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Abstract: This paper deals with observer-based stabilization for a class of Linear Parameter-Varying (LPV) systems in discrete-time case. Two new LMI design methods are proposed to design the observer-based controller gains. The first one is based on the use of a Young's relation in a judicious way, while the second one, which provides more interesting results, is based on the use of a general congruence principle. This use of congruence principle leads to some additional slack matrices as decision variables, which make disappear some bilinear terms leading to less conservative LMI conditions. To the authors' best knowledge, this is the first time the congruence principle is exploited in this way. The effectiveness and superiority of the proposed design techniques, compared to existing results in the literature, are demonstrated through two numerical examples.

Keywords: Linear matrix inequalities (LMIs), linear parameter-varying (LPV) systems, Young's relation based approach, congruence principle, uncertainties.

1. INTRODUCTION

Linear parameter-varying (LPV) modeling has been the subject of increasing interest in the literature with many practical applications in the areas of aerospace, automotive, ...etc.. Indeed, LPV systems offer an alternative to handle the complexity of nonlinear systems and combine the simplicity of linear systems and the real variability of their parameters avoiding the approximation of nonlinear systems by linearization or with a transformation. Although this simplified representation of systems is very useful and practical, but the parameters of an LPV system are generally uncertain and unavailable for measurements. For the case of continuous-time LPV systems, see for instance, Apkarian and Adams (1998), Kose and Jabbari (1999), Balas et al. (2004), Scorletti and El Ghaoui (1995), Wu (2001), Gilbert et al. (2010), Sato (2011), Song and Yang (2011). Also, for the discrete-time dynamic output feedback controller LPV systems, see for instance, Blanchini and Miani (2003), De Caigny et al. (2012), Zhang et al. (2009), Emedi and Karimi (2014), De Oliveira et al. (1999), Oliveira and Peres (2005)). Hence, the investigation of LPV systems with inexact but bounded parameters attracts the attention of many researchers in this field Wu et al. (1997), Kalsi et al. (2010). If the problem of observer design for LPV systems with known and bounded parameters is easy to investigate, the observer-based stabilization problem is difficult from the LMI point of view. Therefore, the stabilization problem becomes more complicated when the parameters are unknown. Indeed, these unknown parameters lead to bilinear matrix coupling, which are difficult to linearize with known mathematical tools. Some design methods have been proposed in this area in the literature, however all the proposed techniques still remain conservative (see for instance Daafouz and Bernussou (2001), Daafouz et al. (2002), Ibrir (2008), Ibrir and Diopt (2008), Kheloufi et al. (2013)). This motivates us to develop new and less conservative LMI conditions by using new mathematical tools.

In this paper we propose new and enhanced LMI conditions to solve the problem of observer-based stabilization for a class of discrete-time LPV systems with uncertain parameters. The proposed technique consists in designing an observer-based controller which stabilizes the LPV system, provided that the difference between the uncertain parameters and their estimates are norm-bounded with bounds not exceeding a tolerated maximum value. Hence, the observer-based gains depend on these bounds. The problem we investigate in this paper is motivated by the work proposed in Heemels et al. (2010), Jetto and Orsini (2010) and recently in Zemouche et al. (2016). One of the contributions of this paper consists in the use of a relaxed reformulation for the parameter uncertainty. This relaxation is inspired from recent design methods for observer design of nonlinear Lipschitz systems Zemouche and Boutayeb (2013), Phanomchoeng et al. (2011). Using this reformulation, two new LMI synthesis methods are developed to design observer-based controllers for a class of LPV systems with inexact but bounded parameters. The approach used a new congruence principle by pre- and post-multiplying the basic BMI condition by new and ingenious slack matrices. Thanks to these matrices, which can be seen as additional decision variables, some bilinear terms vanish from the BMI, which increases flexibility in the to linearization. We show analytically how particular forms of the slack variables reduce the complexity of the bilinear problem. This new use of congruence principle allows avoiding the gridding method as in Kheloufi et al. (2015).

The rest of this paper is organized as follows: after giving the problem formulation in Section 2, we devote Section 3 to our contribution: new LMI synthesis methods to design observer-based controllers for uncertain LPV systems are presented. Section 4 gives simulation examples and comparisons to show the superiority of the proposed design methodology. Finally, some conclusions are reported in Section 5.

2. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

Consider a discrete-time LPV system described by the following state-space equation:

$$\begin{cases} x_{t+1} = A(\rho(t))x_t + Bu_t \\ y_t = C\,x_t \end{cases} \tag{1}$$

where $x_t \in \mathbb{R}^n$ is the state vector, $y_t \in \mathbb{R}^m$ is the measurement vector, $u_t \in \mathbb{R}^p$ is the control signal, for any $t \in \mathbb{Z}^+$. Further, the state matrix $A(\rho(t)) \in \mathbb{R}^{n\times n}$ depends on a bounded time-varying parameter $\rho(t) = [\rho_1(t), \ldots, \rho_N(t)]^T$, which is assumed to be not available in real time, but only an approximated $\hat{\rho}(t) \in \Theta \subset \mathbb{R}^N$, satisfying

$$\sup_{t\in\mathbb{Z}^+} ||\rho(t) - \hat{\rho}(t)|| \leq \Delta \tag{2}$$

is known, where Δ is some nonnegative constant indicating the uncertainty level, and Θ is some bounded subset of $\mathbb{R}^N$. B and C are real matrices the dimension $n \times p$ and $m \times n$, respectively. Throughout the paper, the following assumptions are made:

- The matrix $A(\rho(t))$ lies for each $\rho(t) \in \Theta$ in the convex hull $\mathrm{Co}(A_1, ..., A_N)$, that is there exists a finite sequence $\left(\xi^i(\rho(t))\right)_{i=1}^N$ depending on $\rho(t)$ such that $\xi^i(\rho_t) \geq 0$, $\sum_{i=1}^N \xi^i(\rho(t)) = 1$, and
$$A(\rho(t)) = \sum_{i=1}^N \xi^i(\rho(t))A_i; \tag{3}$$
- The pairs (A_i, B) and (A_i, C), for $i = 1\ldots, N$, are respectively stabilizable and detectable.

The observer-based controller we proposed here is the same as in Heemels et al. (2010); Jetto and Orsini (2010); Zemouche et al. (2016), and described by the following equations:

$$\begin{cases} \hat{x}_{t+1} = A(\hat{\rho}(t))\hat{x}_t + L(\hat{\rho}(t))(y_t - C\hat{x}_t) + Bu_t, \\ \hat{y}_t = C\hat{x}_t. \end{cases} \tag{4}$$

Based on the estimate $\hat{x}_t$, we develop a set of the controllers of the form

$$u_t = K(\hat{\rho}(t))\hat{x}_t. \tag{5}$$

Define $\overline{x}_t = [\hat{x}_t^T\ e_t^T]^T$, where $e_t = \hat{x}_t - x_t$ is the estimate error. Then the closed-loop system of (1), (4) and (5) is described by

$$\overline{x}_{t+1} = \begin{bmatrix} A(\hat{\rho}) + BK(\hat{\rho}) & -L(\hat{\rho})C \\ -\Delta A & A(\hat{\rho}) + \Delta A - L(\hat{\rho})C \end{bmatrix} \overline{x}_t, \tag{6}$$

where, for shortness, we set $\Delta A := A(\rho(t)) - A(\hat{\rho}(t))$, $X(\hat{\rho}) := X(\hat{\rho}(t))$ and $Y(\rho) := Y(\rho(t))$, for any parametric matrices X and Y.

In this paper, the aim is to design a collection of observer-based controller gains $K(\hat{\rho}) = \sum_{j=1}^N \xi^j(\hat{\rho})K_j$ and $L(\hat{\rho}) = \sum_{j=1}^N \xi^j(\hat{\rho})L_j$ such that the closed-loop system (6) is globally asymptotically stable.

We first formulate the non-convex optimization problem that allows us to compute the observer-based controller gains K_j and L_j, for $j = 1, \ldots, N$, using Lyapunov stability. For the stability analysis, we use the same quadratic and parameter dependent Lyapunov function as that in Heemels et al. (2010), namely,

$$V(\overline{x}, \hat{\rho}) := \overline{x}_t^T P(\hat{\rho}(t))\overline{x}_t = \overline{x}_t^T \begin{bmatrix} P_{11}(\hat{\rho}) & P_{12}(\hat{\rho}) \\ P_{12}^T(\hat{\rho}) & P_{22}(\hat{\rho}) \end{bmatrix} \overline{x}_t,$$

where

$$P(\hat{\rho}(t)) = \sum_{j=1}^N \xi^j(\hat{\rho}(t))P_j.$$

One obtains after some calculations (see Zemouche et al. (2016) for more details) that

$$\begin{aligned} \Delta V_t(\overline{x}, \hat{\rho}) &:= V\big(\overline{x}_{t+1}, \hat{\rho}(t+1)\big) - V\big(\overline{x}_t, \hat{\rho}(t)\big) \\ &= \sum_{j,l=1}^N \xi^j\big(\hat{\rho}(t)\big)\xi^l\big(\hat{\rho}(t+1)\big)\overline{x}_t^T \Omega_{jl}\overline{x}_t < 0 \end{aligned}$$

where for each $j, l = 1, 2, \ldots, N$,

$$\Omega_{jl} = -P_j + \Pi_j^T P_l \Pi_j$$

and

$$\Pi_j = \begin{bmatrix} A_j + BK_j & -L_jC \\ -\Delta A & A_j + \Delta A - L_jC \end{bmatrix}. \tag{7}$$

We have that $\Delta V_t(\overline{x}, \hat{\rho}) < 0$ for all $\overline{x}(.) \neq 0$ and $\hat{\rho}(t) \in \Theta$ if

$$\Omega_{jl} = -P_j + \Pi_j^T P_l \Pi_j < 0,\ \forall j, l = 1, 2, \ldots, N, \tag{8}$$

or, equivalently,

$$\begin{bmatrix} -P_j & \Pi_j^T \\ \Pi_j & -P_l^{-1} \end{bmatrix} < 0, \quad \forall j, l = 1, \ldots, N. \tag{9}$$

For simplicity, we set in the rest of the paper $\Lambda = \{1, 2, \ldots, N\}$.

3. TWO NEW LMI DESIGN PROCEDURES

This section proposes new ways to linearize the Lyapunov stability problem (9), which is BMI due to many coupling between the Lyapunov matrices and the observer based controller gains. Our purpose is to give new strategies that give better solutions to (9) in the sense that they tolerate larger uncertainty level Δ. We begin by giving an equivalent reformulation for the parameter uncertainty, and consequently a relaxation of the assumption proposed in Heemels et al. (2010) will be derived.

3.1 About the Assumption (2) on the uncertainty

It is easy to see that condition (2) is equivalent to the existence of two vectors $\Delta^{\min} \in \mathbb{R}^N$ and $\Delta^{\max} \in \mathbb{R}^N$ so that

$$\Delta_k^{\min} \leq \rho_k(t) - \hat{\rho}_k(t) \leq \Delta_k^{\max}, \quad \forall k = 1, \ldots, N. \tag{10}$$

Thanks to this reformulation of (2), it allows us to relax the assumptions proposed in Heemels et al. (2010), namely the matrix ΔA satisfies the following condition:

$$\Delta A^T \Delta A \leq \Delta^2 \Gamma^T \Gamma \tag{11}$$

where Γ is known constant matrix of appropriate dimension. This relaxation, inspired from recent design methods for observer design of nonlinear Lipschitz systems Zemouche and Boutayeb (2013), Phanomchoeng et al. (2011), is based on the use of an LPV approach to treat the uncertainties on the parameters. From now on, the condition (10) will be denoted by (H).

Let us start by checking that under condition (2) or (H), the following result, which is given in Zemouche et al. (2016) without proof, holds.

Proposition 1. Under condition (H), the parametric matrix ΔA belongs to a bounded convex $\mathbb{B}$, for which the set of vertices is defined by :

$$\mathbb{V}_{\mathbb{B}} = \Big\{ \mathcal{A}(\varrho) = \sum_{(i,j,k)\in\mathbb{S}} \varrho_k \lambda_{ij}^k \Lambda_{ij} : \varrho_k \in \{\Delta_k^{min}, \Delta_k^{max}\} \Big\}. \tag{12}$$

The set $\mathbb{S}$ is defined by

$$\mathbb{S} = \Big\{ (i,j,k) : (\rho_k - \hat{\rho}_k)\lambda_{ij}^k \neq 0 \Big\}.$$

The proof is based on the following Lemma:

Lemma 2. (Zemouche and Boutayeb (2013)). Consider a function $\Psi : \mathbb{R}^n \to \mathbb{R}$. Then, for all

$$X = [x_1, \ldots, x_n]^T \in \mathbb{R}^n \text{ and } Y = [y_1, \ldots, y_n]^T \in \mathbb{R}^n$$

there exist functions $\psi_j : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, $j = 1, \ldots, n$ so that

$$\Psi(X) - \Psi(Y) = \sum_{j=1}^{j=n} \psi_j\Big(X^{Y_{j-1}}, X^{Y_j}\Big) e_n^T(j) \Big(X - Y\Big) \tag{13}$$

where $X^{Y_j} = [y_1, \ldots, y_j, x_{j+1}, \ldots, x_n]^T$, $X^{Y_0} = X$ and $e_n(j)$ is the jth vector of the canonical basis of $\mathbb{R}^n$.

Proof of Proposition 1: We have the detailed form of $A(\rho)$:

$$A(\rho) = \sum_{i,j} A_{ij}(\rho)\ e_n(i) e_n^T(j).$$

Thus we can write ΔA under the more suitable form:

$$\begin{aligned} \Delta A &= A(\rho) - A(\hat{\rho}) \\ &= \sum_{i,j=1}^{n} \Big(A_{ij}(\rho) - A_{ij}(\hat{\rho}) \Big)\ e_n(i) e_n^T(j). \end{aligned} \tag{14}$$

By using Lemma 1 on A_{ij}, for each fixed $i, j = 1, \ldots, n$, we obtain the existence of functions $\psi_{ij}^k : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, $k = 1, \ldots, n$ such that

$$A_{ij}(\rho) - A_{ij}(\hat{\rho}) = \sum_{k=1}^{n} \psi_{ij}^k \Big(\rho^{\hat{\rho}_{k-1}}, \rho^{\hat{\rho}_k} \Big)(\rho_k - \hat{\rho}_k) \tag{15}$$

By replacing in (14) the reformulation of $A_{ij}(\rho) - A_{ij}(\hat{\rho})$ given in (15), we get

$$A(\rho) - A(\hat{\rho}) = \sum_{i,j,k} (\rho_k - \hat{\rho}_k) \psi_{ij}^k \Big(\rho^{\hat{\rho}_{k-1}}, \rho^{\hat{\rho}_k} \Big)\ e_n(i) e_n^T(j) \tag{16}$$

Thanks to the following notations

$$\lambda_{ij}^k = \psi_{ij}^k \Big(\rho^{\hat{\rho}_{k-1}}, \rho^{\hat{\rho}_k} \Big), \ \ \Lambda_{ij} = e_n(i) e_n^T(j)$$

we obtain that

$$\Delta A = \sum_{(i,j,k)\in\mathbb{S}} (\rho_k(t) - \hat{\rho}_k(t)) \lambda_{ij}^k \Lambda_{ij}, \tag{17}$$

from which we conclude that ΔA belongs to the bounded convex $\mathbb{B}$. Which completes the proof of Proposition 1.

■

Now, let us return to the linearization problem (9). To our knowledge, the best manner to linearize the Lyapunov inequality in the discrete-time systems is the introduction of slack variables (see for instance De Oliveira et al. (1999), Daafouz et al. (2002)). Since inequality (9) depends on both indices j, l, let us introduce a matrix G_{jl} of adequate dimension that depends on j and l. Firstly, we use the congruence principle as follows: we pre- and post- multiply (9) by $\mathrm{diag}(I, G_{jl})$ and $\mathrm{diag}(I, G_{jl}^T)$ respectively. Secondly we exploit the adapted well-known inequality (see for example (Heemels et al., 2010, Ineq. (12))),

$$-G_{jl} P_l^{-1} G_{jl}^T \leq P_l^{11} - G_{jl} - G_{jl}^T, \ \ \forall j, l \in \Lambda.$$

One obtains that inequalities (9) hold, if the following ones

$$\begin{bmatrix} -P_j & \Pi_j^T G_{jl}^T \\ G_{jl}\Pi_j & P_l - G_{jl} - G_{jl}^T \end{bmatrix} < 0, \quad \forall j, l \in \Lambda \tag{18}$$

are fulfilled. Now, we take the detailed structures of P_l and G_{jl} respectively:

$$P_l = \begin{bmatrix} P_l^{11} & P_l^{12} \\ (\star) & P_l^{22} \end{bmatrix}, G_{jl} = \begin{bmatrix} G_{jl}^{11} & G_{jl}^{12} \\ G_{jl}^{21} & G_{jl}^{22} \end{bmatrix}. \tag{19}$$

Notice that the slack variable G_{jl} that we use here is more general than Heemels et al. (2010); Jetto and Orsini (2010); Zemouche et al. (2016); Alessandri et al. (2013) since it involves both indices j and l and not necessarily of the form $\alpha_j I$. This form comes from the dependency of inequality (9) on both j and l.

3.2 A relaxed LMI Design Procedure

In what follows, we will discuss a new way to choose judiciously the matrix G_{jl} that allows to linearize inequality (20). Our strategy consists in analyzing how to eliminate the bilinear terms coming from a more general structure of G_{jl}, without imposing a diagonal structure. By substituting (19) in inequality (18) and after some mathematical developments, one obtains the following detailed version of (18):

$$\begin{bmatrix} -P_j^{11} & -P_j^{12} & \omega_{13}^{jl} & \omega_{14}^{jl} \\ (\star) & -P_j^{22} & \omega_{23}^{jl} & \omega_{24}^{jl} \\ (\star) & (\star) & \omega_{33}^{jl} & P_l^{12} - G_{jl}^{12} - (G_{jl}^{21})^T \\ (\star) & (\star) & (\star) & P_l^{22} - \mathrm{He}(G_{jl}^{22}) \end{bmatrix} < 0, \tag{20}$$

for all $j, l \in \Lambda$, where

$$\omega_{13}^{jl} = A_j^T (G_{jl}^{11})^T + K_j^T B^T (G_{jl}^{11})^T - \Delta A^T (G_{jl}^{12})^T,$$
$$\omega_{14}^{jl} = A_j^T (G_{jl}^{21})^T + K_j^T B^T (G_{jl}^{21})^T - \Delta A^T (G_{jl}^{22})^T,$$
$$\omega_{23}^{jl} = -C^T L_j^T (G_{jl}^{11} + G_{jl}^{12})^T + (A_j + \Delta A)^T (G_{jl}^{12})^T,$$
$$\omega_{24}^{jl} = (\Delta A + A_j)^T (G_{jl}^{22})^T - C^T L_j^T (G_{jl}^{21} + G_{jl}^{22})^T,$$
$$\omega_{33}^{jl} = P_l^{11} - \mathrm{He}(G_{jl}^{11}).$$

We begin by dealing with terms coupled with L_j, $j \in \Lambda$, namely the bilinear terms in ω_{23}^{jl} and ω_{24}^{jl}. Our first strategy consists in choosing $G_{jl}^{11} = G_l^{11}$ (independent of j) and $G_{jl}^{22} = G_j^{22}$ (independent of l) and eliminate the remining bilinear terms by setting $G_{jl}^{11} + G_{jl}^{12} = 0$ or $G_{jl}^{21} + G_{jl}^{22} = 0$. But since G_{jl}^{21} is also coupled with the matrices K_j, it should be taken null. For instance, the following structure of G_{jl} is convenient.

$$G_{jl} = \begin{bmatrix} G_l^{11} & -G_l^{11} \\ 0 & G_j^{22} \end{bmatrix}. \tag{21}$$

However, for the relaxed LMI design, we will use a more general G_{jl}^{12} instead of $-G_l^{11}$. That is, we take

$$G_{jl} = \begin{bmatrix} G_l^{11} & G_{jl}^{12} \\ 0 & G_j^{22} \end{bmatrix} \tag{22}$$

where G_{jl}^{12} will be selected suitably after using the congruence principle.

Pre- and post- multiply (20) by $\mathrm{diag}\Big((G_j^{11})^{-1}, I, (G_l^{11})^{-1}, I\Big)$ and taking the following change of variables and notations

$$\tilde G_j^{11} = (G_j^{11})^{-1}, \;\; \tilde P_j^{11} = \tilde G_j^{11} P_j^{11} (\tilde G_j^{11})^T,$$
$$\tilde K_j = K_j (\tilde G_j^{11})^T, \tilde P_j^{12} = \tilde G_j^{11} P_j^{12}.$$

we get the following equivalent inequality:

$$\begin{bmatrix} -\tilde P_j^{11} & -\tilde P_j^{12} & \tilde\omega_{13}^{jl} & -\tilde G_j^{11} \Delta A^T (G_j^{22})^T \\ (\star) & -P_j^{22} & \tilde\omega_{23}^{jl} & \tilde\omega_{24}^{j} \\ (\star) & (\star) & \tilde\omega_{33}^{l} & \tilde P_l^{12} - \tilde G_l^{11} G_{jl}^{12} \\ (\star) & (\star) & (\star) & P_l^{22} - (G_j^{22})^T - G_j^{22} \end{bmatrix} < 0, \tag{23}$$

where

$$\tilde\omega_{13}^{jl} = \tilde G_j^{11} A_j^T + \tilde K_j^T B^T - \tilde G_j^{11} \Delta A^T \Big(\tilde G_l^{11} G_{jl}^{12}\Big)^T,$$
$$\tilde\omega_{23}^{jl} = -C^T L_j^T (I + \tilde G_l^{11} G_{jl}^{12})^T + (A_j + \Delta A)^T \Big(\tilde G_l^{11} G_{jl}^{12}\Big)^T,$$
$$\tilde\omega_{24}^{j} = (\Delta A + A_j)^T (G_j^{22})^T - C^T L_j^T (G_j^{22})^T,$$
$$\tilde\omega_{33}^{l} = \tilde P_l^{11} - \tilde G_l^{11} - (\tilde G_l^{11})^T.$$

In order to eliminate the remaining terms in (23), and since the term L_j is coupled with both $I + \tilde G_l^{11} G_{jl}^{12}$ and G_j^{22}, an adequate choice consists in taking

$$G_{jl}^{12} = G_l^{11} G_j^{22} - G_l^{11}.$$

This leads to the following structure of G_{jl}:

$$G_{jl} = \begin{bmatrix} G_l^{11} & G_l^{11} G_j^{22} - G_l^{11} \\ 0 & G_j^{22} \end{bmatrix} \tag{24}$$

Both choices (21) and (24) allow to simplify the complexity of the BMIs (23). To be more general and to reduce the conservatism, the idea consists in combining the two choices (21) and (24) by introducing free scalars α_{jl} as follows:

$$G_{jl}^{12} = \alpha_{jl} G_l^{11} G_j^{22} - G_l^{11} \tag{25}$$

which means that

$$G_{jl} = \begin{bmatrix} G_l^{11} & \alpha_{jl} G_l^{11} G_j^{22} - G_l^{11} \\ 0 & G_j^{22} \end{bmatrix}. \tag{26}$$

Note that if $\alpha_{jl} = 0$, for all $j, l \in \Lambda$, we get the structure (21), and if $\alpha_{jl} = 1$, for all $j, l \in \Lambda$, we get the structure (24).

Now, we complete the design methodology by linearizing the uncertain terms ΔA. From the convexity principle, we deduce that (23) with (26) holds for each $l, j \in \Lambda$, if it holds for each $l, j \in \Lambda$, and each $\mathcal{A}(\varrho) \in \mathbb{V}_{\mathbb{B}}$.

Taking into account (26) and using the notation $\hat L_j = G_j^{22} L_j$, inequality (23) can be rewritten under the following form:

$$\Xi_{jl}(\mathcal{A}(\varrho)) + \mathrm{He}(Z_{1j}^T \;\; Z_{2jl}) < 0, \qquad \forall l, j \in \Lambda, \; \forall \mathcal{A}(\varrho) \in \mathbb{V}_{\mathbb{B}}, \tag{27}$$

$$\Xi_{jl}(\mathcal{A}(\varrho)) = \begin{bmatrix} -\tilde P_j^{11} & -\tilde P_j^{12} & \Omega_{13}^{j} & 0 \\ (\star) & -P_j^{22} & \Omega_{23}^{jl} & \Omega_{24}^{j} \\ (\star) & (\star) & \Omega_{33}^{l} & \tilde P_l^{12} + I - \alpha_{jl} G_j^{22} \\ (\star) & (\star) & (\star) & P_l^{22} - G_j^{22} - (G_j^{22})^T \end{bmatrix},$$

$$\Omega_{13}^{j} = \tilde G_j^{11} A_j^T + \tilde K_j^T B^T,$$
$$\tilde\Omega_{23}^{jl} = -\alpha_{jl} C^T \hat L_j^T + (A_j + \mathcal{A}(\varrho))^T (\alpha_{jl} G_j^{22} - I)^T,$$
$$\tilde\Omega_{24}^{j} = (A_j + \mathcal{A}(\varrho))^T (G_j^{22})^T - C^T \hat L_j^T,$$
$$\tilde\Omega_{33}^{l} = \tilde P_l^{11} - \tilde G_l^{11} - (\tilde G_l^{11})^T,$$
$$Z_{1j} = \left[-\mathcal{A}(\varrho)(\tilde G_j^{11})^T \;\; 0 \;\; 0 \;\; 0\right],$$
$$Z_{2jl} = \left[0 \;\; 0 \;\; (\alpha_{jl} G_j^{22} - I)^T \;\; (G_j^{22})^T\right].$$

By using Young relation, and Schur complement Lemma, we obtain that inequalities (27) hold, if the following inequalities are fulfilled:

$$\begin{bmatrix} \Xi_{jl} & Z_{1j}^T \mathcal{A}^T(\varrho) & Z_{2j}^T \\ (\star) & -\frac{1}{\epsilon_j} I & 0 \\ (\star) & (\star) & -\epsilon_j I \end{bmatrix} < 0, \forall l, j \in \Lambda, \; \forall \mathcal{A}(\varrho) \in \mathbb{V}_{\mathbb{B}}. \tag{28}$$

Then the following Theorem is inferred (and then proved).

Theorem 3. The observer-based controller (5) stabilizes asymptotically the system (1) if, for some scalars fixed a priori $\alpha_{j,l}$, $\epsilon_j > 0$, there exist symmetric positive definite matrices $D_l = \begin{bmatrix} \tilde P_l^{11} & \tilde P_l^{12} \\ (\star) & P_l^{22} \end{bmatrix} \in \mathbb{R}^{2n \times 2n}$, $\tilde G_j^{11}$, G_j^{22} are invertible matrices, and matrices $\tilde K_j$, $\hat L_j$, with $l, j \in \Lambda$, such that LMI (29) holds for all $l, j \in \Lambda$. Hence, the stabilizing observer-based control gains are given by $L_j = (G_j^{22})^{-1} \hat L_j$ and $K_j = \tilde K_j (\tilde G_j^{11})^{-T}$.

Remark 4. Notice that (29) is an LMI if we fix a priori α_{jl}, ϵ_j. Then we use the gridding method with respect to α_{jl}, ϵ_j, for each j and l, to solve inequalities (29). From numerical point of view, interesting results are obtained even when we take α_{jl} and ϵ_j independent of j (see Table 2 in the following section). On the other hand, we can also

$$\begin{bmatrix} -\tilde{P}_j^{11} & -\tilde{P}_j^{12} & \tilde{G}_j^{11} A_j^T + \tilde{K}_j^T B^T & 0 & -\tilde{G}_j^{11} \mathcal{A}^T(\varrho) & 0 \\ (\star) & -P_j^{22} & -\alpha_{jl} C^T \hat{L}_j^T + (A_j + \mathcal{A}(\varrho))^T (\alpha_{jl} G_j^{22} - I)^T & (A_j + \mathcal{A}(\varrho))^T (G_j^{22})^T - C^T \hat{L}_j^T & 0 & 0 \\ (\star) & (\star) & \tilde{P}_l^{11} - \tilde{G}_l^{11} - (\tilde{G}_l^{11})^T & \tilde{P}_l^{12} + I - \alpha_{jl} G_j^{22} & 0 & \alpha_{jl} G_j^{22} - I \\ (\star) & (\star) & (\star) & P_l^{22} - G_j^{22} - (G_j^{22})^T & 0 & G_j^{22} \\ (\star) & (\star) & (\star) & (\star) & -\frac{1}{\epsilon_j} I & 0 \\ (\star) & (\star) & (\star) & (\star) & (\star) & -\epsilon_j I \end{bmatrix} < 0. \tag{29}$$

linearize (29) with respect to ϵ_j by using the inequality $-\frac{1}{\epsilon_j} I \leq -(2 - \epsilon_j) I$.

4. NUMERICAL EXAMPLES

In this section, we provide numerical examples and simulations. The first example is taken from Heemels et al. (2010). The goal of this example is to show that the proposed design methodology tolerates larger uncertainty level Δ. The second example is taken from Jetto and Orsini (2010). We compare the proposed methods to those in Heemels et al. (2010); Jetto and Orsini (2010); Zemouche et al. (2016).

Example 1. Consider the following discrete-time LPV system Heemels et al. (2010)

$$x_{t+1} = \begin{bmatrix} 0.25 & 1 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.6 + \rho(t) \end{bmatrix} x_t + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u_t \tag{30a}$$

$$y_t = [1 \;\; 0 \;\; 2]\, x_t \tag{30b}$$

with $\rho(t) \in [0, 0.5]$, $t \in \mathbb{Z}^+$. In this case, we can take the functions $\xi_1(\rho) = \frac{0.5-\rho}{0.5}$ and $\xi_2(\rho) = \frac{\rho}{0.5}$ and $A_1 = A(0)$, $A_2 = A(0.5)$. The LMIs (29) return simultaneously for $\epsilon_j = 0.03$, $j = 1, 2$ and $(\alpha_j)_{j=1}^2 = (100, 130)$, the controller gains

$$K_1 = 10^{-2} \times [0.0019 \;\;\; 0.0005 \;\;\; -58.8554]\,,$$

$$K_2 = 10^{-2} \times [-0.0001 \;\;\; 0.0006 \;\;\; -133.5613]$$

and the observer gains given by

$$L_1 = \begin{bmatrix} -0.0054 \\ 0.0055 \\ 0.2273 \end{bmatrix}, L_2 = \begin{bmatrix} -0.0284 \\ 0.0112 \\ 0.4598 \end{bmatrix}.$$

These observer-based controller gains are obtained for the largest value of uncertainty level $\Delta_{\max} = 0.4441$, while the other methods in Heemels et al. (2010), Jetto and Orsini (2010) and Zemouche et al. (2016) are found infeasible for this uncertainty level.

Example 2. Now, we consider the DC motor model given example 2 in Jetto and Orsini (2010).

We have compared the feasibility of the proposed design methods and those established in Heemels et al. (2010); Jetto and Orsini (2010); Zemouche et al. (2016), by increasing the uncertainty level Δ until obtaining infeasibility. The superiority of the proposed LMIs (29) is quite clear from the results presented in Table 1.

With $\Delta_{\max} = 1.4$, LMIs (29) return simultaneously the observer-based controller gains

$$K_1 = [0.0100 \;\;\; -4.0310]\,, K_2 = [0.0100 \;\;\; -0.0067]\,,$$

and

$$L_1 = \begin{bmatrix} 0.3031 \\ 1.9758 \end{bmatrix}, L_2 = \begin{bmatrix} 0.2415 \\ -0.0407 \end{bmatrix}.$$

We see through these comparisons that the proposed methods solve the stabilization problem with better uncertainty levels. The gains of the observer-based controller are computed by solving only a single set of LMIs running with only one-step algorithm. This demonstrates the simplicity and the efficiency of the proposed methodology.

5. CONCLUSION

In this paper, we have presented two new LMI synthesis methods to design observer-based controllers for a class of LPV systems with inexact but bounded parameters. The approach used a new congruence principle by pre- and post-multiplying the basic BMI by new and ingenious matrices. Thanks to these matrices, some bilinear terms vanish from the BMI, which becomes more flexible for the linearization. To show the validity and superiority of the proposed design methods, two numerical examples from the literature have been reconsidered in this paper. The comparisons show that the proposed methodologies provide less conservative LMI conditions compared to LMI techniques reported previously in the literature.

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Table 1. $\Delta_{\max}$ tolerated for different methods in Example 1 and 2

Method	$\Delta_{\max}$ in Example 1	$\Delta_{\max}$ in Example 2
LMI in Heemels et al. (2010)	0.1786	1.0855
LMI in Jetto and Orsini (2010)	(!)	0.0800
LMI 18 in Zemouche et al. (2016)	0.2123	0.9999
LMI 24 in Zemouche et al. (2016)	0.3107	0.9999
	$(\alpha_j)_{j=1,2} = (100, 130)$ $(\epsilon_j)_{j=1,2} = 0.03$	$(\alpha_j)_{j=1,2} = 1$ $(\epsilon_j)_{j=1,2} = 24$
LMI (29)	0.4441	1.4

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