Study of the Tuning of Variable Pitch Milling Cutters

Zoltan Dombovari, Alexander Iglesias, Gábor Stépán

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Z. Dombovari[1,2], A. Iglesias[2] and G. Stepan[1]
[1] Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest H-1521, Hungary
[2] Dynamics and Control, IK4-Ideko, 20870 Elgoibar, Basque Country, Spain

Abstract. It is well known that variable pitch cutters may improve stability of a given milling operation. Several methods are known for tuning the consecutive pitch angles to approach idealistic phase shift on regeneration. Most of the tuning procedures are based on the measurement of the chatter frequency. However, these design procedures completely disregard the effect of the harmonics of the milling process. In this study, the efficiency of these methods is verified in milling for an interrupted cutting engagement. The critical analysis of different tuning methodologies are carried out by means of accurate stability charts constructed with semi-discretization. It is shown that the effectiveness of tuning can be improved further with numerical optimization techniques.

Keywords: stability, milling, tool geometry, variable pitch

1 Introduction

The dynamic behavior of regenerative cutting processes can be influenced through the generated cutting force. Apart from the material properties, it is the edge geometry that affects the most the relative magnitude and the regeneration of the instantaneous cutting force. In this manner, any attempt to distort both the strength and the length of the regenerative delays most likely leads to better conditions for stable stationary cutting [1]. As impacts are reduced by introducing helices, regenerative delays can be distorted by irregular spacing of edge portions in the direction of the cutting speed, or by the variation of the spindle speed [2]. Discrete variation of regenerative delays can be introduced by simple uneven pitch angles or by irregularities in local subsequent radii causing missed cuts e.g. in case of serrated cutters [3, 4]. While continuous variation on the delay's weights can be introduced by helix angle variations [5].

Due to the uneven pitch geometry, the corresponding milling processes are subject to the effect of the irregular interaction of the time-periodicity and the distorted constant delays. This special geometry changes the regenerative phases between the past and the present vibration pattern in modulo, which effect was historically modelled by unevenly displaced multi-edge orthogonal planing operation by Slavicek [6]. In this work significant gain was achieved in depth of cut compared to conventional multi-edge-planing. Average directional factors were used by Opitz et al [7] to model the experimentally observed increase on stability by considering the simplest uneven pitch case with two teeth. In terms of rectilinear tool motions, computer simulations were carried out by Vanherck [8] for different set of varying pitch angles showing the behavior of distinct non-conventional tool geometries. Also, applying numerical simulations, stability properties of variable pitch cutters were simulated in the work of Tlusty et al [9]. Significant changes in stability were shown in [10] without mentioning geometrical design strategies: improved mechanical model was used by fully respecting the varying directional orientations of the variable pitch geometry.

Single frequency variable pitch model was presented by Altintas et al [11] where the eigenvalue problem is formulated and the stability behavior is shown with respect to the pitch angles introducing real case dynamics. Based on time averaged single frequency solution Budak [12, 13, 14] introduced an analytical design methodology based on the phase differences between consecutive constant delays, the effectiveness of which was confirmed with in industrial application. In the work of Olgac and Sipahi [15], a unique scheme, the so-called cluster treatment of characteristic roots paradigm was used to investigate the effect of two constant delays on the time-averaged dynamics of milling operation with variable pitch cutters. Time finite element method and time averaged semi-discretization were used by Sims et al [16] to build general models of variable pitch and helix tools, around which optimization process was established in [17, 18] based on genetic algorithms.

The inconsistency of the time period and the regenerative delays in case of variable pitch cutters results in that higher harmonics can have significant strength in the regenerative force even in the case of non-interrupted cutting operations, causing discrepancies in time averaging methods showed in Sellmeier and Denkena [19]. The importance of the optimization of variable pitch was pointed out in the experimental work of [20]. The construction of transition matrix and the unavoidable eigenvalue calculation were joined by subspace iteration in order to decrease the overall order of the numerical methods (see [21]).

Similar effect can also be reached in drilling and in turning processes, when multiple regenerative delays are introduced instead of the conventional single one. In case of turning, optimized multi edge cutting can disturb the regenerative phase between consecutive edges as presented in Budak and Ozturk [22].

2 Model Description

For simplicity, helix is not considered in this study which can practically correspond to inserted milling with slight helix angle. In this case the regenerative delays τ_i can be calculated with the help of the period of the tool $T = 2\pi/\Omega$ and the pitch angles $\varphi_{\mathrm{p},i}$ between consecutive edges as

$$\tau_i = \frac{\varphi_{\mathrm{p},i}}{\Omega}, \qquad (2.1)$$

where the spindle speed is $\Omega\ (\mathrm{rad/s}) = 2\pi n\ (\mathrm{rpm}) / (60\ \mathrm{s/min})$. Due to the successive regenerative waves, phase can be defined absolutely as ε_i and in modulo as ϑ_i in the following way

$$\varepsilon_i := 2\pi l_i + \vartheta_i \qquad (2.2)$$

for each (i mod Z) and (i+1 mod Z) pair of teeth. The integers $l_i = 0, 1, \ldots$ are the number of whole waves copied in the surface originated from the main chatter angular frequency ω of the self-excitation. Thus, the different properties of the regenerative phases can be expressed for uneven pitch cutters as (round down is $\lfloor \bullet \rfloor$)

$$\varepsilon_i = \omega\tau_i\,,\ l_i = \left\lfloor \tfrac{\omega\tau_i}{2\pi} \right\rfloor \text{ and } \vartheta_i = \omega\tau_i \bmod 2\pi\,. \qquad (2.3)$$

Unlike regular cutters, the principle period T_p of the milling process is not the tooth passing period $T_Z = T/Z$. Depending on the actual configuration, it is determined by the natural number divisor

$$N = Z / \operatorname{rank}\left[\varphi_{\mathrm{p},(k+l-1) \bmod Z}\right]_{k,l=1}^{Z}\,. \qquad (2.4)$$

as $T_\mathrm{p} = T / N$.

In this framework, we assume momentary chip thickness variation between two consecutive edges as

$$\begin{aligned} h_i(t) &= \mathbf{n}^\mathsf{T}(\varphi_i(t))(\mathbf{f}_i + \Delta\mathbf{x}_i(t)) \\ &= (f_i + x(t) - x(t-\tau_i))\sin\varphi_i(t) + \\ &\qquad (y(t) - y(t-\tau_i))\cos\varphi_i(t) \end{aligned} \qquad (2.5)$$

for a milling operation with feed $f_i = f\,\varphi_{\mathrm{p},i}/(2\pi)$ in x direction. Thus, $\mathbf{f}_i = \mathrm{col}(f_i, 0, 0)$ and $\mathbf{n}(\varphi_i(t)) = \mathrm{col}(\sin\varphi_i(t), \cos\varphi_i(t), 0)$. The feed per revolution is f (m/rev), while the position angle

$$\varphi_i(t) = \Omega t + \sum_{k=1}^{i-1}\varphi_{\mathrm{p},i} \qquad (2.6)$$

determines the position of the ith tooth geometrically. Assuming (for simplicity) linear cutting force characteristics with edge coefficients $\mathbf{K}_\mathrm{e}$ and cutting coefficients $\mathbf{K}_\mathrm{c} = \mathrm{col}(1, K_{\mathrm{c},r}, K_{\mathrm{c},a})$ (see Table 2.1.), the regenerative cutting force has the form:

$$\begin{aligned} &\mathbf{F}(t, \mathbf{x}(t), \mathbf{x}(t-\tau_k)) \\ &\quad = -a\sum_{i=1}^{Z} g(\varphi_i(t))\mathbf{T}(\varphi_i(t))(\mathbf{K}_\mathrm{e} + K_{\mathrm{c},t}\mathbf{K}_\mathrm{c} h_i(t)) \\ &\quad = \mathbf{G}(t) + a\,K_{\mathrm{c},t}\sum_{i=1}^{Z}\mathbf{A}_i(t)(\mathbf{x}(t) - \mathbf{x}(t-\tau_i)), \end{aligned} \qquad (2.7)$$

for lead angle $\kappa = 90$ (deg), where the transformation matrix $\mathbf{T}$ and the screen function g is defined in [3], and $k = 1, 2, \ldots, N_\tau$. The regenerative cutting force in (2.7) depends on the present $x(t)$ and the delayed $x(t-\tau_k)$ positions of the tool where $\mathbf{x}(t) = \mathrm{col}\,(x(t), y(t), z(t))$. Furthermore, it can be separated into periodic state independent part $\mathbf{G}(t) = \mathbf{G}(t + T_\mathrm{p})$, and linear state dependent part with periodic coefficient matrices $\mathbf{A}_i(t) = \mathbf{A}_i(t + T_\mathrm{p})$ [23].

2.1 Milling Dynamics

By using the same procedure described in [13, 23], the determination of asymptotic stability of the periodic stationary solution with $\Omega_\mathrm{p} = 2\pi / T_\mathrm{p}$ leads to the following characteristic equation

$$\det\left(\mathbf{I} - a\,K_{\mathrm{c},t}\boldsymbol{\Phi}(\omega)\sum_{i=1}^{Z}(\mathbf{I} - \mathrm{e}^{-\mathrm{j}\omega\tau_i}\,\mathbf{E}_i)\,\mathbf{D}_i\right) = 0\,. \qquad (2.8)$$

Considering h harmonics in (2.8), we have

$$\boldsymbol{\Phi}(\omega) = \operatorname*{diag}_{l=-h}^{h}\mathbf{H}(\omega + l\,\Omega_\mathrm{p})\,,\ \mathbf{E}_i = \operatorname*{diag}_{l=-h}^{h}\mathbf{I}\,\mathrm{e}^{-\mathrm{j}l\,\varphi_{\mathrm{p},i}}\,,$$

$$\mathbf{D}_i = [\mathbf{A}_{l-k,i}]_{l,k=-h}^{h} \text{ and } \mathbf{A}_i(t) = \sum_{l=-\infty}^{\infty}\mathbf{A}_{l,i}\,\mathrm{e}^{\mathrm{j}l\,\Omega_\mathrm{p}t}\,. \qquad (2.9)$$

Basically, (2.8) is a truncated Hill's representation by using the frequency response function (FRF) $\mathbf{H}(\omega)$ at the tool tip [24], which describes the dynamics of the milling process. This solution is referred to in the literature as multi-frequency solution [25, 26, 27].

In order to deal with the analytical literary cases, the single frequency solution is introduced here for

$$\det\left(\mathbf{I} - a\,K_{\mathrm{c},t}\mathbf{H}(\omega)\sum_{k=1}^{N_\tau}(1 - \mathrm{e}^{-\mathrm{j}\omega\tau_k})\,\mathbf{A}_0\right) = 0\,. \qquad (2.10)$$

corresponding to the model in [13], where the harmonics

$$\mathbf{A}_0 = \frac{N}{2\pi}\int_{\varphi_{\mathrm{en}}}^{\varphi_{\mathrm{ex}}} \mathbf{T}(\varphi)\mathbf{K}_{\mathrm{c}}\mathbf{n}^{\mathsf{T}}(\varphi)\,\mathrm{d}\varphi = \sum_{i=1}^{Z} \gamma_{k,i}\mathbf{A}_{0,i} \, , \quad (2.11)$$

while $\gamma_{k,i}=1$ if $\tau_k=\tau_i$, otherwise $\gamma_{k,i}=0$ with $k=1, 2, \ldots, N_\tau$. The radial immersion is simply given by the entry φ_{en} and exit φ_{ex} angles (see Fig. 2.1.a).

Generally, the solution of (2.8) and (2.10) can be calculated by solving the eigenvalue problem for the depth of cut a [13] or by using the bisection method [28] in the parameter space (a, Ω, ω).

2.2 Simplified Case

In this work, we deal with the differences between the historical analytical tuning procedures and the possibly best achievable one. Considering that every new pitch angle increases the dimension of the optimization parameter space by one, a simple and practical one-parameter case (with $\varphi_{\mathrm{p},1}$) can be introduced with the symmetric arrangement of $Z=4$ flutes with $\kappa=90$ deg lead angle (see Fig. 2.1.a). In this case the following notation can be introduced $\varphi_{\mathrm{p},i}=\varphi_{\mathrm{p},1}$, $\pi-\varphi_{\mathrm{p},1}$, $\varphi_{\mathrm{p},1}$, $\pi-\varphi_{\mathrm{p},1}$ with $N=2$ in (2.4). Also the notation can be introduced describing the angle differences as $\varphi_{\mathrm{p},2}=\varphi_{\mathrm{p},1}+\Delta\varphi_{\mathrm{p},2}$.

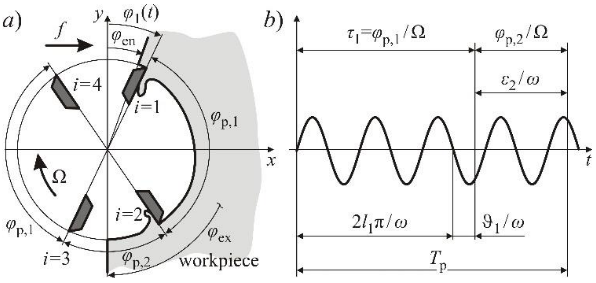

Fig. 2.1. *a*) simplified symmetric teeth arrangement, *b*) phase shifts

As further simplification, only one spatial direction is considered in the *x* direction, which makes the matrices at (2.9) scalars along with the considered single mode (see Table 2.1.) in (2.10), resulting in

$$1 - a K_{\mathrm{c},t}\, H(\omega) \sum_{k=1}^{N_\tau} (1-\mathrm{e}^{-\mathrm{j}\omega\tau_k}) A_0 = 0 \, , \quad (2.12)$$

The milling process is chosen to be interrupted to show the discrepancies on analytical tuning procedures that are all based to some extent in the single frequency solution (2.10). Moreover, this interrupted case is more practical process-wise, since it models finishing operation for which vibration needs to be attenuated to achieve good surface quality. Further details of the considered case can be followed in Table 2.1.

Table 2.1. Technological parameters of the cutting process under consideration

Z	κ (deg)	η (deg)	a_{r} (mm)	feed direction	
4	90	0	1	(1, 0, 0)	
$K_{\mathrm{c},t}$ (MPa)	$K_{\mathrm{c},r}$ (1)	$K_{\mathrm{c},a}$ (1)	$\omega_{\mathrm{n},x}$ (Hz)	ξ_x (%)	k_x (N/μm)
804	0.314	0.150	178	0.54	19.78

3 Different Tuning Schemas

The three main literary analytical tuning techniques are discussed in this section in chronological order. We mention these solutions as Slaviček's [6], Engin's [11] and Budak's [13] tuning methods referring to the main contributors' names. Although strictly speaking, Slaviček's dealt with planing only in his work, his tuning methodology can easily be applied for milling. All methodologies are simplified to the case introduced in Section 2.

3.1 Slaviček's tuning

Originally, this method is a graphical way of tuning based on the different phase shifts in the scalar single frequency formulation introduced in (2.12). Substituting $H(\omega) = r\,(\omega)\mathrm{e}^{\,\mathrm{j}\,\psi(\omega)}$, considering the regenerative phases at (2.3), and keeping in mind that the geometric arrangement introduced in Fig. 2.1.*a*) with ($N_\tau=2$), the following expression is obtained:

$$(N_\tau - 2\,(\mathrm{e}^{-\mathrm{j}\varepsilon_1} + \mathrm{e}^{-\mathrm{j}\varepsilon_2})) = \frac{1}{a\,K_{\mathrm{c},t}\,r\,A_0}\mathrm{e}^{-\mathrm{j}\,\psi} \, . \quad (3.1)$$

Contrary to the original work, the force does not introduce additional phase shift represented by ρ in [6]. By using the intricate trigonometrical manipulations in [6], (3.1) can be rearranged as

$$\mathrm{e}^{\mathrm{j}\delta} := (N_\tau - 4\mathrm{e}^{-\mathrm{j}\varepsilon}\cos\Delta) = \frac{1}{a\,K_{\mathrm{c},t}\,r\,A_0}\mathrm{e}^{-\mathrm{j}\,\psi} \, , \quad (3.2)$$

where the average regenerative phase is $\varepsilon = (\varepsilon_1 + \varepsilon_2)/2$, and the phase difference is $\Delta = (\varepsilon_2 - \varepsilon_1)/2 = \Delta\varepsilon_2$. The resultant phase shift δ in (3.2) has the least chance (see [6]) to "intersect" with $-\psi$ if δ varies in the slightest way, that is $\cos\Delta \approx 0$. Taking into account (2.1) and (2.3), the tuning criterion is given as

$$\Delta = n\frac{\pi}{2} \quad \Rightarrow \quad \frac{\omega}{\Omega}\Delta\varphi_{\mathrm{p},2} = \Delta\varepsilon_2 = \pi + 2k\pi \, , \quad (3.3)$$

where historically n = 1, 3, 5, … and k = 1, 2, 3, … . That is, knowing the main vibration (chatter) frequency ω and the spindle speed $\boldsymbol{\Omega}$ the pitch difference $\Delta\varphi_{\mathrm{p},2}$ can be designed setting k considering geometric design restrictions ($\varphi_{\mathrm{p,min}}$, $\varphi_{\mathrm{p,max}}$ and $\Delta\varphi_{\mathrm{p,min}}$).

Bearing in mind (3.2) and (3.3), the methodology for this simple one dimensional, single frequency and simple pitch arrangement case (Fig. 2.1.*a*) can be generalized for multiple modes, too.

3.2 Engin's tuning

The tuning methodology presented in [11] is based on an initial stability calculation (e.g. with single frequency or semi-discretization) where the regenerative phase is "picked up" from the diagram to ensure the positioning of the stability pockets (sweet spots or stability resonances, e.g. n = 2670 rpm in Fig. 3.1.*d*). A stability pocket actually corresponds to two different regenerative phase shifts. In the simple case presented in Fig. 3.1.*c*), these are actually ϑ_{sp} =180 deg or 360 deg. According to [11] the one which changes less with the spindle speed ensures more robust solution.

In this manner the design criterion is to have the regenerative phase shift (in modulo) for the stability pockets ensured by the first pitch angle

$$\frac{\omega}{\Omega}\varphi_{\mathrm{p},1} = \vartheta_{\mathrm{sp}} + 2k\pi \text{ and } k = 1, 2, 3, \dots . \quad (3.4)$$

This design criterion is taking the complicated dynamics (even multi-dimensional one) into account by an initial calculation, predicting the necessary regenerative phase in modulo ϑ_{sp}. The integer k has to be set to satisfy geometrical design criteria (see previous subsection) and to have $\varphi_{\mathrm{p},1}$ as close to $2\pi/Z$ as it is possible.

3.3 Budak's tuning

This tuning procedure is based on the eigenvalue consideration of single frequency solution at (2.10). This results in parametric solution for the depth of cut a as explained in [13]. Considering 3 spatial dimensions (xyz) in (2.10) the following holds

$$a(\omega) = -\frac{2\pi}{K_{\mathrm{c},t}}\frac{\Lambda_{\mathrm{I}}(\omega)}{S(\omega)}, \; \Lambda(\omega) = \Lambda_{\mathrm{R}}(\omega) + \mathrm{j}\Lambda_{\mathrm{I}}(\omega), \quad (3.5)$$

where

$$\Lambda(\omega) = \frac{a}{2\pi}K_{\mathrm{c},t}\sum_{k=1}^{N_\tau}(1-\mathrm{e}^{-\mathrm{j}\omega\tau_k}) \text{ and } S(\omega) = \sum_{k=1}^{N_\tau}\sin\omega\tau_k \; .$$

The basic idea here is that, the limit depth of cut a at (3.5) is maximal if S is minimal or in extreme cases 0. Applying geometry in Fig. 2.1.*a*) results in the following

$$\begin{aligned} S(\omega) &= \sin\varepsilon_1 + \sin(\varepsilon_1 + \Delta\varepsilon_2) \\ &= 2\cos\frac{\Delta\varepsilon_2}{2}\sin\frac{\omega}{\Omega}\pi = 0 \quad \Rightarrow \cos\frac{\Delta\varepsilon_2}{2} = 0. \end{aligned} \quad (3.6)$$

In this case, this methodology gives the same criterion as Slaviček's tuning at (3.3).

The tuning is general since the eigenvalue solution of (2.10) can be made for multi-dimension and multi modal cases, too. However, the $S(\omega) = 0$ would be true only in limiting case which might interfere with $\Lambda_{\mathrm{I}}(\omega)$.

3.4 Comparison of analytical methods

In this simplified one-parameter optimization case Slaviček's and Budak's tunings are identical. The comparison of these can be followed in Fig. 3.1. where the first pitch angle was designed to be smaller and closer to $2\pi/Z$.

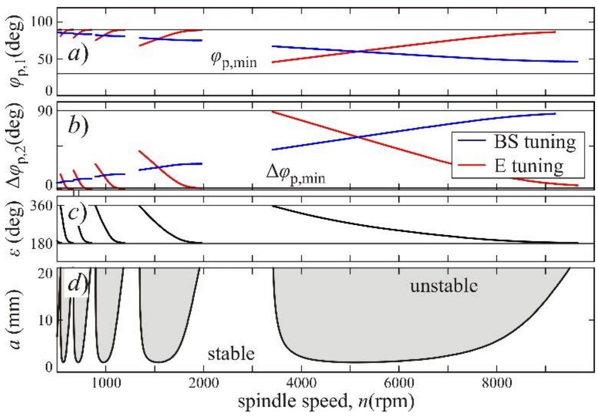

Fig. 3.1. *a*) tuned first pitch angle, *b*) designed pitch differences, *c*) regenerative phase and *d*) ideal single frequency solution for the process in Table 2.1.

This means Engin's and the other two tuning methodologies introduce opposite trends in $\varphi_{\mathrm{p},1}$ seemingly resulting in identical tuning for the minimum in this one DOF single frequency case. Also toward the larger lobe number the modulo of regenerative phase eventuates smaller deviation for $\Delta\varphi_{\mathrm{p},2}$ (see Fig. 3.1.*b*). This makes variable pitch milling tools extremely difficult to accurately tune and produce in low spindle speed zone.

4 Applying semi-discretization

In order to have the accurate stability chart even for the described interrupted case (Table 2.1.), semi-discretization (SD) [29] method is applied. By applying

SD, the prediction of the main vibration frequency is not trivial at all. This is because the critical multiplier μ_c (closest to magnitude 1) refers directly only to the smallest possible modulation (base frequency, ω_b) of the main vibration frequency ω described in [30], that is

$$\lambda_c = \alpha_c + \omega_{c,b}\mathrm{j} \text{ and } \alpha_c T_p = \ln|\mu_c|\,,\ \omega_{c,b} T_p = \arg \mu_c\,. \quad (4.1)$$

The main vibration frequency can be determined by performing low resolution fast Fourier transform (FFT) on the periodic part of the critical eigenvector of the transition matrix [30]. Thus, it can be set by the largest modulation number q on the low resolution FFT as

$$\omega = |\,\omega_{c,b} + q\ \Omega_p\,|\,. \quad (4.2)$$

In this way, it is possible to apply tuning criteria (3.3), (3.4) and (3.6) by using a preliminary calculation determining the corresponding main vibration (chatter) frequency ω. Also it is worth to mention that after the tuning, the ω can slightly change, but in this one DOF case this effect is negligible.

A brute-force iterative (BFI) method is also implemented, which only relies on the SD method. The optimum is achieved by applying basically bisection scanning on $\varphi_{p,1}$ minimizing the magnitude of the corresponding critical multiplier $|\,\mu_c\,|$. Afterwards this method is referred as "BFI tuning".

4.1 Applying for one specific tuning

By using the SD method, the quality of the tuning can be rated by calculating the accurate stability behavior of the introduced interrupted milling process (Table 2.1.). In Fig. 4.1., the different tuning techniques are compared away from the single frequency minimum of the first lobe at n = 1800 rpm. In this case, Budak's and Slaviček's tuning (afterwards "BS-tuning") serve different pitch arrangement than Engin's tuning (afterwards "E-tuning").

Table 4.1. The results of different tuning techniques for n = 1800 rpm

tuning technique	pitch angles, $\varphi_{p,i}$ (deg)
BS tuning	75.4180, 104.5820, 75.4180, 104.5820
E tuning	87.4852, 92.5148, 87.4852, 92.5148
BFI tuning	50.5373, 129.4627, 50.5373, 129.4627

It can be followed in Fig. 4.1. that E-tuning completely fails for this given case, while BS-tuning really improves the stability properties, in spite of the fact that it is "chopped off" by the now more intricate flip instability corresponding to the principle period T_p. The best tuning is given by the BFI tuning that eliminates the effect of the flip instability in this case. The actual tuning results are listed in Table 4.1.

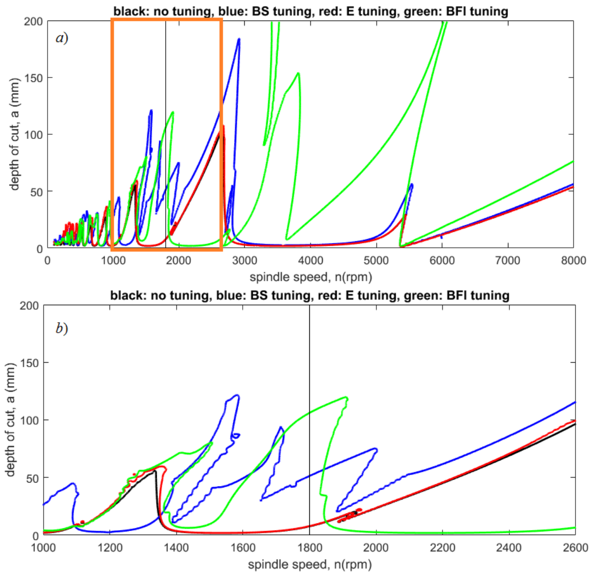

Fig. 4.1. The effect of different tuning methodologies for n = 1800 rpm

4.2 Optimal tuning along the stability limit

This subchapter compares the tuning strategies by their trends along the linear stability limits similarly to Fig. 3.1., in which only the analytical tuning methodologies are compared using ideal single frequency solution.

It can be recognized immediately the E tuning methodology does not serve good geometrical arrangement in pitch spaces to gain stability. The BS tuning results in better optimized geometry, however, the BFI tuning induces even higher stability limits (see Fig. 4.2.).

Similarly to Fig. 3.1. opposite trend between BS and E tunings are appearing. Not surprisingly, the brute force method (BFI tuning) follows different optimized geometric arrangement ensuring the best possible tuning.

5 Conclusions

In this paper, a simple variable pitch tool arrangement was investigated. It was shown, that the single-frequency based analytical solutions of the literature are not efficient enough to find optimal tuning. Based on the semi-discretization method, a brute force methodology was developed that is able to determine the best tuning for each parameter configuration by minimizing the critical characteristic multiplier of the corresponding time-periodic delayed system. The simple practical case-study showed that there is a potential in developing improved tuning algorithms for variable pitch cutters.

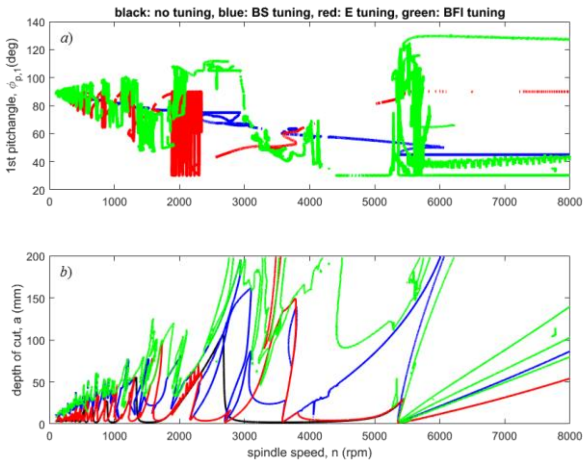

Fig. 4.2. Continuous optimal tuning: *a*) first pitch angle $\varphi_{p,1}$, *b*) corresponding stability charts.

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