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SEPARATING TIME-FREQUENCY SOURCES FROM TIME-DOMAIN CONVOLUTIVE MIXTURES USING NON-NEGATIVE MATRIX FACTORIZATION

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ABSTRACT

This paper addresses the problem of under-determined audio source separation in multichannel reverberant mixtures. We target a semi-blind scenario assuming that the mixing filters are known. Source separation is performed from the time-domain mixture signals in order to accurately model the convolutive mixing process. The source signals are however modeled as latent variables in a time-frequency domain. In a previous paper we proposed to use the modified discrete cosine transform. The present paper generalizes the method to the use of the odd-frequency short-time Fourier transform. In this domain, the source coefficients are modeled as centered complex Gaussian random variables whose variances are structured by means of a non-negative matrix factorization model. The inference procedure relies on a variational expectation-maximization algorithm. In the experiments we discuss the choice of the source representation and we show that the proposed approach outperforms two methods from the literature.

Index Terms— Audio source separation, reverberant mixtures, non-negative matrix factorization, variational inference.

1. INTRODUCTION

Multichannel audio source separation consists in recovering several source signals from the observation of a mixture recorded with multiple microphones. In this paper we consider under-determined mixtures where the number of microphones is lower than the number of sources to be estimated. Moreover we focus on separating reverberant (or convolutive) mixtures, assuming a semi-blind scenario where the mixing filters are known.

In an under-determined context, source separation methods commonly work with a time-frequency (TF) representation of the source signals. Indeed, model-based approaches can take advantage of the very particular structure of audio signals in the TF plane [1]. For example, sparse component analysis methods [2] exploit the sparsity of the source signals in the TF domain. They rely on stationary super-Gaussian priors or they make use of deterministic approaches based on sparsity inducing penalties. Another important trend in audio source separation corresponds to the variance modeling framework [3]. Non-negative matrix factorization (NMF) techniques are especially popular for representing the short-term power spectral density of the source signals [4, 5, 6, 7, 8].

Under-determined source separation becomes even more challenging when the mixtures are reverberant. In that case the time-domain source signals are convolved with mixing filters before being added to produce a mixture. While this convolution is simply expressed in the time domain, it is not straightforward to take it into account when working with a TF representation of the mixture signals. Therefore, it is common to approximate the convolutive mixing process as being instantaneous in each frequency band of the short-time Fourier transform (STFT) [9, 10]. This approximation is considered to be valid when the mixing filters are short compared with the STFT analysis window. Source separation performance under this approximation is thus fundamentally limited when the mixture is highly reverberant. To overcome this limitation some methods have investigated more accurate TF mixture models. For example, time-domain convolution is exactly represented as a two-dimensional filtering in the TF domain in [11]. In [12] it is accurately approximated using a convolutive transfer function model.

Another approach introduced in [13] consists in modeling the sources in the TF domain while keeping a time-domain representation of the convolutive mixture. This is the approach we also followed in [14]. In this previous paper the source signals were characterized using the modified discrete cosine transform (MDCT) which is real-valued and critically sampled. In the present paper we generalize this method to a source representation based on the odd-frequency STFT (OFSTFT) which is redundant and complex-valued. This transform is similar to the STFT except that the discrete Fourier transform (DFT) is replaced by the odd-frequency DFT (OFDFT) [15]. Each source coefficient in this domain is modeled as a centered complex Gaussian random variable whose variance is structured by means of an NMF model. We infer the latent source variables using a variational expectation-maximization (VEM) algorithm. We experimentally study the performance of the method according to the choice of the source representation (MDCT or OFSTFT with different redundancy factors). We also show that the proposed approach outperforms two methods from the literature [16, 13] in a semi-blind setting where the mixing filters are known.

We start by presenting in Section 2 the OFDFT from which the OFSTFT can be constructed. In Section 3 we introduce the model. Section 4 details the VEM algorithm. The experimental evaluation is presented in Section 5 and we finally conclude in Section 6.

2. THE ODD-FREQUENCY DFT

The OFDFT of a signal $z(t)$, $t = 0, ..., T - 1$, is defined for $f = 0, ..., T - 1$ as

$$z_f = \frac{1}{\sqrt{T}} \sum_{t=0}^{T-1} z(t) \exp \left( -i \frac{2\pi}{T} (f + \frac{1}{2}) t \right)$$

[15], where $i = \sqrt{-1}$. It is a particular case of the Generalized DFT [17]. Compared with the standard DFT, we see that it simply corresponds to shifting the frequency index by a factor 1/2. For a real-valued signal $z(t)$, the following symmetry property holds: $z_{-f-1} = z_f^*$ where $^*$ denotes complex conjugation. Compared with the standard DFT where the coefficients at the zero and the Nyquist’s frequencies are real-valued, all coefficients of the OFDFT are complex-valued. It is thus more appropriate when the frequency coefficients are modeled as complex-valued random variables, which is common in audio source separation. Moreover
this symmetry property allows us to write a simple expression for the inverse OFDFT involving only the non-redundant coefficients:
\[ z(t) = \frac{2}{N-w} \Re \left( \sum_{f=0}^{T/2-1} z_f \exp \left( \frac{i}{T} \pi (f + \frac{1}{2}) t \right) \right) , \]
where \( \Re(\cdot) \) denotes the real part. The OFSTFT can then be defined similarly as the standard STFT but using the OFDFT. We can mention that in [18] it has been shown that the MDCT and the OFSTFT are more appropriate than the standard STFT for assuming independent TF coefficients, which is common in audio source separation.

3. MODEL

The signal at each microphone \( i = 1, ..., I \) is represented as a noisy mixture of \( J \) source images \( y_{ij}(t), j = 1, ..., J, \) for \( t = 0, ..., T-1 \):
\[ x_i(t) = \sum_{j=1}^{J} y_{ij}(t) + b_i(t), \]
where \( b_i(t) \sim N(0, \sigma_b^2) \) is a white Gaussian additive noise. The probability density function (pdf) of \( N_b \) is defined in Appendix A.

Each source image \( y_{ij}(t) \) corresponds to the discrete convolution of a source signal \( s_j(t) \in \mathbb{R}, t = 0, ..., L_s - 1 \), with a mixing filter \( a_{ij}(t) \in \mathbb{R} \), \( t = 0, ..., L_a - 1 \), such that \( T = L_s + L_a - 1 \):
\[ y_{ij}(t) = [a_{ij} * s_j](t). \]

Similarly as in [19], a source signal \( s_j(t) \) is represented by a set of TF synthesis coefficients \( \{ s_{j,f,n} \}_{f,n} \) for \( f, n \in B \) with \( B = \{0, ..., F-1\} \times \{0, ..., N-1\} \):
\[ s_j(t) = \frac{2}{\phi} \Re \left( \sum_{(f,n) \in B} s_{j,f,n} \psi_{f,n}(t) \right) , \]
\[ \psi_{f,n}(t) \in \mathbb{C}, t = 0, ..., L_s - 1, \] is a TF synthesis atom and \( \phi = 1 \) if \( K = \mathbb{C} \) or \( \phi = 2 \) if \( K = \mathbb{R} \). In this work we consider either the MDCT [20] if \( K = \mathbb{C} \) or the OFSTFT if \( K = \mathbb{C} \). For the MDCT, the TF synthesis atom is defined as:
\[ \psi_{f,n}(t) = \sqrt{\frac{2}{F}} w(t - nH) \cos \left( \frac{2\pi}{L_w} \left( f + \frac{1}{2} \right) (t - nH) \right) , \]
while for the OFSTFT we have:
\[ \psi_{f,n}(t) = \sqrt{\frac{1}{L_w}} w(t - nH) \exp \left( \frac{2\pi}{L_w} \left( f + \frac{1}{2} \right) (t - nH) \right) . \]

The pdfs of these distributions are provided in Appendix A. The variables \( v_{j,f,n} \in \mathbb{R}_+ \) are finally structured by means of an NMF model of rank \( K_j \), generally chosen such that \( K_j(F + N) \ll FN \):
\[ v_{j,f,n} = [W_j H_j]_{f,n}, \]
with \( W_j \in \mathbb{R}_+^{F \times K_j}, \) \( H_j \in \mathbb{R}_+^{K_j \times N} \).

4. VARIATIONAL INFERENCE

Let \( x = \{ x_i(t) \}_{i,t} \) denote the set of observed variables, \( s = \{ s_{j,f,n} \}_{j,f,n} \) the latent variables and \( \theta = \{ \sigma_j^2 \}, \{ W_j, H_j \}_j \) the model parameters. Remember that the mixing filters \( \{a_{ij}(t)\}_{i,j} \) are assumed to be known. Exact posterior inference of the latent variables is here computationally heavy because the time-domain convolution induces complex posterior dependencies between the latent variables. We thus adopt a variational approach to infer the latent variables and estimate the model parameters. Let \( q \in \mathcal{F} \) be a pdf over \( s \), where \( \mathcal{F} \) is a variational family. Variational inference consists in optimizing a criterion called the variational free energy and defined as [21]:
\[ \mathcal{L}(q; \theta) = \langle \ln ( p(x; s; \theta) / q(s) ) \rangle_q , \]
where \( \langle \cdot \rangle_q \) denotes the mathematical expectation taken with respect to \( q \). More precisely we will use the VEM algorithm that consists in iterating two steps until convergence: the E-step where we compute \( q^* = \arg \max_{q \in \mathcal{F}} \mathcal{L}(q; \theta^*) \) and the M-step where we compute \( \theta^* = \arg \max_{\theta} \mathcal{L}(q^*; \theta) \). In practice we will use the mean-field approximation by constraining the variational family \( \mathcal{F} \) to the set of pdfs that factorize as \( q(s) = \prod_{j,f,n} q_{f,n}(s_{j,f,n}) \).

Under this approximation we can show that the pdf over \( s \in \mathcal{S} \) that maximizes the variational free energy satisfies [21]:
\[ \ln q^*(s) \equiv \langle \ln p(x; s; \theta) \rangle_{q(s \setminus s)}, \]
where \( \equiv \) represents equality up to an additive constant and \( s \setminus s \) denotes the set of all latent variables but \( s \).

**Source estimate.** Under the variational mean-field approximation, the estimate of the \( j \)-th source in the TF domain is given by:
\[ \hat{s}_{j,f,n} = \langle s_{j,f,n} \rangle_q . \]

The time-domain signal \( \hat{s}_j(t) \) is then reconstructed by inverse TF transform and the source image \( y_{ij}(t) \) is obtained by convolution with the corresponding mixing filter: \( y_{ij}(t) = [a_{ij} * \hat{s}_i](t) \).

**Complete-data log-likelihood.** From the model introduced in Section 3, the complete-data log-likelihood \( \ln p(x; s; \theta) = \ln p(x|s; \theta) + \ln p(s; \theta) \) can be expressed as:
\[ \ln p(x; s; \theta) \equiv - \frac{1}{2} \sum_{i=1}^{I} \sum_{t=0}^{T-1} \left[ \ln(\sigma_{t}^2) + \frac{1}{\sigma_{t}^2} \left( x_i(t) - \sum_{j=1}^{J} y_{ij}(t) \right)^2 \right] - \frac{1}{\phi} \sum_{j=1}^{J} \sum_{f=1}^{F} \sum_{n=0}^{N-1} \left[ \ln(v_{j,f,n}) + \frac{|s_{j,f,n}|^2}{v_{j,f,n}} \right] . \]

**E-step.** From (10) and (12) we can show that:
\[ q_{f,n}^{*}(s_{j,f,n}) = \begin{cases} N_{\mathcal{C}}(\mu_{j,f,n}^{*}, \gamma_{j,f,n}^{*}) & \text{if } K = \mathbb{C}; \\ N_{\mathbb{R}}(\mu_{j,f,n}^{*}, \gamma_{j,f,n}^{*}) & \text{if } K = \mathbb{R}, \end{cases} \]

where \( N_{\mathcal{C}}(\mu, \gamma) = \left\{ z \in \mathbb{C} : |z - \mu|^2 < \gamma \right\} \) and \( N_{\mathbb{R}}(\mu, \gamma) = \left\{ z \in \mathbb{R} : z^2 < \gamma \right\} \) denote the Gaussian and exponential distributions respectively.
where these pdfs are defined in Appendix A and for $p \in \{r, t\}$ we have:

$$
\rho_{j, fn} = \left( \frac{2}{\varphi^2} \sum_{i=1}^{l} \sum_{t=0}^{T-1} \Re(g_{j, fn}(t)) \Im(g_{j, fn}(t)) \right) /
$$

$$
\left[ \left( \frac{2}{\varphi^2} \sum_{i=1}^{l} \sum_{t=0}^{T-1} \Re(g_{j, fn}(t))^2 + \frac{1}{\varphi^2} \right) \times \right]
$$

$$
\left( \frac{2}{\varphi^2} \sum_{i=1}^{l} \sum_{t=0}^{T-1} \Im(g_{j, fn}(t))^2 + \frac{1}{\varphi^2} \right) \right]^{0.5}; \quad (14)
$$

$$
\gamma_{j, fn} = \left( 2 - \rho_{j, fn}^2 \right) \left( \frac{2}{\varphi^2} \sum_{i=1}^{l} \sum_{t=0}^{T-1} \Re(g_{j, fn}(t))^2 + \frac{1}{\varphi^2} \right) \right]^{-1}; \quad (15)
$$

$$
\hat{\gamma}_{j, fn} = \gamma_{j, fn} - \frac{\varphi}{\gamma_{j, fn}} \frac{1 - \rho_{j, fn}^2}{\gamma_{j, fn}}, \quad (16)
$$

with $\Re(\cdot)$ denoting the real part $\Re(\cdot)$ (resp. the imaginary part $\Im(\cdot)$) if $p = r$ (resp. $t$) and

$$
\hat{d}_{j, fn} = \hat{d}_{j, fn}^r - \hat{d}_{j, fn}^t \hat{d}_{j, fn}^t,
$$

$$
\begin{aligned}
\hat{d}_{j, fn}^r = \frac{2}{\varphi} \sum_{i} \sum_{t=0}^{T-1} \Re\left( g_{j, fn}(t) \right) \left( x_i(t) - \sum_{j=1}^{J} \hat{y}_{j}(t') \right) \frac{1}{\sigma_i^2} \sum_{t=0}^{T-1} \Re\left( g_{j, fn}(t) \right) \left( x_i(t) - \sum_{j=1}^{J} \hat{y}_{j}(t') \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
\hat{d}_{j, fn}^t = \frac{2}{\varphi} \sum_{i} \sum_{t=0}^{T-1} \Im\left( g_{j, fn}(t) \right) \left( x_i(t) - \sum_{j=1}^{J} \hat{y}_{j}(t') \right) \frac{1}{\sigma_i^2} \sum_{t=0}^{T-1} \Im\left( g_{j, fn}(t) \right) \left( x_i(t) - \sum_{j=1}^{J} \hat{y}_{j}(t') \right) \right]
\end{aligned}
$$

Interestingly, in the complex case ($\mathbb{K} = \mathbb{C}$) $q_{j, fn}(s_{j, fn})$ is the pdf of a complex Gaussian distribution which is not proper (see Appendix A). It means that the real and imaginary parts of the source coefficients are posteriori correlated and have different variances. In the real case ($\mathbb{K} = \mathbb{R}$) we obtain the same results as presented in [14]. We have to mention that update (16) for $p \in \{r, t\}$ holds if the parameters are updated in turn. However, we can show that $d_{j, fn}^p = \partial(-\mathcal{L}(q^p; \theta))/\partial s_{j, fn}$, where $\mathcal{L}(q^p; \theta)$ is given in the next paragraph. Therefore, (16) corresponds to a coordinate ascent of the variational free energy. In practice, as in [14], we will rather use the conjugate gradient method with diagonal preconditioning [22] for optimizing this criterion with respect to the whole set of coefficients $\{s_{j, fn}, \hat{s}_{j, fn}\}$. This choice allows us to make the E-Step more computationally efficient. Further details on the derivation of the E-step and on this conjugate gradient algorithm can be found in the supporting document [23]. The source estimate is finally given by $\hat{s}_{j, fn} = \hat{s}_{j, fn}^r + i\hat{s}_{j, fn}^t$. We also define the following second-order moments that will be used in the sequel:

- **Variance**: $\gamma_{j, fn} = \langle |s_{j, fn} - \hat{s}_{j, fn}|^2 \rangle = \gamma_{j, fn}^r + \gamma_{j, fn}^t$;
- **Pseudo-variance**: $\tilde{\gamma}_{j, fn} = \langle (s_{j, fn} - \hat{s}_{j, fn})^2 \rangle = \gamma_{j, fn}^r - \gamma_{j, fn}^t + 2i\rho_{j, fn} \sqrt{\gamma_{j, fn}^r \gamma_{j, fn}^t}$.

**Variational free energy.** Omitting the terms that are independent of the model parameters, the variational free energy can be written from (9), (12) and the E-step as follows:

$$
\mathcal{L}(q^p; \theta) \doteq - \frac{1}{2} \sum_{i=1}^{l} \sum_{t=0}^{T-1} \ln(\sigma_i^2) + \frac{e_i(t)}{\sigma_i^2} - \frac{1}{2} \sum_{j=1}^{J} \sum_{(f, n) \in B} \ln (v_{j, fn}) + \frac{\bar{\gamma}_{j, fn}}{v_{j, fn}} \right]
$$

where $e_i(t) = \langle (x_i(t) - \sum_{j=1}^{J} \hat{y}_{j}(t))^2 \rangle_{\theta^*}$ can be further expressed from the mean-field approximation and (6) as:

$$
e_i(t) = \left( x_i(t) - \sum_{j=1}^{J} \hat{y}_{j}(t) \right)^2 + \frac{2}{\varphi^2} \sum_{j=1}^{J} \sum_{(f, n) \in B} \left[ \Re(\hat{\gamma}_{j, fn} \bar{g}_{j, fn}(t)) + \gamma_{j, fn} |g_{j, fn}(t)|^2 \right].
$$

**M-step.** The M-step consists in maximizing (or only increasing) $\mathcal{L}(q^p; \theta)$ in (19) with respect to $\theta$. Zeroing the derivative of this criterion with respect to $\sigma_i^2$ leads to the following update:

$$
\sigma_i^2 = \frac{1}{T} \sum_{t=0}^{T-1} e_i(t). \quad (21)
$$

For the NMF parameters, we can recognize in (19) the Itakura-Saito divergence [4] between the posterior mean of the source power spectrogram $\langle |s_{j, fn}|^2 \rangle_{\theta^*} = |\hat{s}_{j, fn}|^2 + \gamma_{j, fn}$ and $\hat{v}_{j, fn} = |\hat{W}_j^T \hat{H}_j|_f / n$ (up to an additive constant). Therefore the NMF parameters can be updated using the multiplicative update rules given in [4].

### 5. Experiments

We perform the experiments using audio source signals provided by the MTG MASS database [24]. We created 8 stereo mixtures sampled at 16 kHz using room impulse responses from the RWCP (JR2) database [25]. These room responses were recorded in a real room with a reverberation time of 470 ms. Each mixture contains between 3 and 5 spatially disjoint sources and the duration ranges from 12 to 28 seconds.

All experiments are performed with the true mixing filters known and fixed. We evaluate the quality of the separation in terms of reconstructed mono sources. We use standard energy ratios defined in [26] and expressed in decibels (dB): the Signal-to-Distortion (SDR), Artifact (SAR) and Interference (SIR) Ratios. These measures are computed using the BSS Eval toolbox [27]. We also consider perceptually motivated objective measures introduced in [28, 29]: the Overall (OPS), Target-related (TPS), Interference-related (IPS) and Artifact-related (APS) Perceptual Scores. They are expressed in percentage and computed using the PEASS toolbox [30]. For all experiments we use an analysis/synthesis sine window of 128 ms. For the methods relying on NMF, the factorization rank is arbitrarily fixed to 10 for all sources.

We refer the reader to our web page for listening to audio examples illustrating the results discussed below. Matlab code implementing the proposed method is also available.

We first evaluate the separation according to the TF source representation used in the proposed framework. More precisely, we study the influence of the redundancy of the transform. The MDCT is critically sampled which means that there are as many TF coefficients as time-domain samples. On the contrary, the OFSTFT is a redundant transform, the redundancy being controlled by the overlap size. For example, a 50% overlap leads to a number of (real) TF coefficients which is twice the number of time-domain samples. The average separation results for the different transforms

are shown in Table 1 (lines 2 to 5), higher overlap means higher redundancy. The VEM algorithm was run for 200 iterations. We first observe that according to the SDR, SIR and SAR, the more redundancy we use, the better the results are. However this improvement is not so clear by listening to the separated sources. Therefore we also computed perceptually motivated objective measures. As can be seen from Table 1, the overall separation quality as measured by the OPS is much less dependent on the TF representation. We even obtain the best performance with the MDCT which is critically sampled. These results seem to be more consistent with the perceived separation quality. Moreover we have to mention that by increasing the redundancy we increase the number of latent TF source variables, so the separation is computationally more expensive. Finally we can see that according to the IPS and APS, increasing the redundancy seems to help reducing interferences to some extent, but it induces more artifacts. The overlap-add of multiple incoherent mixtures, even with the critically sampled MDCT. This is confirmed by the results with this method when using the OFSTFT and an overlap of 50 or 75%.

### Table 1: Source separation results averaged over all the sources in the dataset and normalized computational time (NCT) for one of the mixtures containing 3 sources and lasting for 12 seconds.

<table>
<thead>
<tr>
<th></th>
<th>SDR</th>
<th>SIR</th>
<th>SAR</th>
<th>OPS</th>
<th>TPS</th>
<th>IPS</th>
<th>APS</th>
<th>NCT</th>
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<tbody>
<tr>
<td>MDCT [14]</td>
<td>4.8</td>
<td>10.9</td>
<td>5.2</td>
<td>87.9</td>
<td>66.5</td>
<td>65.7</td>
<td>68.2</td>
<td>1.00</td>
</tr>
<tr>
<td>OFSTFT - overlap 25%</td>
<td>6.5</td>
<td>12.9</td>
<td>9.5</td>
<td>87.7</td>
<td>63.4</td>
<td>67.2</td>
<td>35.7</td>
<td>2.83</td>
</tr>
<tr>
<td>OFSTFT - overlap 50%</td>
<td>7.6</td>
<td>14.9</td>
<td>10.4</td>
<td>79.9</td>
<td>64.6</td>
<td>68.6</td>
<td>35.1</td>
<td>4.07</td>
</tr>
<tr>
<td>OFSTFT - overlap 75%</td>
<td>9.7</td>
<td>17.9</td>
<td>12.1</td>
<td>64.4</td>
<td>62.8</td>
<td>67.4</td>
<td>34.1</td>
<td>7.79</td>
</tr>
<tr>
<td>Özerv and Fétotte [16]</td>
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<td>4.3</td>
<td>2.1</td>
<td>25.5</td>
<td>55.3</td>
<td>63.5</td>
<td>9.1</td>
<td>0.01</td>
</tr>
<tr>
<td>Kowalski et al. [13]</td>
<td>7.5</td>
<td>14.1</td>
<td>10.1</td>
<td>29.1</td>
<td>63.2</td>
<td>60.7</td>
<td>23.1</td>
<td>1.13</td>
</tr>
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</table>

#### 6. CONCLUSION

In this paper we generalized our previous work [14] to the use of a source representation based on the OFSTFT. We experimentally studied the impact of using this complex-valued and redundant TF transform on the source separation performance compared with the use of the MDCT. We also showed that the proposed approach outperforms two standard methods from the literature in a semi-blind setting where the mixing filters are known. This experimental evaluation demonstrated the importance of jointly modeling the convolutive mixing process in the time domain and the source signals in the TF domain by means of an NMF model.

Future work will focus on developing a fully blind source separation method where the mixing filters will also be estimated. Using probabilistic priors on the mixing filters could help us to reach this objective [32]. Indeed, the mixing filters are room responses so they exhibit a simple specific structure in the time domain that could be used to guide their estimation.

### A. GAUSSIAN PROBABILITY DISTRIBUTIONS

Let $\mathcal{N}(x; \mu, \sigma^2)$ denote the Gaussian distribution over a real-valued random variable (r.v.) $x$. Its pdf is given by:

$$N(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right). \quad (22)$$

Let $\mathcal{C}(x; \rho, \mu_x, \mu_z, \sigma_x^2, \sigma_z^2)$ denote the Gaussian distribution over a complex-valued r.v. $x = x_r + jx_i$. Its pdf is given by [33]:

$$\mathcal{N}(x; \rho, \mu_x, \mu_z, \sigma_x^2, \sigma_z^2) = \frac{1}{2\sigma_x \sigma_z \sqrt{1 - \rho^2}} \exp\left[-\frac{1}{2(1 - \rho^2)} \left(\frac{2\rho(x_r - \mu_x) + (x_r - \mu_z)^2}{\sigma_x^2} + \frac{2\rho(x_i - \mu_z) + (x_i - \mu_z)^2}{\sigma_z^2}\right)\right]. \quad (23)$$

where $\rho = \mathbb{E}[(x_r - \mu_x)(x_i - \mu_z)]/(\sigma_x \sigma_z) \in [-1, 1]$. The particular case $\mathcal{N}(x; 0, \mu_x, \mu_z, \sigma_x^2/2, \sigma_z^2/2)$ corresponds to the proper complex Gaussian distribution. It is denoted by $\mathcal{N}_x(x; \mu, \sigma^2)$ where $\mu = \mu_x + j\mu_z$ and $\sigma^2 = 2\sigma_x^2 = 2\sigma_z^2$. In this case the pdf gets simplified to:

$$\mathcal{N}_x(x; \mu, \sigma^2) = \frac{1}{\pi\sigma^2} \exp\left(-\frac{|x|^2}{\sigma^2}\right). \quad (24)$$

Finally, the complex Gaussian distribution is circularly symmetric if it is proper and $\mu = 0$.

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2As proposed later by the authors in [31], the NMF parameters are updated differently from [16] by using multiplicative update rules.
B. REFERENCES