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Abstract:
This paper presents an actuator fault compensation approach for a class of Linear Parameter-Varying (LPV) systems with noisy measurements. The proposed method is based on interval estimation assuming that the fault vector and the external disturbances are unknown but bounded. The main idea consists in designing a control law, based on a linear state feedback, to guarantee closed-loop stability. An additive control, based on fault bounds, is used to compensate the impact of actuator faults on system performances. The closed-loop stability of the robust fault compensation scheme is established in the Lyapunov sense. Finally, the theoretical results are illustrated using a numerical example.

Keywords: Linear Parameter-Varying systems, Actuator fault Compensation, Interval observers, Stability.

1. INTRODUCTION

As the performance requirement, safety and reliability increase, industrial systems can be affected by unexpected events and even catastrophic disaster. Since actuator, sensor or component failures can cause performance degradation and loss of stability, it is important to design a control system to avoid damage and reduce faults effect.

The area of Fault Tolerant Control (FTC) has attracted more and more attention in the past decades. The objective of FTC is to maintain desired performances and preserve stability conditions in presence of faults. An interesting review on the FTC is presented for instance in (Zhang and Jiang [2008]) and (Blanke [1999]). In practical applications, many FTC methodologies have been developed and applied to aircraft (Alwi et al. [2009]), (Jiang and Zhang [2006]), space vehicles (Pirmoradi et al. [2009]), (Varma and Kumar [2010]), power plants (Noura et al. [2000]), (Ye et al. [2001]) and wind turbines (Sloth et al. [2010]).

Generally speaking, FTC techniques are divided into Passive and Active methods. Passive ones are based on robust control techniques which handle only a class of predefined faults using the same controller in faulty-free and faulty cases without requiring information about the faults. In contrast to Passive FTC, the Active ones require a Fault Detection and Diagnosis (FDD) scheme to provide information about the fault and a reconfiguration mechanism in order to design a control law to achieve stability and acceptable closed-loop system performance.

Various approaches of Active FTC methods are devoted to the reconfiguration problem such as: output feedback (Mhaskar et al. [2006]), fuzzy logic systems (Zhang et al. [2005]), eigenstructure assignment (Jiang [1994]), neural networks (Zhang et al. [2004]), multiple-model tracking (Zhang and Jiang [2001]), compensation via additive input design (Efimov et al. [2011]), (Dunia and Joe Qin [1998]), adaptive control (Diao and Passino [2001]) and pseudo-inverse method (Bao et al. [2002]).

Industrial systems are usually affected by uncertainties, noises and disturbances. Conventional observers may not be efficient to solve the estimation problem. Therefore, interval observers, used to compute the set of all admissible values instead of a single point approximation, can be considered as an alternative. An interval observer is composed of two classical ones allowing one to compute two bounds which enclose in a guaranteed way all the feasible states. Recently, several approaches are developed dealing with interval observers based on the assumption that the noise, disturbance, and uncertainties are assumed to be unknown but belongs to a given bounded sets (Raïssi et al. [2010]), (Mazenc and Bernard [2011]), (Efimov et al. [2013]), (Chebotarev et al. [2015]) and (Lamouchi et al. [2016]).

In this paper, an Active FTC of Linear Parameter-Varying systems against actuator faults is studied. The main idea is designing the robust compensation controller based on interval observer estimation to guarantee closed-loop stability and compensate the faults effect. Only the control design scheme is considered in this paper assuming that the FDD is implemented and all the informations of faults are known.

The paper is organized as follows. Preliminaries are described in Section 2. Section 3 presents the problem formulation. The design of interval observer for LPV systems are developed in Section 4. Section 5 is devoted to Fault compensation and stability analysis. To show the effectiveness of the proposed approach, numerical simulations are presented in Section 6. Finally, the conclusions are drawn in Section 7.

2. PRELIMINARIES

In the sequel, the following notations and definitions are used.

The symbol $|.|$ denotes vector or corresponding induced matrix Euclidean norm. For a measurable and locally essentially bounded input $u : \mathbb{N} \to \mathbb{R}$, the symbol $\|u\|_{[t_0,t_1)}$ denotes its $\mathscr{L}_\infty$-norm $\|u\|_{[t_0,t_1)} = \sup\{|u_t|, t \in [t_0,t_1)\}, \|u\| = \|u\|_{[0,+\infty)}$. The set of all inputs u with the property $\|u\| < \infty$ is denoted by $\mathscr{L}_\infty$. For a matrix $P = P^T$, the relation $P \succ 0$ means that P is positive definite. I and E_p denote respectively the $(n \times n)$ identity matrix and the $(p \times 1)$ ones matrix. Given a matrix $A \in \mathbb{R}^{m\times n}$, define $A^+ = \max\{0,A\}$, $A^- = \max\{0,-A\}$ (similarly for vectors). A matrix $A \in \mathbb{R}^{n\times n}$ is called nonnegative if all its elements are nonnegative.
A discrete-time dynamical system $x_{k+1} = f(x_k)$ is nonnegative if for any integer k_0 and any initial condition $x_{k_0} \geq 0$, the solution x satisfies $x_k \geq 0$ for all integers $k \geq k_0$.

A system described by

$$x_{k+1} = Ax_k + u_k,$$

with $x_k \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$, is nonnegative if and only if the matrix A is elementwise nonnegative, $u_k \geq 0$ and $x_{k_0} \geq 0$. In this case the system is also called cooperative.

Lemma 1. Chebotarev et al. [2013]

Let $\underline{x},x,\overline{x} \in \mathbb{R}^n$ if $\underline{x} \leq x \leq \overline{x}$ then,

$$\underline{x}^+ \leq x^+ \leq \overline{x}^+ \textit{ and } \overline{x}^- \leq x^- \leq \underline{x}^- \tag{1}$$

Similarly, let $\underline{A},A,\overline{A} \in \mathbb{R}^{m\times n}$, if $\underline{A} \leq A \leq \overline{A}$ then

$$\underline{A}^+ \leq A^+ \leq \overline{A}^+ \textit{ and } \overline{A}^- \leq A^- \leq \underline{A}^- \tag{2}$$

□

Lemma 2. Chebotarev et al. [2013]
Let $x \in \mathbb{R}^n$ be a vector such that $\underline{x} \leq x \leq \overline{x}$ for some $\underline{x},\overline{x} \in \mathbb{R}^n$.

(1) If $A \in \mathbb{R}^{m\times n}$ is a constant matrix, then

$$A^+\underline{x} - A^-\overline{x} \leqslant Ax \leqslant A^+\overline{x} - A^-\underline{x} \tag{3}$$

(2) If $A \in \mathbb{R}^{m\times n}$ is a matrix satisfying $\underline{A} \leq A \leq \overline{A}$ for some $\underline{A},\overline{A} \in \mathbb{R}^{m\times n}$, then

$$\begin{aligned}\underline{A}^+\underline{x}^+ - \overline{A}^+\underline{x}^- - \underline{A}^-\overline{x}^+ + \overline{A}^-\overline{x}^- &\leq Ax \\ \leq \overline{A}^+\overline{x}^+ - \underline{A}^+\overline{x}^- - \overline{A}^-\underline{x}^+ + \underline{A}^-\underline{x}^-. &\end{aligned} \tag{4}$$

□

3. PROBLEM FORMULATION

Consider a discrete-time LPV system:

$$\begin{cases} x_{k+1} = (A + \Delta A(\rho))x_k + Bu_k + w_k \\ y_k = Cx_k + v_k \end{cases} \tag{5}$$

where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}^q$ is the input, $y_k \in \mathbb{R}^p$ is the output; w_k, v_k are respectively the bounded disturbance and noise. The vector of scheduling parameters $\rho \in \Pi$ is considered unknown and only the set of its admissible values Π is given. $\Delta_A : \Pi \to \mathbb{R}^{n\times n}$ is a known piecewise continuous matrix function.

An actuator fault can be modeled by an additive term in the system (5). Therefore, the faulty system dynamics is given by:

$$\begin{cases} x_{k+1} = (A + \Delta A(\rho))x_k + Bu_k + Ff_k + w_k \\ y_k = Cx_k + v_k \end{cases} \tag{6}$$

where $F \in \mathbb{R}^{n\times q}$ is a known matrix and $f_k \in \mathbb{R}^q$ is the fault vector. In the following, it is assumed that the matrix $\Delta A(\rho)$ belongs to the interval $[\underline{\Delta A}, \overline{\Delta A}]$. The value of the scheduling vector ρ is not available for measurement but it is easy to compute $\underline{\Delta A}$ and $\overline{\Delta A}$ for a given set Π and a known function ΔA : $\Pi \to \mathbb{R}^{n\times n}$. The fault vector f_k belongs into an interval $[\underline{f}_k, \overline{f}_k]$. The disturbances w_k are bounded by two known sequences $\underline{w}_k \leqslant w_k \leqslant \overline{w}_k$ and the measurement noise v_k is bounded by a positive constant $\overline{V}$.

Assumption 1. $\underline{\Delta A} \leqslant \Delta A(\rho) \leqslant \overline{\Delta A}$ for all $\rho \in \Pi$ and some known $\underline{\Delta A}, \overline{\Delta A} \in \mathbb{R}^{n\times n}$. □

Assumption 2. $\underline{f}_k \leqslant f_k \leqslant \overline{f_k}, \forall\, f_k \in \mathbb{R}^q$. □

Assumption 3. There exist $\overline{w}_k$, $\underline{w}_k$ and $\overline{V}$ such that: $\underline{w}_k \leqslant w_k \leqslant \overline{w}_k$ and $|v_k| < \overline{V}$ are satisfied $\forall\, k \in \mathbb{N}$. □.

The goal of this paper is to compute a control law using interval observer in order to compensate the fault effect and to maintain the required performances despite the appearance of actuator faults and external disturbances.

Figure 1 shows a general schematic diagram for the proposed approach. The Diagnosis module is composed of three main sub-modules: fault detection consists to indicate whether a fault has occurred or not, fault isolation is used to determine the location of a fault in the system and fault estimation, consists to estimate precisely the size and nature of a fault. The Interval observer allows to estimate the upper and lower bounds of the system state taking into account the presence of fault in the system. Finally, a control law computation, based on reconfiguration mechanism, consists in changing the control action in order to compensate the fault effect on the system.

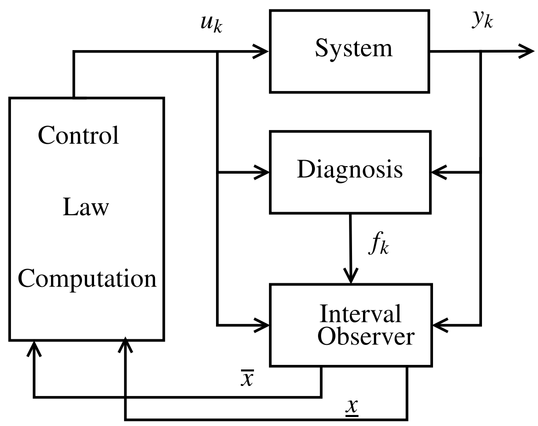

Fig. 1. Active compensation under feedback control based on interval observers.

It should be stressed that the Diagnosis module is not considered in this paper and only the control law computation is considered assuming that the fault information is available.

4. INTERVAL OBSERVERS

Consider the following interval observer structure for (6):

$$\begin{cases} \overline{x}_{k+1} = (A - LC)\overline{x}_k + \overline{\chi}_k + Bu_k + \overline{\varphi}_k + \overline{w}_k \\ \qquad\quad + Ly_k + |L|\overline{V}E_p \\ \underline{x}_{k+1} = (A - LC)\underline{x}_k + \underline{\chi}_k + Bu_k + \underline{\varphi}_k + \underline{w}_k \\ \qquad\quad + Ly_k - |L|\overline{V}E_p \end{cases} \tag{7}$$

with $\overline{\chi}_k = \overline{\Delta A}^+\overline{x}_k^+ - \underline{\Delta A}^+\overline{x}_k^- - \overline{\Delta A}^-\underline{x}_k^+ + \underline{\Delta A}^-\underline{x}_k^-$, $\underline{\chi}_k = \underline{\Delta A}^+\underline{x}_k^+ - \overline{\Delta A}^+\underline{x}_k^- - \underline{\Delta A}^-\overline{x}_k^+ + \overline{\Delta A}^-\overline{x}_k^-$, $\overline{\varphi}_k = F^+\overline{f}_k - F^-\underline{f}_k$ and $\underline{\varphi}_k = F^+\underline{f}_k - F^-\overline{f}_k$.

Denote by $\overline{e}_k = \overline{x}_k - x_k$ and $\underline{e}_k = x_k - \underline{x}_k$ the interval estimation errors; their dynamics follow:

$$\begin{cases} \overline{e}_{k+1} = (A-LC)\overline{e}_k + \overline{\psi}_k \\ \underline{e}_{k+1} = (A-LC)\underline{e}_k + \underline{\psi}_k \end{cases} \tag{8}$$

with $\overline{\psi}_k = \overline{\chi}_k - \Delta A(\rho)x_k + \overline{\varphi}_k - Ff_k + \overline{w}_k - w_k + |L|\overline{V}E_p + Lv_k$ and $\underline{\psi}_k = \Delta A(\rho)x_k - \underline{\chi}_k + Ff_k - \underline{\varphi}_k + w_k - \underline{w}_k + |L|\overline{V}E_p - Lv_k$.

Clearly, $\overline{\psi}_k$ and $\underline{\psi}_k$ are globally Lipschitz, it follows that for $\underline{x}_k \leq x_k \leq \overline{x}_k$ and for a chosen submultiplicative norm $\|.\|$, there exist positive constants a_1, a_2, a_3, b_1, b_2 and b_3 such that (Zheng et al. [2016]):

$$\begin{cases} \|\overline{\psi}_k\| \leqslant a_1\|\overline{x}_k - x_k\| + a_2\|\underline{x}_k - x_k\| + a_3 \\ \left\|\underline{\psi}_k\right\| \leqslant b_1\|\overline{x}_k - x_k\| + b_2\|\underline{x}_k - x_k\| + b_3 \end{cases} \tag{9}$$

Theorem 1. Assume that Assumptions 1-3 are satisfied, $A-LC$ is nonnegative and the initial state x_0 verifies $\underline{x}_0 \leq x_0 \leq \overline{x}_0$. If there exist positive definite and symmetric matrices Q, P and W such that the following Riccati matrix inequality is verified:

$$D^TPD - P + D^TPW^{-1}PD + \alpha(\|W+P\|)I + Q \leq 0 \tag{10}$$

where $D = A-LC$ and $\alpha = 3\,max((a_1{}^2 + b_1{}^2), (a_2{}^2 + b_2{}^2))$, then $\underline{x}_k, \overline{x}_k \in \mathscr{L}_\infty^n$.

Proof. According to Lemma 2 and Assumption 1, we have for $x_k \in \mathbb{R}^n$:

$$\underline{\Delta A}^+\underline{x}_k^+ - \overline{\Delta A}^+\underline{x}_k^- - \underline{\Delta A}^-\overline{x}_k^+ + \overline{\Delta A}^-\overline{x}_k^- \leqslant \Delta A(\rho)x_k \leqslant \overline{\Delta A}^+\overline{x}_k^+ - \underline{\Delta A}^+\overline{x}_k^- - \overline{\Delta A}^-\underline{x}_k^+ + \underline{\Delta A}^-\underline{x}_k^- \tag{11}$$

According to Lemma 2 and Assumption 2, we have for any $f_k \in \mathbb{R}^q$:

$$F^+\underline{f}_k - F^-\overline{f}_k \leqslant Ff_k \leqslant F^+\overline{f}_k - F^-\underline{f}_k \tag{12}$$

Since $A-LC$ is assumed to be nonnegative, and by construction $\overline{\psi}_k$ and $\underline{\psi}_k$ are positive, then if $\overline{x}_0$ and $\underline{x}_0$ are chosen such that $\overline{e}_0$ and $\underline{e}_0$ are positive, the dynamics of interval estimation errors $\overline{e}_k$ and $\underline{e}_k$ stay positive for all $k \in \mathbb{N}$.

Let's show now that the variables $\overline{x}_k, \underline{x}_k$ stay bounded $\forall k \in \mathbb{N}$. Consider the positive definite quadratic Lyapunov function

$$V(\overline{e}_k, \underline{e}_k) = \overline{e}_k{}^TP\overline{e}_k + \underline{e}_k{}^TP\underline{e}_k \tag{13}$$

The increment of ΔV is given by:

$$\begin{aligned} \Delta V &= V_{k+1} - V_k \\ &= \overline{e}_k^T(D^TPD - P)\overline{e}_k + 2\overline{e}_k^TD^TP\overline{\psi}_k + \overline{\psi}_k^TP\overline{\psi}_k \\ &\quad + \underline{e}_k^T(D^TPD - P)\underline{e}_k + 2\underline{e}_k^TD^TP\underline{\psi}_k + \underline{\psi}_k^TP\underline{\psi}_k \end{aligned}$$

Furthermore, from (Zheng et al. [2016]), we have the following inequalities:

$$\begin{aligned} 2\overline{e}_k^TD^TP\overline{\psi}_k &= 2\overline{e}_k^TD^TPW^{-0.5}W^{0.5}\overline{\psi}_k \\ &\leq \overline{e}_k^TD^TPW^{-1}PD\overline{e}_k + \overline{\psi}_k^TW\overline{\psi}_k \\ 2\underline{e}_k^TD^TP\underline{\psi}_k &= 2\underline{e}_k^TD^TPW^{-0.5}W^{0.5}\underline{\psi}_k \\ &\leq \underline{e}_k^TD^TPW^{-1}PD\underline{e}_k + \underline{\psi}_k^TW\underline{\psi}_k \end{aligned}$$

This leads to:

$$\begin{aligned} \Delta V &\leq \overline{e}_k^T(D^TPD - P + D^TPW^{-1}PD)\overline{e}_k + \underline{e}_k^T(D^TPD - P \\ &\quad + D^TPW^{-1}PD)\underline{e}_k + \overline{\psi}_k^T(W+P)\overline{\psi}_k + \underline{\psi}_k^T(W+P)\underline{\psi}_k \\ &\leq \overline{e}_k^T(D^TPD - P + D^TPW^{-1}PD)\overline{e}_k + \underline{e}_k^T(D^TPD - P \\ &\quad + D^TPW^{-1}PD)\underline{e}_k + 3\|W+P\|(a_1^2\|\overline{e}_k\|^2 + a_2^2\|\underline{e}_k\|^2 + a_3^2) \\ &\quad + 3\|W+P\|(b_1^2\|\overline{e}_k\|^2 + b_2^2\|\underline{e}_k\|^2 + b_3^2) \\ &\leq \overline{e}_k^T(D^TPD - P + D^TPW^{-1}PD)\overline{e}_k + \underline{e}_k^T(D^TPD - P \\ &\quad + D^TPW^{-1}PD)\underline{e}_k + \alpha\|W+P\|\overline{e}_k^T\overline{e}_k + \alpha\|W+P\|\underline{e}_k^T\underline{e}_k \\ &\quad + 3\|W+P\|(a_3^2 + b_3^2) \\ &\leq \overline{e}_k^T(D^TPD - P + D^TPW^{-1}PD + \alpha\|W+P\|I)\overline{e}_k \\ &\quad + \underline{e}_k^T(D^TPD - P + D^TPW^{-1}PD + \alpha\|W+P\|I)\underline{e}_k \\ &\quad + 3\|W+P\|(a_3^2 + b_3^2) \\ &\leq -\overline{e}_k^TQ\overline{e}_k^T - \underline{e}_k^TQ\underline{e}_k^T + 3\|W+P\|(a_3^2 + b_3^2). \end{aligned}$$

By using (10) we get $\Delta V \leq -\overline{e}_k^TQ\overline{e}_k^T - \underline{e}_k^TQ\underline{e}_k^T + \beta$ with $\beta = 3\|W+P\|(a_3^2 + b_3^2)$ which provides the boundedness of the dynamics of estimation errors $\overline{e}_k, \underline{e}_k$, therefore the variables $\overline{x}_k, \underline{x}_k$ stay bounded $\forall k \in \mathbb{N}$. □

Since it is not always possible to compute a gain L such as $A-LC$ is nonnegative. This restriction can be relaxed through a change of coordinates $z_k = Rx_k$ with a nonsingular matrix R such that the matrix $E = R(A-LC)S$ is nonnegative where $S = R^{-1}$.(Mazenc et al. [2013], Efimov et al. [2013]).

An interval observer for the system (6) can be written in the new coordinates z as follows:

$$\begin{cases} \overline{z}_{k+1} = E\overline{z}_k + \overline{\chi}_k^z + RBu_k + \overline{\varphi}_k^z + \overline{\rho}_k^z + RLy_k + |F|VE_p \\ \underline{z}_{k+1} = E\underline{z}_k + \underline{\chi}_k^z + RBu_k + \underline{\varphi}_k^z + \underline{\rho}_k^z + RLy_k - |F|VE_p \end{cases} \tag{14}$$

where $\overline{\chi}_k^z = (\overline{\sigma}^+\overline{z}_k^+ - \underline{\sigma}^+\overline{z}_k^- - \overline{\sigma}^-\underline{z}_k^+ + \underline{\sigma}^-\underline{z}_k^-)$, $\underline{\chi}_k^z = (\underline{\sigma}^+\underline{z}_k^+ - \overline{\sigma}^+\underline{z}_k^- - \underline{\sigma}^-\overline{z}_k^+ + \overline{\sigma}^-\overline{z}_k^-)$, $\overline{\sigma} = S^+(R^+\overline{\Delta A} - R^-\underline{\Delta A}) - S^-(R^+\underline{\Delta A} - R^-\overline{\Delta A})$, $\underline{\sigma} = S^+(R^+\underline{\Delta A} - R^-\overline{\Delta A}) - S^-(R^+\overline{\Delta A} - R^-\underline{\Delta A})$, $\underline{\varphi}_k^z = R^+(F^+\underline{f_k} - F^-\overline{f_k}) - R^-(F^+\overline{f_k} - F^-\underline{f_k})$, $\overline{\varphi}_k^z = R^+(F^+\overline{f_k} - F^-\underline{f_k}) - R^-(F^+\underline{f_k} - F^-\overline{f_k})$, $\underline{\rho}^z{}_k = R^+\underline{w}_k - R^-\overline{w}_k$, $\overline{\rho}^z{}_k = R^+\overline{w}_k - R^-\underline{w}_k$, $F = RL$.

Theorem 2. Given a nonsingular matrix R such that $E = R(A-LC)S$ is nonnegative. The initial state z_0 verifies $\underline{z}_0 \leq z_0 \leq \overline{z}_0$. If there exist positive definite and symmetric matrices Q, P and W such that the following Riccati matrix inequality is verified

$$E^TPE - P + E^TPW^{-1}PE + \alpha_z(\|W+P\|)I + Q \leq 0 \tag{15}$$

where $E = R(A-LC)S$ and $\alpha_z = 3\ max((c_1{}^2 + d_1{}^2), (c_2{}^2 + d_2{}^2))$, then $\underline{z}_k, \overline{z}_k \in \mathscr{L}_\infty^n$. □

Proof. The proof is similar to that of Theorem 1 . □

5. FAULT COMPENSATION AND STABILITY ANALYSIS

5.1 Fault Compensation

This section presents the design of an observer-based control law, which can compensate the faults effect and stabilize the systems (6). Since the interval observer (7) is composed of two classical ones, an alternative control law can be computed by using the lower and the upper bound $(\underline{x}_k, \overline{x}_k)$.

The nominal control law is chosen as a state linear feedback:

$$u_{n,k} = -K\frac{(\overline{x}_k + \underline{x}_k)}{2} \tag{16}$$

where K is a feedback gain.

Then, replacing the nominal control law (16) in the system equations leads to the following closed-loop representation:

$$\begin{cases} x_{k+1} = (A + \Delta A(\rho))x_k + Bu_{n,k} + Ff_k + w_k \\ y_k = Cx_k + v_k \end{cases} \tag{17}$$

We propose in this paper to compute a new control law $u_{k,ad}$ to be added to the nominal control law in order to compensate the fault effect on the system. Therefore, the total control law applied to the system is given by:

$$u_k = u_{n,k} + u_{ad,k} \tag{18}$$

Hence, the closed-loop state equation becomes:

$$\begin{cases} x_{k+1} = (A + \Delta A(\rho))x_k + Bu_{n,k} + Bu_{ad,k} + Ff_k + w_k \\ y_k = Cx_k + v_k \end{cases} \tag{19}$$

The additional control law $u_{ad,k}$ must be computed to make the faulty system behavior as close as possible to the nominal system one. In other terms, $u_{ad,k}$ should satisfy:

$$Ff_k + Bu_{ad,k} \approx 0 \tag{20}$$

Using the upper and the lower bounds of the fault described in the previous section, the solution of (20) can be obtained by the following relation if matrix B is of full row rank:

$$u_{ad,k} = -B^* F\,\frac{\underline{f}_k + \overline{f}_k}{2} \tag{21}$$

where B^* is the pseudo-inverse of matrix B.

The fault magnitude is not exactly computed, the compensation is only partially performed.

5.2 Stability analysis

In the presence of additive actuator fault, the closed loop system is defined by:

$$\begin{cases} x_{k+1} = (A + \Delta A(\rho))x_k - \dfrac{BK}{2}(\overline{x}_k + \underline{x}_k) + Ff_k + Bu_{ad,k} + w_k \\ y_k = Cx_k + v_k \end{cases} \tag{22}$$

The interval observer for the system (22) is defined as follows:

$$\begin{cases} \overline{x}_{k+1} = (A - LC - \dfrac{BK}{2})\overline{x}_k - \dfrac{BK}{2}\underline{x}_k + \overline{\chi}_k + \overline{\delta}_k \\ \underline{x}_{k+1} = (A - LC - \dfrac{BK}{2})\underline{x}_k - \dfrac{BK}{2}\overline{x}_k + \underline{\chi}_k + \underline{\delta}_k \end{cases} \tag{23}$$

with $\overline{\delta}_k = \overline{\varphi}_k - \frac{1}{2}F(\overline{f}_k + \underline{f}_k) + \overline{w}_k + Ly_k + |L|\overline{V}E_p$ and $\underline{\delta}_k = \underline{\varphi}_k - \frac{1}{2}F(\overline{f}_k + \underline{f}_k) + \underline{w}_k + Ly_k - |L|\overline{V}E_p$.

Theorem 3. Assume that Assumptions 1-3 are satisfied. The initial state x_0 verifies $\underline{x}_0 \le x_0 \le \overline{x}_0$. Let

$$u_k = u_{n,k} + u_{ad,k} \tag{24}$$

If there exist matrices $P \in \mathbb{R}^{2n\times 2n}, P = P^T \succ 0, Q \in \mathbb{R}^{2n\times 2n}, Q = Q^T \succ 0$ and $W \in \mathbb{R}^{2n\times 2n}, W = W^T \succ 0$ such that following matrix inequality is verified:

$$\Phi = \begin{bmatrix} G^T PG - P + Q & G^T P & G^T P \\ PG & P & P \\ PG & P & P - W \end{bmatrix} \preceq 0,$$

with

$$G = \begin{bmatrix} A - LC - \dfrac{BK}{2} & -\dfrac{BK}{2} \\ -\dfrac{BK}{2} & A - LC - \dfrac{BK}{2} \end{bmatrix},$$

Then $\underline{x}_k, \overline{x}_k$ and $x_k \in \mathscr{L}_n^\infty$. □

Proof.

For the stability analysis, we introduce the auxiliary system

$$\xi_{k+1} = G\xi_k + \chi_k + \delta_k \tag{25}$$

where $\xi_k = \begin{bmatrix} \underline{x}_k \\ \overline{x}_k \end{bmatrix}$, $\chi_k = \begin{bmatrix} \underline{\chi}_k \\ \overline{\chi}_k \end{bmatrix}$, $\delta_k = \begin{bmatrix} \underline{\delta}_k \\ \overline{\delta}_k \end{bmatrix}$

To establish the asymptotic stability of the system (25), consider the positive definite quadratic Lyapunov function:

$$V_k = \xi_k{}^T P \xi_k \tag{26}$$

The increment of ΔV is given by:

$$\begin{aligned} \Delta V &= V_{k+1} - V_k \\ &= \xi_k^T G^T PG\xi_k - \xi_k^T P\xi_k + \xi_k^T G^T P\chi_k + \chi_k^T PG\xi_k \\ &+ \chi_k^T P\chi_k + 2\xi_k^T G^T P\delta_k + 2\delta_k{}^T P\chi_k + \delta_k{}^T P\delta_k \\ &= \xi_k^T (G^T PG - P)\xi_k + \xi_k^T G^T P\chi_k + \chi_k^T PG\xi_k \\ &+ \chi_k^T P\chi_k + 2\xi_k^T G^T P\delta_k + 2\delta_k{}^T P\chi_k + \delta_k{}^T (P - W)\delta_k \\ &+ \xi_k^T Q\xi_k - \xi_k^T Q\xi_k + \delta_k{}^T W\delta_k \\ &= \begin{bmatrix} \xi_k \\ \chi_k \\ \delta_k \end{bmatrix}^T \Phi \begin{bmatrix} \xi_k \\ \chi_k \\ \delta_k \end{bmatrix} - \xi_k^T Q\xi_k + \delta_k{}^T W\delta_k \end{aligned}$$

Then $\underline{x}_k$ and $\overline{x}_k$ stay bounded for all $u_k \in \mathbb{R}^q$, $k \in \mathbb{N}$, which implies the same property for x_k.

6. SIMULATIONS RESULTS

Consider the discrete-time LPV system subject to an additive actuator fault:

$$\begin{cases} x_{k+1} = (A + \Delta A(\rho))x_k + Bu_k + Ff_k + w_k, \\ y_k = Cx_k + v_k, \end{cases} \tag{27}$$

$$A = \begin{bmatrix} 1.1 & -0.1 & 0.35 \\ 0.9 & 0.2 & -0.2 \\ 0.85 & -0.2 & 0.25 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T$$

For simulation we selected:

$$\Delta A(\rho) = \begin{bmatrix} 0.01\,sin(k) & 0 & 0 \\ 0 & 0.02sin(k) & 0 \\ 0 & 0 & 0.05\,cos(k) \end{bmatrix},$$

$\overline{\Delta A} = -\underline{\Delta A} = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}, w_k = [0 \;\; 0.01\, sin(k) \;\; 0]^T,$ $\overline{w}_k = -\underline{w}_k = [0 \;\; 0.01 \;\; 0]^T$, $v_k = 0.01\, cos(k)$ and $\overline{V} = 0.01$.

An actuator time-varying fault signal is set up as:

$$f_k = \begin{cases} 0 & if \;\; k < 400 \\ \sin(0.01\,k) & if \;\; k \geqslant 400 \end{cases}$$

where $F = [0 \;\; 0.05 \;\; 0]^T$, and it is assumed that the diagnosis module deliver an uncertain estimation of $f_k$such that $\underline{f}_k \leq f_k \leq \overline{f}_k$ with $\underline{f}_k = 0.95\, sin(0.01\, k)$ and $\overline{f}_k = 1.05\, sin(0.01\, k)$.

For $L = [0.9 \;\; 1.1 \;\; 0.5]^T$, the matrix $A - LC$ is not nonnegative. Thus, a transformation of coordinates,

$S = \begin{bmatrix} -0.058 & 0.997 & -0.052 \\ 0.134 & -0.044 & -0.99 \\ 0.989 & 0.064 & 0.131 \end{bmatrix}$ is used such that $E = R(A - LC)S$, is nonnegative. Consequently, the dynamic extension (14) is an interval observer for the system (27) when in closed-loop with $u_k = u_{n,k} + u_{ad,k}$ in the coordinates x with $K = [0.9365 \;\; -0.3135 \;\; 1.0157]$ and $u_{ad,k} = -B^* F \frac{\overline{f}_k + \underline{f}_k}{2}$.

The faulty case is considered in the simulations, that is, before 400s, the system operates in normal regime. At 400s, an additive fault occurs in the actuator. Assuming that the fault information is given, the fault compensation is applied at 600s.

The results of simulations are given in Fig.2, Fig.3, Fig.4, Fig.5 and Fig.6.

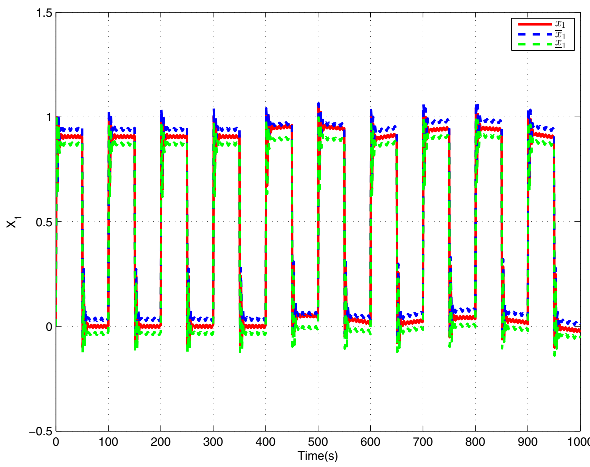

Fig. 2. Evolution of the first component x_1.

Fig.2, Fig.3 and Fig.4 present the evolution of the system states. Fig.5 gives a zoom of the second component x_2. Fig.6 present the error bounds evolution in the faulty and faulty-free cases. These figures allow to compare the system responses to the faults without and with compensation. The results of using the proposed control law show that the fault effect is handled and the robustness of the closed-loop FTC system are guaranteed in the presence of additive actuator faults and even with external disturbances.

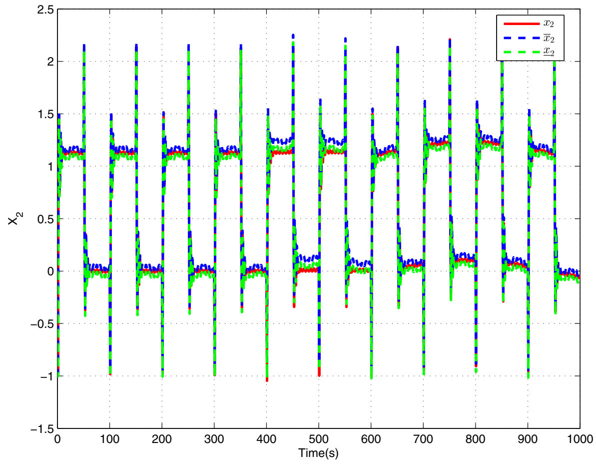

Fig. 3. Evolution of the second component x_2.

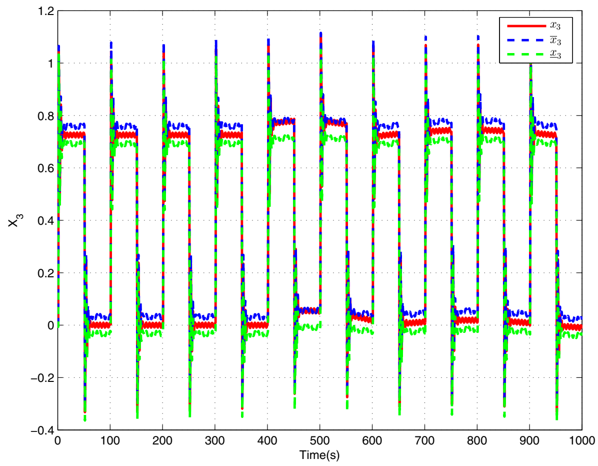

Fig. 4. Evolution of the third component x_3.

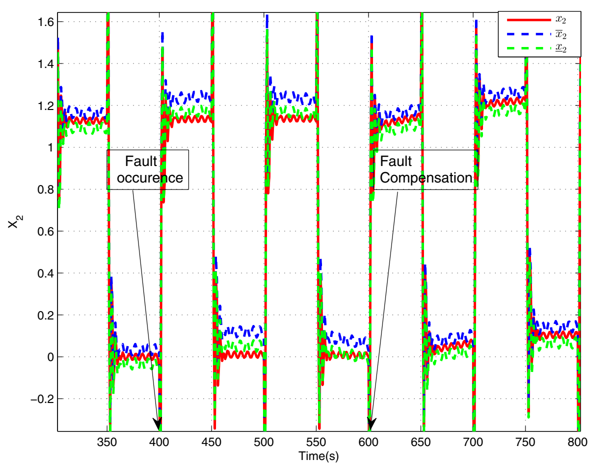

Fig. 5. Zoom on the second component x_2.

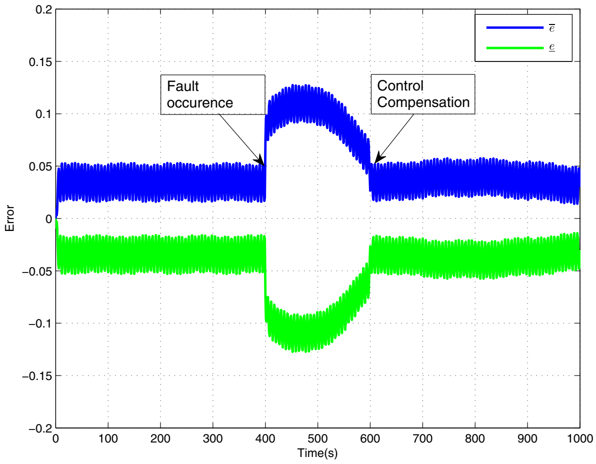

Fig. 6. Evolution of the upper and lower bounds of the error with compensation.

7. CONCLUSION

A methodology for actuator fault compensation for discrete-time LPV systems has been proposed in this paper. The actuator fault is considered as additive term, and the control is designed based on an interval observer so that the closed-loop system is robust against faults as well as the exogenous disturbances. Simulation results show the robustness and effectiveness of the proposed approach.

The design of fault identification module and the interaction analysis with Fault Tolerant Control module will be investigated in further works.

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