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Reactive dynamic assignment for a bi-dimensional traffic flow model

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Abstract. This paper develops a graph-theoretic framework for large scale bi-dimensional transport networks and provides new insight into the dynamic traffic assignment. Reactive dynamic assignment are deployed to handle the traffic contingencies, traffic uncertainties and traffic congestion. New shortest paths problem in large networks is defined and routes cost calculation is provided. Since mathematical modelling of traffic flow is a keystone in the theory of traffic flow management, and then in the traffic assignment, it is convenient to elaborate a good model of assignment for large scale networks relying on an appropriate model of flow related to very large networks. That is the zone-based optimization of traffic flow model on networks developed by [8], completed and improved by [9].

Keywords: reactive assignment; cross-entropy algorithm; traffic control; continuum anisotropic network; zone-based model of flow.

1 Introduction

The assignment is one of the recurring issues in respect of networks operators. Particular attention is taken in the case of transport since such networks allow people to move every day providing to them means of mobility. The government and cities are concerned. There are many static allocation models with respect to the assignment problem in the literature. There are also dynamic allocation models. We are concerning in these second type of assignment. Mainly algorithm for assignment derived from the algorithm of Dijkstra. Versatile algorithms such genetic algorithms, greedy algorithms, revolutionary algorithms, have been developed too in the early century addressing assignment issues in traffic controlling over networks.

These models incorporating such algorithms are of good quality depending on the purposes for which they are deployed, and on particular networks. Although, it is difficult that they represent accurately the dynamic aspects of network flows when very large transport networks are involved.

In this paper we propose a realistic model of dynamic assignment of vehicles flow for the prediction and the estimation of traffic on wide and dense networks. Since the vagaries of traffic in densely populated areas are very numerous, different and varied, we thought it would be more appropriate to use a template stream that already aggregates the roads and the network into zones, say two-dimensional grids or cells with a finite number of directions of the propagation of flow [8, 9]. Using instantaneous travel times of users over networks, a reactive dynamic assignment developped by [4] and applied on networks allow to describe behavioral movement of users over the networks. It allows to compute accurately two-dimensional cells flow of such type of networks. Paths of users are ventilate along such directions of propagation.

2 The bi-dimensional flow model

The bi-dimensional flow model is particularly timely responding to the traffic flow computing problematic over large and dense networks. We mean by dense network, a network with infinite secondary roads and very close to each other. It is the case of the road network of the city of Paris. Its road network forms a spiderweb, ranking in the type of anisotropic networks. For US cities, cities are new cities and their networks are rather orthotropic since roads are not gradually constructed as cities grow.

2.1 Concept of the zone-based traffic flow modeling

At a very large scale, the area of a dense network is well approximated by a continuous medium where vehicles flow corresponds to fluid flow flowing on a surface, with a finite directions of preferred propagation. That is vehicles flow is viewed from a great elevation into the airspace, approximately 100 meters to 500 meters. Our case study concerns transport networks comprising major arteries, secondary urban/suburban roadways. The method is to decompose the surface of such networks in zones in such a way that the principal roads constitute frontiers of certain zones. The zone is meshed in two-dimensional cells. Within cells, are reduced the directions of flow propagation in four preferred outflow directions and four preferred inflow directions. Those directions of propagation of the flow will ventilate the generated traffic demands, from cell to cell. Cells represent edges and relations between cells represent vertices, when we are referring such obtained simplify network in graph theory.

Based on local behavior of flow at a macroscopic scale, a global two-dimensional behavior of flow is easily constructed [8]. It implies flow conservation for the two-dimensional zones both in Eulerian and Lagrangian coordinates. Every cell comprises of course many lanes in the preferred directions of propagation. For intersections of the simplified network that are formed in each zones or cells, intersection traffic flow model rules are applied [5]. It describes the interactions and the dynamic of incoming and outgoing flows at and out of intersections.

The conservation equation is constructed so that it takes into account turning rates at intersections within cells and interactions through interfaces of cells. Interfaces of cells are curved lines in the Euclidian space $\mathbb{R}^2$. We build a corresponding Lagrangian system of the two-dimensional traffic flow for large-scale networks. This allows the estimation of the flow of Lagrangian data. That is in particular the estimation of floating cars flow in any zone of the network over a relatively long time intervals. For sake of clarity, we applied this concept of network flow computing without taking into account difference between major roads and secondary roads. These difference shall be studied in a secondary paper. Every cell is setting by a maximum flow capacity (a free flow capacity), a critical density and a maximal density constraints labeled with directions of propagation. Variables are the 4-cell inflows and 4-cell outflows, traffic demand and traffic supply of cells with respect to the directions.

2.2 Semidiscretized shape of the two-dimensional flow model

Traffic theory on dynamics of vehicles on highways and urban network and the analogy with fluids flowing within two-dimensional domain suggest the formulation of the following physical model (1) for traffic in a cell:

$$N_{c,i}(t+\delta t) = N_{c,i}(t) + \left(Q_{fc}(t) - R_{cg}(t)\right)\delta t + \left(r_{c,i}(t^+) - q_{c,i}(t^+)\right)\delta t \qquad (1)$$

with (i) the direction of propagation of flow inside the cell (c). (f) and (g) are respectively the indexes of zones located at left and right of the target-cell (c).

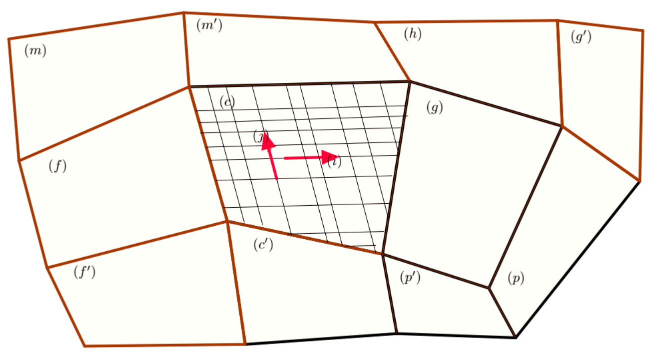

Fig. 1: Surface of the large-network (as the network-domain) is dis-aggregated in $2d$-zones. Zones are meshed in quadrangular cells. Each cell has certain parameters: a finite number of preferred propagation directions of flows, numbers of lanes in each direction, lengths of lanes in each direction.

Cell internal flows control Using intersection traffic flow model rules following [5], we find out that these functions (or variables) are solution of the the below (2) linear-quadratic optimization problem:

$$\max_{(q,r)} \left(\sum_{i=1}^{4} \Phi(q_i) + \sum_{j=1}^{4} \Psi(r_j) \right)$$

$$\textbf{s.t.} \left| \begin{array}{ll} 0 \le q_i \le \Delta_{ci}^{t+1/2}, & \forall i \in \{1,2,3,4\}, \\ 0 \le r_j \le \Sigma_{cj}^{t+1/2}, & \forall j \in \{1,2,3,4\}, \\ -r_j + \sum_{i=1}^{4} q_i \Gamma_{c,ij}^{t} = 0, & \forall j \in \{1,2,3,4\}. \end{array} \right. \qquad (2)$$

with $q = (q_1, q_2, q_3, q_4)$ and $r = (r_1, r_2, r_3, r_4)$ internal vectors flows which expresses traffic states inside cells. Functions $\Psi(\vartheta_\ell) \doteq \Phi(\vartheta_\ell)$ are defined by (3). They are assumed equal, concave, increasing and they describe interactions of vehicles inside cell. The optimization problem (2) results in an intersection model similar to the intersection models of [3, 10].

$$\Psi(\vartheta_\ell) \stackrel{\text{def}}{=} -\frac{1}{2}\vartheta_\ell^2 + \vartheta_\ell.\vartheta_{\ell,max}. \tag{3}$$

Notations and definitions are the following.

- $\forall i \in \{1, 2, 3, 4\}$, q_i is the incoming vehicles flow in the direction i;
- $\forall j \in \{1, 2, 3, 4\}$, r_j is the outgoing vehicles flow in the direction j;
- $\ell = i, j \in \{1, 2, 3, 4\}$, ϑ_ℓ referring to q_i or r_j, $\vartheta_{\ell,max}$ denotes $q_{i,max}$ or $r_{j,max}$ which is the maximum flow constraint in the direction i or j;
- $\Gamma^t_{c,ij}$, assignment coefficients of flows within cell c, from direction i to direction j, at instant time t;
- μ_{ci}, number of lanes in cell (c) in the direction i;
- ν_{ci}, number of exiting lanes in the cell (c) respect to direction i;
- $\delta_i = \Delta_{ci}(\rho^t_{ci})$, lane supply in direction i;
- $\sigma_i = \Sigma_{ci}(\rho^t_{ci})$, lane demand in direction i;
- $\Delta^{t+1/2}_{ci} = \mu_{ci}\delta_i$, vehicles demand in c to direction i, at time t^+ $(t + 1/2)$;
- $\Sigma^{t+1/2}_{cj} = \mu_{ci}\sigma_i$, cell c supply on line j, at time t^+ $(t + 1/2)$;
- $r^{t+1/2}_{cj}, q^{t+1/2}_{ci}$, $\forall i, j$ denote the solution of the above convex optimization problem (2).

The obtained optimization model is easily solved with the python-cvxopt solver. The turning rates $\Gamma^t_{c,ij}$ can be considered as assignment coefficients; they are updated at each time-step $t,\ t \in [0, T]$ in the designed reactive dynamic assignment engine. Furthermore, having the number of vehicles in every $2d$-cell, matrix of turning rates at time t, minimum travel paths, and directional outflows of zones of each path are easily identifiable and calculable.

Cell inflows and cell outflows Flows through cells denote by Q_{fc} and R_{cg} for $f \in \mathfrak{V}(c)$ are governed by a such semidiscretized model (1). $\mathfrak{V}(c)$ denotes the neighboring of the cell (c) comprising only adjacent cells that share an edge the cell (c). Q_{fc} and R_{cg} are respectively inflow and outflow of the cell (c) (see figure 1). There are define as following.

$$Q_{fc}(t) = \min\left(\delta_{f,i'}(t), \sigma_{c,i}(t)\right) \text{ and } R_{cg}(t) = \min\left(\delta_{c,k}(t), \sigma_{g,k'}(t)\right). \tag{4}$$

where i lies in the sense of lanes on $f \to c$, from the cell (f); i' lies in the sense of lanes on $f \to c$, from the cell (f); k lies in the sense of lanes of (c) which flows flow directly into (g); k' lies in the sense of lanes on $c \to g$, within (g).

2.3 Computational aspect : methodology

Using a finite volume mesh of a transportation network area (which can be obtained easily by any mesh software for finite volume methods), we deduce a graph of the simplified network obtained at the two-dimensional scale. To compute bi-dimensional cells flows, the general structure of the algorithm is shown schematically in Fig. 2.

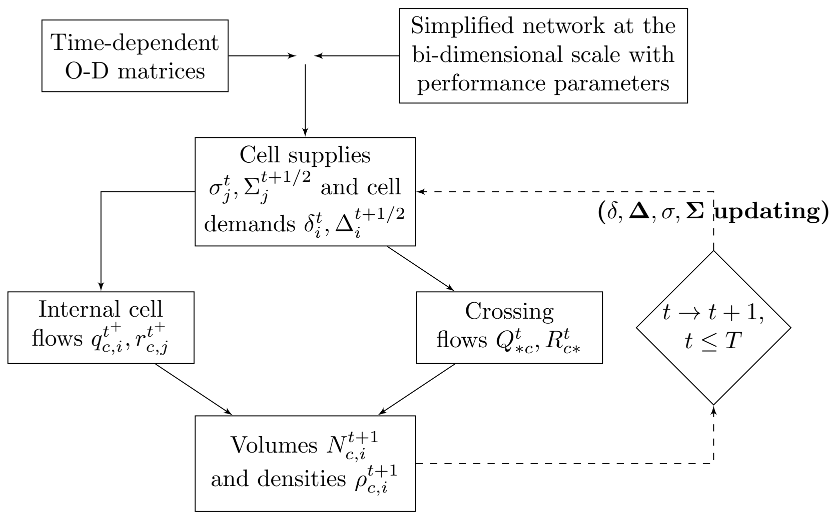

Fig. 2: Bi-dimensional network flows computing engine.

Flows Q_{*c}^t and R_{c*}^t denote the directional inflow $Q_{fc}(t)$ and directional outflow $R_{cg}(t)$ respectively. They are the flows that cross interfaces of cell (c). We name them crossing flows which are at the opposite of the cell internal flows $q_{c,i}^{t^+}, r_{c,j}^{t^+}$. These are computed with the linear-quadratic optimization problem (3). At each time step in the computation, the number of vehicles $N_{c,i}(t+1)$ in cell $c \in \mathfrak{C}$ is calculated by just applying the semidiscretized bi-dimensional formula (1), with $\mathfrak{C}$ the set of all nodes of the simplified network.

3 Reactive dynamic assignment

In this section we develop a reactive dynamic traffic assignment model applying to the semidiscretized traffic flow model (1). The model derives from works of [11, 2]. Thought we use rather instantaneous travel times instead of predictive travel time or experienced travel time. The instantaneous travel time allow to capture rapid changes in flow when traffic breakdown occurs. [1] give computational procedures for instantaneous travel times within macroscopic approximation of interrupted traffic flow. By analogy, we give definition of instantaneous travel time in the two-dimensional traffic flow modeling theory, and provide a diagram of the reactive assignment over large network (see Figure 4).

Notation:

- π_c^d : the weight of path of minimum cost that reached the destination point (the node (d)) from the node (c).
- $\pi_c^{d,k}$: the weight of path of minimum cost that reached the destination point (the node (d)) from the node (c), consisting in k-arcs.
- $\Gamma_{c,ij}(t)$: turning rate movements of vehicles within cell (c), from direction (i) to direction (j), at time instant t.
- $\Gamma^d_{c,ij}(t)$: at time instant t turning rates, of incoming flow in the direction (i) in the cell (c), and that going to (c)-direction (j), in order to reach the destination cell (d).
- $\varpi_{cc'}$: the cost of the arc (c, c').

3.1 Instantaneous travel time

Let $(c) \in \mathfrak{C}$ a cell. For (i) a direction, we denote by $V^t_{cg,i}$ the cell exit speed of (c) in the direction $(i) = c \to g$. We are defining instantaneous travel time (ITT) for cell links: that is the links in $2d$-cell that lie in the preferred directions of flow propagation. It is a good approximation since flows will assign through these preferred directions of propagation. This is even the main expected in two-dimensional modeling: reduce the infinite links and nodes of dense network in a simplified network while still ensure a way of providing almost perfect information about network traffic states. Let us mention that instantaneous travel time in two-dimension space shall be describe as an integral along the path a user or vehicle will follow with respect to its velocity. A formal definition of ITT is the following (see [7]).

$$ITT(a, b, t) = \int_a^b d\chi / V(\chi, t) \tag{5}$$

$T(x, t) \stackrel{def}{=} ITT(x, b; t)$ is instantaneous travel time from x to b estimated at time t, labeled backward in [7]. This formula is valid in non-interrupted traffic flow, particularly when velocity is always bound by a strictly positive lower speed.

The authors of [7] have give clear computational definition of the ITT in interrupted traffic. The cell exit speed defined as $V^t_{cg,i} = R_{cg} L_{c,i} / N^t_{c,i}$ by authors is a CFL condition: It permits emulation of FIFO behavior within each $2d$-cell. Proper discretization constraint such $R_{cg}\delta t \leq N^t_{c,i}$ is set. Introducing the cell travel time $ITT^t_{c,i} = T^t_{c,i} - T^t_{f,i}$, these below (6) formulas hold:

$$\left[\begin{array}{l} ITT^{t+1}_{c,i} - L_{c,i}/V^t_{c,i} = \left(1 - \frac{\alpha_{c,i}\nu^t_{c,i}}{1-\nu^t_c}\right)\left(ITT^t_{c,i} - L_{c,i}/V^t_{c,i}\right) - \left(T^{t+1}_{f,i} - T^t_{f,i}\right), \\ \hfill \text{if } \nu^t_{c,i} \leq \frac{1}{1+\alpha_{c,i}} \\ \\ ITT^{t+1}_{c,i} - L_{c,i}/V^t_{c,i} = -\frac{1-\alpha_{c,i}}{\alpha^t_{c,i}\nu^t_{c,i}}\left(T^{t+1}_{f,i} - T^t_{f,i}\right) \ \text{ if } \nu^t_{c,i} \geq \frac{1}{1+\alpha_{c,i}} \end{array}\right. \tag{6}$$

where coefficients $\alpha_{c,i}$ and $\nu^t_{c,i}$ are defined such as:

$$\alpha_{c,i} \stackrel{def}{=} V_{c,i,max}\delta t/L_{c,i} \text{ and } \nu^t_{c,i} \stackrel{def}{=} V^t_{c,i}/V_{c,i,max} = R^t_{c,g}\delta t/\big(\alpha_{c,i}N^t_{c,i}\big). \quad (7)$$

$V_{c,i,max}$ is the maximal exit speed of vehicles in the cell (c) and in the direction (i).

3.2 Travel cost

The cost of an arc ϖ_{cf} can be estimated in the framework of the proposed model by the instantaneous travel time, which itself can be estimated at each time-step by the following:

$$\varpi_{cg} \approx N_{c,i}/R_{cg} \quad (8)$$

if the cell (g) lies in direction (i) with respect to cell (c).

3.3 Logit algorithm

Let us introduce a Logit model for the choice of neighboring nodes or cells, and address shortest paths computation. From a cell, vehicles have 4 possible choices for their next motion since there are 4 outflow directions. Due to target cell (the destination of vehicles), in a cell vehicles have generally just 2 possible directions that they may take when they are going out of the cell (a simplifying assumption). Therefore, the weight of path of minimum cost π^d_c can be decomposed as below:

$$\pi^d_c \quad \rightarrow \quad \begin{cases} \varpi_{cf} + \pi^d_f &= C^d_f \\ \varpi_{cg} + \pi^d_g &= C^d_g \end{cases} \quad (9)$$

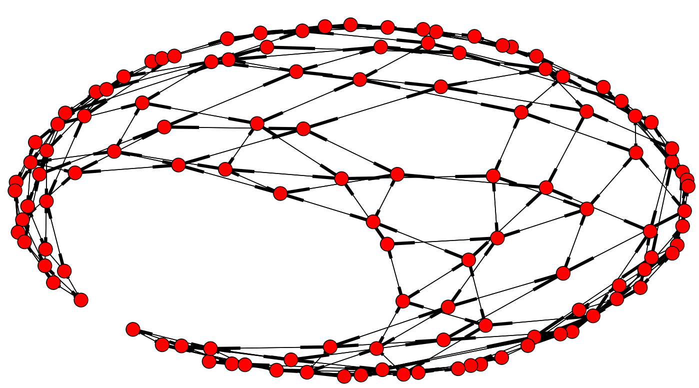

Fig. 3: Every node represents a zone. Every arc is a connection between two adjacent zones. The zones are two-dimensional computing cells, with at more 4-directions of propagation of the vehicles flow: (4-inflows and 4-outflows for each node/zone).

We can determine the probability of choice of users for choosing either the one cell between the neighboring cells of the cell they are located, at a fixed time t. The formulation of this probability of user cell-choice is given by (10):

$$\left[\begin{array}{l} \mathrm{P}\,(\text{choice} = (f)/\text{Dest.} = d) = \dfrac{\exp(-\theta C_f^d)}{\exp(-\theta C_f^d) + \exp(-\theta C_g^d)} = \mathcal{F}_{cf}^d \\ \\ \mathrm{P}\,(\text{choice} = (g)/\text{Dest.} = d) = \dfrac{\exp(-\theta C_g^d)}{\exp(-\theta C_f^d) + \exp(-\theta C_g^d)} = \mathcal{F}_{cg}^d \end{array}\right. \tag{10}$$

Parameters $\mathcal{F}_{cf}^d$ and $\mathcal{F}_{cg}^d$ allow the calculation of the coefficients of turning rates. We can easily compute π_c^d by the below algorithm (min,+ type, which can be improved as a Dijkstra algorithm):

- $\pi_c^{d,1} = 0$.
- If $c \neq d$, $\pi_c^{d,1} = \varpi_{cd}$ if exists arc (c, d), or $= \infty$ if not.
- $\pi_c^{d,k+1} = \min\left(\pi_c^{d,k}\ ,\ \min_{c' \in Succ(c)} \left(\varpi_{cc'} + \pi_{c'}^{d,k}\right)\right)\ ,\ \forall t.$

Therefore, the traffic assignment model identifies the minimum cost travel paths, and the directional outflow within cells of each path. That is discussed in the Section 3. The general structure of the reactive algorithm is shown in Fig. 4.

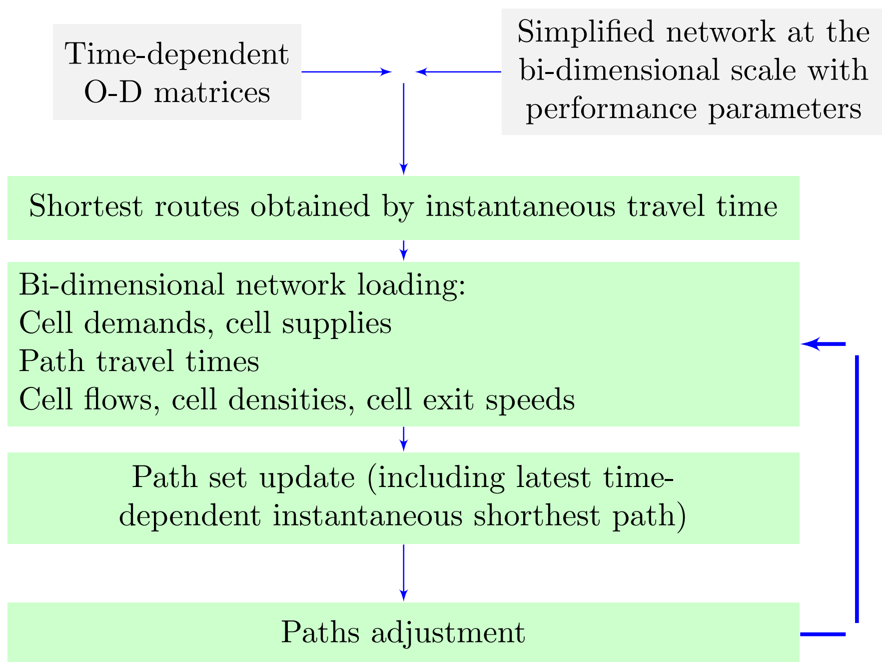

Fig. 4: Structure of solution algorithm to the RDA (Reactive Dynamic Assignment) problem.

The constructed assignment model enables flow assignment of big cities, if their transportation networks are approximated by two-dimensional media (orthotropic or anisotropic), completed by their main arteries network. For instance, we look at the road networks of Paris, Atlanta, Tokyo, Chicago, Manhattan, and Minneapolis. The presented assignment model allows computing paths of given OD pairs, and then successive cell-flows of the cells of the paths.

4 Recommendations

The main reasons of this bi-dimensional and zone-based approach is that it is difficult in practice to secure traffic data for all links of a dense network, even in a context of streaming data (portable, GPS), and that using traditional models (microscopic or macroscopic traffic models) requires cumbersome computational calculations. Further, very macroscopic management decisions do not necessarily require a very high level of detail of the traffic on the network.

Our zone-based approach results in a high level of aggregation of links flow. Hence the zone-based (or the two-dimensional scale framework) of traffic modeling requires less information than the traditional network approaches and makes possible to model traffic flow of transportation systems of large surface networks with few network sensors of traffic counts. It optimizes traffic zone-flows well and then provides a good traffic flow management.

Further issues are flow modeling and optimization per transportation mode within large-scale network. The bi-dimensional traffic flow model can be interfaced with a GSOM model ([6]) of the main motorways and arteries. In this perspective the bi-dimensional model will mainly describe dense networks of secondary roads. This modeling framework is also compatible with vehicular multimodality (distinguishing between private cars, taxis, electric vehicles, demand responsive systems etc).

The reactive assignment introduced allows to calculate the exact flow on roads networks since dynamic user equilibrium is no longer valid in a very inhomogeneous transport network. A reactive assignment requires instantaneous travel time and capture perfectly vagaries of the traffic.

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