AJAE appendix to
Trade Policy Coordination and Food Price Volatility*

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November 20, 2015

Computational details

This section gathers all the equations that define the cooperative policies and describes how the model is solved numerically. Since all the variables on the interior Nash equilibrium are characterized analytically by equations (1), (6)–(7), and (12)–(14), this appendix focuses on the dynamic game. The problem is implemented numerically in GAMS version 24.5.4 and solved on a PC running Windows 7 (64 bit) using the mixed complementarity solver `PATH` (Dirkse and Ferris, 1995), with a precision of 10^{-7}.[1] Since the model has no intrinsic dynamics, there is no need to consider several periods. However, the model has to be solved over various supply shocks to allow for the calculation of expectations. The shocks on which the model is solved are chosen through 55-node Gaussian quadratures,[2] which define sets of pairs $\{(\varepsilon_i, \varepsilon_i^*), w_i\}$ in which $(\varepsilon_i, \varepsilon_i^*)$ represents a possible realization of shocks and w_i the associated probability. In the following, except for the time index which is substituted by i or j, the mathematical notations mostly follow the article. i and j index possible shocks realizations. The superscript D is used to designate situations of deviation from the cooperative policies.

The expected welfare under Nash ($\mathrm{E}W_N$ and $\mathrm{E}W_N^*$) is calculated by replacing the expectations operators by sums using the Gaussian quadrature and using the analytical expressions of welfare, (6) and (7), and Nash trade policies, (12) and (13). Other variables solve the following set of complementarity equations, in which for compactness some functions are introduced and defined later:[3]

$$\text{(S1)} \qquad P_i^f : P_i^f = \frac{a}{b} - \frac{\varepsilon_i + \varepsilon_i^*}{2b},$$

$$\text{(S2)} \qquad V_i^f : V_i^f = \frac{\varepsilon_i - \varepsilon_i^*}{2},$$

*The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE).

[1] Programs are available as online appendix.

[2] Gaussian quadrature are generated using the functions available in the MATLAB toolbox `CompEcon` (Miranda and Fackler, 2002).

[3] Complementarity conditions in what follows are written using the "perp" notation (⊥). This means that both inequalities must hold, and at least one must hold with equality.

(S3) $\quad P_i^w : P_i^w = P_i^f - \dfrac{\tau_i + \tau_i^*}{2},$

(S4) $\quad P_i : P_i = P_i^w + \tau_i,$

(S5) $\quad P_i^* : P_i^* = P_i^w + \tau_i^*,$

(S6) $\quad W_i : W_i = \displaystyle\int_{P_i}^{a/b} D(p)\,\mathrm{d}p + P_i\varepsilon_i - \tau_i\left[\varepsilon_i - D(P_i)\right] - K\frac{\left(P_i - \bar{P}\right)^2}{2},$

(S7) $\quad W_i^* : W_i^* = \displaystyle\int_{P_i^*}^{a/b} D^*(p)\,\mathrm{d}p + P_i^*\varepsilon_i^* - \tau_i^*\left[\varepsilon_i^* - D^*(P_i^*)\right] - K\frac{\left(P_i^* - \bar{P}\right)^2}{2},$

(S8) $\quad \tau_i : \tau_i \leq 0 \perp (1+\mu_i)\dfrac{\partial W(s_i,\tau_i,\tau_i^*)}{\partial \tau_i} + (1+\mu_i^*)\dfrac{\partial W^*(s_i,\tau_i,\tau_i^*)}{\partial \tau_i} \geq \mu_i^*\dfrac{\partial W_D^*(s_i,\tau_i)}{\partial \tau_i},$

(S9) $\quad \tau_i^* : \tau_i^* \geq 0 \perp (1+\mu_i^*)\dfrac{\partial W^*(s_i,\tau_i,\tau_i^*)}{\partial \tau_i^*} + (1+\mu_i)\dfrac{\partial W(s_i,\tau_i,\tau_i^*)}{\partial \tau_i^*} \leq \mu_i\dfrac{\partial W_D(s_i,\tau_i^*)}{\partial \tau_i^*},$

(S10) $\quad \mu_i : \mu_i \geq 0 \perp W_i + \dfrac{\beta}{1-\beta}\displaystyle\sum_j w_j W_j \geq W_i^D + \frac{\beta}{1-\beta}\mathrm{E}W_N,$

(S11) $\quad \mu_i^* : \mu_i^* \geq 0 \perp W_i^* + \dfrac{\beta}{1-\beta}\displaystyle\sum_j w_j W_j^* \geq W_i^{*D} + \frac{\beta}{1-\beta}\mathrm{E}W_N^*,$

(S12) $\quad W_i^D : W_i^D = \displaystyle\int_{P_i^D}^{a/b} D(p)\,\mathrm{d}p + P_i^D\varepsilon_i - \tau_i^D\left[\varepsilon_i - D\left(P_i^D\right)\right] - K\frac{\left(P_i^D - \bar{P}\right)^2}{2},$

(S13) $\quad W_i^{*D} : W_i^{*D} = \displaystyle\int_{P_i^{*D}}^{a/b} D^*(p)\,\mathrm{d}p + P_i^{*D}\varepsilon_i^* - \tau_i^{*D}\left[\varepsilon_i^* - D^*\left(P_i^{*D}\right)\right] - K\frac{\left(P_i^{*D} - \bar{P}\right)^2}{2},$

(S14) $\quad \tau_i^D : \tau_i^D \leq 0 \perp \tau_i^D \leq 2\dfrac{K\left(\bar{P} - P_i^f\right) - V_i^f}{K+3b} + \dfrac{K+b}{K+3b}\tau_i^*,$

(S15) $\quad \tau_i^{*D} : \tau_i^{*D} \geq 0 \perp \tau_i^{*D} \geq 2\dfrac{K\left(\bar{P} - P_i^f\right) + V_i^f}{K+3b} + \dfrac{K+b}{K+3b}\tau_i,$

(S16) $\quad P_i^D : P_i^D = P_i^{wD} + \tau_i^D,$

(S17) $\quad P_i^{*D} : P_i^{*D} = P_i^{w*D} + \tau_i^{*D},$

(S18) $\quad P_i^{wD} : P_i^{wD} = P_i^f - \dfrac{\tau_i^D + \tau_i^*}{2},$

(S19) $\quad P_i^{w*D} : P_i^{w*D} = P_i^f - \dfrac{\tau_i + \tau_i^{*D}}{2}.$

From equations (6) and (7), we have

(S20) $\quad \dfrac{\partial W(s_i,\tau_i,\tau_i^*)}{\partial \tau_i} = \dfrac{-K\left(P_i - \bar{P}\right) - b\left(P_i - P_i^w\right) - \varepsilon_i + D(P_i)}{2},$

(S21) $\quad \dfrac{\partial W^*(s_i,\tau_i,\tau_i^*)}{\partial \tau_i} = \dfrac{K\left(P_i^* - \bar{P}\right) + b\left(P_i^* - P_i^w\right) - \varepsilon_i^* + D^*(P_i^*)}{2},$

(S22) $\quad \dfrac{\partial W(s_i,\tau_i,\tau_i^*)}{\partial \tau_i^*} = \dfrac{K\left(P_i - \bar{P}\right) + b\left(P_i - P_i^w\right) - \varepsilon_i + D(P_i)}{2},$

$$\text{(S23)}\quad \frac{\partial W^*(s_i,\tau_i,\tau_i^*)}{\partial \tau_i^*} = \frac{-K(P_i^* - \bar{P}) - b(P_i^* - P_i^w) - \varepsilon_i^* + D^*(P_i^*)}{2}.$$

Using the envelope theorem

$$\text{(S24)}\quad \frac{\partial W_D^*(s_i,\tau_i)}{\partial \tau_i} = \frac{\partial W^*(s_i,\tau_i,\tau_R^*(s_i,\tau_i))}{\partial \tau_i},$$

$$\text{(S25)}\quad = \frac{K\left(P_i^{*D} - \bar{P}\right) + b\left(P_i^{*D} - P_i^{w*D}\right) - \varepsilon_i^* + D^*\left(P_i^{*D}\right)}{2},$$

and similarly

$$\text{(S26)}\quad \frac{\partial W_D(s_i,\tau_i^*)}{\partial \tau_i^*} = \frac{K\left(P_i^{D} - \bar{P}\right) + b\left(P_i^{D} - P_i^{wD}\right) - \varepsilon_i + D\left(P_i^{D}\right)}{2}.$$

The expectations of welfare under cooperation should be consistent with the cooperative trade policies actually chosen. This is ensured numerically by equations (S10) and (S11), where the expressions $\sum_j w_j W_j$ and $\sum_j w_j W_j^*$ represent the welfare expectations discretized by the Gaussian quadrature. So in the solution process the expectations change endogenously with the cooperative trade policies.

Sensitivity analysis

The parameter characterizing the preference for price stability, K, has important implications for the quantitative analysis. It governs the importance of the smoothing motivation for trade policies and so how much the policies react to movements in world price. In the article, it is calibrated at $K = 0.3$ by assuming a 15% budget share, a relative risk aversion equal to 2, and a null income elasticity and applying a formula from Turnovsky, Shalit, and Schmitz (1980). Here we analyze the sensitivity of the results to two different values of K: 0.15 and 0.6, i.e., halving and doubling the risk aversion.

Figures S1 and S2 correspond to figure 2 and present the trade policies under a symmetric price distribution for the two different values of K. Figures S3 and S4 correspond to figure 3 and present the trade policies under a positively skewed price distribution.

According to equations (12) and (13), decreasing K has two effects on Nash trade policies: it decreases the slope of trade policies with respect to the world price, and increases the slope of trade policies with respect to trade volume (the intercept at $\bar{P}$ in the figures). So it makes the trade policies flatter. If there is no concern over price volatility, at the extreme of $K = 0$, policies would be constant and equal in Nash in absolute value to $V^f/(2b) = 0.5$. Because trade policies are flatter with a lower K, the region where the trade policies of the exporting and importing countries overlap is larger. In figure S3 for example, the exporting country applies export taxes in almost all of the price distribution. Given that the cooperative policies qualitatively follow the behavior of Nash policies, the effects on them of K are the same.

In contrast, for a high K as in figures S2 and S4, the slope of trade policies is higher; the smoothing motivation becomes dominant, and the region where both countries apply trade policies is very limited. This region is symmetric about $\bar{P}$. When K increases, this region decreases and at the limit would collapse to just one point: $\bar{P}$. Since it increases the welfare cost of deviating from the target price, increasing K has important effects on the reaction of the exporting country policy to high prices when prices have an asymmetric distribution.

References

Dirkse, S.P., and M.C. Ferris. 1995. The PATH Solver: A Non-monotone Stabilization Scheme for Mixed Complementarity Problems. *Optimization Methods and Software* 5:123–156.

Miranda, M.J., and P.L. Fackler. 2002. *Applied Computational Economics and Finance*. Cambridge, US: MIT Press.

Turnovsky, S.J., H. Shalit, and A. Schmitz. 1980. Consumer's Surplus, Price Instability, and Consumer Welfare. *Econometrica* 48(1):135–152.

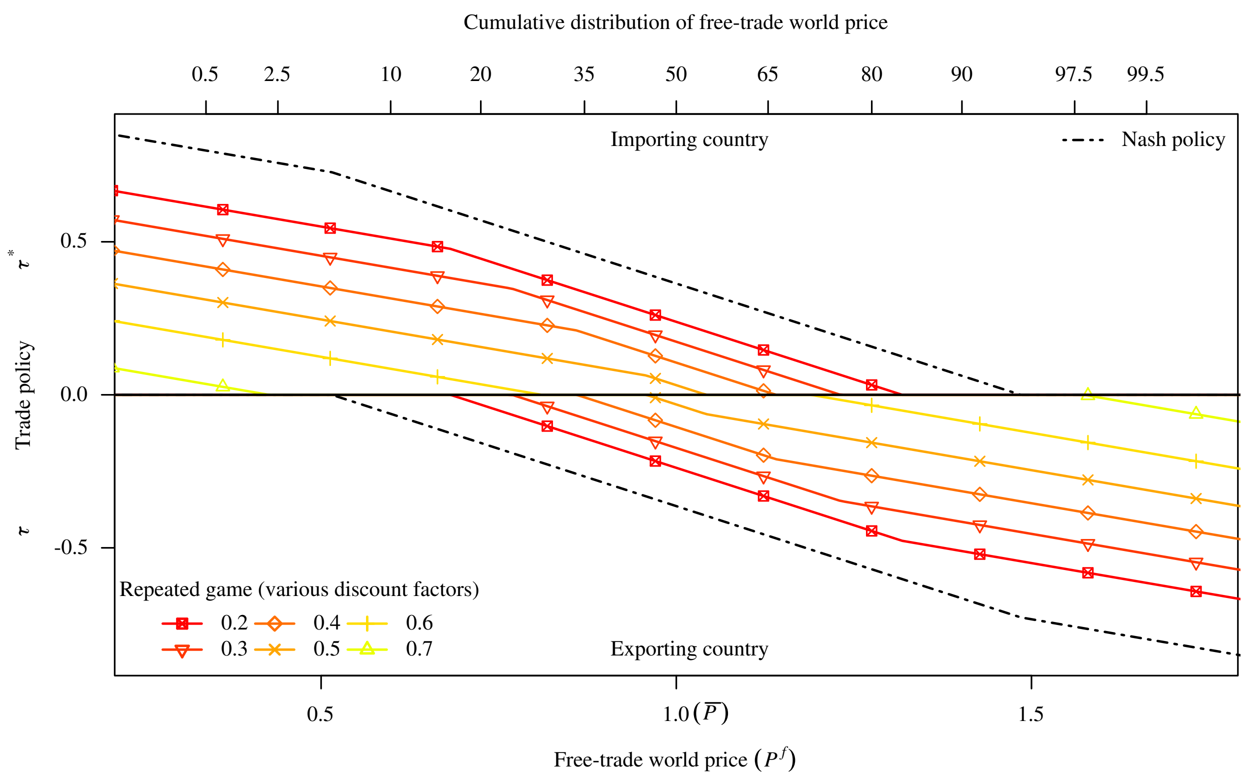

Figure S1. Cooperative and non-cooperative trade policies under a symmetric price distribution if $K = 0.15$

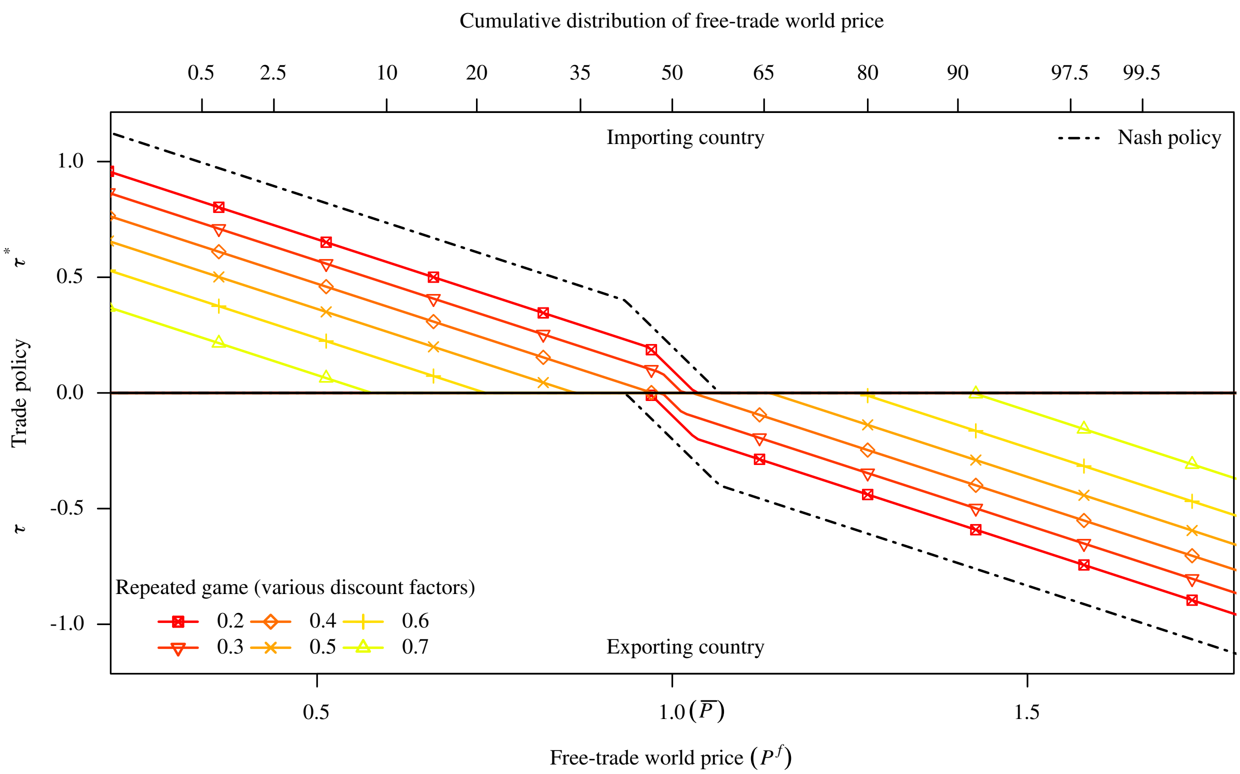

Figure S2. Cooperative and non-cooperative trade policies under a symmetric price distribution if $K = 0.6$

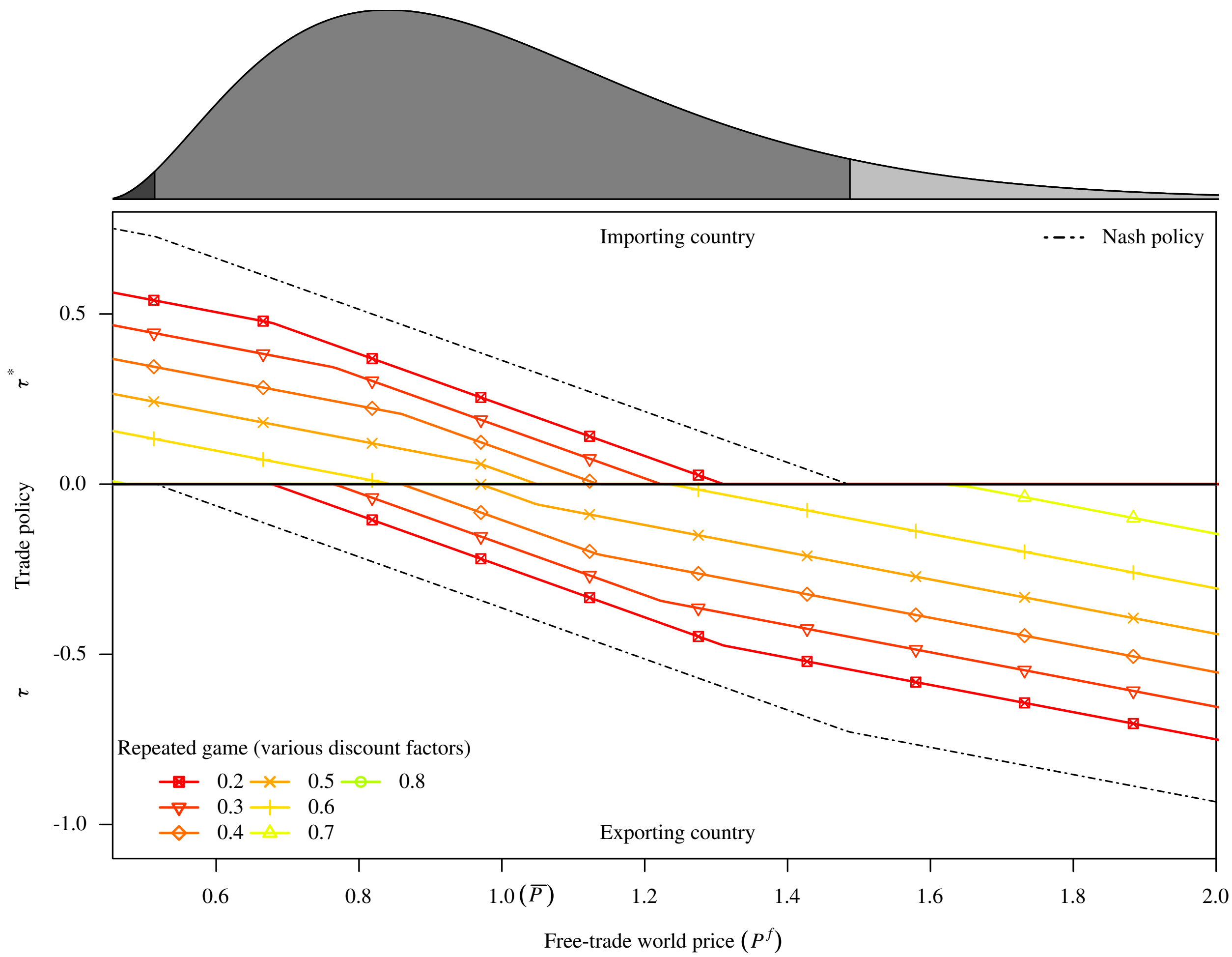

Figure S3. Trade policies under a positively skewed price distribution if $K = 0.15$
Note: Density of free-trade world price above the plot, with a distinction between the regions where, in the Nash, only the importing country intervenes, where countries both intervene, and where only the exporting country intervenes.

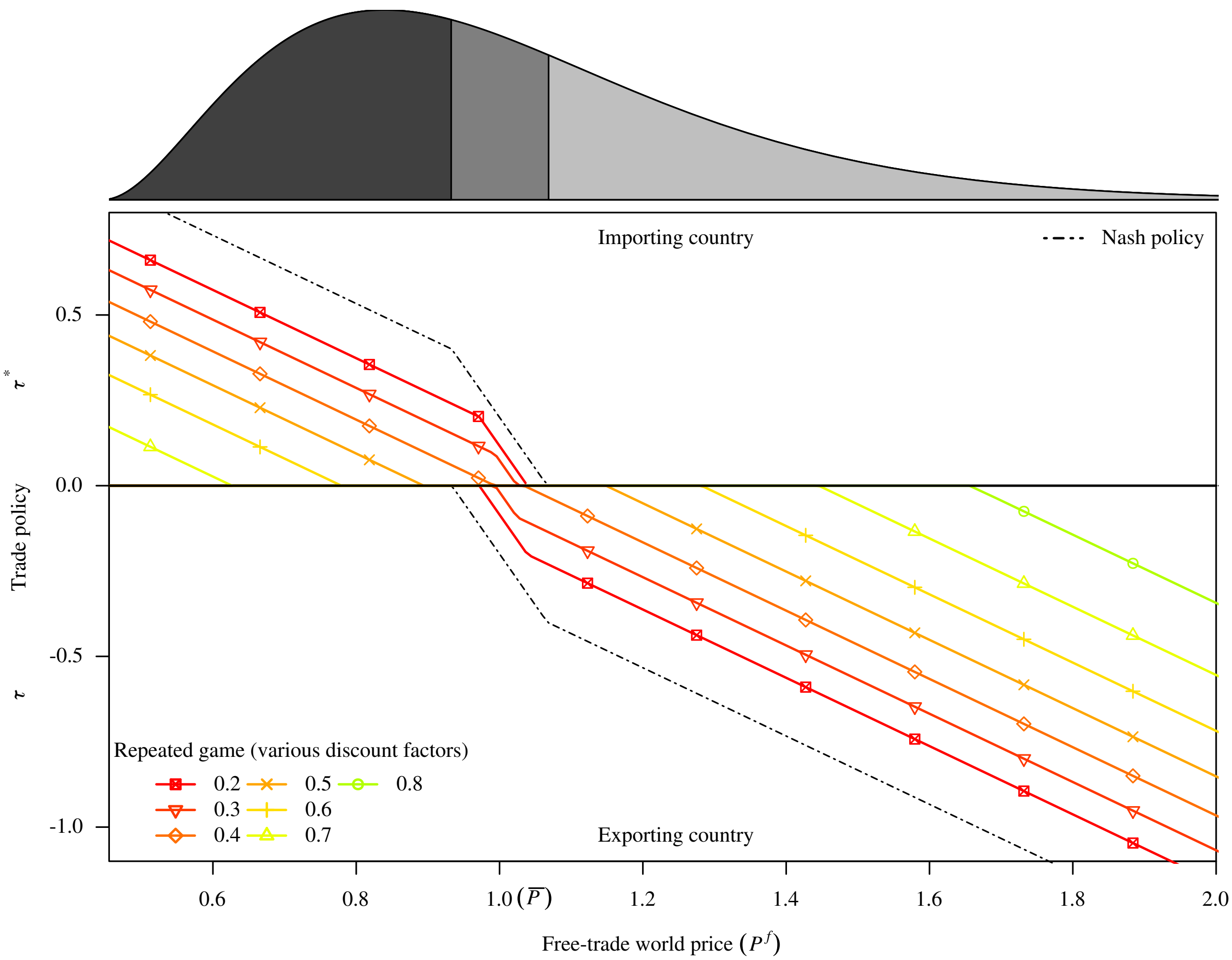

Figure S4. Trade policies under a positively skewed price distribution if $K = 0.6$

Note: Density of free-trade world price above the plot, with a distinction between the regions where, in the Nash, only the importing country intervenes, where countries both intervene, and where only the exporting country intervenes.