Can we estimate the Young's modulus of the space-time from simple analogies based on the concepts of the strength of materials? Reflection and proposals

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Reflection and proposals

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Juillet 2017

Summary

Einstein has shown that the matter curves the space-time. In its model, the space-time is a mathematical conceptualization unifying the space and the time that accounts for measured physical phenomena (delay of the perihelion of Mercury, deviation, curvature of light by a large mass, relativity of length and time, propagation of gravitational waves, caused in particular by coalescence of black holes, which materialize by infinitesimal deformations ΔL / L of space-time (10^{-21})), etc. The question that then comes to mind is whether the mathematical conceptualization of space-time has a physical reality? In this case, what is the material of the fabric of the space-time that undulates or bends under gigantic masses in motion? Where, from a mechanical point of view, can we find an equivalent Young's modulus E to the fabric of the space-time? A first clue is the Casimir's force, which tells us that the vacuum of space is not empty ... From recent publications on the subject and simple analogies based on the strength of materials, we will show that if the E-module of space-time exists, it seems incommensurably large whatever the approach considered (elastic plates in 2 dimensions, plates analyzed by a tensor calculation in 4 dimensions, strings, Casimir force). In any case, only a measure of the latter would make it possible to decide definitively.

1) Determination of the young modulus of the fabric of the space time by analogy with the classical mechanic then with a tensorial calculation in 4 dimensions

1.1) Determination of the base equations

We have shown in previous publications that the relation between the curvature and the energy can be written in 2 dimensions (plate theory) and in 4 dimensions (General relativity theory) [23].

We have so:

- In plate theory in two dimensions:

$$\Delta U = \frac{D}{2}\left[\left(\frac{\partial^2 w_{(x,y)}}{\partial x^2}\right)^2 + 2\nu\frac{\partial^2 w_{(x,y)}}{\partial x^2}\frac{\partial^2 w_{(x,y)}}{\partial y^2} + 2(1-\nu)\left(\frac{\partial^2 w_{(x,y)}}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 w_{(x,y)}}{\partial y^2}\right)^2\right]\Delta x \Delta y \quad (1)$$

  Or again, if we put following a more clearly way the curvature in one side and the energy on the other side of the equation, we obtain:

$$\boxed{\left[\left(\frac{1}{R_x}\right)^2 + \left(\frac{1}{R_y}\right)^2 + 2(1-\nu)\left\{\left(\frac{1}{R_{xy}}\right)^2\right\} + 2\nu\left\{\frac{1}{R_x}\frac{1}{R_y}\right\}\right] = \frac{24(1-\nu^2)}{Eh^3} \times \frac{\Delta U}{\Delta x \Delta y} \quad (2)}$$

- Following the general relativity, if we put following a more clearly way the curvature of the space time in one side and the stress energy tensor on the other side of the equation, we obtain:

$$\boxed{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad (3)}$$

With for memory for the stress energy tensor:

$$T_{\mu\nu} = \begin{bmatrix} \frac{mc^2}{V} & \rho c v_x & \rho c v_y & \rho c v_z \\ \rho c v_x & \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \rho c v_y & \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \rho c v_z & \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \quad (4)$$

If we suppose that the space-time fabric is a superposition of thin sheets of thickness L_p (with L_p the length of Planck), we obtain a multi sandwich universe.
It is then possible by analogy between the two models (Einstein's general relativity and the plate theory in strength of materials) to find an evaluation of the Young's modulus E of the space-time E (N / m²).

To do this, it is sufficient, to compare the curvature of the two equations (2) and (3). We obtain then:

$$\frac{24(1-\nu^2)}{Eh^2} \times \frac{U}{abh} \approx \frac{8\pi G}{c^4} \times \frac{U}{V} \quad (5)$$

By simplifying by U / V on each side of the equation, we obtain:

$$\frac{8\pi G}{c^4} \approx \frac{24(1-\nu^2)}{Eh^2} \quad (6)$$

The dimensional equation allows us to verify that this equivalence is reasonable:

$$\frac{L^3}{MT^2\frac{L^4}{T^4}} => \frac{1}{M\frac{L}{T^2}} = \frac{1}{\frac{M}{L^2}\frac{L}{T^2}L^2} => \frac{1}{M\frac{L}{T^2}} = \frac{s^2}{kgm} \quad (7)$$

We can extract from the equation (6) an estimation of the Young's modulus E of the space-time fabric..

$$E_{fabric\ of\ the\ space\ time} = \frac{24(1-\nu^2)c^4}{h^2 8\pi G} \quad (8)$$

By introducing $\kappa = \frac{8\pi G}{c^4}$ (9)

We obtain the Young modulus E of the space-time fabric by analogy with a thin sheet of thickness h of the space-time in bending:

$$E_{fabric\ of\ the\ space\ time} = \frac{24(1-\nu^2)}{h^2\kappa} \quad (10)$$

v is the Poisson's ratio of the space-time .

If we assume that the thickness of the thin sheet is equal to the length of Planck L_p (smaller than it is possible to reach), we obtain:

$$E_{fabric\ of\ the\ space\ time} = \frac{24(1-\nu^2)}{{L_p}^2\kappa} \quad (11)\ with\ L_p = \sqrt{\frac{\hbar G}{c^3}} \quad (12) \quad and \quad \kappa = \frac{8\pi G}{c^4}$$

$$E_{fabric\ of\ the\ space\ time} = \frac{6(1-\nu^2)c^7}{G^2 h} = \frac{3(1-\nu^2)c^7}{G^2\hbar\pi} \quad (13)$$

It will be noted that this last expression associates the 3 fundamental constants of physics, c, G and $\hbar$.

1.2) Numerical application

We apply the formula (11) from the following data:

v = 0 (hypothesis)

G = 6.6726x10^{-11} m^3/kg/s^2

c = 299792458 m/s

h =6.62607004x10^{-34} m^2 kg/s

κ =2.07651x10^{-43} s^2/(kg . m)

L_p = 1.62 x10^{-35} m

This results for the Young's modulus E is so:

$E_{space\ time\ fabric}$ = 4.4247E^{+113} N/m²

We can see the extremely high value of this young modulus following this approach.

1.3) Comparison of the obtained results with the state of the art– tensorial calculation of a plate in 4 dimensions

In the publication [5]: “The Mechanics of Spacetime {A Solid Mechanics Perspective on the Theory of General Relativity} T G Tenev and M F Horstemeyer” we have an expression of young modulus.

The same approach was developed, but this time considering a plate in 4 dimensions and performing the tensor calculations. The authors arrive at the following expression:

$Y = 2(1+v)\mu$ With $\mu = \frac{12}{L_p^2\kappa}$ and with $\kappa = \frac{8\pi G}{c^4}$ and $L_p = \sqrt{\frac{\hbar G}{c^3}}$ and v = 1

See. Formula 3.12 or 4.45 following the date of publication [5] considered with v = 1:

$$Y = E_{space\ time\ fabric} = \frac{48}{{L_p}^2\kappa} = \frac{24(1+v)}{{L_p}^2\kappa} \quad (14)$$

$E_{space\ time\ fabric\ (v=1)}$ = 8.8495E^{+113} N /m²

By taking v = 0 (Poisson’s ratio) to compare with our results, we obtain:

$$E_{space\ time\ fabric} = \frac{24}{{L_p}^2\kappa} = \frac{24(1+v)}{{L_p}^2\kappa} \quad (15)$$

$E_{space\ time\ fabric\ (v=0)}$ = 4.43E^{+113} N /m²

2) Determination of the Young modulus of the space time by an approach of wave transmit by the elastic material constituting the fabric of the universe

2.1) Determination of the base equations

We can consider the space-time fabric as an assembly of extremely fine and short strings (see Figure 1).

We also consider that it is this fabric (invisible to our eyes since it is made up of elements of the size of Planck's length) which transmits the gravitational waves and bends in the presence of matter.

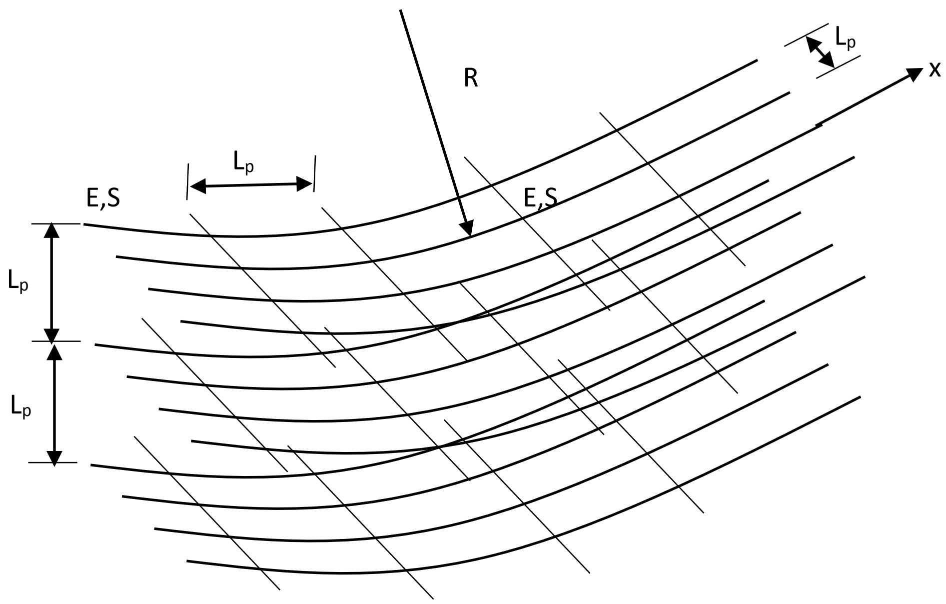

Figure 1 – Fabric of the space time made of an assembly of thin strings that have a young modulus E and a section S –

By definition of a force we have:

$$F = m\gamma \quad (16)$$

By denoting by S the section of the string and ρ the density of the material constituting it, we denote by $u_{(x,t)}$ the displacement of a point of a string :

$$\rho S \times \Delta x \times \frac{\partial^2 u(x,t)}{\partial t^2} = F(x+dx) - F(x) \quad (17)$$

In addition:

$$F(x+\Delta x) = F(x) + \frac{\partial F(x)}{\partial x}\Delta x \quad (18)$$

By introducing the expression (18) into the expression (17) we obtain:

$$\rho S \times \frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial F(x)}{\partial x} \quad (19)$$

In addition, the Hooke law allows to write:

$$F(x) = \sigma S = E\varepsilon S \quad (20)$$

By definition of a strain we have:

$$\varepsilon = \frac{\partial u(x)}{\partial x} \quad (21)$$

$$F(x) = \sigma S = E\varepsilon S = ES\frac{\partial u(x)}{\partial x} \quad (22)$$

So for a plane wave by introducing the equation (22) in the equation (19) we obtain:

$$\rho S \times \frac{\partial^2 u(x,t)}{\partial t^2} = ES\frac{\partial^2 u(x)}{\partial x^2} \quad (23)$$

We obtain the d'Alembert's equation:

$$\boxed{\frac{\rho}{E} \times \frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial^2 u(x)}{\partial x^2} \quad (24)}$$

By definition of a strain, we obtain for a plane wave:

$$\frac{\partial u(x,t)}{\partial x} = \varepsilon(x,t) = -\frac{a}{C}f'\left(t - \frac{x}{C}\right) \quad (25)$$

And by the Hook law we obtain also:

$$F(x) = \sigma S = E\varepsilon S$$

$$\sigma(x,t) = -\frac{aE}{C}f'\left(t - \frac{x}{C}\right) \quad (26)$$

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{a}{C^2}f''\left(t - \frac{x}{C}\right) \quad (27)$$

In addition we have:

$$\frac{\partial u(x,t)}{\partial t} = a \times f'\left(t - \frac{x}{C}\right) \qquad (28)$$

$$\frac{\partial^2 u(x,t)}{\partial t^2} = a \times f''\left(t - \frac{x}{C}\right) \qquad (29)$$

If we introduce these two equations (27) and (29) in the d'Alembert's formula we obtain:

$$\frac{\rho}{E} a \times f''\left(t - \frac{x}{c}\right) = \frac{a}{C^2} f''\left(t - \frac{x}{c}\right) \qquad (30)$$

And finally we obtain:

$$\boxed{\frac{\rho}{E} = \frac{1}{c^2} \qquad (31)}$$

We can so deduct from the expression (31) the equivalent young modulus of the fibers that constitute the fabric of the space time.

$$\frac{\rho}{E} = \frac{1}{c^2} => E = \rho c^2 \qquad (32)$$

If we consider that the space-time fabric consists of an infinity of these fibers / cords and that the gravitational waves propagate at the speed of light, we can evaluate the Young-E modulus of the space-time.

2.2) Numerical application (cf. [30])

We envisage two hypotheses, because the state of the art of current physics gives two values of the energy density of the vacuum.

a) <u>Hypothesis n°1 : the vacuum energy comes of the cosmologic constant</u>

The vacuum energy density: $\rho_{cosmologic}$ is 1E^{-9} J/m^3

The vacuum density from $E_v = mc^2 => E_v/c^2 = m$ that imply $\rho_{cosmologic}$ = 1.11E^{-26} kg/m^3 or 10^{-29} g/cm^3

The equivalent Young modulus E is so 1.11E^{-26} x 299792458^2 = 1E^{-9} N/m^2

$E_{\text{fabric of the space time}}$ = 1E^{-9} N/m^2

b) <u>Hypothesis n°2 : the vacuum energy comes of the quantum mechanics (field theory)</u>

The vacuum energy density: $\rho_{quantique}$ is 1E^{+113} J/m^3

The vacuum density from $E_v = mc^2 => E_v/c^2 = m$ that imply $\rho_{quantic}$ = 1.11E^{+96} kg/m^3

The equivalent Young modulus E is so 1.11E^{96} x 299792458 ^{2}= 1E^{113} N/m^2

$E_{\text{fabric of the space time}}$ = 1E^{113} N/m^2

The hypothesis of the quantum mechanics seems more realistic than the cosmologic approach by comparison with the previous results obtained.

3) Determination of the young modulus of the space time by a quantic approach– field theory and Casimir force

3.1) Determination of the base equations

The principle of the Casimir force is given figure 2 below. The vacuum energy of each side of the two plates create a force F which brings them closer together:

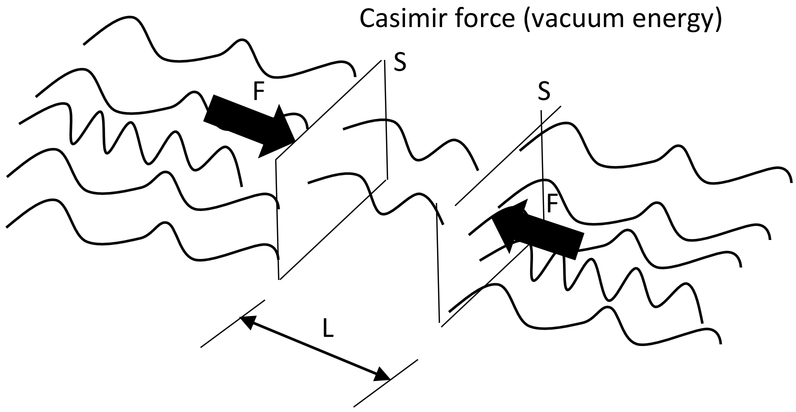

Figure 2 – Definition of the Casimir force confirmed experimentally –

In the case of an approach in one dimension, the energy due to the Casimir force takes the following expression (see [25] and [29]):

$$U = \frac{\pi \hbar c}{24L} \quad (33)$$

To obtain the associated force, we use the fact that a force derives of a potential:

$$F = grad\,(U) \quad (34)$$

We obtain by deriving the energy by L:

$$F = -\frac{\pi \hbar c}{24L^2} \quad (35)$$

Moreover, we can assimilate the displacement of the two plates due to the Casimir force F to an application of Hooke's law to the space-time fabric located between these two plates (see Figure 3).

In this case, it is possible to consider, between the two surface plates S of a length L, an elastic material of Young's modulus E.

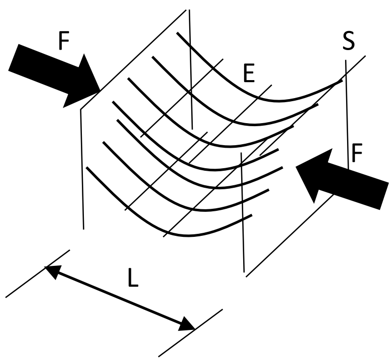

Figure 3 – Equivalent material (fabric of the space time) located between the two plates loaded by the Casimir Force F–

The strain energy of this type of system loaded with a normal effort F supposed constant for L fixed is the following:

$$U_{elastic\ strain} = \frac{1}{2}\int_0^L \frac{\left(N_{(x)}\right)^2}{ES}dx = \frac{1}{2}\times\frac{F^2L}{ES} \quad (36)$$

Assuming that U = $U_{strain\ energy}$ and considering the force F of Casimir we obtain for the elastic deformation energy of the fabric of the space time located between the two plates:

$$U_{elastic\ strain} = U_{cas} = \frac{\hbar c\pi}{24L} = \frac{1}{2}\times\frac{\left(-\frac{\hbar c\pi}{24L^2}\right)^2 L}{ES} \quad (37)$$

So:

$$\frac{\hbar c\pi}{24L} = \frac{1}{2}\times\frac{\hbar^2c^2\pi^2L}{24^2ESL^4} \quad (38)$$

Or again:

$$\frac{\hbar c\pi}{24L} = \frac{1}{ES}\times\frac{1}{2}\times\frac{\hbar^2c^2\pi^2}{24^2}\times\frac{1}{L^3}$$

After simplification we obtain:

$$\frac{1}{L} = \frac{1}{ES}\times\frac{\hbar\ \ c\pi}{24L}\times\frac{1}{2L^2} \ (39)$$

With:

$$U = \frac{\pi\hbar c}{24L} \quad (40)$$

So we obtain:

$$\frac{1}{L} = \frac{1}{2}\times\frac{1}{ES}\times\frac{U}{L^2} \quad (41)$$

The dimensional equation of 1/ES is:

$$\frac{1}{ES} => \frac{1}{MPa.m^2} = \frac{1}{\frac{MN}{m^2}m^2} = \frac{1}{MN} = \frac{1}{kg\frac{m}{s^2}} = \frac{s^2}{kgm} = \frac{T^2}{ML} \quad (42)$$

From the expression (39), we can extract the equivalent Young's modulus E from the space-time fabric, which is also here that of the energy of the vacuum created by the disintegrating creations of the particles continuously and instantaneously (see quantum field theory).

$$\boxed{E = \frac{\hbar\ \ c\pi}{48SL^2} \qquad (43)}$$

$$E = \frac{N.m\times s\times m}{s\times m^2\times m^2} = \frac{N}{m^2} \quad (44)$$

3.2) Numerical application:

By hypothesis we consider for L the Planck distance L_p in view to have the possibility to compare the results with the previous results obtained ($S = L^2 = L_p^2$).

ħ =1.05457E^{-34} J.s

L_p = 1.616252E^{-35} m

c = 299792458 m/s

$E_{\text{tissu de l'espace-temps}}$ **= 3.03 E^{112} N/m²**

The results obtained is completely in connection with the previous results obtained.

4) Conclusions

The results are therefore similar regardless of the simplified approaches envisaged (see Table 1 below). The Young-E modulus of space-time fabric therefore seems extremely large. E seems to be of the order 1E^{+113} **N / m²**. This remains to be confirmed by experience and measurement.

Hypothesis	**Theory followed**	**Equivalent Young modulus E obtained**	**Numerical application at the Planck scale (E in N/m²)**
Fabric of the space time in compression between the two plates	Casimir force developed in one direction and use of the Hook law	$E = \frac{\hbar \quad c\pi}{48SL^2}$	$L = L_p$ $S = L_p^2$ E = **3.03 E^{112}**
Waves propagation in the fabric of the space time	d'Alembert's Equation	$E = \rho c^2$	ρ issued of the quantum field theory E = **1E^{113}**
The space time is a multi-sandwich constituted of a multitude of thin plates of Planck thickness L_p	By analogy with the relation between the curvature and energy issued of the plate theory and the formula of Einstein general relativity	$E = \frac{24(1-\nu^2)}{h^2\kappa}$	$h = L_p$ if v = 0 **E = 4.43E^{+113}**
Elastic theory and tensor approach of the plates theory in 4 dimensions	See [5]	$E = \frac{48}{{L_p}^2\kappa} = \frac{24(1+\nu)}{{L_p}^2\kappa}$	$h = L_p$ If v=1 **E = 8.8495E^{+113}** If v=0 (added by DI) **E = 4.43E^{+113}**

Table 1 – Comparison of different approaches and values obtained from the young modulus E of the space time fabric –

Bibliographic references

For the strength of materials (frame mechanics):

[1] S. Timoshenko,(1949) « Résistance des matériaux Traduit de l'anglais sur la 2e édition par Ch. Laffitte »
[2] D. Izabel (2007) « Formulaire de résistance des matériaux tome 1 à 5 » Sebtp

[3] S. Timoshenko, (1951) « Théorie des plaques et des coques, Traduit de l'anglais par L. Vial Reliure inconnue » (1951) p 46 chap 12 and p37 chap 10

[4] Ménad CHENAF Jean-Vivien HECKCSTB Guide PPRT (2007)
Compléments technique relatif a l'effet de surpression
Recommandations et précautions en vue de réduire les risques – rapport d'étude

For the general relativity:

[5] T.G. Tenev M.F. Horstemeyer (2016) « The Mechanics of Spacetime { A Solid Mechanics Perspective on the Theory of General» Foundations of Physics manuscript arXiv:1603.07655v3 » p 24
[6] M. R. Beau (2014) « Théorie des champs des contraintes et des déformations en relativité générale et expansion cosmologique ». - Foundations of Physics manuscript arXiv:1209.0611v2 p4 and Annales de la Fondation Louis de Broglie, Volume 40, 2015
[7] T. Damour (2006) – « La Relativité Générale aujourd'hui »
Institut des Hautes Etudes Scientifiques Bures-sur-Yvette, France - Séminaire Poincaré 1-40 p
[8] T. Damour (2005) « Einstein et la physique du vingtième siècle » Membre de l'Académie des sciences p 4
[9] M. R. Beau (2014) « Sur l'hypothèse concernant l'existence d'un champ de gravité de type vectoriel couplant avec l'accélération des masses » Hal archive ouvertes 00683021v2 point c p13
[10] Collective (2016) Observation of Gravitational Waves from a Binary Black Hole Merger physical review letter

[11] Cours de physique « tenseur énergie impulsion »
http://physique.coursgratuits.net/relativite-generale/tenseur-d-energie-impulsion.php

[12] Cours en ligne de richard Taillet univ grenoble– initiation à la relativité générale et restreinte

For the quantum mechanics:

[13] C. De Beule (2016) "Graphical interpretation of the Schrodinger equation" university antwerpen
[14] Michael Fowler (2007) "Schrödinger's Equation in 1-D: Some Examples Curvature of Wave Functions"

[15] Steven Errede (2016) UIUC Physics 406 Acoustical Physics of Music, Department of Physics, University of Illinois at Urbana-Champaign, IL,
[16] Xinzhong Wu (2015) Probability and Curvature in Physics School of History and Culture of Science, Shanghai Jiaotong University, Shanghai, China Journal of Modern Physics, 2015, 6, 2191-2197
[17] Dalibard : Cour en ligne de mécanique quantique X polytechnique
[18] Cours en ligne : calcul variational et relativité restreinte X polytechnique

For all the topics :

[19] Feynman (1963) The Feynman Lectures on Physics, Addison-Wesley 1963, vol.II, p.38-10.
[20] C.W. Misner, K.S. Thorne, J.A. Wheeler, Gravitation, W.H. Freeman 1973; x17.5, pp.426-428
[21] Paty Michel (2005) HAL L'espace-temps de la théorie de la relativité
[22] Bernard Guy (2016) Relier la mécanique et la relativité générale ? Réflexions et propositions
[23] David Izabel (2017) HAL Peut-on comprendre et unifier les fondamentaux de la relativité générale et de la mécanique quantique à partir des concepts de la résistance des matériaux ? Réflexions et propositions

[24] David Izabel (2017) HAL Prise en compte de la rigidité de l'espace-temps dans la relativité générale - Compréhension par une analogie simple basée sur les concepts de la résistance des matériaux

For the Casimir force:

[25] Gert-Ludwig Ingold (2014) Casimir effect from a scattering approach Gert-Ludwig Ingold Institut fur Physik, Universit• at Augsburg, Universitatsstra_e 1, D-86135 Augsburg, Germany arXiv:1404.6919v1 [quant-ph] 28 Apr 2014 p5 chapter 5
[26] G. Bressi,1 G. Carugno,2 R. Onofrio,2,3 and G. Ruoso2,3, (2002) "Measurement of the Casimir Force between Parallel Metallic Surfaces" VOLUME 88, NUMBER 4 PHYSICAL REVIEW LETTERS p 3
[27] Mubassira Nawaz, Remco J. Wiegerink, Theo S. J. Lammerink, Miko Elwenspoek "PARALLEL PLATE STRUCTURES FOR OPTICAL MODULATION AND CASIMIR FORCE MEASUREMENT" Paper ID : 142
[28] Matthew Schwartz (2008) lecture 14 the casimir effect
[29] Joseph P. Straley (University of Kentucky),Luke S. Langsjoen (University of Virginia),Hussain Zaidi (University of Virginia) "Casimir effect due to a single boundary as a manifestation of the Weyl problem" arXiv:1002.1762v1
[30] http://physiquereussite.fr/energie-du-vide/