



**HAL**  
open science

## Interference phenomena and long-range proximity effect in superconductor/ferromagnet systems

Alexandre I. Buzdin, A. V. Samokhvalov, R. I. Shekhter

► **To cite this version:**

Alexandre I. Buzdin, A. V. Samokhvalov, R. I. Shekhter. Interference phenomena and long-range proximity effect in superconductor/ferromagnet systems. *Journal of Physics: Conference Series*, 2014, 27th International Conference on Low Temperature Physics (LT27) 6–13 August 2014, Buenos Aires, Argentina, 568, Part 2, pp.022007. 10.1088/1742-6596/568/2/022007 . hal-01555872

**HAL Id: hal-01555872**

**<https://hal.science/hal-01555872>**

Submitted on 21 Feb 2018

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

## Interference phenomena and long-range proximity effect in superconductor/ferromagnet systems

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2014 J. Phys.: Conf. Ser. 568 022007

(<http://iopscience.iop.org/1742-6596/568/2/022007>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 147.210.24.83

This content was downloaded on 10/06/2015 at 08:28

Please note that [terms and conditions apply](#).

# Interference phenomena and long-range proximity effect in superconductor/ferromagnet systems

**A.I. Buzdin**

University of Bordeaux, LOMA UMR-CNRS 5798, F-33405, France

**A.V. Samokhvalov**

Institute for Physics of Microstructures, Russian Academy of Sciences, 603950, Nizhny Novgorod, Russia  
Lobachevsky State University of Nizhny Novgorod, Nizhny Novgorod 603950, Russia

**R.I. Shekhter**

Department of Physics, University of Gothenburg, SE-412 96 Goteborg, Sweden  
E-mail: a.bouzdine@loma.u-bordeaux1.fr

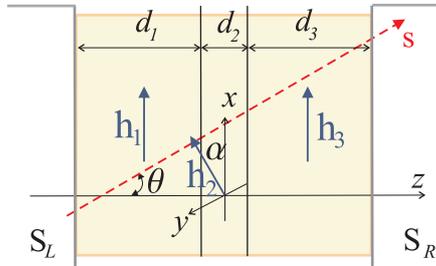
**Abstract.** We study the novel interference mechanism of the long-range proximity effect in superconductor/ferromagnet/superconductor (SFS) structures in the ballistic regime. Even a small non-collinear magnetic domain near the center of a ferromagnetic weak link is shown to restore the singlet supercurrent inherent to the normal metal. The underlying physics of the effect is the magnetic scattering of the Cooper pair by the domain, which reverses total momentum of the pair in the ferromagnet and thus compensates the phase gain before and after the spin-flip scattering. The above phenomenon opens a way to easily control the properties of SFS junctions and, inversely, to manipulate the magnetic moment via the Josephson current.

## 1. Introduction

Josephson junctions with a ferromagnetic (F) metal weak link is known to reveal a very strong decrease of the critical current compared to a normal metal weak link (see Refs. [1, 2] for review). This behavior is induced by the destructive effect of the exchange field  $h$  which leads to phase difference  $\gamma \sim L/\xi_h$  accumulated between the electron- and hole- like parts of the total wave function while propagating along a path of the length  $L$  in the ferromagnet [3, 4]. Here  $\xi_h = \hbar V_F/2h$  is a characteristic length determined by the exchange field, and  $V_F$  is the Fermi velocity. Measurable quantities should be calculated as the superpositions of fast oscillating contributions  $e^{i\gamma}$  from different trajectories and, thus, rapidly vanish with the increasing distance  $L$  and displays a short-range ( $\sim \xi_h$ ) behavior. It should be noted though, that a simple domain structure consisting of two F layers with opposite orientations of exchange field cancels the phase gain  $\gamma$  and suppresses the destructive effect of an exchange field [3, 5]. Another way to produce long-range proximity effect in SFS (superconductor-ferromagnet-superconductor) weak links is a composite F layers comprising three non-collinear domains, was suggested theoretically [6, 7, 8] and realized in recent experiments [9, 10]. In such a case the triplet Cooper pairs of electrons



with aligned spins (with equal-spin pairs) are generated by a thin ferromagnetic domain [11, 12], located between superconducting lead and a thick central non-collinear domain. Since these triplet pairs bind electrons with exactly the same de-Broglie wave length, they do not dephase, thereby leading to long-range proximity effect.



**Figure 1.** The schematic sketch of the SFS Josephson junction containing three ferromagnetic layers (domains) with a stepwise profile of the exchange field (1). Linear quasiparticle trajectory is shown by the red dashed line.

Here we suggest a new way of manipulating the Cooper pairs flow through a clean ferromagnet, which consists in using the composite F layer with a thin non-collinear domain  $d_2 \sim \xi_h \ll d_1, d_3$ , located near the center of the F barrier ( $d_1 \simeq d_3$ ), as shown in Fig. 1. In contrast with the situation  $d_1, d_3 \sim \xi_h \ll d_2$  analyzed in Refs. [6, 7, 8], this setup does not support the long-range triplet superconducting current. The thin central domain induces a special scattering of Cooper pairs which corresponds to exchange spins of two electrons forming a singlet Cooper pair (spin-flip scattering). Since modulus of the Fermi wave-vector for electrons with a spin polarized along the field is larger  $|\mathbf{k}_\uparrow| > |\mathbf{k}_\downarrow|$  due to the ferromagnetic spin-splitting effect, the pair have a non-zero total momentum  $\hbar\mathbf{q} = \hbar\mathbf{k}_\uparrow - \hbar\mathbf{k}_\downarrow$  ( $|\mathbf{q}| \sim 1/\xi_h$ ). The spin-flip scattering by the domain changes spin arrangement of a pair so that the new total momentum of the Cooper pair  $\hbar\mathbf{q}'$  is reversed:  $\hbar\mathbf{q}' = -\hbar\mathbf{q}$ . At a symmetric position of the scatterer ( $d_1 \simeq d_3$ ) the total phase gain  $\gamma \sim (d_1 - d_3)/\xi_h$  for a singlet Cooper pair should be cancelled and the long-range singlet superconducting proximity in SFS link becomes possible.

The paper is organized as follows. In Sec. 2, we briefly discuss the basic equations and methods used in calculations and calculate the long-range component of the Josephson current at the first harmonics of the current–phase relation. We discuss our results in Sec. 4.

## 2. Model and Methods

Let us consider the Josephson transport through a ballistic SFS junction containing three ferromagnetic layers (domains) with a stepwise profile of the exchange field

$$h(z) = \begin{cases} h\mathbf{x}_0, & \text{in domains } d_1, d_3 \\ h(\mathbf{x}_0 \cos \alpha + \mathbf{y}_0 \sin \alpha), & \text{in domain } d_2, \end{cases} \quad (1)$$

where  $\alpha$  is the angle of the exchange field rotation in the central domain  $d_2$  (see Fig. 1). The total length of the constriction  $d = d_1 + d_2 + d_3$  is assumed to be large compared to the magnetic coherence length  $\xi_h = \hbar V_F/2h$ :  $d \gg \xi_h$ . Here we consider the limit of the short junction  $d \ll \xi_n$ , where  $\xi_n = \hbar V_F/T_c$  is the coherence length of normal metal and  $T_c$  is the critical temperature of the S layer.

The current–phase relation of SFS Josephson junction is determined by the quasiclassical relation [4, 5]

$$I = \sum_n I_n = \sum_n a_n \sin n\varphi \frac{\langle (\mathbf{n}, \mathbf{n}_F) \cos n\gamma \rangle}{\langle (\mathbf{n}, \mathbf{n}_F) \rangle}, \quad (2)$$

where  $\mathbf{n}$  is the unit vector normal to the junction plane,  $\mathbf{n}_F$  is the unit vector along the trajectory, and  $a_n$  are the coefficients of the Fourier expansion for the current–phase relation for superconductor–normal metal junction of the same geometry. The angular brackets denote the

averaging over different quasiclassical trajectories characterized by a given angle  $\theta$  and a certain starting point at the superconductor surface, and for 3D constriction looks as

$$\frac{\langle(\mathbf{n}, \mathbf{n}_F) \cos(n\gamma)\rangle}{\langle(\mathbf{n}, \mathbf{n}_F)\rangle} = 2 \int_0^{\pi/2} d\theta \sin \theta \cos \theta \cos(n\gamma), \quad (3)$$

where  $\cos \theta = (\mathbf{n}, \mathbf{n}_F)$ . At temperatures  $T$  close to  $T_c$  the current–phase relation (2) is sinusoidal, and the coefficient  $a_1$  is determined by the following simple relations [5]:

$$a_1 = \frac{eT_c}{8\hbar} N \left( \frac{\Delta}{T_c} \right)^2. \quad (4)$$

Here  $\Delta$  is the temperature dependent superconducting gap, The factor  $N$  is determined by the number of transverse modes in the junction:  $N = s_0^{-1} \int ds \int d\mathbf{n}_F (\mathbf{n}_F, \mathbf{n}) \sim S/s_0$ , where  $S$  is the junction cross–section area, and  $s_0^{-1} = (k_F/2\pi)^2$ , where  $k_F$  is the Fermi momentum.

### 2.1. Transfer–matrix formalism for Eilenberger Equations

The phase gain  $\gamma(\theta)$  along a trajectory  $\mathbf{s} = s \mathbf{n}_F$  (see Fig. 1) is determined by the singlet part  $f_s$  of the anomalous quasiclassical Green function  $f = f_s + \mathbf{f}_t \hat{\sigma}$  taken at the right superconducting electrode ( $s = s_R = d/\cos\theta$ ) [5]:  $\cos \gamma = f_s(s_R)$ . Here  $\hat{\sigma}$  is a Pauli matrix vector in the spin space. The singlet (triplet) parts  $f_s, (\mathbf{f}_t)$  of the Green function  $f$  satisfy the linearized Eilenberger equations [6] written for zero Matsubara frequencies

$$-i\hbar V_F \partial_s f_s + 2\mathbf{h} \mathbf{f}_t = 0, \quad -i\hbar V_F \partial_s \mathbf{f}_t + 2f_s \mathbf{h} = 0, \quad (5)$$

with the boundary conditions  $f_s(s_L) = 1, \mathbf{f}_t(s_L) = 0$  at the left superconducting electrode ( $s = s_L = 0$ ).

To consider the Josephson transport through ferromagnetic layer with an arbitrary non-collinear distribution of the magnetizations  $\mathbf{M}$  and the exchange field  $\mathbf{h}$  it is convenient to utilize the transfer–matrix formalism. For this, we need to solve the equations (5) for the case when the quantization axis is taken arbitrarily in the ferromagnetic layer of a thickness  $d_i = z_i - z_{i-1}$ . We assume that a quasiclassical trajectory  $\mathbf{s}$  is characterized by a given angle  $\theta$  with respect to the  $z$ -axis and exchange field  $\mathbf{h} = h(\mathbf{x}_0 \cos \alpha_i + \mathbf{y}_0 \sin \alpha_i)$  lie in the plane  $(x, y)$ , as shown in Fig. 1. The triplet part  $\mathbf{f}_t$  consists of two nonzero components and can be written as  $\mathbf{f}_t = f_{tx} \mathbf{x}_0 + f_{ty} \mathbf{z}_0$ . Defining the transfer–matrix  $\hat{T}_{\alpha_i}(d_i, \theta)$  that relates the components of the Green function  $\hat{f}(s) = \{f_s(s), f_{tx}(s), f_{ty}(s)\}$  at the left ( $s = s_{i-1} = z_{i-1}/\cos \theta$ ) and right ( $s = s_i = z_i/\cos \theta$ ) boundaries of the F layer,

$$\hat{f}(s_i) = \hat{T}_{\alpha_i}(d_i, \theta) \hat{f}(s_{i-1}). \quad (6)$$

we get the following expression:

$$\hat{T}_{\alpha_i}(d_i, \theta) = \begin{pmatrix} \cos(qs_{d_i}) & -i \cos \alpha_i \sin(qs_{d_i}) & -i \sin \alpha_i \sin(qs_{d_i}) \\ -i \cos \alpha_i \sin(qs_{d_i}) & \sin^2 \alpha_i + \cos^2 \alpha_i \cos(qs_{d_i}) & \sin \alpha_i \cos \alpha_i (\cos(qs_{d_i}) - 1) \\ -i \sin \alpha_i \sin(qs_{d_i}) & \sin \alpha_i \cos \alpha_i (\cos(qs_{d_i}) - 1) & \cos^2 \alpha_i + \sin^2 \alpha_i \cos(qs_{d_i}) \end{pmatrix}, \quad (7)$$

where  $q \equiv 1/\xi_h = 2\hbar/\hbar V_F$  and  $s_{d_i} = d_i/\cos \theta$ .

### 2.2. The long-range phase gain

Solving the equations (5) by the transfer matrix method for the stepwise profile of the exchange field (1), the anomalous quasiclassical Green function  $\hat{f}(s_R) = \{f_s(s_R), f_{tx}(s_R), f_{ty}(s_R)\}$  at the right superconducting electrode ( $s = s_R = d/\cos\theta$ ) can be easily expressed via the boundary conditions  $\hat{f}(0) = (1, 0, 0)$  at the left superconducting electrode ( $s = 0$ ) as follows:

$$\hat{f}(s_R) = \hat{T}_0(d_3, \theta) \hat{T}_\alpha(d_2, \theta) \hat{T}_0(d_1, \theta) \hat{f}(0), \quad (8)$$

where  $d = d_1 + d_2 + d_3$  is the total thickness of the ferromagnetic barrier, and the transfer-matrix  $\hat{T}_\alpha(d_i, \theta)$  is determined by the expression (7). As a result, the singlet part  $f_s(s_R)$  responsible for the Josephson current through the junction, can be written in the form:

$$\begin{aligned} \cos \gamma &= \cos \delta_2 \cos(\delta_1 + \delta_3) - \cos \alpha \sin \delta_2 \sin(\delta_1 + \delta_3) \\ &- \sin^2 \alpha \sin \delta_1 \sin \delta_3 (1 - \cos \delta_2), \end{aligned} \quad (9)$$

where  $\cos \theta = (\mathbf{n}, \mathbf{n}_F)$  and  $\delta_i = d_i/\xi_h \cos \theta$  ( $i = 1, 2, 3$ ). Introducing the shift  $z_0 = (d_1 - d_3)/2$  of the central domain with respect to the weak link center it is convenient to rewrite the expression (9) in the following equivalent form:

$$\cos \gamma(\theta) = b_1 \cos \delta_d + b_2 \sin \delta_d - b_3 \cos 2\delta_z, \quad (10)$$

where  $\delta_d = d/\xi_h \cos \theta$ ,  $\delta_z = z_0/\xi_h \cos \theta$ , and the coefficients  $b_n$  ( $n = 1, 2, 3$ ) don't depend on the total length of the junction  $d = d_1 + d_2 + d_3$ :

$$\begin{aligned} b_1 &= \cos^2 \delta_2 + \cos \alpha \sin^2 \delta_2 + b_3 \cos \delta_2, \\ b_2 &= \cos \delta_2 \sin \delta_2 - \cos \alpha \cos \delta_2 \sin \delta_2 + b_3 \sin \delta_2, \\ b_3 &= \sin^2 \alpha (1 - \cos \delta_2)/2. \end{aligned}$$

Averaging the expression (10) over the trajectory direction  $\theta$  and neglecting the first two terms proportional to  $\xi_h/d \ll 1$ , which decrease just as for the case of homogeneous ballistic 3D SFS junction, one arrives at the following expression

$$(\cos \gamma)^{LR} = -\frac{1}{2} \sin^2 \alpha (1 - \cos \delta_2) \cos 2\delta_z, \quad (11)$$

### 3. Results

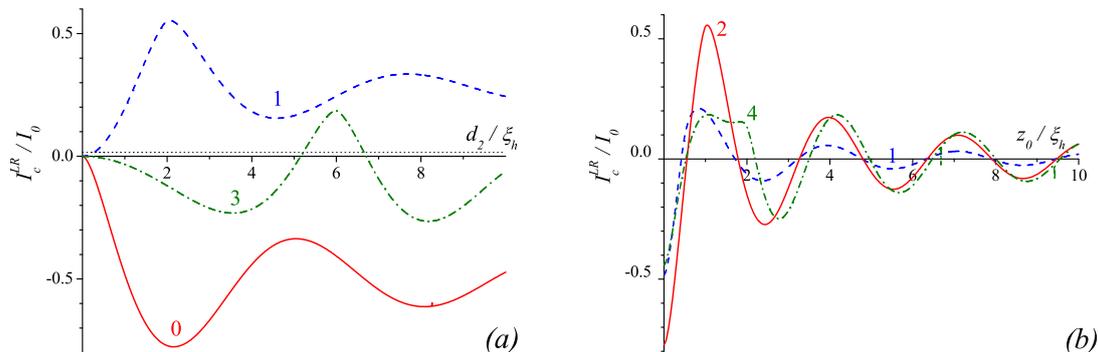
The long-range component of the Josephson current at the first harmonic is determined by the relation:

$$I^{LR} = I_c^{LR} \sin \varphi = a_1 T_1^{LR} \sin \varphi, \quad T_1^{LR} = 2 \int_0^{\pi/2} d\theta \sin \theta \cos \theta (\cos \gamma)^{LR}. \quad (12)$$

For a very thin central domain  $d_2 \ll \xi_h$  in the center of the NW ( $z_0 = 0$ ) one can easily estimate from (11, 12) the critical current of the SFS junction

$$I_c^{LR} \approx \frac{I_0}{2} \sin^2 \alpha \left( \frac{d_2}{\xi_h} \right)^2 \ln \frac{\xi_h}{d_2}, \quad (13)$$

where  $I_0 = (eT_c N/8\hbar) (\Delta/T_c)^2$  – is the critical current of the SNS junction for zero exchange field ( $\gamma = 0$ ). Figure 2a shows the dependence of the maximal Josephson current  $I_c^{LR} = a_1 T_1^{LR}$



**Figure 2.** (a) The dependence of  $I_c^{LR}$  on the thickness  $d_2$  of the  $90^\circ$  domain ( $\alpha = \pi/2$ ) for different values of the shift of the domain  $z_0$ :  $z_0 = 0$  - red solid line;  $z_0 = \xi_h$  - blue dashed line;  $z_0 = 3\xi_h$  - green dash-dotted line. Dotted line shows the value of  $I_c = \max\{I_1\}$  in absence of domain  $d_2$ . (b) The dependence of maximal Josephson current  $I_c^{LR}$  on the shift of the central domain  $z_0$  for different values of the  $d_2$ :  $d_2 = \xi_h$  - blue dashed line;  $d_2 = 2\xi_h$  - red solid line;  $d_2 = 4\xi_h$  - green dash-dotted line. We have set  $T = 0.9T_c$ ;  $d = 50\xi_h$  [ $I_0 = (eT_c N/8\hbar) (\Delta/T_c)^2$ ].

on the thickness  $d_2$  of the  $90^\circ$  domain ( $\alpha = \pi/2$ ) for different positions of the domain with respect to the weak link center. The amplitude of  $I^{LR}$  oscillates with varying the thickness of the central domain  $d_2$  and has the first maximum at  $d_2 \simeq 2.5\xi_h$ . Naturally, when the central domain disappears ( $d_2 \rightarrow 0$ ), the long-range effect vanishes. Comparison of the long-range Josephson current  $I^{LR}$  with the total supercurrent across the junction (2) shows that the difference is negligible until the outer domains are long enough:  $d_1, d_3 \gg \xi_h$  [13]. Figure 2b shows the dependences of the maximal Josephson current  $I_c^{LR}$  on the position of the central domain  $z_0$  for different values of the domain  $d_2$  thickness. We may see that the critical current is very sensitive to the position of the central domain and the first zero of  $I_1$  occurs already at  $z_0 \simeq 0.5\xi_h$ . Interestingly, that the long-range supercurrent  $I^{LR}$  is negative at a symmetric position ( $z_0 = 0$ ) of the domain  $d_2$ . This means that the spin-flip scatterer produces the  $\pi$  shift effect and generates a  $\pi$  junction. With a displacement of the central domain the SFS junction can be switched from  $\pi$  to 0 state.

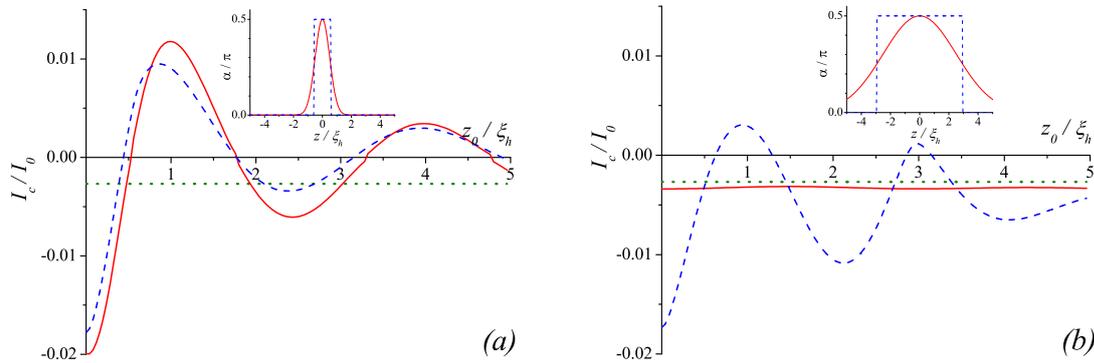
### 3.1. Arbitrary ferromagnetic barrier

The transfer-matrix formalism can be easily generalized for a layered ferromagnetic barrier with an arbitrary non-collinear distribution of the exchange field  $\mathbf{h}$  which is described by the dependence  $\alpha(z)$ . Splitting the barrier on  $N$  thin layers of the thickness  $d_i = z_i - z_{i-1}$  ( $i = 1 \div N$ ) one consider the exchange field to be constant inside each layer  $i$ . Application of the transfer-matrix  $\hat{T}_{\alpha_i}(d_i, \theta)$  (7) "layer by layer" results in the following relation between the components of the Green function  $f(0)$  and  $f(s_R)$  at the left and right superconducting electrodes, respectively:

$$\hat{f}(s_R) = \hat{T}_{\alpha_N}(d_N, \theta) \dots \hat{T}_{\alpha_2}(d_2, \theta) \hat{T}_{\alpha_1}(d_1, \theta) \hat{f}(0). \quad (14)$$

We apply the described transfer-matrix formalism to study the effect of smooth in the SFS constriction shown in Fig. 1a of the Letter. As the dependence  $\alpha(z)$  we use a draft model of  $90^\circ$ -domain described by Gaussian function:

$$\alpha(z) = \frac{\pi}{2} \exp\left(-\frac{(z - z_0)^2}{2w^2}\right), \quad (15)$$



**Figure 3.** The dependence of  $I_c^{LR}$  on the shift  $z_0$  of the  $90^\circ$ -domain (15) ( $\alpha = \pi/2$ ) for different values of the width of the transition region  $w$  (solid red line): (a)  $w = 1$ ; (b)  $w = 5$ . The blue dashed line shows the dependence of  $I_c^{LR}$  on the shift  $z_0$  of stepwise  $90^\circ$ -domain for comparison. The dotted line shows the critical current  $I_c$  in the absence of the domain ( $d_2 = 0$ ). Inset shows the coordinate dependence of the rotation angle  $\alpha$  (15) (red solid line) and the in the relevant stepwise domain (blue dashed line). We have set  $d = 20\xi_h$ ;  $T = 0.9T_c$  [ $I_0 = (4eT_cN/\hbar) (\Delta/T_c)^2$ ].

where  $z_0$  is the shift of the domain with respect to the weak link center, and  $w$  describes the width of the domain. Figure 3 shows the dependences of the critical current of the SFS junction on position  $z_0$  of the  $90^\circ$ -domain (15) for different values of the domain width  $w$ . We may see that the long-range effect seems to be completely disappeared if  $w \gg \xi_h$  (see Fig. 3c), but it is quite robust for the domain width smaller than  $2 - 3\xi_h$  and only weakly depends on the exact form of the transition region. So, the smooth (on the scale  $\xi_h$ ) profile of the magnetization decreases the long-range effect and the proposed mechanism occurs to be most efficient for  $d_2 \sim \xi_h$ .

Certainly, the above long-range effect in the first harmonic describes the properties of the SFS constriction if the contribution of higher harmonics in the current-phase relation (2) is negligible. This approach is usually applicable everywhere, except very close to the  $0 - \pi$  transition ( $T_1 = 0$ ), because  $|a_n| (n \geq 2) \ll |a_1|$  at  $T \approx T_c$ . At this  $0 - \pi$  transition the contribution of the second harmonic  $I_2$  becomes dominant. For all considered cases we obtained the positive amplitude of the second harmonic in the vicinity of these transitions, which means that they occur discontinuously by a jump between  $0$  and  $\pi$  phase states [13].

#### 4. Discussion

In order to elucidate the peculiarities of the Cooper pairs scattering with a spin-flop transition of electrons it is convenient to introduce the new functions  $f_\pm = f_s \pm f_{tx}$  which describes the pairs with zero spin projection and a reversed spin arrangement. The equations (5) can be drastically simplified if the direction of the exchange field coincides with a spin quantisation axis  $x$ :

$$\mp i\hbar V_F \partial_s f_\pm + 2\hbar f_\pm = 0. \quad (16)$$

In this case  $f_\pm(s_R) = e^{\mp iqs_R} f_\pm(0)$ ,  $f_{ty}(s_R) = f_{ty}(0) = 0$ ,  $f_{tz}(s_R) = f_{tz}(0) = 0$ . Calculating the superconducting current at the right electrode  $s_R$  we readily see that it results from the interference with the singlet component coming from the left electrode  $f_s(s_R) = (f_+(s_R) + f_-(s_R))/2$  (triplet components are absent and irrelevant because the right electrode provides only the singlet component). The oscillating factors  $e^{\mp iqs_R}$  in  $f_\pm(s_R)$  produce, after the averaging over the trajectories directions (angle  $\theta$ ), a strong damping of the critical current compared to the normal metal (where these factors are absent).

Now we may easily understand the mechanism of the singlet long-range proximity effect in the presence of a small region with a non-collinear magnetization near the center of a ferromagnetic weak link. Indeed after coming through the first F layer  $d_1$  an extra phase factor appears in  $f_{\pm}$  functions:  $f_{\pm}(s_{d_1}) = e^{\mp iqs_{d_1}} f_s(0)$ . In the absence the middle layer  $d_2$ , the  $f_{\pm}$  at the right electrode would be  $f_{\pm}(s_{d_1} + s_{d_3}) = e^{\mp iq(s_{d_1} + s_{d_3})} f_s(0)$  and the oscillating factors will strongly damp critical current. The additional non-collinear middle layer  $d_2$  (spin-flip scatterer) will mix up the components  $f_+$  and  $f_-$  so that

$$f_{\pm}(s_{d_1} + s_{d_2}) = a_{\pm} e^{-iqs_{d_1}} + b_{\pm} e^{+iqs_{d_1}}.$$

Then the resulting  $f_{\pm}$  functions at the right electrode should be

$$f_{\pm}(s_{d_1} + s_{d_2} + s_{d_3}) = a_{\pm} e^{-iq(s_{d_1} \pm s_{d_3})} + b_{\pm} e^{+iq(s_{d_1} \mp s_{d_3})}$$

and for  $d_1 = d_3$  the oscillating factor at the first (second) term vanishes. This means the emergence of the long-range singlet proximity effect discussed in the present paper. Note that the additional noncollinear F layer  $d_2$  may strongly increase the critical current, provided that it is placed at the center of the structure.

The very important property of the discussed system is that it provides a direct mechanism of the coupling between supercurrent and magnetic moment, similar to the situation discussed in Refs. [14, 15]. Since the long-range critical current  $I_c^{LR}$  depends on the profile of the magnetization, the superconducting current acts as a direct driving force on the magnetic moment and can change its orientation. Inversely, the precession of the magnetic moment shall modulate the critical current.

To summarize, we studied the interference phenomena and long-range proximity effect originated by the spin-exchange scattering in ferromagnetic ballistic weak link. In contrast to the widely discussed triplet long-range proximity effect we elucidate a new singlet long-range proximity effect. We demonstrated that this phenomenon provides an efficient way to control the Josephson current and to couple it with a magnetic moment. This offers the means for spintronic manipulation of superconducting weak links by external magnetic inhomogeneity located on a submicron length scale [13].

The authors thank A.S. Mel'nikov for stimulating discussions. This work was supported, in part, by European IRSES program SIMTECH (contract n.246937), by French ANR grant "MASH", by the Swedish VR, by the RFBR, by the program "Quantum Mesoscopic and Disordered Structures" of RAS, and by Ministry of Education and Science of RF and Lobachevsky State University (agreement 02.B.49.21.0003).

## References

- [1] Buzdin A I 2005 *Rev. Mod. Phys.* **77** 935
- [2] Bergeret F S, Volkov A F and Efetov K B 2005 *Rev. Mod. Phys.* **77** 1321
- [3] Blanter Ya M and Hekking F W J 2004 *Phys. Rev. B* **69** 024525
- [4] Buzdin A I, Melnikov A S, and Pugach N G 2011 *Phys. Rev. B* **83** 144515
- [5] Melnikov A S, Samokhvalov A V, Kuznetsova S M, and Buzdin A I 2012 *Phys. Rev. Lett.* **109** 237006
- [6] Houzet M and Buzdin A I 2007 *Phys. Rev. B* **76** 060504(R)
- [7] Alidoust M and Linder J, Rashedi G, Yokoyama T, and Sudbo A. 2010 *Phys. Rev. B* **81** 014512
- [8] Volkov A F and Efetov K B 2010 *Phys. Rev. B* **81** 144522
- [9] Robinson J W A, Witt J D S, and Blamire M G 2010 *Science* **329** 59
- [10] Khaire T S, Khasawneh M A, Pratt W P (Jr.), and Birge N O 2010 *Phys. Rev. Lett.* **104** 137002
- [11] Bergeret F S, Volkov A F, and Efetov K B 2001 *Phys. Rev. Lett.* **86** 4096
- [12] Kadigrobov A, Shekhter R I and Jonson M 2001 *Europhys. Lett.* **54** 394
- [13] Samokhvalov A V, Shekhter R I and Buzdin A I 2014 *Sci. Rep.* in press
- [14] Buzdin A I 2008 *Phys. Rev. Lett.* **101** 107005
- [15] Konschelle F and Buzdin A 2009 *Phys. Rev. Lett.* **102** 017001