement

From the previous relations, we will be able to set up an ***iterative process*** to study the motion of the two bodies in interaction as a function of the absolute time T.

The derivation of the relation [3], taking account of the relation [2], leads to the following relation **[6]**:

$$\frac{d\rho}{dT}=\left[\frac{\cos\beta_1\sin\beta_2\left(\sin\beta_2-\dfrac{Gh}{c^3\rho\Delta w_{02}}\right)}{\sin^2\beta_2+\cos^2\beta_2\cos^2\psi}+\frac{\cos\beta_2\sin\beta_1\left(\sin\beta_1-\dfrac{Gh}{c^3\rho\Delta w_{01}}\right)}{\sin^2\beta_1+\cos^2\beta_1\cos^2\psi}\right](1+\cos\beta_1\cos\beta_2)\cos\psi.$$

With $\rho_1=\rho\dfrac{\Delta w_{01}\sin\beta_1}{\Delta w_{01}\sin\beta_1+\Delta w_{02}\sin\beta_2}$ and $\rho_2=\rho\dfrac{\Delta w_{02}\sin\beta_2}{\Delta w_{01}\sin\beta_1+\Delta w_{02}\sin\beta_2}$

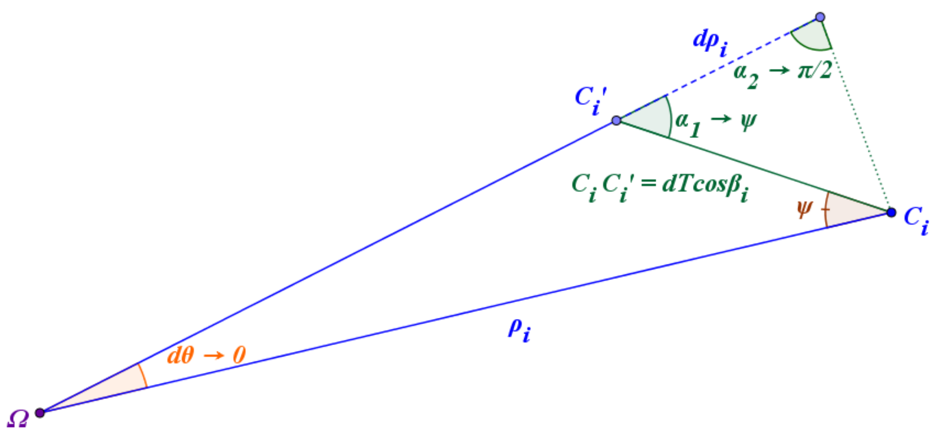
(see previous pages), we also have the relation **[7]**:

$$\frac{d\rho_i}{dT}=\frac{\rho\Delta w_{01}\Delta w_{02}\left(\cos\beta_i\sin\beta_j\dfrac{d\beta_i}{dT}-\sin\beta_i\cos\beta_j\dfrac{d\beta_j}{dT}\right)+\dfrac{d\rho}{dT}\left(\Delta w_{01}\Delta w_{02}\sin\beta_1\sin\beta_2+\Delta w_{0i}{}^2\sin^2\beta_i\right)}{\left(\Delta w_{01}\sin\beta_1+\Delta w_{02}\sin\beta_2\right)^2}.$$

Thus, the derivation of the relation [4] leads to:

$$\frac{d\psi}{dT}=\frac{\tan\psi}{\cos\beta_1+\cos\beta_2}\left[\frac{1}{\sin\beta_1}\frac{d\beta_1}{dT}-\frac{\cos\beta_1}{\rho_1}\frac{d\rho_1}{dT}+\frac{1}{\sin\beta_2}\frac{d\beta_2}{dT}-\frac{\cos\beta_2}{\rho_2}\frac{d\rho_2}{dT}\right] \qquad [8].$$

The presence of the quantity $\tan\psi$, which is problematic for the current measure $\psi=\pm\dfrac{\pi}{2}$, leads to seeking a simplified expression of the quantities $\dfrac{d\rho_i}{\cos\psi}$ that it generates implicitly. The diagram below makes it easy to obtain this simplification:



We know that $\cos\beta_i$ corresponds to the absolute velocity of the body C_i.
After an absolute time dT, the latter has therefore traveled the distance $C_iC_i' = dT\cos\beta_i$.
With $d\theta \to 0$, the distance ΩC_i has thus varied by a quantity $d\rho_i \to dT\cos\beta_i\cos\psi$.
Hence the relation $\dfrac{d\rho_i}{\cos\psi} \to dT\cos\beta_i$.
And therefore the relation [8] leads to the relation **[9]** below, after the complementary simplification of $d\beta_1$ and $d\beta_2$ by $\cos\psi$:

$$\frac{d\psi}{dT} = \frac{\sin\psi}{\cos\beta_1+\cos\beta_2}\left[\frac{Gh\left(1+\cos\beta_1\cos\beta_2\right)}{c^3\rho^2}\left(\sum_{i=1}^{i=2}\frac{\sin\beta_i}{\Delta w_{0i}\left(\sin^2\beta_i+\cos^2\beta_i\cos^2\psi\right)}\right)-\sum_{i=1}^{i=2}\frac{\cos^2\beta_i}{\rho_i}\right].$$

Finally, the variation of the angle θ is easy to estimate, starting from $\rho_i d\theta = dT\cos\beta_i$ (since $\cos\beta_i$ corresponds to the absolute velocity of the body C_i).
We have thus, with the relation [5]:

$$\frac{d\theta}{dT} = \frac{\cos\beta_i}{\rho_i} = \frac{\Delta w_{01}\sin\beta_1+\Delta w_{02}\sin\beta_2}{\rho\Delta w_{01}\tan\beta_1} = \frac{\Delta w_{01}\sin\beta_1+\Delta w_{02}\sin\beta_2}{\rho\Delta w_{02}\tan\beta_2}. \qquad \textbf{[10]}$$

(**N.B.** : to avoid factors $\tan\beta_i$, prefer **[10bis]** : $\dfrac{d\theta}{dT} = \dfrac{\cos\beta_i}{\rho}\left(1+\dfrac{\Delta w_{0j}\sin\beta_j}{\Delta w_{0i}\sin\beta_i}\right)$.)

Insofar as the mass differences chosen are large and the speeds involved are small compared with the speed of light, ***the position of the barycenter Ω of the system does not vary very much*** (its fluctuations may initially be neglected relative to the distance considered in the examples in paragraph 9).
Therefore, the results **[2]**, **[6]**, **[7]**, **[9]** and **[10]** can already allow us to set up an ***iterative process*** to study the motion of two bodies in interaction as a function of the absolute time *T*.
We will choose Ω as the origin of the reference frame.

- The masses of the bodies C_i and C_j are in a first approximation equal to the sums of the masses of the particles which constitute them.
  The stratification distances Δw_{0i} and Δw_{0j} taken into account in the calculations on the basis of the relation $\Delta w_0 = \dfrac{h}{cm_{abs}\sin\beta}$ are fictitious distances which make it possible to generate the occurrences of the mass waves corresponding to the accumulations of the occurrences generated by all of these particles.
  The masses of these bodies are to be indicated in ***cells C4*** and ***E4***.

- The initial value of ρ is the distance at the periapsis (to be indicated in ***cell G5***).

- The eccentricity e of the elliptic trajectories is to be indicated in the ***cell G4***.

- The initial values of the angles β_1 and β_2 are estimated from the velocity v_{2p} at the periapsis of the body C'_2 and from the relations:

$$v_{abs2} = \cos\beta_2 = \frac{v_{2p}}{c} \text{ ; then, according to [5] : } v_{abs1} = \cos\beta_1 = \frac{\Delta w_{01}}{\Delta w_{02}} \cos\beta_2 \text{ .}$$

- The initial value of θ_1 is 0, that of θ_2 is π and that of ψ is $\frac{\pi}{2}$.

- The variations ΔW_1 and ΔW_2 of the coordinates W_1 and W_2 of the bodies C_1 and C_2 are implemented by the increments $dW_1 = dT \sin\beta_1$ and $dW_2 = dT \sin\beta_2$.
This, beyond the situation of these bodies in the Platonic space, makes it possible to evaluate the proper durations Δt_1 and Δt_2 elapsed for C_1 and C_2, from the relation $\Delta t_i = \frac{\Delta W_i}{c}$.

- In order for these tables to give usable results, ***the absolute time step dT of the cell J5 must be adjusted*** so that the measurement of the angle θ_{1max} appearing in the ***cell I16*** is close to 2π .
This setting requires some successive tests (... and a little patience ...).

The iterative process has been applied to the following situations (see paragraph 9):

- ✓ C'_1 : Sun and C'_2 : Earth,
- ✓ C'_1 : Sun and C'_2 : Mercury,
- ✓ C'_1 : Sun and C'_2 : Neptune,
- ✓ C'_1 : Earth and C'_2 : Moon,
- ✓ the Pulsar PSR B1913+16.

These situations were first calculated using the classical laws of gravitation (the formulas used are given in Appendix 9); which gives in the Excel tables of paragraph 9 the quantities referred to as "theoretical".
They were then simulated as indicated in this paragraph.
Finally, the results of these simulations and the "theoretical" data were compared.

At the margins of errors due to the numerous iterations (here 20 000) and the software used (which only retains 15 significant digits for each calculation), this process allows us to find the main characteristics of each of the different orbits established according to the laws of classical mechanics.

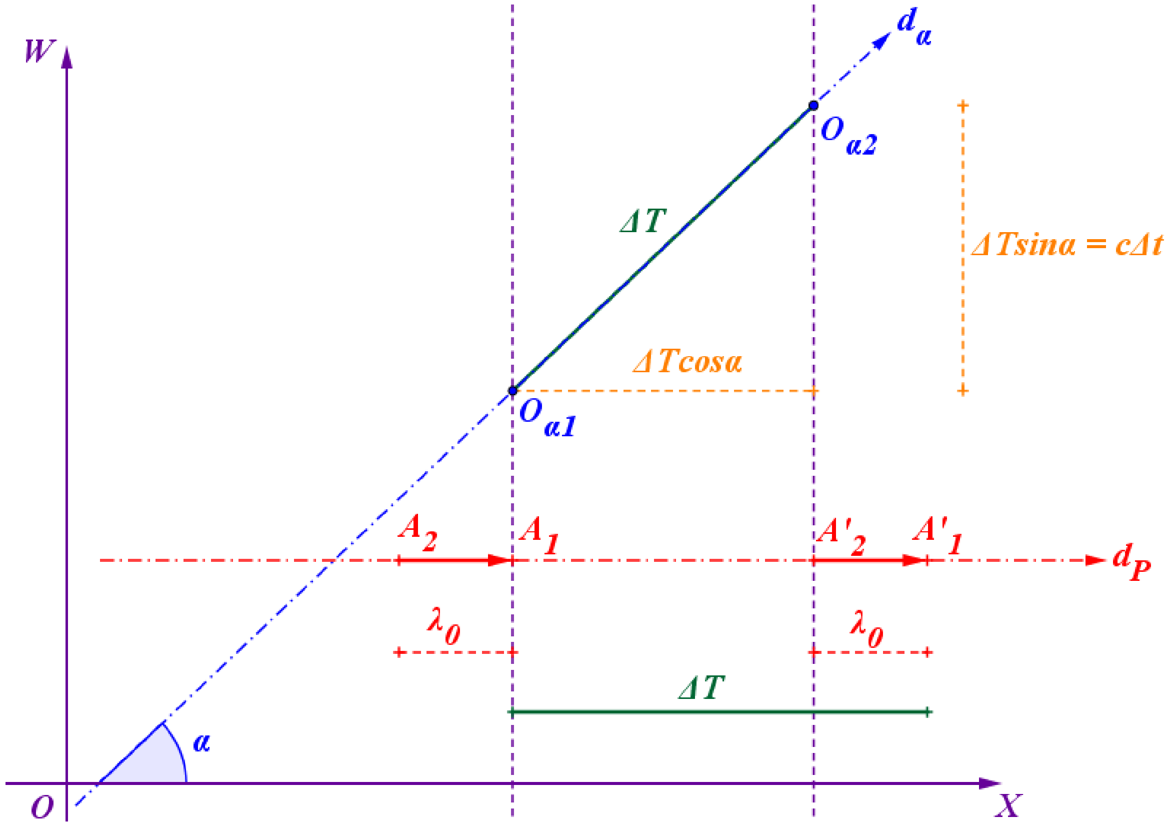
By way of illustration, the maximum errors recorded for the Sun-Earth system are of the order of 0.008% for the speeds, 0.0017% for the distances and 0.00298% for the period of revolution.

When the exentricity *e* increases, the maximum errors on the velocities and distances remain low, but the error on the revolution period increases (it becomes close to 0.8% for *e* close to 0.2).

The "Excel" file corresponding to this iterative process is attached.

7. Case of an electromagnetic wave in a gravitational field, half-radius of Schwarzschild

7.1 Absolute wave length of an electromagnetic wave and relativistic Doppler effect



In the plane (XOW), let us consider an observer O_α of any arbitrary reference frame R_α moving along the direction d_α and an electromagnetic signal moving along the direction d_P, orthogonal to $\vec{h}$.

A_1 and A_2 represent two consecutive peaks of this signal, separated by a distance λ_0.

We shall call this distance λ_0 the ***absolute wavelength*** of this signal.

For the observer O_α, the ***perceived frequency*** f_α of this signal is given by $f_\alpha = \dfrac{c}{\lambda_0}\dfrac{1-\cos\alpha}{\sin\alpha}$.

Indeed, if O_α perceives the peak A_1 while it is at $O_{\alpha 1}$ and the peak A_2 (then located at A'_2) when it is at $O_{\alpha 2}$, the absolute duration ΔT separating these two events satisfies the equation $\Delta T = \Delta T\cos\alpha + \lambda_0$, which has for solution $\Delta T = \dfrac{\lambda_0}{1-\cos\alpha}$.

The relative proper time Δt measured by O_α satisfying $\Delta t = \dfrac{\Delta T \sin\alpha}{c}$, the ***period*** of this signal measured in R_α is therefore $\Delta t = \dfrac{\lambda_0 \sin\alpha}{c(1-\cos\alpha)}$, whence the conclusion, with $f_\alpha = \dfrac{1}{\Delta t}$.

The well-foundedness of this concept of absolute wavelength of an electromagnetic wave appears immediately, for example, through the ***relativistic Doppler effect***.

Indeed, if two referentials R_α and R_β observe an electromagnetic wave of absolute wavelength λ_0, the corresponding proper frequencies f_α and f_β measured by these referentials are given by $f_\alpha = \dfrac{c}{\lambda_0}\dfrac{1-\cos\alpha}{\sin\alpha}$ and $f_\beta = \dfrac{c}{\lambda_0}\dfrac{1-\cos\beta}{\sin\beta}$.

We thus obtain: $\dfrac{f_\alpha}{f_\beta} = \dfrac{1-\cos\alpha}{\sin\alpha}\dfrac{\sin\beta}{1-\cos\beta}$.

This leads, with $\sin\alpha > 0$, $\sin\beta > 0$, $\sin\alpha = \sqrt{1-\cos^2\alpha}$ and $\sin\beta = \sqrt{1-\cos^2\beta}$, to :

$$\frac{f_\alpha}{f_\beta} = \frac{\sqrt{(1-\cos\alpha)(1+\cos\beta)}}{\sqrt{(1+\cos\alpha)(1-\cos\beta)}}.$$

Now, the relative velocity v of R_β measured in R_α is given by : $\dfrac{v}{c} = \dfrac{\cos\beta-\cos\alpha}{1-\cos\alpha\cos\beta}$.

Consequently, we have: $\dfrac{1+\dfrac{v}{c}}{1-\dfrac{v}{c}} = \dfrac{(1-\cos\alpha)(1+\cos\beta)}{(1+\cos\alpha)(1-\cos\beta)}$, hence the relation ("***Doppler factor***") :

$$\boxed{\frac{f_S}{f_O} = \sqrt{\frac{1+\dfrac{v}{c}}{1-\dfrac{v}{c}}},}$$

where the observer moves in the direction d_α, the source in the direction d_β, f_S denotes the frequency of the wave measured by the source and f_O the frequency of the wave measured by the observer.

7.2 Absolute energy of a photon and half-radius of Schwarzschild

The concept of absolute wavelength of an electromagnetic wave leads us to the relation $E = hf_\alpha = \frac{hc}{\lambda_0}\frac{1-\cos\alpha}{\sin\alpha}$ for the measurement of the ***energy*** of a photon in a reference frame R_α.

Consequently, the ***absolute concepts*** being obtained for $\alpha = \frac{\pi}{2}$, we shall define the ***absolute energy of a photon*** by: $E_{abs} = \frac{hc}{\lambda_0}$ (λ_0 representing its ***absolute wavelength***).

From this notion, in the particular case of the interaction of a massive body C_1 and a photon, the relation [3] can be written, considering as the "relativistic mass" of the photon the quantity $m_{abs} = \frac{h}{c\lambda_0}$:

$$\frac{hc}{\lambda_0} + \frac{hc}{\Delta w_{01}\sin\beta_1} - \frac{G}{\rho}\frac{h^2}{c^2\lambda_0\Delta w_{01}\sin\beta_1} = k_1 \qquad [3^{bis}] ;$$

whose derivation with respect to the absolute time T leads to (considering that, in a gravitational field, the absolute wavelength λ_0 varies as a function of T):

$$\frac{Gh^2}{\rho^2c^2\lambda_0\Delta w_{01}\sin\beta_1}\frac{d\rho}{dT} + \left(\frac{Gh^2\cos\beta_1}{\rho c^2\lambda_0\Delta w_{01}\sin^2\beta_1} - \frac{hc\cos\beta_1}{\Delta w_{01}\sin^2\beta_1}\right)\frac{d\beta_1}{dT}$$

$$+\left(\frac{Gh^2}{\rho c^2{\lambda_0}^2\Delta w_{01}\sin\beta_1} - \frac{hc}{{\lambda_0}^2}\right)\frac{d\lambda_0}{dT} = 0 .$$

Now, the quantity $\frac{d\beta_1}{dT}$ is equal to ***zero***. Indeed, the absence of mass of the photon (the concept of "relativistic mass" mentioned above is only an artifact used to introduce the formula chosen for the potential energy of the system) is equivalent to the fact that its displacement in the Platonic space does not generate a De Broglie mass wave and therefore does not modify the direction angle β_1 of the massive body C_1.

Therefore, we have:

$$\frac{Gh^2}{\rho^2c^2\lambda_0\Delta w_{01}\sin\beta_1}\frac{d\rho}{dT} + \left(\frac{Gh^2}{\rho c^2{\lambda_0}^2\Delta w_{01}\sin\beta_1} - \frac{hc}{{\lambda_0}^2}\right)\frac{d\lambda_0}{dT} = 0 ,$$

from where it comes:

$$\frac{d\rho}{dT} = \frac{\rho}{Gh\lambda_0}\left(\rho c^3\Delta w_{01}\sin\beta_1 - Gh\right)\frac{d\lambda_0}{dT} .$$

We note that this quantity is equal to zero for: $\rho c^3\Delta w_{01}\sin\beta_1 - Gh = 0$,

i.e. :

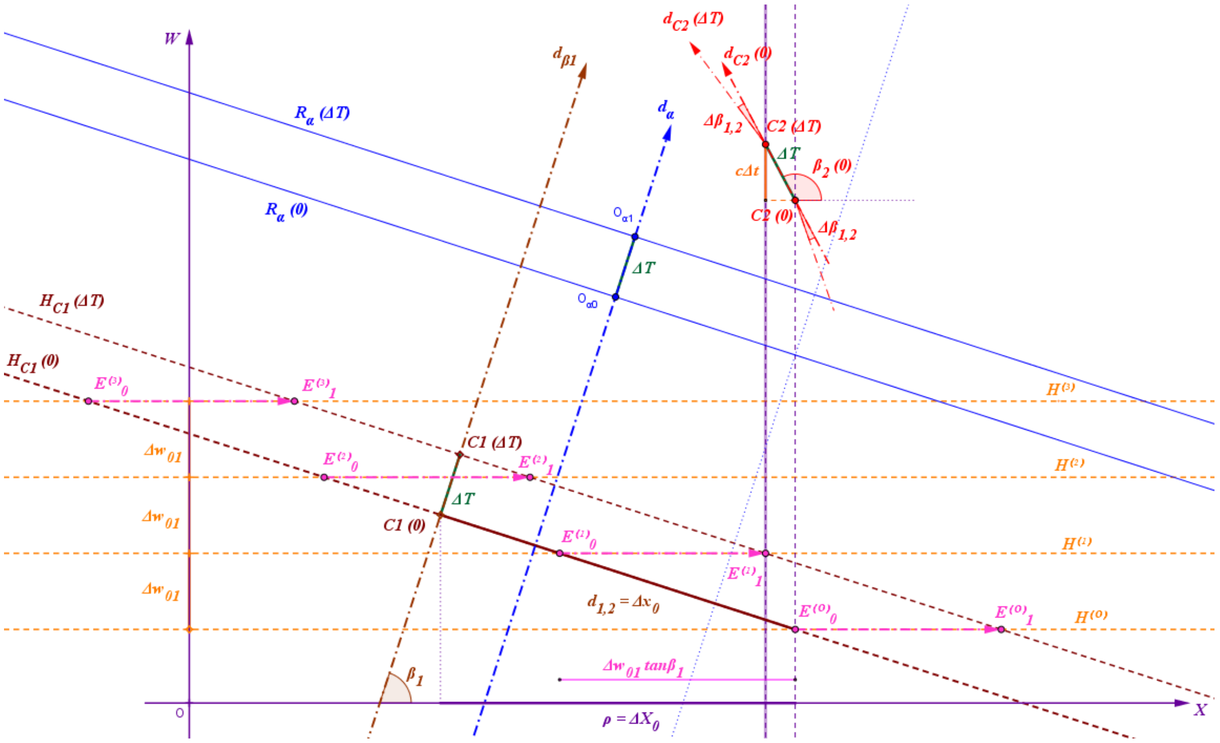
$$\rho = \frac{Gh}{c^3\Delta w_{01}\sin\beta_1} = \frac{G}{c^2}m_{abs1} .$$

Here we recognize the value of the ***Schwarzschild half-radius*** for which the trajectory of the photon is circular, in other words for which we have $\frac{d\rho}{dT}=0$.

N.B : for a massive body C_2 whose direction angle β_2 of the trajectory tends to 0, the relation [6] leads to: $$\frac{d\rho}{dT}\to\left[\frac{\cos\beta_2\sin\beta_1\left(\sin\beta_1-\frac{Gh}{c^3\rho\Delta w_{01}}\right)}{\sin^2\beta_1+\cos^2\beta_1\cos^2\psi}\right]\left(1+\cos\beta_1\cos\beta_2\right)\cos\psi\,,$$

which is also equal to zero for: $\sin\beta_1-\frac{Gh}{c^3\rho\Delta w_{01}}=0$, i.e. $\rho=\frac{Gh}{c^3\Delta w_{01}\sin\beta_1}=\frac{G}{c^2}m_{abs1}$.

8. Acceleration of a body at rest in a gravity field



The diagram above reproduces the notations and concepts developed in the document hal-1207447, v1 : *Towards a modeling of De Broglie waves in a platonic quadridimensional space.*

In this diagram is a massive punctual body C_2, observed at $C_2(0)$ and $C_2(\Delta T)$, subjected to the gravitational field generated by the massive body C_1.

The observation frame R_α is a reference frame linked to C_1 (i.e. $\alpha = \beta_1$).

We assume here that the mass of C_2 is very small in relation to that of C_1 and consequently that the change in the trajectory of C_1 generated by the gravitational field of C_2 is negligible for the absolute time intervals considered.

In order to preserve the notations used and the results obtained in the article hal-01207447, v1 and in the book " *De l'Allégorie de la Caverne à la Relativité Restreinte (From the Allegory of the Cave to the Special Relativity)*", it should be noted that the angles β_i considered in this paragraph are measured differently from those considered in the preceding paragraphs.

So, we have here $\beta_i = \left(\vec{i}, \overrightarrow{d_{\beta_i}}\right)$ instead of the measure β_i used in the preceding paragraphs (measurement of the angle formed by the hyperplane (H_0) generated by $\left(\vec{i}, \vec{j}, \vec{k}\right)$ and the velocity vector $\overrightarrow{v_i}$ of the body C_i considered). The formulas are therefore slightly different, but consistent with the results obtained previously.

Thus, the frequency $f_{1,2}$ of the mass wave perceived by C_2 is written here:

$$f_{1,2} = \frac{c}{\Delta w_{01}} \cdot \frac{1 - \cos\beta_1 \cos\beta_2}{\sin\beta_1 \sin\beta_2} .$$

For a duration dt measured in a rest reference frame of C_2, the number $n = f_{1,2}dt$ of events $E_r^{(n)}$ perceived by C_2 becomes, with $dT = \dfrac{cdt}{\sin\beta_2}$ (elementary displacement of C_2 during dt):

$$n = \frac{1}{\Delta w_{01}} \cdot \frac{1-\cos\beta_1\cos\beta_2}{\sin\beta_1} dT \ .$$

During an infinitesimal displacement dT (considering that the elementary variation $\Delta\beta_{1,2}$ is negligible before β_2), the modification $d\beta_{1,2}$ of the angle of the trajectory of C_2 can be estimated by $d\beta_{1,2} = n\times\Delta\beta_{1,2}$.

From the relations [1] and [1bis]:

$$\Delta\beta_{1,2} = \frac{Gh}{c^3\left(d_{1,2}\right)^2}\sin\beta_2\cos\psi = \frac{Gh}{c^3\rho^2}\frac{\sin^2\beta_1\sin\beta_2\cos\psi}{\sin^2\beta_1+\cos^2\beta_1\cos^2\psi}$$ and the fact that $\psi = 0$,

we obtain the following first result:

$$\Delta\beta_{1,2} = \frac{Gh}{c^3\rho^2}\sin^2\beta_1\sin\beta_2 .$$

From where we have : $d\beta_2 = n\times\Delta\beta_{1,2} = \dfrac{1}{\Delta w_{01}} \cdot \dfrac{1-\cos\beta_1\cos\beta_2}{\sin\beta_1} \cdot \dfrac{Gh}{c^3\left(\Delta x_0\right)^2}\sin\beta_2 dT$,

or, with $\rho = \Delta x_0 \sin\beta_1$: $\dfrac{d\beta_2}{dT} = \dfrac{Gh}{c^3\rho^2\Delta w_{01}}\left(1-\cos\beta_1\cos\beta_2\right)\sin\beta_1\sin\beta_2$.

The acceleration a_α of C_2 measured into R_α being given by (cf. the book « *De l'allégorie de la caverne à la relativité restreinte* »): $a_\alpha = -c^2\dfrac{d\beta_2}{dT} \cdot \dfrac{\sin^3\alpha\sin\beta_2}{\left(1-\cos\alpha\cos\beta_2\right)^3}$;

one arrives therefore to (with here $\alpha = \beta_1$): $a_\alpha = -\dfrac{Gh}{c\Delta w_{01}\rho^2} \cdot \dfrac{\sin^4\alpha\sin^2\beta_2}{\left(1-\cos\alpha\cos\beta_2\right)^2}$.

i.e. , with $\rho = \Delta x_0\sin\alpha$ and $\dfrac{\sin^2\alpha\sin^2\beta_2}{\left(1-\cos\alpha\cos\beta_2\right)^2} = 1-\dfrac{v^2}{c^2}$:

$$a_\alpha = -\frac{Gh}{c\Delta w_{01}\left(\Delta x_0\right)^2} \cdot \frac{\sin^2\alpha\sin^2\beta_2}{\left(1-\cos\alpha\cos\beta_2\right)^2} = -\frac{Gh}{c\Delta w_{01}\left(\Delta x_0\right)^2} \cdot \left(1-\frac{v^2}{c^2}\right) ;$$

i.e. $$a_\alpha = -\frac{Gm_{01}}{\left(\Delta x_0\right)^2}\left(1-\frac{v^2}{c^2}\right) ,$$

where $m_{01} = \dfrac{h}{c\Delta w_{01}}$ denotes the mass of the body C_1 into its rest reference frame R_α (cf. the article hal-01213062, v1).

Thus, if we consider a body C_2 initially at rest into R_α, we then have initially $\beta_2 = \alpha$ (i.e. $v = 0$), and the previously calculated acceleration leads to the classical relation:

$$a_\alpha = -\frac{Gh}{c\Delta w_{01}\rho^2}.\sin^2\alpha = -\frac{Gh}{c\Delta w_{01}\left(\Delta x_0\right)^2}\ , \quad \text{i.e.} \quad a_\alpha = -\frac{Gm_{01}}{\left(\Delta x_0\right)^2}.$$

N.B. : from a simple situation, this paragraph has, among other things, supported the fact that the quantum of declination $\Delta\beta_{i,j}$ is indeed ***independent of the observation frame*** R_α.

9. Related documents

9.1. Calculation of the distance d1,2

The quantities used in this paragraph refer to the diagrams in paragraphs 2 and 3.
Since the movements considered here are assumed to take place with a third constant coordinate ($Z = Z_0$) in the space $\left(O,\vec{i},\vec{j},\vec{k},\vec{h}\right)$, the velocity vector of the body C_1 is given, with β_1 not multiple of π, by:

$$\vec{v_1} = \begin{pmatrix} -\cos\beta_1\cos\left(\psi-\theta\right) \\ \cos\beta_1\sin\left(\psi-\theta\right) \\ 0 \\ \sin\beta_1 \end{pmatrix} \text{(cf. } \textbf{note} \text{ below).}$$

Let H_{C1} be the hyperplane associated with $C_1\left(X_1,Y_1,Z_1,W_1\right)$ (hyperplane orthogonal to $\vec{v_1}$ passing through C_1) and let H_2 be the projection of C_2 onto H_{C1}.

We thus have the equivalence: $M\left(X,Y,Z,W\right) \in H_{C1}$ if and only if

$$-\cos\beta_1\cos\left(\psi-\theta\right)\left(X-X_1\right)+\cos\beta_1\sin\left(\psi-\theta\right)\left(Y-Y_1\right)+\sin\beta_1\left(W-W_1\right)=0.$$

As $H_2\left(X_2,Y_2,Z_2,W_2\right) \in H_{C1}$, we have :

$$-\cos\beta_1\cos\left(\psi-\theta\right)\left(X_2-X_1\right)+\cos\beta_1\sin\left(\psi-\theta\right)\left(Y_2-Y_1\right)+\sin\beta_1\left(W_2-W_1\right)=0,$$

from which emerges: $W_2 - W_1 = -\dfrac{\cos\beta_1}{\sin\beta_1}\left[\cos\left(\psi-\theta\right)\left(X_2-X_1\right)-\sin\left(\psi-\theta\right)\left(Y_2-Y_1\right)\right].$

On the other hand, by definition, we have:

$$(d_{1,2})^2 = (X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (W_2 - W_1)^2 = \rho^2 + (W_2 - W_1)^2,$$

with : $X_2 - X_1 = \rho\cos\theta$ and $Y_2 - Y_1 = \rho\sin\theta$.

We thus obtain: $$(W_2 - W_1)^2 = \rho^2\left(\frac{\cos\beta_1}{\sin\beta_1}\right)^2\left[\cos(\psi-\theta)\cos\theta - \sin(\psi-\theta)\sin\theta\right]^2,$$

i.e. $$(W_2 - W_1)^2 = \rho^2\left(\frac{\cos\beta_1}{\sin\beta_1}\right)^2\cos^2\psi.$$

Finally, we arrive at: $$(d_{1,2})^2 = \rho^2 + (W_2 - W_1)^2 = \rho^2\left(1 + \frac{\cos^2\beta_1\cos^2\psi}{\sin^2\beta_1}\right),$$

i.e. $$(d_{1,2})^2 = \rho^2\,\frac{\cos^2\beta_1\cos^2\psi + \sin^2\beta_1}{\sin^2\beta_1}.$$

Note on the concept of ***velocity*** of a punctual object M in the Platonic space :

given the definition of the absolute time T (in m), the ***norm of the speed vector*** of all the mobiles ***is equal to 1*** and the ***velocity vector*** $\vec{v} = \frac{\overrightarrow{dM}}{dT}$ of any mobile M is given by:

$$\vec{v} = \begin{pmatrix} \cos\beta\cos\gamma\cos\varphi \\ \cos\beta\cos\gamma\sin\varphi \\ \cos\beta\sin\gamma \\ \sin\beta \end{pmatrix},$$

with any φ,

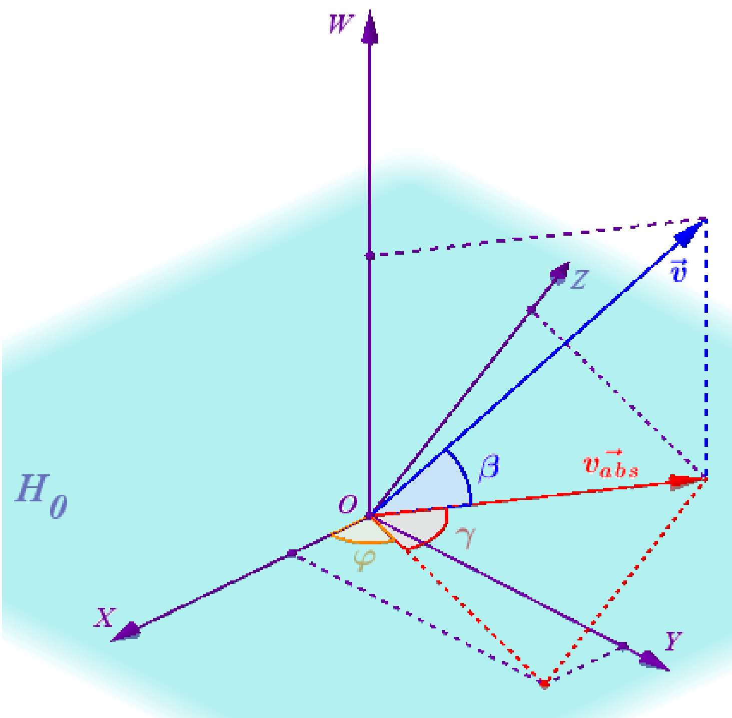
$\gamma \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ and $\beta \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$.

Its ***absolute speed*** is given by:

$$\overrightarrow{v_{abs}} = \begin{pmatrix} \cos\beta\cos\gamma\cos\varphi \\ \cos\beta\cos\gamma\sin\varphi \\ \cos\beta\sin\gamma \\ 0 \end{pmatrix},$$

whose ***norm is equal to*** $\cos\beta$.

For the vector $\overrightarrow{v_1}$ in this paragraph, we have $\gamma = 0$ and $\varphi = \theta + \pi - \psi$.



9.2. Formulas used to obtain theoretical reference data in simulations

The results of the simulations are compared to the theoretical elliptic trajectories of two bodies C'_1 and C'_2 in gravitational interaction. Given the velocities considered, which are low compared to the speed of light, the calculations are made from the laws of classical mechanics.

The data used are: the distance at the periapsis d_p, the common eccentricity e, the masses at rest m_1 and m_2.

From these elements, we obtain:

- the speeds of the two bodies at the periapsis:

$$v_{1p} = m_2\sqrt{\frac{G(1+e)}{d_p(m_1+m_2)}} \quad , \quad v_{2p} = m_1\sqrt{\frac{G(1+e)}{d_p(m_1+m_2)}} \quad ;$$

- the speeds of the two bodies at the apoapsis:

$$v_{1a} = m_2(1-e)\sqrt{\frac{G}{d_p(m_1+m_2)(1+e)}} \, , \quad v_{2a} = m_1(1-e)\sqrt{\frac{G}{d_p(m_1+m_2)(1+e)}} \quad ;$$

- the distance at the apoapsis : $d_a = d_p \dfrac{1+e}{1-e}$;

- the period of revolution (in seconds): $p_r = 2\pi\sqrt{\dfrac{d_p^{\,3}}{G(m_1+m_2)(1-e)^3}}$;

- the half-axes of the ellipses traveled by C'_1 and C'_2:

$$a_1 = \frac{m_2}{m_1+m_2}\frac{d_p}{1-e} \quad \text{and} \quad b_1 = \frac{m_2}{m_1+m_2} d_p\sqrt{\frac{1+e}{1-e}} \, ,$$

$$a_2 = \frac{m_1}{m_1+m_2}\frac{d_p}{1-e} \quad \text{and} \quad b_2 = \frac{m_1}{m_1+m_2} d_p\sqrt{\frac{1+e}{1-e}} \, .$$

The results of the simulations and the comparisons between the reference results and the results of the simulations are carried out in the following paragraph.

9.3. Results of simulations and comparisons with reference results

The following screenshots were obtained from the attached Excel file.
Only the data, final results, checks and calculation steps 0 and 1 are displayed here.
The numbers in brackets (**[10]**, **[2]**, ...) on line 17 refer to the formulas used. The related comments are in paragraph 6.

9.3.1. Sun-Earth Sytem

	A	B	C	D	E	F	G	H	I	J	K
1											
2		Data and Theoretical calculations									
3		Body C'1		Body C'2		System		Constants		Absolute Time dT	
4		Mass (kg) ("_m1") :	1,9884E+30	Mass (kg) ("_m2") :	5,9722E+24	Eccentricity ("e") :	0,01671022	G ("G")	c ("_c")	(m) ("dTa")	
5		Δw01 (m) ("_w1") :	1,11156E-72	Δw02 (m) ("_w2") :	3,70085E-67	Periapsis (m) ("dp") :	1,47098E+11	6,67428E-11	299792458	4,73033E+11	
6		v1 periapsis (m/s) ("_v1p") :	0,090966006	v2 periapsis (m/s) ("_v2p") :	30286,46162	Apoapsis (m) ("da") :	1,52098E+11	h ("h")	6,62607E-34		
7		v1 apoapsis (m/s) ("_v1a"):	0,087975848	v2 apoapsis (m/s) ("_v2a"):	29290,91062	Period rev. (days) ("pr"):	365,2578657	Observer			
8		_v1p/c :	3,0343E-10	_v2p/c :	0,000101025			α ("alp")	dt (days) ("dt")		
9								1,570796327	0,018262349		
10											
11											
12											
13											
14											
15									θ1 max (6,2831853)		
16									6,2831853		
17		Number of steps	Number of days	β1	β2	ρ	ρ1	ρ2	θ1	θ2	ψ
18		0	0	1,570796326	1,570695302	1,47098E+11	441810,7418	1,47098E+11	0	3,141592654	1,570796327
19		1	0,018262349	1,570796326	1,570695302	1,47098E+11	441810,7418	1,47098E+11	0,000324873	3,141917526	1,570790987

	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1														
2	Simulation results checks													
3	System									Bodies C'1 & C'2				
4	Simul. Eccent.	Theoretical distances (m)		Simulated distances (m)		Errors (Δ%)	Period of revolution (days)			Theoretical Speeds (m/s)		Simulated Speeds (m/s)		Errors (Δ%)
5	0,016722729	Periapsis (m)	1,47098E+11	ρ min =	1,47096E+11	-0,001705197	Theoretical	Simulated	Error (Δ%)	v1 periapsis(m/s) =	0,090966006	v1 max =	0,090967293	0,001414983
6	Error (Δ%)	Apoapsis (m)	1,52098E+11	ρ max =	1,52099E+11	0,000797222	365,2578656526	365,2469781340	-0,002980776	v1 apoapsis(m/s) =	0,087975848	v1 min =	0,087968503	-0,008349226
7	0,074856312									v2 periapsis(m/s) =	30286,46162	v2 max =	30286,96554	0,001663833
8										v2 apoapsis(m/s) =	29290,91062	v2 min =	29290,67105	-0,000817904
9				Proper time Δt1 elapsed for C1 (days):			365,246978133959			Theoretical half-axes (m)		Simulated half-axes (m)		Errors (Δ%)
10				Proper time Δt2 elapsed for C2 (days):			365,246976331368			a1 =	449318,9605	a1 =	449317,0146	-0,000433079
11										b1 =	449256,2241	b1 =	449254,1845	-0,000453995
12										a2 =	1,49597E+11	a2 =	1,49597E+11	-0,000433079
13										b2 =	1,49577E+11	b2 =	1,49576E+11	-0,000453995
14														
15														
16														
17	dθ1 [10]	dθ2	dβ1 [2]	dβ2 [2]	dρ [6]	dρ1 [7]	dρ2 [7]	dψ [9]	v1 (m/s)	v2 (m/s)	X1	Y1	ΔW1	X2
18	0,000324873	0,000324873	5,93927E-30	1,97744E-24	2,92738E-09	8,79242E-15	2,92737E-09	-5,33947E-06	0,090966028	30286,46162	441810,7418	0	0	-1,47098E+11
19	0,000324873	0,000324873	5,17693E-19	1,72362E-13	255,1634281	0,000766386	255,1626618	-5,33947E-06	0,090966028	30286,46162	441810,7185	143,5322866	4,73033E+11	-1,47098E+11

9.3.2. Sun-Mercury System

	A	B	C	D	E	F	G	H	I	J	K
1											
2		Data and Theoretical calculations									
3		Body C'1		Body C'2		System		Constants		Absolute Time dT	
4		Mass (kg) ("_m1") :	1,9884E+30	Mass (kg) ("_m2") :	3,3011E+23	Eccentricity ("e") :	0,20563069	G ("G")	c ("_c")	(m) ("dTa")	
5		Δw01 (m) ("_w1") :	1,11156E-72	Δw02 (m) ("_w2") :	6,6954E-66	Periapsis (m) ("dp") :	46001272000	6,67428E-11	299792458	1,13014E+11	
6		v1 periapsis (m/s) ("_v1p") :	0,009791094	v2 periapsis (m/s) ("_v2p") :	58976,13115	Apoapsis (m) ("da") :	69817079542	h ("h")	6,62607E-34		
7		v1 apoapsis (m/s) ("_v1a"):	0,006451183	v2 apoapsis (m/s) ("_v2a"):	38858,3577	Period rev. (days) ("pr"):	87,96969311	Observer			
8		_v1p/c :	3,26596E-11	_v2p/c :	0,000196723			α ("alp")	dt (days) ("dt")		
9								1,570796327	0,004363143		
10											
11											
12											
13											
14											
15									θ1 max (6,2831853)		
16									6,2831853		
17		Number of steps	Number of days	β1	β2	ρ	ρ1	ρ2	θ1	θ2	ψ
18		0	0	1,570796327	1,570599604	46001272000	7637,033484	46001264363	0	3,141592654	1,570796327
19		1	0,004363143	1,570796327	1,570599604	46001272000	7637,033484	46001264363	0,000483305	3,142075959	1,570713895

	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1														
2	Simulation results checks													
3	System									Bodies C'1 & C'2				
4	Simul. Eccent.	Theoretical distances (m)		Simulated distances (m)		Errors (Δ%)	Period of revolution (days)			Theoretical Speeds (m/s)		Simulated Speeds (m/s)		Errors (Δ%)
5	0,205657949	Periapsis (m)	46001272000	ρ min =	46001272000	0	Theoretical	Simulated	Error (Δ%)	v1 periapsis(m/s) =	0,009791094	v1 max =	0,009791136	0,000431751
6	Error (Δ%)	Apoapsis (m)	69817079542	ρ max =	69821053962	0,005692618	87,9696931052	87,2628566803	-0,803499933	v1 apoapsis(m/s) =	0,006451183	v1 min =	0,006472224	0,32614967
7	0,013256107									v2 periapsis(m/s) =	58976,13115	v2 max =	58976,13115	-1,18066E-11
8										v2 apoapsis(m/s) =	38858,3577	v2 min =	38854,87164	-0,0089712
9				Proper time Δt1 elapsed for C1 (days):			87,262856680325			Theoretical half-axes (m)		Simulated half-axes (m)		Errors (Δ%)
10				Proper time Δt2 elapsed for C2 (days):			87,262855572398			a1 =	9613,958379	a1 =	9614,288291	0,003431598
11										b1 =	9408,504858	b1 =	9408,772649	0,002846269
12										a2 =	57909166157	a2 =	57911153367	0,003431598
13										b2 =	56671627821	b2 =	56673240848	0,002846269
14														
15														
16														
17	dθ1 [10]	dθ2	dβ1 [2]	dβ2 [2]	dρ [6]	dρ1 [7]	dρ2 [7]	dψ [9]	v1 (m/s)	v2 (m/s)	X1	Y1	ΔW1	X2
18	0,000483305	0,000483305	8,02E-31	4,83081E-24	1,36191E-09	2,26101E-16	1,36191E-09	-8,24315E-05	0,009791136	58976,13115	7637,033484	0	0	-46001264363
19	0,000483305	0,000483305	1,07922E-18	6,50061E-12	1832,663928	0,000304255	1832,663624	-8,24315E-05	0,009791136	58976,13115	7637,032592	3,691018624	1,13014E+11	-46001258990

9.3.3. Sun-Neptune System

Data and Theoretical calculations

Body C'1		Body C'2		System	
Mass (kg) ("_m1") :	1,9884E+30	Mass (kg) ("_m2") :	1,0243E+26	Eccentricity ("e") :	0,00858587
Δw01 (m) ("_w1") :	1,11156E-72	Δw02 (m) ("_w2") :	2,15778E-68	Periapsis (m) ("dp") :	4,45294E+12
v1 periapsis (m/s) ("_v1p") :	0,282422503	v2 periapsis (m/s) ("_v2p") :	5482,465154	Apoapsis (m) ("da") :	4,53007E+12
v1 apoapsis (m/s) ("_v1a"):	0,277614102	v2 apoapsis (m/s) ("_v2a"):	5389,12311	Period rev. (days) ("pr"):	60088,11528
_v1p/c :	9,4206E-10	_v2p/c :	1,82875E-05		

Constants		Absolute Time dT (m) ("dTa")
G ("G")	c ("_c")	7,78161E+13
6,67428E-11	299792458	
h ("h")	6,62607E-34	
Observer		
α ("alp")	dt (days) ("dt")	
1,570796327	3,004244579	

θ1 max (6,2831853): 6,2831853

Number of steps	Number of days	β1	β2	ρ	ρ1	ρ2	θ1	θ2	ψ
0	0	1,570796326	1,570778039	4,45294E+12	229375998,1	4,45271E+12	0	3,141592654	1,570796327
1	3,004244579	1,570796326	1,570778039	4,45294E+12	229375998,1	4,45271E+12	0,000319595	3,141912249	1,570793606

Simulation results checks

System

Simul. Eccent.	Theoretical distances (m)		Simulated distances (m)		Errors (Δ%)
0,008592268	Periapsis (m)	4,45294E+12	ρ min =	4,4529E+12	-0,000862503
Error (Δ%)	Apoapsis (m)	4,53007E+12	ρ max =	4,53009E+12	0,00041727
0,07452256					

Period of revolution (days)		
Theoretical	Simulated	Error (Δ%)
60088,1152773364	60084,8915884434	-0,005364936

Proper time Δt1 elapsed for C1 (days):	60084,891588423200
Proper time Δt2 elapsed for C2 (days):	60084,891578567700

Bodies C'1 & C'2

Theoretical Speeds (m/s)		Simulated Speeds (m/s)		Errors (Δ%)
v1 periapsis(m/s) =	0,282422503	v1 max =	0,282424294	0,000634141
v1 apoapsis(m/s) =	0,277614102	v1 min =	0,277633246	0,006896017
v2 periapsis(m/s) =	5482,465154	v2 max =	5482,511835	0,000851456
v2 apoapsis(m/s) =	5389,12311	v2 min =	5389,100327	-0,000422762
Theoretical half-axes (m)		**Simulated half-axes (m)**		**Errors (Δ%)**
a1 =	231362445,9	a1 =	231361943,6	-0,000217122
b1 =	231353918,1	b1 =	231353403,1	-0,000222618
a2 =	4,49127E+12	a2 =	4,49126E+12	-0,000217122
b2 =	4,49111E+12	b2 =	4,4911E+12	-0,000222618

dθ1 [10]	dθ2	dβ1 [2]	dβ2 [2]	dρ [6]	dρ1 [7]	dρ2 [7]	dψ [9]	v1 (m/s)	v2 (m/s)	X1	Y1	ΔW1	X2
0,000319595	0,000319595	1,82863E-29	3,54978E-25	8,71778E-08	4,49063E-12	8,71733E-08	-2,72064E-06	0,282422497	5482,465154	229375998,1	0	0	-4,45271E+12
0,000319595	0,000319595	8,12153E-19	1,57657E-14	3871,854822	0,199443603	3871,655379	-2,72064E-06	0,282422497	5482,465154	229375986,3	73307,48325	7,78161E+13	-4,45271E+12

9.3.4. Earth-Moon System

Data and Theoretical calculations

Body C'1		Body C'2		System	
Mass (kg) ("_m1") :	5,9736E+24	Mass (kg) ("_m2") :	7,3477E+22	Eccentricity ("e") :	0,0549
Δw01 (m) ("_w1") :	3,69998E-67	Δw02 (m) ("_w2") :	3,00804E-65	Periapsis (m) ("dp") :	363104000
v1 periapsis (m/s) ("_v1p") :	13,15742816	v2 periapsis (m/s) ("_v2p") :	1069,684566	Apoapsis (m) ("da") :	405288762,7
v1 apoapsis (m/s) ("_v1a"):	11,7879281	v2 apoapsis (m/s) ("_v2a"):	958,3457043	Period rev. (days) ("pr"):	27,2596999
_v1p/c :	4,38885E-08	_v2p/c :	3,56808E-06		

Constants		Absolute Time dT (m) ("dTa")
G ("G")	c ("_c")	35280194500
6,67428E-11	299792458	
h ("h")	6,62607E-34	
Observer		
α ("alp")	dt (days) ("dt")	
1,570796327	0,001362061	

θ1 max (6,2831853): 6,2831853

Number of steps	Number of days	β1	β2	ρ	ρ1	ρ2	θ1	θ2	ψ
0	0	1,570796283	1,570792759	363104000	4412014,699	358691985,3	0	3,141592654	1,570796327
1	0,001362061	1,570796283	1,570792759	363104000	4412014,699	358691985,3	0,000350949	3,141943603	1,570778062

Simulation results checks

System

Simul. Eccent.	Theoretical distances (m)		Simulated distances (m)		Errors (Δ%)
0,054941705	Periapsis (m)	363104000	ρ min =	363082375,3	-0,005955516
Error (Δ%)	Apoapsis (m)	405288762,7	ρ max =	405298532,1	0,002410481
0,075964578					

Period of revolution (days)		
Theoretical	Simulated	Error (Δ%)
27,2596999009	27,2412179556	-0,067799519

Proper time Δt1 elapsed for C1 (days):	27,241217955573
Proper time Δt2 elapsed for C2 (days):	27,241217955413

Bodies C'1 & C'2

Theoretical Speeds (m/s)		Simulated Speeds (m/s)		Errors (Δ%)
v1 periapsis(m/s) =	13,15742816	v1 max =	13,1581533	0,005511235
v1 apoapsis(m/s) =	11,7879281	v1 min =	11,78761604	-0,002647277
v2 periapsis(m/s) =	1069,684566	v2 max =	1069,74352	0,005511306
v2 apoapsis(m/s) =	958,3457043	v2 min =	958,3204584	-0,002634322
Theoretical half-axes (m)		**Simulated half-axes (m)**		**Errors (Δ%)**
a1 =	4668304,622	a1 =	4668232,597	-0,001542871
b1 =	4661264,155	b1 =	4661181,529	-0,001772605
a2 =	379528076,7	a2 =	379522221,1	-0,001542871
b2 =	378955694,4	b2 =	378948977	-0,001772605

dθ1 [10]	dθ2	dβ1 [2]	dβ2 [2]	dρ [6]	dρ1 [7]	dρ2 [7]	dψ [9]	v1 (m/s)	v2 (m/s)	X1	Y1	ΔW1	X2
0,000350949	0,000350949	8,94421E-28	7,27155E-26	7,8061E-12	9,48506E-14	7,71125E-12	-1,82644E-05	13,15742818	1069,684566	4412014,699	0	0	-358691985,3
0,000350949	0,000350949	2,66679E-16	2,16807E-14	2,32745201	0,028280472	2,299171538	-1,82644E-05	13,15742818	1069,684566	4412014,427	1548,393242	35280194500	-358691963,2

9.3.5. Pulsar PSR B1913+16 (version A)

These two systems have been simulated in order to show that the quantified modeling proposed in this article remains consistent with the classical gravitational laws for high velocities at the periapsis (close to 0.0015c for the body C'_2).

The eccentricity is about 0.627 in version A (eccentricity close to that actually measured) and, artificially, 0.01 in the second. This second version emphasizes the influence of eccentricity on the margin of error associated with the period of revolution (approximately 2.58% for version A but only 0.0027% for version B).

	A	B	C	D	E	F	G	H	I	J	K
1											
2		Data and Theoretical calculations									
3		Body C'1		Body C'2		System		Constants		Absolute Time dT	
4		Mass (kg) ("_m1") :	2,86608E+30	Mass (kg) ("_m2") :	2,75731E+30	Eccentricity ("e") :	0,62712	G ("G")	c ("_c")	(m) ("dTa")	
5		Δw01 (m) ("_w1") :	7,71165E-73	Δw02 (m) ("_w2") :	8,01585E-73	Periapsis (m) ("dp") :	765270000	6,67428E-11	299792458	440346368	
6		v1 periapsis (m/s) ("_v1p") :	438017,7101	v2 periapsis (m/s) ("_v2p") :	455295,8299	Apoapsis (m) ("da") :	3339374926	h ("h")	6,62607E-34		
7		v1 apoapsis (m/s) ("_v1a"):	100378,6099	v2 apoapsis (m/s) ("_v2a"):	104338,1613	Period rev. (days) ("pr"):	0,34900613	Observer			
8		_v1p/c :	0,00146107	_v2p/c :	0,001518703			α ("alp")	dt (days) ("dt")		
9								1,570796327	1,70004E-05		
10											
11											
12											
13											
14											
15									θ1 max (6,2831853)		
16									6,2831853		
17		Number of steps	Number of days	β1	β2	ρ	ρ1	ρ2	θ1	θ2	ψ
18		0	0	1,569335256	1,569277623	765270000	375234224	390035776	0	3,141592654	1,570796327
19		1	1,70004E-05	1,569335256	1,569277623	765270000	375234224	390035776	0,0017146	3,143307254	1,570135493

	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1														
2	Simulation results checks													
3	System									Bodies C'1 & C'2				
4	Simul. Eccent.	Theoretical distances (m)		Simulated distances (m)		Errors (Δ%)	Period of revolution (days)			Theoretical Speeds (m/s)		Simulated Speeds (m/s)		Errors (Δ%)
5	0,626891717	Periapsis (m)	765270000	ρ min =	765270000	0	Theoretical	Simulated	Error (Δ%)	v1 periapsis(m/s) =	438017,7101	v1 max =	438017,7101	2,47173E-12
6	Error (Δ%)	Apoapsis (m)	3339374926	ρ max =	3336863540	-0,07520527	0,3490061296	0,3400086524	-2,578028421	v1 apoapsis(m/s) =	100378,6099	v1 min =	100399,7792	0,02108943
7	-0,036401737									v2 periapsis(m/s) =	455295,8299	v2 max =	455295,8299	2,33958E-12
8										v2 apoapsis(m/s) =	104338,1613	v2 min =	104360,1686	0,021092302
9				Proper time Δt1 elapsed for C1 (days):			0,340008575706			Theoretical half-axes (m)		Simulated half-axes (m)		Errors (Δ%)
10				Proper time Δt2 elapsed for C2 (days):			0,340008569540			a1 =	1006313624	a1 =	1005697921	-0,061184
11										b1 =	783840481,6	b1 =	783545681,5	-0,037609708
12										a2 =	1046008839	a2 =	1045368849	-0,061184
13										b2 =	814759984,3	b2 =	814453555,4	-0,037609708
14														
15														
16														
17	dθ1 [10]	dθ2	dβ1 [2]	dβ2 [2]	dρ [6]	dρ1 [7]	dρ2 [7]	dψ [9]	v1 (m/s)	v2 (m/s)	X1	Y1	ΔW1	X2
18	0,0017146	0,0017146	9,43134E-23	9,80337E-23	8,03778E-11	3,94116E-11	4,09662E-11	-0,000660834	438017,7101	455295,8299	375234224	0	0	-390035776
19	0,0017146	0,0017146	1,01744E-09	1,05757E-09	867,1013339	425,1650773	441,9362566	-0,000660834	438017,7101	455295,8299	375233672,5	643376,4696	440345898	-390035202,6

9.3.6. Pulsar PSR B1913+16 (version B)

	A	B	C	D	E	F	G	H	I	J	K
1											
2		Data and Theoretical calculations									
3		Body C'1		Body C'2		System		Constants		Absolute Time dT	
4		Mass (kg) ("_m1") :	2,86608E+30	Mass (kg) ("_m2") :	2,75731E+30	Eccentricity ("e") :	0,01	G ("G")	c ("_c")	(m) ("dTa")	
5		Δw01 (m) ("_w1") :	7,71165E-73	Δw02 (m) ("_w2") :	8,01585E-73	Periapsis (m) ("dp") :	765270000	6,67428E-11	299792458	104478399	
6		v1 periapsis (m/s) ("_v1p") :	345098,098	v2 periapsis (m/s) ("_v2p") :	358710,8952	Apoapsis (m) ("da") :	780730000	h ("h")	6,62607E-34		
7		v1 apoapsis (m/s) ("_v1a"):	338264,4723	v2 apoapsis (m/s) ("_v2a"):	351607,7092	Period rev. (days) ("pr"):	0,080674057	Observer			
8		_v1p/c :	0,001151123	_v2p/c :	0,001196531			α ("alp")	dt (days) ("dt")		
9								1,570796327	4,03359E-06		
10											
11											
12											
13											
14											
15									θ1 max (6,2831853)		
16									6,2831853		
17		Number of steps	Number of days	β1	β2	ρ	ρ1	ρ2	θ1	θ2	ψ
18		0	0	1,569645203	1,569599796	765270000	375234224	390035776	0	3,141592654	1,570796327
19		1	4,03359E-06	1,569645203	1,569599796	765270000	375234224	390035776	0,000320513	3,141913167	1,570793154

	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1														
2	Simulation results checks													
3	System									Bodies C'1 & C'2				
4	Simul. Eccent.	Theoretical distances (m)		Simulated distances (m)		Errors (Δ%)	Period of revolution (days)			Theoretical Speeds (m/s)		Simulated Speeds (m/s)		Errors (Δ%)
5	0,010006103	Periapsis (m)	765270000	ρ min =	765262293,6	-0,00100702	Theoretical	Simulated	Error (Δ%)	v1 periapsis(m/s) =	345098,098	v1 max =	345101,5221	0,000992216
6	Error (Δ%)	Apoapsis (m)	780730000	ρ max =	780731668,1	0,000213654	0,0806740570	0,0806718579	-0,002725825	v1 apoapsis(m/s) =	338264,4723	v1 min =	338263,7241	-0,000221164
7	0,061027862									v2 periapsis(m/s) =	358710,8952	v2 max =	358714,4544	0,000992213
8										v2 apoapsis(m/s) =	351607,7092	v2 min =	351606,9323	-0,000220957
9				Proper time Δt1 elapsed for C1 (days):			0,080671805550			Theoretical half-axes (m)		Simulated half-axes (m)		Errors (Δ%)
10				Proper time Δt2 elapsed for C2 (days):			0,080671801335			a1 =	379024468,7	a1 =	379022988,3	-0,00039058
11										b1 =	379005517	b1 =	379004013,6	-0,000396685
12										a2 =	393975531,3	a2 =	393973992,5	-0,00039058
13										b2 =	393955832	b2 =	393954269,2	-0,000396685
14														
15														
16														
17	dθ1 [10]	dθ2	dβ1 [2]	dβ2 [2]	dρ [6]	dρ1 [7]	dρ2 [7]	dψ [9]	v1 (m/s)	v2 (m/s)	X1	Y1	ΔW1	X2
18	0,000320513	0,000320513	2,23772E-23	2,32599E-23	1,50251E-11	7,36727E-12	7,65788E-12	-3,17296E-06	345098,098	358710,8952	375234224	0	0	-390035776
19	0,000320513	0,000320513	1,15907E-12	1,2048E-12	0,778260098	0,381603628	0,39665647	-3,17296E-06	345098,098	358710,8952	375234204,8	120267,5224	104478329,8	-390035755,9

10. Conclusion

Surprisingly, the few absolute elementary principles on which this study is based offer, in the relativistic framework of the Platonic model, a fairly correct quantified approach to the classical laws of gravitation.

Initially, these results and their review can certainly be quickly refined using more efficient and more sophisticated computer tools than those used here.

As for them, the principles retained deserve to be enriched and deepened in order to propose, in a more general framework, a much richer and complete approach to a quantum theory of gravitation (available for any frame references, taking into account barycentric fluctuations in the case of higher absolute velocities of the interacting bodies, search for a coupling with the standard model of particle physics, etc.).

In any case, the original way proposed in this article seems, at the very least, to be able to favor the emergence of new and numerous questions, promising, in directions still unexplored.

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