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Abstract. – A summary of experimental results on structure functions obtained using ex-tended self-similarity in various flow configurations (jet, grid, mixing layer, duct flow, cylinder) at Reynolds numbers ranging between 30 and 5000 is presented.

On October 7th 1994, a meeting was held between various European groups involved in experimental studies of 3D homogeneous turbulence. The aim of the meeting was to confront results obtained independently and see whether a general consensus on some properties of the velocity structure functions could be obtained. It turned out that agreement has been obtained on several characteristics of such functions, in particular on the values of scaling exponents (determined by using the technique described below), up to order 7. The participants thought that this fact was interesting to be reported. This does not mean that all the authors of the present letter agree on the significance of the result. In this letter, we essentially report facts and do not favour any particular interpretation.

Structure functions $F_p(l)$ of order p [1] are defined via the velocity increments over a distance l,

$$\Delta v(l) = v(x+l) - v(x),$$

as

$$F_p(l) = \langle \Delta v(l)^p \rangle.$$

It is also useful to introduce at this stage the structure functions of the absolute values of the velocity increments, defined by

$$G_p(l) = \langle |\Delta v(l)|^p \rangle.$$

TABLE I. – *Experimental conditions. Λ is the integral scale, η the Kolmogorov length, R_λ the microscale Reynolds number, u'/U the fluctuation rate (u' is the* rms *fluctuation velocity, and U the mean velocity), l_w the wire length, f_a the low-pass band filter frequency placed before the* a/d *converter and $f_\eta = U/2\pi\eta$ the Kolmogorov frequency.*

Exp.	Configuration	Λ	η	R_λ	u'/U (%)	l_w/η	f_a/f_η	Ref.
1	swirling flow	10 cm	2.5-50 µm	200-5000	20-40	0.1-3	0.5-5	[2]
2a	jet	20 cm	0.28 mm	428	26	2.5	7	[3]
2b	wind tunnel	10 cm	0.35 mm	3050	7	1.2	3	
3	jet	1 cm	7 µm	580	25	3	7	[4]
4a	cylinder	6-10 cm	0.2-0.5 mm	100-300	15	1-2.5	7	[5]
4b	jet	10 cm	0.1 mm	800	30	5	7	
5a	jet	7.5 cm	0.095 mm	810	16	2	1	[6]
5b	grid	17 cm	0.19 mm	530	8	1	1	
6	jet	4-8 cm	22-48 µm	240-330	20-25	0.6-1.3	–	[7]
7	grid	4 mm-1 cm	100-250 µm	35-110	1.5-8	3-10	1-3	[8]

The experimental parameters corresponding to the results presented are summarized in table I. It can be seen that the experiments cover a wide range of Reynolds numbers (between 30 and 5000) and flow configurations. Most of the velocity structure functions were obtained using Taylor's hypothesis, except for the experiments 6 (thanks to RELIEF method [7]). All the results were displayed according to a common format, so as to enable graphics overlap and make quantitative comparison easier.

A first series of results was discussed: the evolution of the longitudinal structure functions of order 3, 4 and 6 with the separation scale l/η (where η is the Kolmogorov scale), or experiments 1, 2, 3, 4, 5 and 7. We do not show the graphs in this report, but only summarize some conclusions. One may fairly say that they approximately follow power laws in the inertial range (defined in the usual way, *i.e.* as the limits beyond which F_3 ceases to be a linear function of l). The quality of the power law behaviour can be qualified as modest in the general case, even at large Reynolds numbers ([1]); in particular, on a log-log plot, a curvature

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([1]) The fact that, in general, the structure functions do not clearly exhibit power laws is an issue. One can argue that it is important to investigate this issue before further processing (this point is stressed by W. Van de Water).
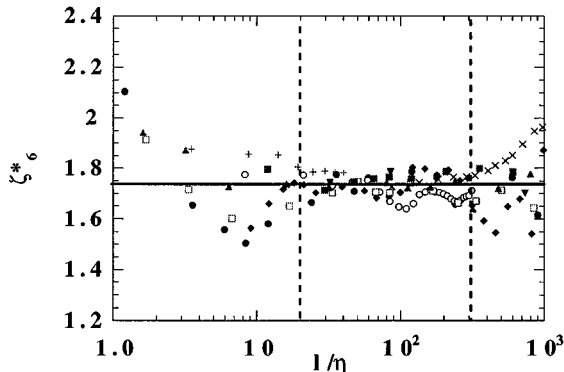
Fig. 1. – Evolution of the local slope of the curve $F_6(l)$ *vs.* $G_3(l)$ (on a log-log plot) with l/η, for different experiments: □ exp. 1, $R_\lambda = 550$, ▲ exp. 1, $R_\lambda = 1264$, × exp. 2a (using wavelet transform), ● exp. 2b, ♦ exp. 3, ■ exp. 5a, ▼ exp. 5b, ○ exp. 6, + exp. 7, $R_\lambda = 109$.

of the structure functions is generally visible in the inertial range of scales. Similar comments also apply for the transverse structure functions (experiment 6). In experiment 2 b (the wind tunnel experiment), oscillations on the structure function plots have been observed.

One can now briefly comment on the second series of results, which was related to the probability density functions (pdf) of the velocity increments $\Delta v(l)$ for two particular values of l/η: 10 and 50; here again, we do not reproduce the graphs, but summarize some conclusions. The general features which have been found are the stretched exponential-like form of the tails of such pdf, and their tendency to form a Gaussian distribution as the separation scale increased. This finding is in agreement with previously reported results [9], [10].

The most striking results were obtained while comparing the scaling properties of the velocity structure functions $F_p(l)$ following the Extended-Self-Similarity (denoted as ESS) concept introduced by Benzi *et al.* [11]. In that case, the scaling behaviour of the $F_p(l)$ is not investigated with respect to l but according to the absolute third-order velocity structure function

$$G_3(l) = \langle |\Delta v(l)|^3 \rangle .$$

The corresponding scaling properties can be called "relative". One must point out that this approach raises a question: how the scaling properties found on plots using $G_3(l)$ as the variable compare with those using l or $F_3(l)$ [12]? The comment of some of the participants was that, at the present time, there is no strong justification showing that the scaling properties (relative and absolute) are the same; moreover, they noted that using $G_3(l)$ instead of $F_3(l)$ (*i.e.* without the absolute values) or l may lead to different values of the exponents of the structure functions. This difference can be typically 10% on the sixth order when $F_3(l)$ is taken instead of $G_3(l)$ (see [13]).

Having these comments in mind, we proceeded to the comparison of the plots $G_p(l)$ *vs.* $G_3(l)$ for all the experiments. The result which appeared is the existence of clear "relative" scaling laws of the form

$$G_p(l) \sim G_3(l)\zeta_p^* , \tag{1}$$

for all experiments, even those with no obvious inertial range. The existence of relative scaling is thus a common feature of all experimental results; even of those that do not show (clear)
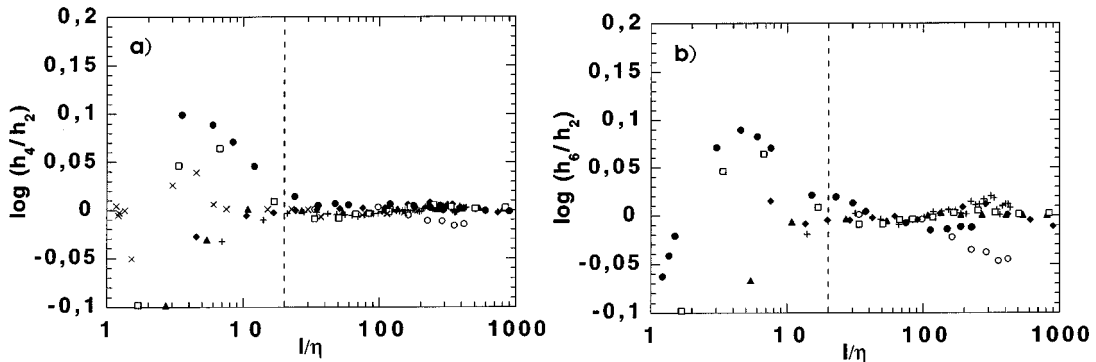
Fig. 2. – Evolution of the quantity h_4/h_2 (a)), and h_6/h_2 (b)) *vs.* l/η, for different experiments: □ exp. 1, $R_\lambda = 550$, ▲ exp. 1, $R_\lambda = 1548$, × exp. 2a, • exp. 2b, ♦ exp. 3, ○ exp. 6, + exp. 7, $R_\lambda = 109$.

absolute scaling. It is then possible to extract from each experiment relative exponents we shall denote by ζ_p^* to recall that, strictly speaking, they are not necessarily identical to those relying on the existence of absolute power laws. This notation was already used in a previous work (see [14]). This being said, we now present the detailed comparison which has been made between the 7 experiments, concerning the exponent of order 6. Figure 1 shows ζ_6^*, the local slope of the sixth-order structure function, as a function of l/η obtained in all the experiments. The local slope is defined by

$$\zeta_6^* = \frac{\mathrm{d}\log G_6}{\mathrm{d}\log G_3} \,.$$

Several comments on fig. 1 are possible and we suggest here the following one: one can see that almost all curves converge, at least for l/η ranging between 20 and 300, towards a single value which is

$$\zeta_6^* = 1.74 \pm 0.04$$

(the standard deviation reflects the scatter of the data around the mean value). The deviation represents, in relative value, an error of $\pm 2.3\%$, which is remarkably small $(^2)$. This result can be understood as supporting the idea of universality for the exponents ζ_p^* of the structure functions.

The value of the lower cut-off characterizing the "relative" scaling region is also an interesting information: it appears to depend on the experimental configuration, and shows no obvious correlation with the Reynolds number. The general trend is that below $l/\eta = 20$, the local exponent deviates from its asymptotic value. This cut-off turns out to correspond to the lower limit of the inertial range, as defined classically (*i.e.* as the value of l/η below which deviations from absolute scaling on F_3 have grown sizeable). This result holds for almost all the experiments, except experiment 1 (with $R_\lambda = 1264$), where deviations are observed only for l/η smaller than a few units. Several comments are possible, and the two reported herein reflect the general discussions: the first one is that if we emphasize the general trends, we obtain that relative scaling holds in the inertial range, but not in the intermediate dissipative range $l/\eta < 20$. In this sense, there is no extension in scale of the "relative" scaling laws, compared to the "absolute" ones. Another comment (which tends to weaken the previous

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$(^2)$ Note that it still represents 15% of the deviation from the Kolmogorov value.
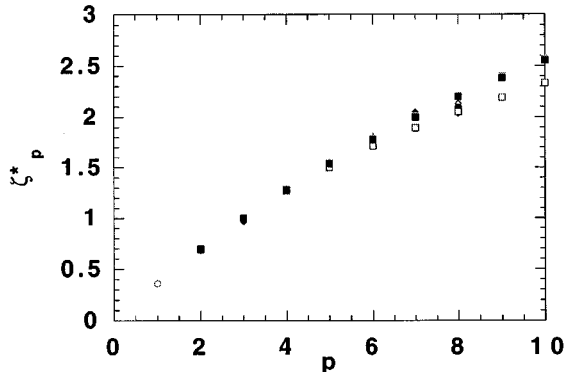
Fig. 3. – Evolution, with p, of the structure function exponents ζ_p^*, for different experiments: □ exp. 1 (the exponents are found independent of R_λ), × exp. 2a, • exp. 2b, ♦ exp. 3, ■ exp. 5a, ▲ exp. 5b, ○ exp. 6, + exp. 7.

statement) is that any discussion on the range of existence of a scaling law (relative or not) must incorporate the definition of a measure of the deviation from a regime where the scaling law applies, and an estimate for the error bar. One can argue that, since these quantities have not been discussed, it is difficult to define a value of l/η below which ESS ceases to apply ([3]). Another remark is that since a local shear can induce violation in ESS [15], [16], the value of the cut-off has no reason to be "universal", and may depend on the experimental configuration or on the location of the wire. It remains that, in the opinion of several participants, the present confrontation does not offer any evidence that the absence of extension in scale is due to anisotropy (as suggested by the previous remark).

Beyond the problem of the lower limit of the domain where relative scaling law applies, one can see, in fig. 1, the existence of a scatter in the data in the intermediate dissipative range. Depending on the configuration, and the Reynolds number, one has different evolutions of the local exponents with the separation scale. This may be an indication that, in conflict with the original Kolmogorov's ideas, the intermediate dissipative range does not display any universal features (but, in the absence of a discussion on the experimental error in this domain, one must be careful in drawing out any conclusion at this stage).

The quality of the estimate of the scaling exponents can be checked by plotting the function $h_p(l/\eta)/(h_2(l/\eta))$, where

$$h_p = G_p^{1/\zeta_p^*},$$

as a function of l/η. If the scaling exponent has been correctly estimated, one should get a flat function extending over the scaling range (see eq. (1)). This check was performed in all the experiments for $p = 4$ and $p = 6$. The result is displayed in fig. 2. All the experiments are characterized by a similar scatter, which turns out to be small in all cases. Clearly, a much larger scatter would have been obtained if the scaling exponents had been measured using the standard technique (*i.e.* determining directly, on a log-log plot, the local slope of the structure function). Another observation, which closely follows the discussion of fig. 1, is that the ratio $h_p(l/\eta)/(h_2(l/\eta))$ is a constant above $l/\eta > 20$; however, for lower values, this ratio depends both on l/η and p. This seems to indicate that ESS does not apply in this range (however, one

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([3]) This argument is proposed by S. Ciliberto and R. Benzi.

must have in mind the previous comment on the definition of the plateau and the experimental error).

Finally, we have compared the values of the exponents ζ_p^* —characterizing the relative scaling of the structure function—, from $p = 2$ to $p = 10$, between the different experiments: the results are displayed in fig. 3. Reliable higher-order scaling exponents could not be obtained in all the experiments due to statistical limitations. All scaling exponents lay within, say, 2.5% of each other up to $p = 6$, and within $\pm 6\%$ at higher values. Anyhow, they are substantially lower than the K41 non-intermittent prediction, $\zeta_p^* = \frac{p}{3}$ (we assume here that K41 prediction also applies for the ζ_p^*), but also slightly higher that those reported in the boundary layer [16].

In summary, we may fairly say that experiments with different flow configurations and Reynolds number ranging between 20 and 5000 can be characterized by anomalous scaling exponents ζ_p^*, following the technique of extended self-similarity. Within, say, $\pm 5\%$, these scaling exponents —at least up to order 6— appear independent of the Reynolds number (provided that it is not too small) and of the configuration. This result indeed sets limitations on the possible Reynolds number dependence of these anomalous exponents. Finally, let us mention again that the objective of the present letter was to establish facts and we must admit that, at the present time, it would be difficult to find a consensus, among the authors, for the significance of this result.

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