Supplemental Appendix to
Managing Food Price Volatility in a Large Open Country
The Case of Wheat in India

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A Estimation of Insulation Achieved by Current Trade Policies

To estimate the degree of insulation achieved by current trade policies, excluding trade costs (for which time-series data are not available) and assuming that at least some trade takes place between India and the RoW, eqs (18)—(21) can be simplified to $P^i = \alpha {P^w}^{1+\beta}$. Since these two prices are likely to be cointegrated, estimating this relationship in levels may capture their long-run dynamics; given the model's focus on dynamics around a steady state, however, the primary interest is in the short-run price transmission elasticity.

India's prices are annual producer prices from FAOSTAT,[1] converted to US dollars, and world prices are US prices of wheat as reported in the International Monetary Fund's International Financial Statistics. All prices are converted to real terms using the US consumer price index. The exchange rate and the consumer price index are from the International Financial Statistics. The sample period is from 1966 to 2008.

An augmented Dickey-Fuller test is used to test for the presence of unit root in the price series. The values of the augmented Dickey-Fuller test are presented in Table A1 for the logarithm of prices. The tests are conducted on equations with a constant (first column) and with a linear trend (second column). The null hypothesis of a unit root cannot be rejected for either of the price series. Differencing prices, the null is rejected, and prices are integrated of order 1. The long-run equilibrium relationship are then estimated; the ordinary least squares estimates are:

$$\ln P_t^i = \underset{(0.525)}{0.138} + \underset{(0.092)}{0.996^{***}} \ln P_t^w, \qquad \text{Adj-R}^2\text{: } 0.73,$$

where the numbers in parentheses are the standard errors. The augmented Dickey-Fuller tests for cointegration are reported in Table A1. The unit root null can be rejected at the 5% significance level. Thus prices are cointegrated.

[1] Available at http://faostat.fao.org/, accessed March 21, 2014.

Using the residuals of the long-run equilibrium equation (noted EC below), the error correction model is estimated:

$$\underset{}{\Delta \ln P_t^i} = \underset{(0.019)}{-0.021} + \underset{(0.106)}{0.244^{**}} \; \Delta \ln P_t^w - \underset{(0.080)}{0.145^{*}} \; EC_{t-1}, \quad \text{Adj-R}^2\text{: 0.11; DW: 2.21.}$$

The speed of adjustment parameter is negative, as expected, but significant only at the 10% level. The coefficient of short-run price transmission is significant at the 5% level and indicates a short-run elasticity of 0.24, implying the coefficient of trade insulation to be –0.76. Adding lags of $\Delta \ln P^i$ and $\Delta \ln P^w$ would increase the statistical significance of the coefficients of short-run transmission and speed of adjustment but would not affect the estimated values.

Table A1: Augmented Dickey-Fuller unit root test statistics

Variable	Constant	Trend
Price		
India	-1.49 (1)	-0.73 (1)
US	-1.42 (2)	-3.15 (1)
Price differential		
India	-5.24*** (1)	-5.57*** (1)
US	-4.85*** (1)	-4.79*** (1)
Residual from cointegration equation	-3.58** (1)	-4.00* (1)

Notes: Critical values (from MacKinnon, 2010) for 43 observations for testing for variable stationarity are for 10%, 5%, and 1%, respectively, -2.60, -2.93, and -3.59 with a constant, and -3.19, -3.52, and -4.19 with a trend. For testing cointegration, they are -3.14, -3.48, and -4.16 with a constant and -3.66, -4.01, and -4.71 with a trend. Number of lags in parenthesis. Lag selection is achieved according to the Akaike information criteria, considering a maximum of 3 lags. *, **, and *** indicate statistical significance at 10%, 5%, and 1% levels, respectively.

One can note that neglecting the trade costs, because of the lack of data, in the estimation of the price insulation parameter can lead to an upward bias in the estimation of the coefficient of trade insulation. Indeed, when India switches from being an importer to an exporter because of a higher world price, there is a change in the trade margin which goes from positive to negative values. This change in the trade margin would be capture in our specification as trade insulation. To correct for this potential bias, we estimated the model also with dummies indicating the trade regime in the long-run equation and with dummies indicating any changes in the trade regime in the short-run equation. This does not significantly affect the results.

B The optimal policy problem

To identify the optimal policy, complementarity equation (3) must be expressed as a combination of inequalities and equalities, because complementarity equations cannot be included directly as constraints in a maximization problem. A positive slack variable, ϕ, is introduced with its associated complementarity slackness conditions:

$$\phi_t = P_t^w + k^w - \delta E_t P_{t+1}^w, \tag{S1}$$

$$S_t^w \phi_t = 0. \tag{S2}$$

Following Marcet and Marimon (2011), the optimal policy problem under commitment can be expressed as a saddle-point functional equation problem:

$$\begin{aligned}
&J\left(A_t^i, A_t^w, \mu_{t-1}^i \epsilon_t^i, \lambda_{t-1} + \mu_{t-1}^w \epsilon_t^w\right) \\
&\quad = \min_{\Gamma_t} \max_{\Omega_t} \left\{ -d^i \frac{{P_t^i}^{1+\alpha^i}}{1+\alpha^i} + P_t^i A_t^i - a^i \bar{H}_t^i - \frac{b^i {\bar{H}_t^i}{}^2}{2} - \left(k^i + P_t^i\right) S_t^i - X_t^i\left(\theta_{i,w} + P_t^i - P_t^w\right) \right. \\
&\quad - X_t^w\left(\theta_{w,i} + P_t^w - P_t^i\right) - \frac{K}{2}\left(P_t^i - \bar{P}^i\right)^2 + \chi_t^i\left[A_t^i + X_t^w - D^{\mathrm{i}}\left(P_t^{\mathrm{i}}\right) - S_t^{\mathrm{i}} - X_t^{\mathrm{i}}\right] \\
&\quad + \chi_t^w\left[A_t^w + X_t^i - D^{\mathrm{w}}\left(P_t^{\mathrm{w}}\right) - S_t^{\mathrm{w}} - X_t^{\mathrm{w}}\right] + \lambda_t\left(\phi_t - P_t^w - k^w\right) + \left(\lambda_{t-1} + \mu_{t-1}^w\right) P_t^w \\
&\quad + \xi_t S_t^w \phi_t + \mu_{t-1}^i \epsilon_t^i P_t^i - \mu_t^i\left(a^i + b^i \bar{H}_t^i\right) - \mu_t^w\left(a^w + b^w \bar{H}_t^w\right) \\
&\quad \left. + \delta E_t\left[J\left(S_t^i + \bar{H}_t^i \epsilon_{t+1}^i, S_t^w + \bar{H}_t^w \epsilon_{t+1}^w, \mu_t^i \epsilon_{t+1}^i, \lambda_t + \mu_t^w \epsilon_{t+1}^w\right)\right] \right\},
\end{aligned} \tag{S3}$$

where $\Omega_t = \{S_t^r \geq 0, P_t^r, \bar{H}_t^r, X_t^r \geq 0, \phi_t \geq 0\}$ and $\Gamma_t = \{\chi_t^r, \mu_t^r, \lambda_t, \xi_t\}$. This problem gives the following first-order conditions (after substitution of the expressions given by envelop theorem):

$$S_t^i \geq 0 \perp P_t^i + k^i + \chi_t^i - \delta E_t\left(P_{t+1}^i + \chi_{t+1}^i\right) \geq 0, \tag{S4}$$

$$S_t^{\mathrm{w}} \geq 0 \perp \chi_t^w - \xi_t \phi_t - \delta E_t(\chi_{t+1}^w) \geq 0, \tag{S5}$$

$$\mu_t^r b^r = \delta E_t(\epsilon_{t+1}^r \chi_{t+1}^r), \tag{S6}$$

$$K\left(P_t^i - \bar{P}^i\right) + \chi_t^i D^{i\prime}\left(P_t^i\right) = \mu_{t-1}^i \epsilon_t^i, \tag{S7}$$

$$\chi_t^w D^{w\prime}(P_t^w) + \lambda_t - \lambda_{t-1} - X_t^i + X_t^w = \mu_{t-1}^w \epsilon_t^w, \tag{S8}$$

$$X_t^r \geq 0 \perp P_t^r - P_t^s + \theta_{r,s} + \chi_t^r - \chi_t^s \geq 0 \text{ for } s \neq r, \tag{S9}$$

$$A_t^r + X_t^s = D^r(P_t^r) + S_t^r + X_t^r \text{ for } s \neq r, \tag{S10}$$

$$\phi = P_t^w + k^w - \delta E_t P_{t+1}^w, \tag{S11}$$

$$\phi_t \geq 0 \perp -\lambda - \xi_t S_t^w \geq 0, \tag{S12}$$

$$\phi_t S_t^w = 0, \tag{S13}$$

$$\delta E_t(P_{t+1}^{\mathrm{r}} \epsilon_{t+1}^r) = \Psi^{\mathrm{r}\prime}(\bar{H}_t^{\mathrm{r}}). \tag{S14}$$

C Computational details

The rational expectations storage model does not allow a closed-form solution; it must be approximated numerically. The numerical algorithm used here is based on a projection method with a collocation approach. The results were obtained using MATLAB R2013b and solved using the rational expectations solver RECS v0.6 (Gouel, 2013). Policy functions were approximated using cubic spline with 15 nodes for availability in India and in the RoW, 9 nodes for public stocks, and 7 nodes for co-state variables in the optimal policy problem. For terms with expectations, shocks were discretized using 5-point Gauss-Hermite quadratures. For further technical details, see the program code available upon request.

Two nested algorithms were used to find the parameters defining the optimal simple rules. The outer algorithm adjusts the policy parameters of the rules to maximize intertemporal social welfare given by eq. (29) by applying an optimization solver. The optimization solver is the nonlinear programming solver *fmincon* available in MATLAB. Its interior-point algorithm is used along with a gradient calculated by central finite differences. For each iteration of the outer algorithm, the inner algorithm (described above) solves the rational expectations problem for the new set of policy parameters.

Two types of results are produced in the study: statistics on the asymptotic distribution and welfare results. Statistics on the asymptotic distribution are calculated over 100,000 observations from random outcomes of the stochastic variables, obtained by simulating 500 paths for 220 periods and after discarding for each path the first 20 observations as burn-in period. The random shocks are the same for all policies. The simulations are done following the approach proposed in Wright and Williams (1984): for any given value of the state variables, the approximated policy rules obtained by solving for the rational expectations are used to approximate expectations, and using these approximated expectations, the equilibrium equations are solved for the value of the response variables. This time-consuming method yields results that are much more precise than those obtained by directly using the approximated policy functions. Since all welfare terms correspond to discounted infinite sums, such as eq. (24), they are calculated by transformation to a recursive formulation and value function iteration. To evaluate welfare, it is assumed that in the initial period the economy is at the deterministic steady state.

References

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