Decoupled–dynamics distributed control for strings of nonlinear autonomous agents

Serban Sabau, Irinel-Constantin Morarescu, Lucian Busoniu, Ali Jadbabaie

Decoupled–Dynamics Distributed Control for Strings of Nonlinear Autonomous Agents

Şerban Sabău[♯], Irinel–Constantin Morărescu[⋆], Lucian Buşoniu[‡] and Ali Jadbabaie[†]

Abstract— **We introduce a novel distributed control architecture for a class of nonlinear dynamical agents moving in the "string" formation, while guaranteeing trajectory tracking and collision avoidance. Each autonomous agent uses information and relative measurements only with respect to its predecessor in the string. The performance of the scheme is entirely scalable with respect to the number of agents in formation. The scalability is a consequence of the "decoupling" of a certain bounded approximation of the closed–loop equations, entailing that individual, local analyses of the closed–loops stability at each agent will in turn guarantee the aggregated stability of the entire formation. An efficient, practical method for compensating communications induced delays is also presented.**

I. Introduction

The distributed control of autonomous agents moving in the "string" (or "line") formation, also know as platooning is a fifty years old problem, going back to the work [1] of Levine & Athans on *intelligent highway systems*. Since no existing control solution was deemed satisfactory from an applications standpoint, consistent research efforts are still being invested for developing practical platooning control schemes. The difficulty of the problem stems from the notorious lack of scalability of networks of dynamical agents, which causes the performance of existing control schemes to depend not only on the number of vehicles in formation but on the vehicle's position in formation, as well [7]. Platooning may also be seen as a particular instance of flocking for multi-agent systems. The flocking problem has been intensively studied in the last decade for agents with linear [3], [4], [15], [12] or nonlinear [2], [16] dynamics. Generally speaking, two important features characterize the flocking behavior of autonomous agents: cohesion of the formation and collision avoidance. In multi-agent systems they are implemented as connectivity/topology preservation [18], [17], [14], [13] and collision avoidance [3], [4], [2], respectively.

Just as for platooning applications (and more generally, in distance based formation control) the regulated measures are the relative (interspacing) distances between consecutive agents, for several other meaningful applications in networks of dynamical agents, the regulated signals in trajectory tracking or synchronization problems represent relative measurements such as relative velocities (with respect to neighboring agents), local clocks offsets or phase differences between (neighboring) coupled oscillators. In this paper, we present preliminary results on a class of novel distributed control policies where the relative measurements (with respect to the neighboring agents) are used by the local sub–controllers in conjunction with the knowledge of the control actions of the sub-controllers at the neighboring agents. It turns out that the performance of the resulted distributed control schemes considerably outperforms the distributed architectures based solely on relative measurements.

The problem considered in this paper can be rephrased as a multi-agent flocking problem with collision avoidance. The literature on this topic is very rich and considers both directed or undirected, fixed or time-varying interconnection graphs. The objective of the control scheme is to achieve the synchronization of the trajectories of all agents in the formation with the trajectory of the leader agent. Such trajectory tracking must be achieved while ensuring zero (steady–state) errors of the regulated measures (in our case the relative speed between two consecutive agents) and while avoiding collisions, *i.e.* performing the needed longitudinal steering (brake/throttle) maneuvers that guarantee the avoidance of collision with the preceding vehicle.

A. Contributions of this Paper

The approach taken here is based on the use of Artificial Potential Functions [3], [2].When compared to the state-of-the-art, our results represent a consistent extension of existing methods [3], [2], from at least three perspectives. Firstly, it guarantees stability, velocity matching (trajectory tracking) and collision avoidance even for *directed topologies*, as illustrated by the distributed control scheme reported here. Secondly, it achieves complete *scalability* with respect to the number of agents in formation [2] and also with respect to the connectivity of the inter-agents communications graph. Finally, a practical adaptation of the distributed controller is able to completely compensate the wireless communications induced time delays.

By comparison, the main result in [2] imposes that all the minor matrices of the weighted Laplacian matrix associated with the interconnection graph are positive definite and lower bounded by a certain constant. This basically requires the

[♯]Şerban Sabău is with the Electrical and Computer Engineering Dept., Stevens Institute of Technology, Hoboken, New Jersey, U.S.A. email: ssabau@stevens.edu

[⋆] I.-C. Morărescu is with Université de Lorraine, CRAN, UMR 7039 and CNRS, CRAN, UMR 7039, 2 Avenue de la Forêt de Haye, 54506 Vandœuvre-lès-Nancy, France. email: constantin.morarescu@univ-lorraine.fr

[‡] L. Buşoniu is with Technical University of Cluj–Napoca, Romania. email:lucian@busoniu.net

[†] Ali Jadbabaie is with the Sociotechnical Systems Research Center, Massachusetts Institute of Technology. email: jadbabai@mit.edu.

maximization of the eigenvalues of the weighted Laplacian matrix, which can be interpreted as the maximization of the number of interconnections in the underlined graph (see [10]) together with the maximization of its diagonal elements (see Geršgorin disk theorem [11]). It is worth noting that the first requirement involves the transmission of the exact state of the leader to many agents in the formation, while the second requirement represents high local control gains.

The most interesting feature of our proposed scheme is the fact that it achieves complete *scalability* with respect to the number of vehicles in the string. Such scalability features come as a consequence of the complete "decoupling" of a certain bounded approximation of the closed–loop equations, such that by performing solely individual, local analyses of the stability at each agent, will in turn guarantee the stability of the aggregated formation.

B. Paper Organization

The paper is organized as follows: in Section II we introduce the general framework and we formulate the platooning control problem. Section III provides a preliminary description of the novel distributed control architecture introduced in this work along with a first glimpse at the closed-loop dynamics "decoupling" featured by the control scheme. Section IV contains the main result as it delineates the guarantees for stability, velocity matching and collision avoidance. Finally, Section V outlines a practical delays compensation mechanism while Section VI provides an illustrative numerical example, worked out on a simplified dynamical model for road vehicles.

II. General Framework and Problem Statement

A. Preliminaries

Definition 2.1: The σ–norm of a vector x is defined as

$$\|x\|_\sigma \overset{def}{=} \frac{1}{\sigma}\left[\sqrt{1+\|x\|_2^2}-1\right] \tag{1}$$

with σ is a positive constant. This is a class $\mathcal{K}_\infty$ function of $\|x\|_2^2$ and is differentiable everywhere.

Definition 2.2: A set Ω is said to be *forward invariant* with respect to an equation, if any solution $x(t)$ of the equation satisfies: $x(0) \in \Omega \Longrightarrow x(t) \in \Omega$, $\forall t > 0$.

Definition 2.3: Artificial Potential Function (APF). The function $V_{k,k-1}(\cdot)$ is a differentiable, nonnegative, radially unbounded function of $\|z\|_\sigma$ satisfying the following properties:

(i) $V_{k,k-1}(\|z\|_\sigma) \to \infty$ as $(\|z\|_\sigma) \to 0$,

(ii) $V_{k,k-1}(\|z\|_\sigma)$ has a unique minimum, which is attained at $\|z\|_\sigma = \delta_k$, with δ_k being a positive constant.

B. The Problem: Trajectory Tracking of the String Formation

We consider a *homogeneous* group of n agents (*e.g.* autonomous road vehicles) moving along the same (positive) direction of a roadway, with the origin at the starting point of the leader. The dynamical model for the agents, relating the control signal $u_k(t)$ of the k–th vehicle to its position $y_k(t)$ on the roadway, is given by

$$\dot{y}_k(t) = v_k(t), \quad \dot{v}_k(t) = f(v_k(t)) + u_k(t)\ ; \tag{2a}$$

$$y_k(0) = -\sum_{j=0}^{k} \ell_j, \quad v_k(0) = 0. \tag{2b}$$

where $v_k(t)$ is the instantaneous speed of the k–th agent, $u_k(t)$ is its command signal and ℓ_k is the initial interspacing distance between the k–th agent and its predecessor in the string. Throughout the sequel we will use the notation

$$y_k = G_k \star u_k \tag{3}$$

to denote (especially for the graphical representations) the input–output operator G_k of the dynamical system from (2a), with the initial conditions (2b).

Assumption 2.4: The index "0" is reserved for the *leader vehicle*, the first vehicle in the string, for which we assume that there is no controller on board and consequently the command signal $u_0(t)$ will represent a *reference* signal for the entire formation.

In the rest of the paper we often forget the time argument of the involved variables. It is worth noting that when the argument is missing it has to be thought as t. Let us further define

$$z_k \overset{def}{=} y_{k-1}-y_k, \quad z_k^v \overset{def}{=} v_{k-1}-v_k \quad \text{for} \quad 1 \le k \le n, \tag{4}$$

to be the interspacing and relative velocity error signals respectively (with respect to the predecessor in the string). By differentiating the first equation in (4) it follows that $\dot{z}_k(t) = z_k^v(t)$, therefore implying that constant interspacing errors (in steady state) are equivalent with zero relative velocity errors and also allowing to write the following time evolution for the relative velocity error of the k–th vehicle

$$\dot{z}_k^v = f(v_{k-1}) - f(v_k) + u_{k-1} - u_k. \tag{5}$$

III. A Novel Distributed Control Architecture

The inherent difficulty in platooning control is rooted in the *nested* nature of the interdependencies between the regulated signals. Specifically, the regulated errors (*e.g.* interspacing errors or relative velocity errors) at the k–th agent depend on the regulated errors of its predecessor (the $(k-1)$–th agent) and so on, such that by a recursive argument – going through all the predecessors of the k–th agent – they ultimately depend on the trajectory of the leader vehicle, which represents the reference for the entire formation.

We introduce a novel control architecture featuring a certain beneficial "decoupling" properties of the closed–loop dynamics that avoid the pitfalls of the aforementioned *nested* interdependencies. The distributed control policies rely only on information locally available to each vehicle. For the scope of this paper, we consider non–linear controllers built on the so-called Artificial Potential Functions (APF), in particular we will look at control laws of the type

$$u_k = u_{k-1}+\beta_k(v_{k-1}-v_k)-\nabla_{y_k} V_{k,k-1}(\|y_{k-1}-y_k\|_\sigma) \tag{6}$$

with $k \ge 1$, where each of the $V_{k,k-1}(\cdot)$ functions is an Artificial Potential Function [2, Definition 7] with β_k being a proportional gain to be designed for supplemental performance requirements. With the notation from (4), the control policy (6) for the k–th vehicle becomes

$$u_k = u_{k-1} + \beta_k z_k^v - \nabla_{y_k} V_{k,k-1}(\|z_k\|_\sigma) \tag{7}$$

and it can further be written as the sum of the following two components: firstly, the control signal u_{k-1} of the preceding vehicle, which is received onboard the k–th vehicle via wireless communications (*e.g.* digital radio). Secondly, the *local* component, which we denote with

$$u_k^\ell \stackrel{def}{=} \beta_k z_k^v - \nabla_{y_k} V_{k,k-1}(\|z_k\|_\sigma) \tag{8}$$

and which is based on the measurements (4) which are *locally* available to the k–th vehicle, as they can be acquired via onboard LIDAR sensors. Thus, the k–th control law reads:

$$u_k = u_{k-1} + u_k^\ell.$$

The control policy (6) entails a highly beneficial "decoupling" feature of the closed–loop dynamics at each agent, as we simply illustrate next. Firstly, note that by plugging (6) into (5) we obtain the following closed–loop error equations at the k–th agent:

$$\dot{z}_k^v = f(v_{k-1}) - f(v_k) - \beta_k z_k^v + \nabla_{y_k} V_{k,k-1}(\|z_k\|_\sigma). \tag{9}$$

The following result will be instrumental in the sequel. Consider the following Lyapunov candidate functions:

$$L_k\big(z_k(t), z_k^v(t)\big) \stackrel{def}{=} \frac{1}{2}\Big(V_{k,k-1}(\|z_k(t)\|_\sigma) + \\ + z_k^{v\top}(t) z_k^v(t)\Big), \text{ with } 1 \le k \le n. \tag{10}$$

Lemma 3.1: The differential of the Lyapunov candidate function $L_k(\cdot,\cdot)$ introduced in (10) along the trajectories of (2) and (6) is given by

$$\frac{d}{dt} L_k(z_k(t), z_k^v(t)) = z_k^{v\top}(t)\Big(f\big(v_{k-1}(t)\big) - f\big(v_k(t)\big)\Big) \\ - \beta_k z_k^{v\top}(t) z_k^v(t)\,, \tag{11}$$

and does not depend on the choice of the APFs $V_{k,k-1}(\cdot)$.

IV. Decoupling control design

As we will show in this section, in the proposed method the control actions of each agent are based exclusively on receiving information from its predecessor. While the control gains are strictly related to the reactivity of the system (*i.e.* faster systems needs higher controller gains) our scheme does not require making the leader's information (instantaneous speed or acceleration) available to other agents in the string (the virtual leaders from [2]). Our directed communications scheme, necessitates a minimal information exchange and sensing radius for all agents (each agent performs measurements and receives information only with respect to its predecessor).

The following result is the main result of this Section, as it delineates a "decoupling" property of the closed–loop dynamics, achieved by the (6) type control policy along with velocity matching and collision avoidance. Due to space limitations we do not provide proofs in this paper.

Theorem 4.1: If the function $f(\cdot)$ from (2a) satisfies the global Lipshitz–like condition [2, Assumption 1]

$$(v_2 - v_1)^\top \big(f(v_2) - f(v_1)\big) \le \alpha \|v_2 - v_1\|_2^2, \quad \forall\, v_1,\, v_2 \tag{12}$$

then for all type (6) control laws with that $\beta_k > \alpha$ the following hold:
(A) Given the Lyapunov function L_k introduced in (10), *local* to the k-th agent, the sub–level sets $\Omega_c^k \stackrel{def}{=} \{(z_k, z_k^v) | L_k \le c,\ \text{with}\, c > 0\}$ of L_k are compact and they represent forward invariant sets for the *local* closed–loop dynamics (9) of the k–th vehicle.
(B) The controller (6) guarantees velocity matching and collision avoidance. Furthermore, considering $c = 2L_k(z_k(0), z_k^v(0))$ there exists η_c such that

$$\|y_k - y_{k-1}\|_2 > \eta_c, \forall t \ge 0.$$

Therefore, a pre–specified safety distance can be imposed by the initial conditions.

Proposition 4.2: Given $L_k(\cdot,\cdot)$ as introduced in (10), the string formation's steady–state configuration is attained at the minimum of the following formation-level Lyapunov function:

$$L(z(t), z^v(t)) \stackrel{def}{=} \frac{1}{2}\sum_{k=1}^{n} L_k(z_k(t), z_k^v(t)) \tag{13}$$

which coincides component–wise with the minima of the Lyapunov functions (10) *local* to the k-th agent. Furthermore, the level sets of L given by $\Omega_c \stackrel{def}{=} \{(z, z^v) | L \le c,\ \text{with}\, c > 0\}$ are compact and they represent forward invariant sets for the closed–loop dynamics of the entire formation, as given in (2) and (6) with $1 \le k \le n$. Consequently, velocity matching and vehicles' collision avoidance are achieved, *without the need for inserting exact leader information in the formation* (the "virtual leaders" from [2]) while maintaining a safe interspacing distance.

V. Delay compensation mechanism

The difficulties caused for networked systems by the communications induced delays and time jittering have been a topic of intensive study for decades. For platooning practical applications it has been argued in [6] that the (low latency) time delays induced by the wireless communications, even if assumed time-invariant and homogeneous[1], irremediably alter the performance of the control scheme. The cause of this this phenomenon is the well understood fact that the delays propagate through the closed-loops towards the back of the platoon and furthermore they *accumulate* in a manner depending on the number of vehicles in formation, ultimately leading to string instability [6]. For the case of linear dynamical agents, the very recent results from [5] provide a functional scheme for compensating the effect of the communications delays by employing GPS time based synchronization mechanisms. However, the aforementioned

[1]For digital radio wireless systems such as WiFi, Bluetooth or Zigbee, the corresponding time-delays have low latencies but they are time-varying, taking values around a nominal delay of about 20 ms.

method [5, Section VI] of essentially "incorporating" the *synchronization delay* in the model of the plant cannot be adapted to nonlinear dynamical models. In this section we introduce a practical method for adapting the distributed controller of Section IV to be able to compensate the communications delays, while essentially preserving all the performance features from the delay free case.

A. An Adaptation of Time-Headways for Velocity Matching

Classical results in platooning control [19], [20], [21] proved that a considerable improvement of performance can be obtained by adequately modifying the regulated interspacing distance (for each vehicle k) $z_k = y_{k-1} - y_k$ such as to include a factor $-h\dot{y}_k(t)$ proportional with the speed of the current vehicle. The resulted interspacing policies (dubbed time headways) become $z_k = y_{k-1}(t) - y_k(t) - h\dot{y}_k(t)$ and provide a spacing in time rather than distance (between two consecutive vehicles). Up until the recent distributed scheme introduced in [5] - for linear dynamical agents and linear controllers, string stability could only be achieved via the use of time headways policies [7]. Furthermore, relatively large values of the time headway $h > 0$ were necessary in order to guarantee both string stability and the improvement of disturbances attenuation at mid and high frequency[2]. The generally adopted value for highway platooning (which became standardized at some point) is $h = 1$ second. The main drawback of such large time headways is that they drastically impair the tightness of the formation, reducing the traffic throughput and any potential fuel savings achievable by the air drag reduction.

We introduce next a novel method for delays compensation that combines the GPS time based synchronization mechanism from [5, Section VI] with an adaptation of the time headways to the velocity matching objective of our distributed controller from Section IV. Firstly, let us revamp as follows the definitions (4) of the interspacing distances z_k and of the regulated relative speeds z_k^v respectively at the k-th vehicle:

$$\begin{aligned} \widetilde{z}_k(t) &\stackrel{def}{=} y_{k-1}(t-\theta) - y_k(t-\theta) - \theta\dot{y}_k(t-\theta), \qquad (14)\\ \widetilde{z}_k^v(t) &\stackrel{def}{=} v_{k-1}(t-\theta) - v_k(t-\theta) - \theta\dot{v}_k(t-\theta), 1 \le k \le n, \end{aligned}$$

where the positive constant $\theta > 0$ will be taken to be equal with the communications delay and will be considered to be the same for all vehicles in formation[3]. It can be seen that the signals defined in (14) are merely θ delayed version of (4), with an additional θ time headway added to the expression of the interspacing errors $\widetilde{z}_k$. The fact that in (14) at the current moment in time t, we regulate the measurements taken at moment $(t-\theta)$ is a limitation imposed by the communications delay (which are relatively very small, though) and it entails some loss in performance which was to be expected. The inclusion of the θ time headway results in a slightly more conservative policy, since it induces slightly larger[4] interspacing distances as the speed increases. The same conservative effect (of the θ time headway) occurs with respect to the regulated relative speeds $\widetilde{z}_k^v(t)$ during the transient regime when the acceleration $\dot{v}_k$ is sizable.

Remark 5.1: For all practical applications related to platooning, the value of θ will be taken to be equal to a worst case scenario value of the latency of the wireless communication systems, which is about 2×10^{-2} seconds for digital radio systems such as WiFi, Bluetooth or Zigbee. Furthermore, the GPS time based synchronization mechanism described in [5, Section VI] used in conjunction with time stamping protocols at the transmission of the predecessor's control signal u_{k-1} is able to emulate and implement time invariant and heterogeneous communications time delays through the entire formation.

B. Analysis

Next, note that by performing a Taylor series expansion it follows that

$$\begin{aligned} y_k(t) &= y_k(t-\theta) + \theta\dot{y}_k(t-\theta) + \mathcal{O}(\theta^2),\\ v_k(t) &= v_k(t-\theta) + \theta\dot{v}_k(t-\theta) + \mathcal{O}(\theta^2), \end{aligned} \qquad (15)$$

therefore an $\mathcal{O}(\theta^2)$ approximation of the measurements from (14) is given by

$$\begin{aligned} z_k(t) &\stackrel{def}{=} y_{k-1}(t-\theta) - y_k(t),\\ z_k^v(t) &\stackrel{def}{=} v_{k-1}(t-\theta) - v_k(t) \quad \text{for} \quad 1 \le k \le n. \end{aligned} \qquad (16)$$

We assume the following initial conditions

$$y_k(t) = -\sum_{j=0}^{k} \ell_j, \quad v_k(t) = 0, \ \forall t \in (-\theta, 0].$$

Remark 5.2: Writing the Taylor series expansion with an integral rest, we obtain the following equivalent expression for (16)

$$\begin{aligned} z_k(t) &= y_{k-1}(t-\theta) - y_k(t-\theta) - \int_{t-\theta}^{t} \dot{y}_k(\tau)d\tau,\\ z_k^v(t) &= v_{k-1}(t-\theta) - v_k(t-\theta) - \int_{t-\theta}^{t} \dot{v}_k(\tau)d\tau \end{aligned} \qquad (17)$$

and so it becomes apparent that the signals introduced in (16) can be measured on board the k-th vehicle via (17), using only onboard ranging sensors[5] and high accuracy longitudinal speedometers in conjunction with a mere integrator. Specifically, the first term in (17) consists of the θ-delayed measurement of the interspacing distance minus the integration of the absolute speed (measurable on board) over a θ-length interval. The second term in (17) consists of the θ-delayed measurement of the relative speed[6] minus the $(v_k(t) - v_k(t-\theta))$ term, comprised of absolute speeds

[2]It's worth mentioning that the control scheme proposed in [5] achieves without time headways the same disturbances attenuation at low and mid frequency as the attenuation achievable with the use of time headways.

[3]See also Remark 5.1 in the following paragraph

[4]The effect is directly proportional with the value of θ which is very small in practice.

[5]Preferably very low latency LIDAR sensors, which are already affordable and widely available commercially

[6]The relative speed with respect to the preceding vehicle, which is also measurable onboard.

measurable onboard. Finally, the entire history on the interval $[(t-\theta),\ t]$ of the ranging sensors (17) must be stored in a memory buffer, in order to be used by the distributed controller we will introduce next.

Given the values of θ that appear in practice (see Remark 5.1) and given the worst case scenario of breaking decelerations $|\ddot{y}_k(t)|$ that could occur during highway traffic, it follows from (15) that the signals from (16) are such an accurate approximation of (14), that the order of the approximation falls way below the tolerated measurement errors of the most performant ranging sensors. That is to say that choosing between two controllers that regulate either the (14) signals or the (16) signals respectively, has considerably less influence on the resulted scheme than the measurement noise of an highly accurate LIDAR. Consequently, we can choose to regulate (16). Considering the definition of z_k and z_k^v as in (16), we will prove that the distributed control policies given next are able to entirely compensate the communication induced delays:

$$\begin{cases} u_k(t) = u_{k-1}(t-\theta) + \beta_k z_k^v(t) + \nabla_{y_k} V_{k,k-1}(\|z_k(t)\|_\sigma) \\ u_k(t) = 0, \quad \forall t \in (-\theta, 0] \end{cases} \tag{18}$$

Remark 5.3: Note that for the real time implementation of type (18) control policies onboard the k-th vehicle, two pieces of information are needed: **(i)** the command signal of the predecessor, received on board a with θ - delay, via wireless communications and **(ii)**, the (16) sensor measurements z_k, z_k^v which are on board measurable (according to the considerations of Remark 5.2). A GPS time-base synchronization of these two pieces of information may be performed as in [5, Section VI] in order to ensure time invariant, point-wise delays of value exactly θ, homogenously throughout the entire formation.

With this controller at hand we obtain the following closed–loop error equations at the k–th agent:

$$\begin{aligned} \dot{z}_k^v(t) &= f(v_{k-1}(t-\theta)) - f(v_k(t)) \\ &\quad - \beta_k z_k^v(t) + \nabla_{y_k} V_{k,k-1}(\|z_k(t)\|_\sigma). \end{aligned} \tag{19}$$

We will use the Lyapunov function defined in (10) keeping in mind that the definitions of $z_k(t), z_k^v(t)$ are in accordance to (16). Consequently, the time delays adaptation for the main result of Section IV reads:

Theorem 5.4: If the function $f(\cdot)$ from (2a) satisfies the global Lipshitz–like condition (12) then for all type (18) control laws with that $\beta_k > \alpha$ the following hold:

(A) The differential of the Lyapunov candidate function $L_k(\cdot,\cdot)$ introduced in (10) along the trajectories of (2) and (18) is given by

$$\begin{aligned} \tfrac{d}{dt} L_k(z_k(t), z_k^v(t)) &= z_k^{v\top}(t)\Big(f\big(v_{k-1}(t-\theta)\big) - f\big(v_k(t)\big)\Big) \\ &- \beta_k z_k^{v\top}(t) z_k^v(t)\,, \end{aligned} \tag{20}$$

and does not depend on the choice of the APFs $V_{k,k-1}(\cdot)$.

(B) Given the Lyapunov function L_k introduced in (10), *local* to the k-th agent, the sub–level sets $\Omega_c^k \stackrel{def}{=} \{(z_k, z_k^v) | L_k \leq c, \text{ with} c > 0\}$ of L_k are compact and they represent forward invariant sets for the *local* closed–loop dynamics (19) of the k–th vehicle.

(C) The controller (18) guarantees velocity matching and a lower bound on the absolute values of the interspacing distance $z_k(t)$. Furthermore, considering $c = 2L_k(z_k(0), z_k^v(0))$ there exists η_c such that

$$\|z_k(t)\|_2 > \eta_c, \forall t \geq 0.$$

Therefore, a pre–specified safety distance can be imposed by the initial conditions.

Remark 5.5: The scheme proposed above is able to regulate $v_{k-1}(t-\theta) - v_k(t)$ in the presence of communications delays. Therefore, as far as the leader's velocity profile is slowly varying relatively to the order of magnitude of the communications delays, the scheme does regulate an accurate approximation of $v_{k-1}(t) - v_k(t)$. Nevertheless, oscillations of the leader's velocity at a frequency that is of the same order of magnitude with the θ time delay cannot be efficiently compensated and the accordion effect will appear. These assumptions are very well satisfied in the platooning setting, but they may not be valid for other applications. The conclusion is in line with the well known fact that for the validity of the control scheme it is always necessary that the time delays that propagate through the controller are smaller than those propagating through the given plant.

VI. A Numerical Example

In this section we illustrate the distributed controller introduced earlier for the case dynamical agents (2), where the function $f(\cdot)$ taken to be a quadratic form $f(v) = -\gamma g - \ell v^2$, in accordance with the dynamical model of road vehicles from [22, (1)/pp. 1]. Here, $\gamma = 0.011$ is the tyre rolling resistance coefficient, $g = 9.81 m/s^2$ the gravitational acceleration, and $\ell = 0.463\, kg/m$ the air drag constant. The dynamics (2) become

$$\dot{y}_k = v_k \tag{21a}$$

$$\dot{v}_k = -\gamma g - \ell v_k^2 + \frac{\eta}{R} w_k \tag{21b}$$

with $\eta = 1.8$ being the gear ratio and $R = 0.5$ being the wheel radius. The command signal ω_k is the engine's torque, and its linear transformation $\frac{\eta}{R}\omega_k$ corresponds to the input u_k in (2). Note that $f(\cdot)$ in (21) satisfies the global Lipschitz–like condition [2, Assumption 1] $(v_2 - v_1)^\top\big(f(v_2) - f(v_1)\big) \leq \alpha \|v_2 - v_1\|_2^2$, for any two vectors v_1, v_2 in the domain of $f(\cdot)$.

The control law is designed using APFs (Definition 2.3) of the following form [2, Fig. 1/ pp.197]

$$V_{k,k-1}(\|z_k\|_\sigma) = \frac{\eta}{R}\left[\ln(\|z_k\|_\sigma)^2 + \frac{100}{\|z_k\|_\sigma^2}\right] \tag{22}$$

and an empirically tuned gain $\beta = 25\frac{\eta}{R}$. The reference signal for the entire formation will be the control control of the leader vehicle, namely $u_0(t)$.

We look at a speed profile of the leader consisting of two smoothed rectangular pulses. The vehicles start at relatively small separations, of about 2 m. The plots for the delay free case, using the baseline controller from Section IV are shown in Figure 1. The controller will cause an initial increase in the interspacing distances, at the expense of delaying velocity matching. Once a sufficient interspacing distance has been achieved (in this case, around 5 m, related to the minimum of the APFs), the velocities are brought together.

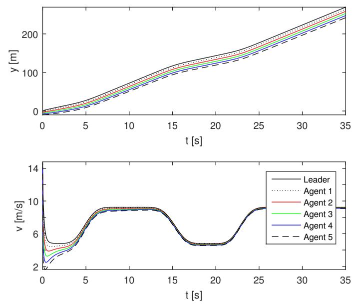

Fig. 1. Trajectories of vehicles - the delay free case

If we consider a time delay of $\theta = 0.02\,\mathrm{s}$ (which is the common nominal value for wireless communications) and we apply the delay compensation mechanism from Section V, then Figure 2 below exhibits practically the same wave forms as Figure 1, hence the effects of the delays are entirely compensated by the controller.

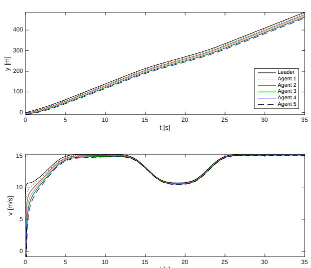

Fig. 2. Trajectories of vehicles with a typical $\theta = 0.02$ s. time delay

Finally, to illustrate the point made by Remark 5.5, we look at the situation when the leader's velocity profile is a fast-varying sine while considering a large time delay $\theta = 0.5\,\mathrm{s}$, comparable to the period of the sine reference. The examination of Figure 3 below, shows that velocity agreement cannot be achieved, and there is an "accordion" effect in the positions (undamped oscillations). However, even in this challenging situation the controller manages to avoid collisions.

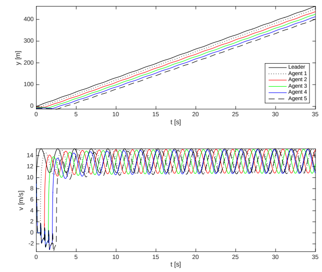

Fig. 3. Trajectories with a fast varying leader reference and large time delay.

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