COMBINING SHORT AND LONG TERM RESERVOIR OPERATION USING INFINITE HORIZON MODEL PREDICTIVE CONTROL

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ABSTRACT

Model Predictive Control (MPC) can be employed for optimal operation of adjustable hydraulic structures. MPC selects the control to apply to the system by solving in real-time an optimal control problem over a finite horizon. The finiteness of the horizon is both the reason of MPC's success and its main limitation. MPC has been in fact successfully employed for short-term reservoir management. Short-term reservoir management deals effectively with fast processes, such as floods, but it is not capable of looking sufficiently ahead to handle long-term issues, such as drought.

We propose an Infinite Horizon MPC solution, tailored for reservoir management, where input signal is structured by use of basis functions. Basis functions reduce the optimization argument to a small number of variables, making the control problem solvable in a reasonable time. We tested this solution on a test case adapted from Manantali Reservoir, on the Senegal River. The long-term horizon offered by IH-MPC is necessary to deal with the strongly seasonal climate of the region for both flood and drought prevention.

Keywords: Model Predictive Control, Reservoir Operation, Infinite Horizon, Manantali

INTRODUCTION

Optimal reservoir operation can be framed as a control problem (Soncini-Sessa et al. 2007), which, for reservoir operation, has been typically solved using methods from the

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dynamic programming family. Stochastic Dynamic Programming (SDP) (Stedinger et al. 1984; Trezos and Yeh 1987) solve a control problem that is markov. SDP, however, suffers from the so-called “curse of dimensionality,, (Bellman and Dreyfus 1966) and “curse of modelling,, (Tsitsiklis and Van Roy 1996; Bertsekas and Tsitsiklis 1995). The curse of dimensionality limits SDP application to simple systems, made of few variables. Curse of modeling implies the demand of modeling the inflow to the reservoir as a stochastic dynamic system.

Model Predictive Control (MPC) is a real-time control technique (Morari et al. 1999; Mayne et al. 2000) suffering neither the curse of dimensionality nor the curse of modelling, as intended for SDP. MPC has been extensively applied on water systems (van Overloop 2006), mostly for canals (Malaterre et al. 1998; Malaterre and Rodellar 1997; van Overloop et al. 2014; Horváth et al. 2014), river delta (Dekens et al. 2014; Tian et al. 2015b), also considering quality (Xu et al. 2013; Xu et al. 2010), transport of water and over water (Tian et al. 2013; Tian et al. 2015a) and reservoir operation (Raso et al. 2014b; Schwanenberg et al. 2015; Ficchi et al. 2015; Galelli et al. 2012; Schwanenberg et al. 2014).

In reservoir operation, MPC proved to be effective for short-term objectives, such as flood prevention. Short term objectives, however, must be balanced with long-term ones, as drought prevention, among others. MPC, in fact, finds a control action optimal for a finite horizon, but large reservoirs have often a slow dynamic, and effect of control actions are mutually interdependent on a long period. In this case, classic MPC can be employed for short-term optimal control method, but it does not ensure long-term optimality, as effects after the optimization horizon are not included.

Methods to integrate the long-term effects within the MPC optimal control problem refer to as Infinite Horizon MPC (Maciejowski 2002). Among these, a suitable approach is input structuring by use of basis function (Wang 2001). We propose here an innovative use of input structuring for Infinite Horizon MPC applied to reservoir operation, and we tested some triangular basis functions. This work extends and generalizes some initial results

initially presented in (Raso et al. 2014a). Constraints on inputs can be easily included. We show an application on a test case adapted from Manantali Reservoir, on the Senegal River.

METHOD

Consider a water system composed of N_x reservoirs that is operated by N_u discharge decisions. Discharge decisions are diversions from rivers and releases from reservoirs. A reservoir may have multiple releases (by different structures or for different users). The system is influenced by N_d streamflows.

We start from framing the reservoir operation problem in control terms. Problem (1) define the optimal control problem for a reservoir system.

$$\min_{\{\pi_t\}_{t=1}^H} \sum_{t=1}^{H+1} \mathbb{E}_{\mathbf{d}_t} \left[g_t(\mathbf{x}_t, \mathbf{u}_t, \mathbf{d}_t) \right] \quad (1a)$$

Subject to:

$$\mathbf{x}_t = \mathbf{x}_{t-1} + \Delta t \cdot (I \cdot [\mathbf{u}_t, \mathbf{d}_t] - O \cdot [\mathbf{u}_t, \mathbf{d}_t]) \quad (1b)$$

$$0 \leq \mathbf{u}_t \leq \mathbf{u}_{max} \quad (1c)$$

$$\mathbf{x}_{min} \leq \mathbf{x}_t \leq \mathbf{x}_{max} \quad (1d)$$

$$\mathbf{c}_t(\mathbf{x}_t, \mathbf{u}_t, \mathbf{d}_t) \leq 0 \quad (1e)$$

$$\mathbf{d}_t \in \mathbf{D}_t \quad (1f)$$

$$\mathbf{x}_{t=0} \text{ given} \quad (1g)$$

In problem (1), vectors $\mathbf{x}_t \in \mathbb{R}^{N_x}$, $\mathbf{u}_t \in \mathbb{R}^{N_u}$, $\mathbf{d}_t \in \mathbb{R}^{N_d}$, represent reservoir volumes, discharge decisions, stochastic streamflow scenarios at instant t for stocks and in the period $[t - \Delta t, t)$ for flows; $g_t(\cdot)$ is a $\mathbb{R}^N$ to $\mathbb{R}$ function, representing the system step-cost function at t , and $N = N_x + N_u + N_d$; $\mathbb{E}[\cdot]$ is the average operator. In some cases different criteria other than the average may be used, such as the max operator. In Expression (1a), π_t is the release policy, which gives the optimal release decision in function of to the system

state, such that $\mathbf{u}_t^* = \pi_t(\mathbf{x}_t)$. Equation (1b) is the continuity equation, represented by the reservoirs mass balance, where Δt is the time-step length, I and O are the input and output matrix, of dimension $N_x \times (N_u + N_d)$, associating at each scenario and discharge decision to its reservoir. $O(i, j)$ and $I(i, j)$ is 1 if the i variable is input or output of reservoir j , 0 elsewhere. Hydrological inflow are hydrological scenarios extracted from $\mathbf{D}_t$, as in Expression (1f), where $\mathbf{D}_t$ is a stochastic variable representing all possible future discharge scenarios. In Inequality (1e), $\mathbf{c}_t$ defines other constraints that apply to the system, such as physical constraints, or other legal or environmental requirements treated as constraints. For example, discharge decision can be limited by water availability within the reservoir. H is the length of simulation, or closed-loop, horizon, on which the system is tested.

Solving problem (1) is finding the control strategy π_t , be either a function mapping observed state to optimal control, or a tree of decisions according to the observed discharge (Shapiro and Andrzej 2003). Different methodologies try to tackle the optimal release policy identification problem for reservoir operation. Simulation-based methods (Sulis and Sechi 2013), known in the operational research community as policy function approximations (Powell and Meisel 2015), are often used by analysts having their main expertise in hydrology, where the class of functions, or set of rules, is defined a priori, and some parameters are adjusted according to simulation results. Apart from simulation-based rules, methods from the dynamic programming family, typically Stochastic Dynamic Programming (SDP) (Stedinger et al. 1984), has been extensively employed to solve Problem (1) by taking advantage of its markov structure. SDP, however, suffers from the curse of dimensionality and the curse of modelling: the SDP functional optimization is particularly complex to solve numerically, therefore application are limited to systems made of few variables, and state transitions must be defined explicitly, requiring a stochastic representation of the inflow process. Stochastic Dual Dynamic Programming (Pereira and Pinto 1991; Tilmant et al. 2008) attenuates the curse of dimensionality (Shapiro 2011), and Sampling Stochastic dynamic Programming (Kelman et al. 1990; Faber and Stedinger 2001) tackles the curse of modelling, but

no methods from the dynamic programming family overcomes effectively both limitations. Evolutionary algorithms for reservoir operation are methods for non-linear optimization used to optimise some parameters that define the release policies (Nicklow et al. 2009; Reed et al. 2013), but their application to large systems has been little tested.

Model Predictive Control (MPC) is an alternative control method to tackle Problem (1). In MPC, at each control instant t , the control actions are obtained by solving on-line, i.e. at each control time-step, the following optimal control problem.

$$\min_U \left[\sum_{k=1}^h g_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{d}_k) + g_{h+1}(\mathbf{x}_h) \right] \quad (2a)$$

Subject to:

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \Delta t \cdot \left(I \cdot [\mathbf{u}_k, \mathbf{d}_k] - O \cdot [\mathbf{u}_k, \mathbf{d}_k] \right) \quad (2b)$$

$$0 \leq \mathbf{u}_k \leq \mathbf{u}_{\max} \quad (2c)$$

$$\mathbf{x}_{\min} \leq \mathbf{x}_k \leq \mathbf{x}_{\max} \quad (2d)$$

$$\mathbf{c}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{d}_k) \leq 0 \quad (2e)$$

$$\mathbf{x}_{k=0} = \mathbf{x}_t, \{\mathbf{d}_k\}_{k=1}^h \text{ given} \quad (2f)$$

In Equations (2), $U = \{\mathbf{u}_k\}_{k=1}^h$, where k is the time index, going from 1 to the final time-step of the optimization, or open-loop, horizon, $h \ll H$; g_h the final penalty that sums up all the future costs beyond the control horizon.

MPC uses the system model in Equations (2b-2e) to predict the system behavior in response to the control actions over a finite future horizon. The model takes the current state of the system as initial state, and a deterministic forecasts of disturbances as uncontrolled input, as in Equations (2f). Once system model, cost function, initial state and forecasted disturbance are given, MPC solves problem (2) and finds the optimal control trajectory for the future prediction horizon. At each time-step, only the first value of the optimal control

trajectory is applied to the real system, i.e. $\mathbf{u}_t^* = \mathbf{u}_{k=1}$; then the horizon is shifted ahead and the procedure is repeated at the next controlling instant using the latest up-to-date information.

MPC is an on-line, or “real-time”, technique, meaning that Problem (2) is solved contemporaneously to the system operation. At every control instant, MPC uses the most up-to-date system state and disturbance forecast. In MPC, a control policy $\mathbf{u}_t^* = \pi(\mathbf{x}_t, \{\mathbf{d}_k\}_{k=1}^h)$ is found on-line by solving a deterministic optimization problem as defined in Equations (2), which is much easier to solve than its stochastic equivalent. MPC is not affected by the limitations of SDP, and it can be applied to much larger systems, using discharge forecasts not influenced by release decisions as input to MPC.

Robustness to uncertainty is a key question in MPC research literature (Morari et al. 1999). At each decision instant, MPC uses the most up-to-date information. This feedback mechanism due to the continuous system update gives to MPC a form of ‘inherent robustness’ (Mayne et al. 2000), which may be sufficient to produce satisfactory results in the face of the present uncertainty. If this is not the case, synthetic robust MPC (Bemporad and Morari 1999) methods can augment the system robustness to the desired level, generally at the cost of additional computational complexity (Muñoz de la Peña et al. 2005).

In MPC, the cost-to-go function g_h should theoretically sum up all the costs, from instant h to infinite, for having left the system in x_h at the end of the control horizon. In practice, however, this function is difficult to obtain. If g_t is a Lyapunov function, and the control horizon is sufficiently long, MPC ensures stability (Maciejowski 2002), even without g_h . An example of Lyapunov function widely used in MPC for trajectory following problems is a quadratic penalty on the state deviance from the optimal trajectory (van Overloop 2006; Xu et al. 2010; Negenborn et al. 2009; van Overloop et al. 2008). This property is extensively used in MPC applications for canal control, where the objective is trajectory tracking, but reservoir objectives are rarely well represented by Lyapunov functions.

An alternative way to guarantee stability is adding a constraint on the final state (De Nicolao

et al. 1996). However, this solution requires the identification of a desired final state, which can be unknown. This is often the case in reservoir operation, where MPC applications use historical final penalties (Ficchi et al. 2015) which does not guarantee optimality. Moreover, if the horizon is too short, this MPC configuration runs the risk of having an infeasible problem.

Infinite horizon MPC is a family of solutions dealing with the finiteness of the optimization horizon. Within this family, input structuring (Wang 2001) seems to be particularly suited for reservoir operation. In input structuring, the control are not optimized directly, but they are arranged according to a convenient form. Among the different forms of input structuring we selected basis functions. Heuberger et al. (2005) offers a clear and accurate description of basis functions and their use for system identification.

Equation (3) shows input structuring using basis function.

$$\mathbf{u}_k = \sum_{i=1}^N \boldsymbol{\lambda}_i \cdot f_i(k) \quad (3)$$

where $f_i(k)$ are fixed time-variant functions and $\boldsymbol{\lambda}_i \in \mathbb{R}^{N_u}$ are their weights, selected with an optimization procedure.

Basis functions can represent a smooth signal using few parameters λ_i , being therefore a potential appropriate approach for input structuring in reservoir operation. Reservoirs, in fact, filter out the high frequency variability of inflow. Consequently, the control signal (i.e. the releases) varies slowly too. Moreover, periodic basis functions can follow the yearly periodicity of natural systems.

If input structuring is to be used in optimization, optimizing $\boldsymbol{\lambda}_i$ instead of $\mathbf{u}_k$ reduces the degrees of freedom from $h \times N_u$ to $N \times N_u$. In a rolling horizon optimization problem, reducing the degrees of freedom allows the use of a much larger h , i.e. extending the control horizon without having an explosive growth in computational complexity. Input structuring reduces the computational complexity related to the horizon length, not affecting or being affected by other sources of complexity related to the system size or the number of objectives.

For this reason the proposed methodology is applicable to large systems in the same way as MPC when applied for short-term operation.

Using input structuring in MPC, it is particularly important that the optimal control sequence is well represented in proximity of the first control value, which will be eventually applied to the system. Therefore, basis functions must be selected such that the control signal at the initial part of the horizon is regulated by a larger degree of freedom, i.e. a larger number of bases. Control values far ahead in the horizon have less influence on the first control value and can be represented by relatively less basis. Influences of control at instant k on first control value shades as k get larger with no clear boundary. Selection of basis functions shapes must follow this regression.

Basis functions have been extensively used for system identification (Van Den Hof et al. 1995; Van den Hof and Ninness 2005; Heuberger et al. 1995). However, in MPC, constraints on $\mathbf{u}_k$ imply constraint on $\boldsymbol{\lambda}_i$. In Equations (4) we define the infinite horizon MPC problem with input structuring by basis functions.

$$\min_{\boldsymbol{\Lambda}} \sum_{k=1}^{h-1} e^{-r \cdot k} \left[\cdot g_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{d}_k) + c_k(\mathbf{x}_k) \right] \quad (4a)$$

Subject to:

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \Delta t \cdot (I \cdot [\mathbf{u}_k, \mathbf{d}_k] - O \cdot [\mathbf{u}_k, \mathbf{d}_k]) \quad (4b)$$

$$U = M \cdot \boldsymbol{\Lambda} \quad (4c)$$

$$0 \leq M \cdot \boldsymbol{\Lambda} \leq \mathbf{u}_{\max} \quad (4d)$$

$$\mathbf{x}_{k=0} = \mathbf{x}_t, \{\mathbf{d}_k\}_{t=1}^h \text{ given} \quad (4e)$$

In Equation (4), $\boldsymbol{\Lambda} = [\boldsymbol{\lambda}_1], \dots, \boldsymbol{\lambda}_N]$, r is the discount rate, M is a $N \times h$ vector defined by the basis functions, such that $M(k, i) = f_i(k)$. Note that Equation (4c) is a linear transformation, implying that the problem stays linear in $\boldsymbol{\Lambda}$, no matter whether the basis

functions are linear or not.

State constraints, as in inequalities (1d) are integrated as soft constraints, such that

$$c_k(\mathbf{x}_k) = \max\{0, w_x \cdot (\mathbf{x}_k - \mathbf{x}_{\max}), w_x \cdot (\mathbf{x}_{\min} - \mathbf{x}_k)\} \quad (5)$$

where $w_x \gg 0$.

Extending the long term far beyond the horizon where forecast are reliable requires the inclusion of climatic information and add large uncertainty (Zhao et al. 2011). Uncertainty that jeopardizes MPC robustness can be dealt with method for synthetic robust methods, such as Multiple MPC (van Overloop et al. 2008), Tree-Based MPC (Raso et al. 2014b; Maestre et al. 2012a; Maestre et al. 2012b) or others (Bemporad and Morari 1999; Muñoz de la Peña et al. 2005; Muñoz de la Peña 2005).

Triangular basis function and triangles selection

We use triangular basis function because of their simplicity to be communicated and to be defined from few parameters. Equations (6) define a generic triangular basis function i .

$$f_i(k) = \begin{cases} 1 - \frac{T_i+k}{L_i} & \text{if } T_i + L_i < k \leq T_i \\ 1 + \frac{T_i+k}{R_i} & \text{if } T_i < k \leq T_i + R_i \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Each triangle i is determined by its peak instant, T_i , its left base, L_i , and its right base R_i . Figure 1 shows a graphical visualization of the triangles and their parameters. An alternative family of basis function could be a combination of exponential functions with different decay rate, and a sum of sines and cosines with difference frequency.

Basis function accuracy must be progressive going ahead on time, this progression depending on the system characteristics. We give here some general indications for triangular functions, highlighting the advantages of some specific shapes.

We suggest selecting progressive triangles, i.e. $L_i < R_i, L_i + 1 > L_i$, in the early stage of the

horizon. In MPC in fact, only the first control value will be applied to the systems. The first control is more sensitive to controls that are closer in time; therefore it is better to have a higher degree of freedom in the initial part of the horizon. The first triangle should have its peak T at the initial time-step.

Sufficiently far from present condition, periodicity becomes dominating. For $t > P$, where P is the system periodicity, triangles having L_i and R_i equal to $P/2$ are able to follow the periodic trend. In this part of the horizon, T should be equal to $P \times j$ and multiple of $P \times (j + 1/2)$, where j is an integer going from zero to the number of years contained in the control horizon. Selection of independent triangles, such that $L_{i+1} = R_i$, and $T_{i+1} = T_i + L_i$, makes constraints independent.

TEST CASE

The method is tested on a system adapted from Manantali reservoir case. Manantali is located in Mali, on the Senegal River, presently used mainly for electricity production. Plans for agro-business on the Senegal River valley could change the management in the short future (Fraval et al. 2002). In this case, the objective of energy production must be balanced with flood and drought prevention. The hydrology on the Senegal River is strongly seasonal, influenced by the tropical rainy season in the upper basin.

The reservoir is modeled by the continuity equation as in Equation (4b). The system disturbance is the uncontrolled inflow, d_t , which is the observed discharge at Soukoutali. The system controls are the release through the turbines, u_t^r , and the release through the spillages, u_t^s . Controls are constrained between zero and maximum release through turbines, $u_{\max}^r$, and maximum release through spillages, $u_{\max}^s$. The operational volume is constrained between $x_{\min}$ and $x_{\max}$. In this experiment, the operational volume is reduced to increase the difficulty of the reservoir operation. Evaporation from the reservoir and other losses are neglected.

The hydrological input d_t uses both real-time forecast and climatic information, gliding from the real-time information into the climatic one going ahead on time: d_t is the Bayesian

Model Averaging (Raftery et al. 2005) of the forecasted inflow, d^{fr} , and the climatic one, d^{cl} , weighted by their reliability.

$$d_k = B_k \cdot d_{t+k} + (1 - B_k) \cdot D \quad (7)$$

Where B_t , representing the forecast reliability, is the product of the inflow autocorrelation lag 1 ϕ_τ , from t to $t + k$, such that $B_k = \prod_t^{t+k} \phi_\tau$. This is equivalent to use a Periodic Autoregressive lag 1 model (Bartolini et al. 1988) as forecast model. Using of an average climatic year as climatic disturbance would filter out extremes; we use instead an observed inflow at each control time-step, randomly extracted from the observed inflow data, $\{d^{cl}\}_{k=1}^h \in D^{cl} = \{d^{obs}\}_{\tau=k}^{k+h}$. When the reservoir residence time is large enough, its slow dynamic will serve as low pass filter, which will average out the effects of different inflow scenarios used at each time-step. This is expected to have little effect on the reservoir volume signal. In this experiment, we consider reservoir management having three objectives: flood and drought prevention, and energy production. Flood and drought prevention are represented by the cost function, gt^{tg} , in Equation (8b): keeping the total discharge as close as possible to the target flow, $q_{tg} = 200m^3/s$, attains both flood and drought prevention. The electricity production is proportional to the product of hydraulic head into discharge through the turbines, $\propto \Delta h_t \cdot u_t^r$. This Equation is a convex function that must be maximized. We cannot use this function directly as objective within the optimization problem because we use a convex optimization method, namely the interior-point method, which does not guarantee, in this case, the convergence to the global optimum (Boyd and Vandenberghe 2004). The Objective function for energy production will be, instead, Equation (8a), which is the function for energy production linearized as in (Raso et al. 2015). In Equation (8a), $\Delta h_0 = 52.5 \text{ m}$ is the nominal hydraulic head, $u_0^r = 500 \text{ m}^3/s$ the nominal release through turbines, and $A_0 = 4.6e8 \text{ m}^2$ the nominal reservoir surface. The negative sign means that its value must be maximized.

$$g_t^e = -(\Delta h_0 \cdot u_t^r + u_0^r / A_0 \cdot v_t) \quad (8a)$$

$$g_t^{tg} = (u_t^r + u_t^s - q_{tg})^2 \quad (8b)$$

$$g_t = w_e \cdot g_t^e + w_{tg} \cdot g_t^{tg} \quad (8c)$$

The aggregated objective function g_t , in Equation (8c), is the weighted sum of g_t^e and g_t^{tg} . Flood and drought prevention objectives have higher priority on energy production, therefore w_{tg} is larger than w_e , being 0.8 and 0.2, respectively; the decaying factor r in Equation (4a) is set to 0.973, selected to be close to zero at the end of the 3 year horizon. The reservoir average residence time is in fact about one year. The system state, at the end of the 3 year optimization horizon, contains a negligible trace of the initial system state. The final state, having little influence on the first release decision, can be weighted much less in the optimization. Other values may be tested to analyse the results sensitivity to this parameter. We use 10 independent triangles, defined by T_i , S_i , and L_i as in Table 1. Triangles are selected so that the resulting composition has a higher degree of freedom, therefore a higher accuracy, at beginning of the control horizon. In this case we selected five asymmetric triangles with increasing left and right base length as the peak time T gets larger. Other symmetric triangles with a larger base length are used to catch the system periodicity.

Results

To evaluate the proposed method, we analyze both the role of input structuring and that of uncertainty, isolating their effects in departing from the optimal solution. We consider three solutions: i) Infinite Horizon MPC using triangular input structuring and realistic forecast, ii) Infinite Horizon MPC using triangular input structuring and perfect forecast, iii) Infinite Horizon MPC with no input structuring and perfect forecast. Comparing first and second case shows the loss due to uncertainty; comparing second and third shows the loss

due to input structuring. In the third case, solving the optimal control problem requires a large computation time, and it is not applicable in reality. This experiment serves, however, as upper boundary of system performance. We use some indicators to measure performance: i) Average yearly energy production, for electricity production, ii) days per year when flow is lower than $100 \text{ m}^3/\text{s}$, for drought prevention, iii) days per year when flow is larger than $800 \text{ m}^3/\text{s}$, for flood prevention, and iv) the quadratic distance from the target discharge, as used within the objective function, for both drought and flood prevention. The first indicator is to be maximized, the others to be reduced. We run a four-year simulation, from the 1st January 2005 to the 31st December 2008.

Table 2 presents a summary of simulation results for the three cases under evaluation, for the considered indicators. This table shows that both uncertainty and input structuring leads to a reduction of system performance. Performance loss is relatively small if compared to the loss due to the presence of a relevant uncertainty for energy production, and comparable for flood and drought prevention indicator. Simulation using input structuring and realistic forecast, if compared to simulation using input structuring and perfect forecast, shows a small deterioration on drought prevention, which is a slow, predictable process. On the other hand energy production, which is a combination of short-long term goals decreases moderately. Flood prevention, being the effect of a faster and less predictable process, shows a major worsening. Results from simulation using structured and un-structured inputs, both using perfect forecast, are nearly equivalent, suggesting that input structuring can be applied with little effect on results. The performance loss can be reduced by increasing the number of basis function (i.e. triangles), even if this will lead to an increase of computational time. Using the interior point method in a Matlab[®] optimizer, on a processor 2,9 GHz Intel Core i7, the computation time required to find a solution was 12-20 seconds for the case using input structuring, and about 4 hours for the case without input structuring. The latter is patently unacceptable for practical application.

Figure 3 shows discharge decisions and reservoir volume for the first year of closed-loop simulation. Discharge decision on simulation using real forecast is noisy: decision is influenced by the random extraction of a future discharge scenario. Discharge increases in early august, as precautionary measure, in anticipation to an high flow which eventually does not occurs. The reservoir filter out the high frequency variability of release decisions and inflow. Reservoir volume on simulation using real forecast is, on the rising part, lower than volume on simulation using perfect forecast, using less efficiently the reservoir capacity. Figure 3 shows the presence of few small violations on volume constraint, due to the implementation of volume constraints as soft ones. In this system, in fact, constraints on the reservoir volume represent a legal, rather than a physical condition, therefore small violations are acceptable.

Figure 4 shows open-loop optimization results at a specific decision instant: plot (a) shows the input and output discharges, and plot (b) the resulting reservoir volume. For both plots we show the nominal inflow, for which the release decisions are optimised, and the observed inflow, that will actually happen, for the first year of open-loop simulation. The controller tries to balance the hydrological variability to keep the total discharge as close as possible to the target discharge. In the dry season the outflow is higher than the inflow, and the reservoir is drawn down, keeping a low water volume until the rising part of the hydrograph, in preparation of the peak. The reservoir is eventually filled, and spillages are minimized. The plot below shows state constraints violation, at around $t = 150$ and $t = 220$. These constraints violation are small, being about 1% of the reservoir volume, and sufficiently far in time from the initial release decision. Their influence on the latter is very likely to be minimal, and therefore they do not affect the control quality. If this is not the case, weight w_x in Expression (5) can be increased. For $w_x \rightarrow \infty$, in fact, the soft constraint $c_k(\mathbf{x}_k)$ “approaches, the behavior of a hard constraint.

Effects of input structuring are evident in plot (a), where a single triangle take into account the entire high flow period. Plot (b) shows a large divergence between the effects on reservoir volume of observed and nominal discharge, which adds evidence that robustness to

uncertainty is a relevant issue for the proposed method.

CONCLUSIONS

This paper presented an Infinite Horizon Model Predictive Control method specifically designed for reservoir operations. Input structuring can be employed thanks to smoothness of the control signal. The control smoothness is related to the slow dynamic of reservoir systems: the reservoir filter out the high variability of inflow, therefore the control signal (i.e. the releases) varies slowly too. Basis functions, often employed in system identification, were used here for control. Input structuring reduces the computational complexity related to the horizon length, and not to other sources of complexity, such as the system size or the number of objectives. For this reason the proposed methodology is applicable to large systems as MPC when applied for short-term operation.

We selected triangular basis function for their simplicity to be communicated and defined. Triangular basis function can handle hypercube constraints on inputs, and we gave some indication on how to select these triangles. Alternative families of basis function that could have been employed are, among others, a combination of sines and cosines with different frequencies, or a combination of exponential functions with different decay rates. We leave to further research the exploration of effective basis functions.

We suggested selecting progressive independent triangles in the early stage, and periodic ahead on time. In water systems, in fact, both water demand and hydrological processes are periodic. The proposed method largely reduces the number of variables to be optimized, reducing the optimization problem complexity. We tested the proposed method for the operational management of Manantali reservoir, on the Senegal River, with the objective of flood and drought prevention, and energy production. Analysis shows that input structuring may have a negative effect on the system performance, mostly related to fast, uncertain processes. The extent of performance loss depends on which indicator is considered, being small or, for some indicator, equivalent to the performance loss due to the presence of inflow uncertainty.

Selecting the proper number of basis functions is the result of a trade-off between system performance and computation time. A larger number of triangles would increase both the computation time and the performance. The latter, however, will saturate. Further research could explore how performance and computation time change in function of number of basis functions.

The question on how to deal with forecast uncertainty is still open in Infinite Horizon MPC using input structuring. We suggest using the proposed method in combination with a compatible synthetic robust MPC algorithm, selected from the vast control literature on the topic. Notwithstanding this limitation, the method we propose can potentially handle large systems, made of multiple reservoirs or routing downstream of the reservoir, offering an optimal compromise between short and long term objectives.

ACKNOWLEDGMENTS

Luciano Raso's work is funded by the AXA Research Fund.

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List of Tables

1	Triangular function specification: Peak time (T), Left base (L), and right side (R) defining the 10 triangles.	24
2	Results for the analyzed configurations	25

TABLE 1. Triangular function specification: Peak time (T), Left base (L), and right side (R) defining the 10 triangles.

i	1	2	3	4	5	6	7	8	9	10
T	1	7	18	41	87	178	269	360	543	726
L	0	6	11	23	46	91	91	91	182	182
R	6	11	23	46	91	91	91	182	182	182

TABLE 2. Results for the analyzed configurations

Indicator	Electricity production	Drought prevention	Flood prevention	Quadratic cost (Flood and Drought)
$\uparrow \max / \downarrow \min$ [<i>Unit</i>]	$\uparrow [\times 10e5 MWh/year]$	$\downarrow [d/year]$	$\downarrow [d/year]$	$\downarrow [(m^3/s)^2] \times 10e7$
Basis functions, real forecast	9.1	65	7	6.3
Basis function, perfect forecast	9.5	75	0	5.5
No structuring, perfect forecast	9.6	71	0	4.3

List of Figures

1	Example of basis triangular functions. For triangle 3, the peak time	
	T , the left base L and the right base R are highlighted.	27
2	Inflow at Soukoutali, from 1 January 1950 to 31 December 2013, daily time-step.	28
3	Simulation results, from 1 January 2005 to 31 December 2005	29
4	Open loop results at $t = 12$ March 2005, for k from 1 to 365	30

FIG. 1. Example of basis triangular functions. For triangle 3, the peak time T , the left base L and the right base R are highlighted.

FIG. 2. Inflow at Soukoutali, from 1 January 1950 to 31 December 2013, daily time-step.

(a) Release

(b) Volume

FIG. 3. Simulation results, from 1 January 2005 to 31 December 2005

FIG. 4. Open loop results at $t = 12$ March 2005, for k from 1 to 365