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To cite this version:
Clément Albert, Anne Dutfoy, Stéphane Girard. On the relative approximation error of extreme quantiles by the block maxima method. 10th International Conference on Extreme Value Analysis, Jun 2017, Delft, Netherlands. hal-01571047

HAL Id: hal-01571047
https://hal.archives-ouvertes.fr/hal-01571047
Submitted on 1 Aug 2017

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On the relative approximation error of extreme quantiles by the block maxima method

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This study takes place in the context of extreme quantiles estimation by the block maxima method. We investigate the behaviour of the relative approximation error of a quantile estimator dedicated to the Gumbel maximum domain of attraction. Our work is based on a regular variation assumption on the first derivative of the logarithm of the inverse cumulative hazard rate function, introduced by de Valk (2016) [Approximation of high quantiles from intermediate quantiles. Extremes 19(4), 661-686].

Let us denote by $X_{m,m}$ the maximum of $m$ iid observations from a distribution function $F$, where $m$ is referred to as the block size. We focus on an extreme quantile associated with $F$ defined by $x_{pm} = F^{-1}(1 - pm) = H^{-1}(-\log pm)$, where $H$ is the cumulative hazard rate function and $pm = m^{-\tau_m}$ with $\tau_m \geq 1$. Assume that $(X_{m,m} - b_m)/a_m$ converges to a Gumbel distribution for some normalizing constants $a_m > 0$ and $b_m \in \mathbb{R}$. The approximation $\tilde{x}_{pm}$ of $x_{pm}$ by the block maxima method is given by

$$\tilde{x}_{pm} = b_m - a_m \log(mp_m)$$

and the associated relative approximation error is

$$\epsilon_{app_m} = (x_{pm} - \tilde{x}_{pm})/x_{pm}.$$

Our main result is:

$$\epsilon_{app_m} \xrightarrow{m \to +\infty} 0 \iff (\tau_m - 1)^2 K_2(\log m) \xrightarrow{m \to +\infty} 0,$$

where $K_2(t) = t^2(H^{-1})''(t)/H^{-1}(t)$, $t > 0$. This result exhibits three families of distributions according to the limit of $K_2$ which can be either zero, a constant or infinite. We also provide a first order approximation of the relative approximation error when the latter converges towards zero. Our results are illustrated on simulated data.

**Key Words:** Extreme quantiles estimation, relative approximation error, asymptotic properties, regular variation.