On the relative approximation error of extreme quantiles by the block maxima method

Clément Albert, Anne Dutfoy, Stéphane Girard

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This study takes place in the context of extreme quantiles estimation by the block maxima method. We investigate the behaviour of the relative approximation error of a quantile estimator dedicated to the Gumbel maximum domain of attraction. Our work is based on a regular variation assumption on the first derivative of the logarithm of the inverse cumulative hazard rate function, introduced by de Valk (2016) [Approximation of high quantiles from intermediate quantiles. *Extremes* 19(4), 661-686].

Let us denote by $X_{m,m}$ the maximum of $m$ iid observations from a distribution function $F$, where $m$ is referred to as the block size. We focus on an extreme quantile associated with $F$ defined by $x_{p_m} = F^{-1}(1 - p_m) = H^{-1}(-\log p_m)$, where $H$ is the cumulative hazard rate function and $p_m = m^{\tau_m}$ with $\tau_m \geq 1$. Assume that $(X_{m,m} - b_m)/a_m$ converges to a Gumbel distribution for some normalizing constants $a_m > 0$ and $b_m \in \mathbb{R}$. The approximation $\tilde{x}_{p_m}$ of $x_{p_m}$ by the block maxima method is given by

$$\tilde{x}_{p_m} = b_m - a_m \log(mp_m)$$

and the associated relative approximation error is

$$\epsilon_{app_m} = \frac{x_{p_m} - \tilde{x}_{p_m}}{x_{p_m}}.$$

Our main result is:

$$\epsilon_{app_m} \xrightarrow{m \to +\infty} 0 \iff (\tau_m - 1)^2 K_2(\log m) \xrightarrow{m \to +\infty} 0,$$

where $K_2(t) = t^2(H^{-1})''(t)/H^{-1}(t)$, $t > 0$. This result exhibits three families of distributions according to the limit of $K_2$ which can be either zero, a constant or infinite. We also provide a first order approximation of the relative approximation error when the latter converges towards zero. Our results are illustrated on simulated data.

**Key Words:** Extreme quantiles estimation, relative approximation error, asymptotic properties, regular variation.