Infinite Horizon Impulse Control Problem with Jumps and Continuous Switching Costs

Hani Hani Abidi, Rim Amami, Monique Pontier

Infinite Horizon Impulse Control Problem with Jumps and Continuous Switching Costs

Hani Abidi [1], Rim Amami [2], Monique Pontier [3]

January 8, 2021

Abstract

Existence results for adapted solutions of infinite horizon doubly reflected backward stochastic differential equations with jumps are established. These results are applied to get the existence of an optimal impulse control strategy for an infinite horizon impulse control problem. The properties of the Snell envelope reduce the problem to the existence of a pair of right continuous left limited processes. Finally, some numerical results are provided.

Keywords: Impulse control, infinite horizon, jumps, reflected backward stochastic differential equations, double barrier, constructive method of the solution.
MSC Classification: 60G40, 35R60, 93E20.

The main motivation of this paper is to prove the existence of an optimal strategy which maximizes the expected profit of a firm in an infinite horizon problem with jumps.
More precisely, let a Brownian motion $(W_t)_{t\geq 0}$ and an independent Poisson measure $\mu(dt, de)$ defined on a probability space $(\Omega, \mathcal{A}, \mathbb{P})$ and let $\mathbb{F}$ be the right continuous complete filtration generated by the pair (W, μ). Assume that a firm decides at stopping times to change its technology to determine its maximum profit. Let $\{1, 2\}$ be the possible technologies set.
A right continuous left limited stochastic process X models the firm log value and a process $(\xi_t, t \geq 0)$ taking its values in $\{1, 2\}$ models the state of the chosen technology. The firm net profit is represented by a function f, the switching technology costs are represented by $c_{1,2}$ and $c_{2,1}$, $\beta > 0$ is a discount coefficient. Then, the problem is to find an increasing sequence of stopping times $\widehat{\alpha} := (\widehat{\tau}_n)_{n\geq -1}$, where $\widehat{\tau}_{-1} = 0$, optimal for the following impulse control problem

$$K(\widehat{\alpha}, i, x) := \operatorname*{ess\,sup}_{\alpha\in\mathcal{A}} \mathbb{E}_{i,x}\left[\int_0^{+\infty} e^{-\beta s} f(\xi_s, X_s)ds - \sum_{n\geq 0}\left\{e^{-\beta\tau_{2n}}c_{1,2} + e^{-\beta\tau_{2n+1}}c_{2,1}\right\}\right],$$

where $\mathcal{A}$ denotes the set of admissible strategies. The Snell envelope tools show that the problem reduces to the existence of a pair of right continuous left limited processes (Y^1, Y^2). This idea originates from Hamadène and Jeanblanc [19]. Their results are extended to infinite horizon case and mixed processes (namely jump-diffusion with a Brownian motion and a Poisson measure). In [19] the authors considered a power station which has two modes: operating and closed. This is an impulse control problem with switching technology without jump of the state variable. They solved the starting and stopping problem when the dynamics of the system are the ones of general adapted stochastic processes.

The existence of (Y^1, Y^2) is established via the notion of doubly reflected backward stochastic differential equation. In this context, another interest of our work is to extend to the infinite horizon case the results of doubly reflected backward stochastic differential equations with jumps. Specifically, a solution for the doubly reflected backward stochastic differential equation associated to a stochastic coefficient g, a null terminal value and a lower (resp. an upper) barrier $(L_t)_{t\geq 0}$ (resp. $(U_t)_{t\geq 0}$) is a quintuplet of $\mathbb{F}$-progressively measurable processes $(Y_t, Z_t, V_t, K_t^+, K_t^-)_{t\geq 0}$ which satisfies

[1] abidiheni@gmail.com, University of Tunis El Manar, Faculty of Sciences of Tunis, Tunisia.
[2] rabamami@iau.edu.sa, Department of Basic Sciences, Deanship of Preparatory Year and Supporting Studies, Imam Abdulrahman Bin Faisal University, P.O. Box 1982, Dammam 34212, Saudi Arabia.
[3] monique.pontier@math.univ-toulouse.fr, institut de mathématiques de Toulouse, France.

$$
(1) \qquad \begin{cases} Y_t = \int_t^{+\infty} e^{-\beta s} g(s)ds + \int_t^{+\infty} dK_s^+ - \int_t^{+\infty} dK_s^- - \int_t^{+\infty} Z_s dW_s - \int_t^{\infty}\int_E V_s(e)\tilde{\mu}(ds,de), \\ L_t \le Y_t \le U_t, \ \forall t \ge 0 \\ \int_0^t (Y_s - L_s)dK_s^+ = \int_0^t (Y_s - U_s)dK_s^- = 0, \quad \mathbb{P}\text{-a.s.} \end{cases}
$$

where $\tilde{\mu}$ is the compensated measure of μ.

Another specificity of this paper is to promote a constructive method of the solution of a BSDEs with two barriers. Specifically, we do not assume the so called Mokobodski's hypothesis. Indeed this one is not so easy to check (see e.g. [21] in finite horizon and continuous case). Our assumptions are more natural and easy to check on the barriers in practical cases.

The notion of backward stochastic differential equation (BSDE) was studied by Pardoux and Peng [27] (meaning in such a case $L = -\infty$, $U = +\infty$ and $K^{\pm} = 0$). To our knowledge, they were the first to prove the existence and uniqueness of adapted solutions, under suitable square-integrability and Lipschitz-type condition assumptions on the coefficients and on the terminal condition. Several authors have been attracted by this area that they applied in many fields such as Finance [6, 12, 13, 19], stochastic games and optimal control [16, 17, 18, 20], and partial differential equations [28].

The existence and the uniqueness of BSDE solutions with two reflecting barriers and without jumps have been first studied by Cvitanic and Karatzas [6] (generalization of El Karoui et al. [12]) applied in Finance area by El Karoui et al. [13]. There is a lot of contributions on this subject since then, consisting essentially in weakening the assumptions, adding jumps, and considering an infinite horizon.

The extension to the case of BSDEs with one reflecting barrier and jumps has been studied by Hamadène and Ouknine [17] considering a finite horizon $T = 1$. The authors show the existence and uniqueness of the solution using the penalization scheme and the Snell envelope tools. They stress the connection between such reflected BSDEs and integro-differential mixed stochastic optimal control. The authors' assumptions are: the terminal value is a square integrable random variable, the drift coefficient function $g(t, \omega, y, z, v)$ is uniformly Lipschitz with respect to (y, z, v) and the obstacle $(S_t)_{t\le 1}$ is a right continuous left limited process whose jumps are totally inaccessible. Hamadène and Ouknine [22] deal with reflected BSDEs in finite horizon, the barrier being right continuous left limited and progressively measurable. Hamadène and Hassani [18] proved existence and uniqueness results of local and global solutions for doubly reflected BSDEs driven by a Brownian motion and an independent Poisson measure in finite horizon. The authors applied these results to solve the related zero-sum Dynkin game.

Here the model is inspired from the papers [12, 17, 18, 20, 22]. But their results do not apply directly to the situation which here requires an infinite horizon. Moreover we connect the reflected BSDE with the impulse control problem. All these papers provide a solution to the reflected BSDE problem which are here extended to the case of infinite horizon by adding a discount coefficient and imposing admissibility conditions of strategies. In this paper, the drift function is assumed to be Lipschitz and non increasing in y. It is proved that the reflected BSDE solutions are limit of Cauchy sequences in appropriate complete metric spaces. Another interesting area is the one of oblique reflections, meaning a multimodal switching problem, see for instance [4, 10, 23]. El Asri [10] considers the same problem proposed by Hamadène and Jeanblanc [19] and extends it to the infinite horizon case without jump of the state variable, namely a power station which produces electricity and has several modes of production (the lower, the middle and the intensive modes). Naturally, the switching from one mode to another induces costs. The optimal switching problem is solved by means of probabilistic tools such as the Snell envelop of processes and reflected backward stochastic differential equations. Moreover their proofs are based on the verification theorem and the system of variational inequalities that we do not use.

Our purpose is similar to the one in [1], but instead of using Snell envelope and fixed point theorem as they do, here the two barriers case is solved using comparison theorem in one barrier case and adding some assumptions on the drift coefficient g.

This paper is composed of six sections. Section 1 presents the impulse control problem and describes the corresponding model. Section 2 introduces a pair of right continuous left limited processes (Y^1, Y^2) that allows one to exhibit an optimal strategy. Section 3 extends the doubly reflected BSDEs tools in the infinite horizon setting with jumps: firstly the case of a single barrier with general Lipschitz drift is solved, then a comparison theorem is proved, finally the uniqueness and the existence of solution for the doubly reflected BSDE under suitable assumptions are proved in case of drift non depending on state (y, z, v). Section 4 proves the existence of the required pair (Y^1, Y^2), and provides an application of these doubly reflected BSDE to a switching problem. Finally, with some simulations, the results allow to define an optimal strategy in Section 5. An appendix is devoted to an extension of Gronwall's lemma and some technical results.

References

[1] **Akdim K., Ouknine Y.** *Infinite horizon reflected backward SDEs with jumps and right continuous left limited obstacle.* Stochastic Analysis and Appl., 24 , 1239-1261 (2006).

[2] **Amami R.** *Application of doubly reflected BSDEs to an impulse control problem.* Optimization: A Journal of Mathematical Programming and Operations Research, 62:11, 1525-1552 (2013).

[3] **Abidi H., Amami R. Pontier M.** *Infinite horizon impulse control problem with continuous costs, numerical solutions.* Stochastics, Taylor & Francis VOL. 89, NOS. 6-7, 1039-1060 (2017).

[4] **Chassagneux J.F., Elie R., Kharroubi I.** *Discrete time approximation of multidimensional BSDEs with oblique reflections,* Ann. Appl. Probab. 22, 971-1007 (2012).

[5] **Chesney M., Jeanblanc M., Yor M.** Mathematical Methods for Financial Markets, *Springer* (2009).

[6] **Cvitanic J., Karatzas I.** *Backward stochastic differential equations with reflection and Dynkin games.* The Annals of Probability, 24-4, 2024-2056 (1996).

[7] **Dellacherie C., Meyer P.A.** *Probabilités et Potentiel.* Chap. I-IV. Hermann, Paris (1980).

[8] **Dellacherie C., Meyer P.A.** *Probabilités et Potentiel.* Chap. V-VIII. Hermann, Paris (1982).

[9] **Dumitrescu R., Labart C.** *Numerical approximation of doubly reflected BSDEs with jumps and RCLL obstacles.* Journal of Math (2016).

[10] **El Asri B.** *Optimal multi-modes switching problem in infinite horizon.* Stochastics and Dynamics, 10-2, 231-261 (2010).

[11] **Essaky E. H.** *Reflected backward stochastic differential equation with jumps and right continuous left limited obstacle,* Bull. Sci. math. 132, 690-710 (2008).

[12] **El Karoui N., Pen S., Quenez M.C.** *Backward Stochastic Differential Equations in Finance.* Mathematical Finance, 7-1, 1-71 (1997).

[13] **N. El Karoui, C. Kapoudjian, E, Pardoux, S. Peng, M.C. Quenez (1997).** *Reflected solutions of backward SDE's and related obstacle problems for SDE's.* The Annals of Probability, 25, No. 2, 702-737.

[14] **N. El Karoui (1981).** *Les Aspects Probabilistes du Contrôle Stochastique.* Lecture Notes in Mathematics 876, Springer-Verlag, Berlin.

[15] **Hamadène S., Lepeltier J.P., MatoussiA.** *Double barrier reflected backward SDE's with continuous coefficient.* Pitman Research Notes in Math. Series, 364. 161-175 (1997).

[16] **Hamadène S., Lepeltier J.P., Wu Z.** *Infinite horizon reflected backward stochastic differential equations and applications in mixed control and game problems.* Probability and Mathematical Statistics, 19, No. 2, 211-234 (1999).

[17] **Hamadène S., Ouknine Y.** *Reflected backward stochastic differential equation with jumps and random obstacle*, Electronic Journal of Probability. 8 -2, 1-20 (2003).

[18] **Hamadène S., Hassani M.** *BSDEs with two reflecting barriers driven by a Brownian and a Poisson noise and related Dynkin game*, Electronic Journal of Probability, 11, Paper No. 5, 121-145 (2006).

[19] **Hamadène S., Jeanblanc M.** *On the starting and stopping problem: Application in reversible investments.* Mathematics of Operations Research, 32-1, 182-192 (2007).

[20] **Hamadène S., Wang H.** *BSDE's with two RCLL obstacles driven by Brownian motion and Poisson measure and a related mixed zero-sum game.* Stochastic Processes and their Applications, 119, 2881-2912 (2009).

[21] **Hamadène S., Hassani M., Ouknine Y.** *Backward SDEs with two rcll reflecting barriers without Mokobodski's hypothesis,* Bull. Sci. Math., 134, 874-899 (2010).

[22] **Hamadène S., Ouknine Y.** *Reflected backward SDEs with general jumps.* Teor. Veroyatnost. i Primenen., 60-22, 357-376 (2015).

[23] **Hu Y. and Tang S.** *Multi-dimensional bsde with oblique reflection and optimal switching,* Probability Theory and Related Fields, 147, 89-121 (2010).

[24] **Huynh H.T., Lai V.S., Soumaré I.** *Stochastic Simulation and Applications in Finance with Matlab Programs.* John Wiley & Sons Ltd, England (2008).

[25] **Jacod J., Shiryaev A.N.** *Limit Theorems for Stochastic Processes,* Second Edition, Springer-Verlag (2003).

[26] **Longstaff F., Schwartz E.** *Valuing American options by simulation: A simple least-squares,* Review of Financial Studies, 1-14, 113-147 (2001).

[27] **Pardoux E., Peng S.G.** *Adapted solution of a backward stochastic differential equation,* System and Control Letters 14, 55-61(1990).

[28] **Pardoux E., Peng S.G.** *Backward stochastic differential equations and quasilinear parabolic partial differential equations.* Stochastic Differential Equations and their Applications (B. Rozovskii and R. Sowers, eds.), Lect. Not. Cont. Inf. Sci., 176, Springer, 200-217 (1992).