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LÉVY NMF FOR ROBUST NONNEGATIVE SOURCE SEPARATION

Paul Magron,1,2 Roland Badeau,2 Antoine Liutkus,3

1 Signal Processing Laboratory, Tampere University of Technology (TUT), Finland, paul.magron@tut.fi
2 LTCI, Télécom ParisTech, Université Paris-Saclay, Paris, France, roland.badeau@telecom-paristech.fr
3 Inria, Nancy Grand-Est, Multispeech team, LORIA UMR 7503, France, antoine.liutkus@inria.fr

ABSTRACT

Source separation, which consists in decomposing data into meaningful structured components, is an active research topic in many fields including music signal processing. In this paper, we introduce the Positive $\alpha$-stable (PoS) distributions to model the latent sources, which are a subclass of the stable distributions family. They notably permit us to model random variables that are both nonnegative and impulsive. Considering the Lévy distribution, the only PoS distribution whose density is tractable, we propose a mixture model called Lévy Nonnegative Matrix Factorization (Lévy NMF). This model accounts for low-rank structures in nonnegative data that possibly has high variability or is corrupted by very adverse noise. The model parameters are estimated in a maximum-likelihood sense. We also derive an estimator of the sources, which extends the validity of the Wiener filtering to the PoS case. Experiments on synthetic data and realistic music signals show that Lévy NMF compares favorably with state-of-the-art techniques in terms of robustness to impulsive noise and highlight its potential for decomposing nonnegative data.

Index Terms— Lévy distribution, Positive alpha-stable distribution, nonnegative matrix factorization, audio source separation.

1. INTRODUCTION

Source separation consists in extracting underlying components called sources that add up to form an observable signal called mixture. It has applications in many fields, including data analysis or audio processing [1].

A groundbreaking idea presented in [2] is to exploit the fact that the observations are often nonnegative, and to decompose them as a sum of nonnegative terms only, permitting only constructive interactions of the latent factors. This Nonnegative Matrix Factorization (NMF) has shown successful in many fields of audio signal processing such as automatic transcription [3] and source separation [4]. NMF, originally introduced as a rank-reduction method, approximates a nonnegative data matrix $X$ as a product of two low-rank nonnegative matrices $W$ and $H$. The factorization can be obtained by optimizing a cost function measuring the error between $X$ and $WH$, such as the Euclidean, Kullback-Leibler (KL [2]) and Itakura-Saito (IS [4]) cost functions. This may often be framed in a probabilistic framework, where the cost function appears as the negative log-likelihood of the data, e.g. Gaussian [4] or Poisson [5]. Addressing the estimation problem in a Maximum A Posteriori (MAP) sense makes it possible to incorporate some prior distribution over the parameters $W$ and $H$ in order to enforce desirable properties for the parameters such as harmonicity, temporal smoothness [6] or sparsity [7].

However, the above-mentioned distributions fail to provide good results when the data is very impulsive or contains outliers. This comes from their rapidly decaying tails, that cannot account for really unexpected observations. The family of heavy-tailed stable distributions [8] was thus found useful for robust signal processing [9]. A subclass of this family, called the Symmetric $\alpha$-stable (SoS) distributions, has been used in audio [10, 11] for modeling complex-valued Short-Term Fourier Transform (STFT). An estimation framework based on Markov Chain Monte Carlo (MCMC) methods has been proposed [12] to perform the separation of SoS mixtures. The common ground of these methods is to assume all observations as independent but to impose low-rank constraints on the nonnegative dispersion parameters of the sources, and not on their actual outcomes.

In this paper, we address the problem of modeling and separating nonnegative sources from their mixture, while still constraining their dispersion to follow an NMF model. To do so, we consider another subclass of the stable family which models nonnegative random variables: the Positive $\alpha$-stable (PoS) distributions. They also benefit from being heavy-tailed and are thus expected to yield robust estimates. Since the Probability Density Function (PDF) of those PoS distributions does not admit a closed-form expression in general, we study more specifically the Lévy case, which is a particular analytically tractable member of the family. We introduce the Lévy NMF model, where the dispersion parameters of the sources are structured through an NMF model and where realizations are necessarily nonnegative. The parameters are then estimated in a Maximum Likelihood (ML) sense by means of a Majorize-Minimization (MM) approach. We also derive an estimator of the sources which extends the validity of the generalized Wiener filtering to the PoS case. Several experiments conducted on synthetic and realistic music signals show the potential of this model for a nonnegative source separation task and highlight its robustness to impulsive noise.

This paper is structured as follows. Section 2 introduces the Lévy NMF mixture model. Section 3 details the parameters estimation and presents an estimator of the sources. Section 4 experimentally demonstrates the denoising ability of the model and its potential in terms of source separation for musical applications.

2. LÉVY NMF MODEL

2.1. Positive $\alpha$-stable distributions

Stable distributions, denoted $\alpha S(\mu, v = \sigma^\alpha, \beta)$, are heavy-tailed distributions parametrized by four parameters: a shape parameter $\alpha \in [0; 2]$ which determines the tails thickness of the distribution...
Each entry of $V$, for instance, the modulus of the STFT of an audio signal). $v$ which preserves the additive property of the model as in [11]: $v = \sum_{k} X_k (f, t) \sim \alpha S(\mu, v, \beta)$, with $\mu = \sum_{k} \mu_k$ and $v = \sum_{k} v_k$.

Stable distributions do not in general have a nonnegative support. However, it can be shown [13] that when $\beta = 1$ and $\alpha < 1$, the support of the distribution is $[\mu: +\infty[$. In this paper, we consider that $\mu = 0$ thus the support is $\mathbb{R}_+$. The PoS distributions are therefore such that $P \alpha S(v) = \alpha S(0, v, 1)$ with $\alpha < 1$.

2.2. Lévy NMF mixture model

The only case for which the PDF of a PoS distribution can be expressed in closed form is the Lévy case $L(v) = \mathcal{P}_{\frac{2}{\alpha}} S(v)$:

$$p(x \mid v = \sqrt{\sigma}) = \begin{cases} \frac{1}{2\pi x^{3/2}} e^{-x/\sigma} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

As it can be seen in Fig. 1, the Lévy distribution has a heavier tail than the most commonly used distributions for nonnegative data modeling (the Rayleigh distribution is non-other that the distribution of the modulus of a complex Gaussian variable).

Let us consider a nonnegative data matrix $X \in \mathbb{R}^{F \times T}$ (it can be, for instance, the modulus of the STFT of an audio signal). Each entry of $X$ is modeled as the sum of $K$ independent Lévy-distributed components $X_k (f, t) \sim \mathcal{L}(\nu_k (f, t))$. Note that all entries are independent, which allows us to extend the standard notation to matrices. Then, $X \sim \mathcal{L}(v)$ with $v = \sum_{k} \nu_k$.

The scale parameters are structured by means of an NMF [2], which preserves the additive property of the model as in [11]: $v = WH$, with $W \in \mathbb{R}^{F \times K}$ and $H \in \mathbb{R}^{K \times T}$. We then refer to this model as the Lévy NMF model.

3. PARAMETERS ESTIMATION

The parameters $W$ and $H$ are estimated in an ML sense, which is natural in a probabilistic framework. The log-likelihood of the data is given by:

$$L(W, H) \triangleq \frac{1}{2} \sum_{f,t} \log([WH](f, t)^2) - \frac{[WH](f, t)^2}{X(f, t)}$$

where $\triangleq$ denotes equality up to an additive constant which does not depend on the parameters, $\log^2$ is entry-wise exponentiation and $d_{IS}$ denotes the IS divergence [4]. We thus remark that maximizing the log-likelihood of the data in the Lévy NMF model is equivalent to minimizing the IS divergence between $[WH]^{\otimes 2}$ and $X$, which boils down to minimizing the following cost function:

$$C(W, H) = \sum_{f,t} \frac{[WH](f, t)^2}{X(f, t)} - 2 \log([WH](f, t)).$$

3.1. Naive multiplicative updates

The cost function (2) can be minimized with the same heuristic approach that has been pioneered in [2] and used in many NMF-related papers in the literature. The gradient of $C$ with respect to the parameter $\theta$ (W or H) is expressed as the difference between two nonnegative terms: $\nabla_{\theta} C = \nabla^+ - \nabla^-$, which leads to the multiplicative update rules:

$$\theta \leftarrow \Theta \odot \nabla^- \theta = \Theta \odot a_{\theta}$$

where $\odot$ (resp. the fraction bar) denotes the element-wise matrix multiplication (resp. division). For Lévy NMF:

$$a_W = \frac{[WH]^{\otimes -1} H^T}{([WH] \odot X^{\otimes -1}) H^T},$$

$$a_H = \frac{W^T [WH]^{\otimes -1}}{W^T (WH) \odot X^{\otimes -1}}.$$
Since $\sum_k \rho_k(f, t) = 1$, we can apply the Jensen’s inequality to the convex function $x \mapsto x^2$:

$$[W H](f, t)^2 = \left( \sum_k \rho_k(f, t) \frac{W(f, k) H(k, t)}{\rho_k(f, t)} \right)^2 \leq \sum_k \rho_k(f, t) \left( \frac{W(f, k) H(k, t)}{\rho_k(f, t)} \right)^2,$$

(9)

which leads to the majorization of the first term of $C$ in (2):

$$\sum_{f, t} [W H](f, t)^2 \leq \sum_{f, t} \frac{V(f, t)}{X(f, t)} \sum_k W(f, k)^2 H(k, t).$$

(10)

In a similar fashion, we majorize the second term in (2) ($x \mapsto -\log(x)$ is also a convex function) and therefore we obtain the following auxiliary function $G$:

$$G(W, H, \overline{W}) = \sum_{f, t} \frac{V(f, t) H(k, t)}{X(f, t) W(f, k)} W(f, k)^2$$

$$- 2 \frac{W(f, k) H(k, t)}{V(f, t)} \log \left( \frac{W(f, k) V(f, t)}{V(f, k)} \right).$$

(11)

We then set the partial derivative of $G$ with respect to $W$ to zero, which leads to an update rule on $W$. The update rule on $H$ is obtained in exactly the same way. Thus, the updates are:

$$\theta \leftarrow \theta \circ a_\theta^{\alpha/2},$$

(12)

where $a_W$ and $a_H$ are given by (4) and (5). A MATLAB implementation of this algorithm can be found at [17].

Remark: Let us assume that $K = 1$ and that $\forall f, W(f) = 1$. Then the Lévy NMF and ISNMF [4] updates on $H$ rewrite:

$$H_{\text{Lévy}}(t) \leftarrow \frac{1}{\sum f X(f, t)}$$

and $H_{\text{IS}}(t) \leftarrow \frac{1}{\sum f X(f, t)}.

(13)

Therefore, the update on $H$ in the Lévy NMF (resp. ISNMF) model boils down to a harmonic (resp. arithmetic) mean on $X$. Since the harmonic mean is known to attenuate the impact of strong outliers (compared to the arithmetic mean), the corresponding Lévy NMF procedure is expected to yield robust estimates.

### 3.3. Estimator of the components

For a source separation task, it can be useful, once the model parameters are estimated, to derive an estimator $\hat{X}_k$ of the isolated components $X_k$. In a probabilistic framework, a natural estimator is given by the posterior expectation of the source given the mixture $\mathbb{E}(X_k | X)$. For Slow random variables, such an estimator is provided by a generalized Wiener filtering [10]. One contribution of this paper is to prove that this result still holds for the ProS random variables. Derivations may be found in the companion technical report for this paper [18]. For Lévy-distributed random variables (with $\alpha = 1/2$), we then have:

$$\hat{X}_k = \mathbb{E}(X_k | X) \equiv \sum_t \frac{v_k}{v_t} \odot X = \frac{W_k H_k}{WH} \odot X.$$
We propose to test the performance of the Lévy NMF model for enhancing the accompaniment in music signals. We consider 50 music excerpts of various genres from the DSD100 database (a remastered version of the database used for the SiSEC 2015 campaign [26]). Each track consists of a singing voice signal and a musical accompaniment signal. While musical accompaniment is assumed to be well represented by a low-rank NMF model, voice is assumed to be similar to impulsive noise. Thus, when fitting a model that is robust to such a noise on the data, we expect that the voice will be treated as noise, therefore leading to enhance the musical accompaniment.

The data $X$ is obtained by taking the magnitude spectrogram of the STFT of the mixture signals, computed as in the previous experiment. We then run 200 iterations of the algorithms with a rank of factorization $K = 30$, and we estimate the sources by means of the generalized Wiener filtering (14). The phase of the mixture is then assigned to the estimated musical accompaniment source in order to resynthesize time signals. The quality of the enhancement is measured by means of the SDR between the original and estimated accompaniment tracks and presented in Fig. 4.

Musical accompaniment enhancement performed with Lévy NMF leads to better results than traditional NMF techniques. An informal subjective evaluation of the synthesized signals (sounds examples available at [25]) shows that ISNMF and KLNMF methods do not lead to a significant attenuation of the voice, while with the other techniques, the voice is hardly perceptible, at the cost of very few artifacts. Lévy NMF competes with other robust methods such as Cauchy NMF and RPCA, and thus appears to be a good candidate for robust musical applications.

5. CONCLUSION

In this paper, we introduced the Lévy NMF model, which structures the dispersion parameters of PoS distributed sources when $\alpha = 1/2$. Experiments have shown the potential of this model for robustly decomposing realistic nonnegative data.

Such a model could be useful in many other fields where the source separation issue frequently occurs and where the Lévy distribution finds applications, such as optics [27]. Future work could focus on novel estimation techniques for the Lévy NMF model, using for instance a MAP estimator, which would permit us to incorporate some prior knowledge about the parameters [6]. Besides, drawing on [12], the family of techniques based on MCMC could be useful to estimate the parameters of any PoS distribution. Alternatively, one could extend the Lévy NMF model to the family of inverse gamma distributions it belongs to. Although this would result in losing the additivity property and thus the theoretical foundation for $\alpha$-Wiener filtering, this would allow for convenient analytical derivations thanks to tractable likelihood functions. This strategy is reminiscent of recent work [28] where the tractable Student-t distribution is used instead of the ScS one.
6. REFERENCES


