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To cite this version:
Mathieu Fontaine, Charles Vanwynsberghe, Antoine Liutkus, Roland Badeau. Scalable Source Localization with Multichannel Alpha-Stable Distributions. 25th European Signal Processing Conference (EUSIPCO), Aug 2017, Kos, Greece. pp.11-15. hal-01531252

HAL Id: hal-01531252
https://hal.archives-ouvertes.fr/hal-01531252
Submitted on 14 Jun 2017

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SCALABLE SOURCE LOCALIZATION WITH MULTICHANNEL $\alpha$-STABLE DISTRIBUTIONS

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ABSTRACT

In this paper, we focus on the problem of sound source localization and we propose a technique that exploits the known and arbitrary geometry of the microphone array. While most probabilistic techniques presented in the past rely on Gaussian models, we go further in this direction and detail a method for source localization that is based on the recently proposed $\alpha$-stable harmonizable processes. They include Cauchy and Gaussian as special cases and their remarkable feature is to allow a simple modeling of impulsive and real world sounds with few parameters. The approach we present builds on the classical converitve mixing model and has the particularities of requiring going through the data only once, to also work in the underdetermined case of more sources than microphones and to allow massively parallelizable implementations operating in the time-frequency domain. We show that the method yields interesting performance for acoustic imaging in realistic simulations.

Index Terms—source localization, acoustic modeling, $\alpha$-stable random variables, spectral measure, sketching

I. INTRODUCTION

Over the past few years, many studies have been conducted on the localization of audio objects, called sources, in an acoustic environment. The purpose of this trend of research may for instance be to recover speech from a noisy [1], [2], [3], [4] or reverberant [5], [2] environment, while knowing the geometry of the microphone array. In the scientific community, the incoming direction of propagating waves originating from a source is called a direction of arrival (DoA). Its knowledge is useful in a wide range of applications: sonar [6], [7], wireless communications [8], or medical imaging [9], [10] to cite a few. For the estimation of the DoAs, an efficient approach is to consider several candidate directions and the corresponding wave propagation as given by the physical model. In practice, this approach makes it possible to build the propagation operator from all candidate DoAs through the so-called mixing matrix, encapsulating the physical model. Several methods were proposed for estimating the DoAs. Among those, we may mention the celebrated MUSIC algorithm [11], which assumes that the number of sources is equal to the rank of the signal subspace and uses its orthogonal subspace called noise subspace for localization. The ESPRIT algorithm [12] is similar to MUSIC but exploits further rotational invariance properties for the signal subspace. Both algorithms achieve good localization, even though ESPRIT proved to be more robust against noise than MUSIC [13]. Another path of research in the case of audio localization is based on the Local Gaussian Model introduced in [14]. In this model, the dependences between the observations at different microphones are not modeled deterministically using classical features such as inter-channel time differences, but rather directly as frequency-dependent correlations embedded in the so-called spatial covariance matrices. In this context, the spatial covariance matrices corresponding to each DoA may be learned beforehand and used afterwards for localization as in [15].

The aforementioned methods for localization all exploit second-order statistics for the sources or observation signals, which are commonly modeled as wide-sense stationary (WSS) processes. From this perspective, it is common to pick a Gaussian process model for the signals [16], to be used along with the assumption of independent sources. Model parameters are then usually estimated by means of an expectation-maximization (EM) algorithm.

However, several studies pinned down the empirical fact that the marginal distributions of the signals of interest work poorly in accordance to the Gaussian model. In other terms, the histogram of the signals of interest is rarely correctly modeled by a Gaussian or a mixture of just a few Gaussian distributions. A classical option to address this problem is to adopt an heteroscedastic Gaussian model, meaning that we adopt varying mean and covariance parameters. In audio, this is notably the case of the Local Gaussian Model (LGM [14]), which operates on the Short Term Fourier Transform (STFT) and models all its entries as independent and centered Gaussian with different variances. As shown, e.g. in [17], this is equivalent to assuming the processes to be locally WSS. This approach yields good performance at the cost of requiring much more parameters to estimate than a simpler marginal model. That said, even when sticking to this local stationarity idea, the Gaussian model was recently outperformed in the context of single channel enhancement by its heavy-tailed $\alpha$-stable generalization [18], coined in [19] as the $\alpha$-harmonic model. This framework encompasses the Cauchy ($\alpha = 1$) and Gaussian ($\alpha = 2$) special cases and leads to better filtering performance for the same computational cost.

Picking an $\alpha$-stable model for modeling time series has the advantage of better accounting for the impulsivity and noisiness of real-world signals, and the particular $\alpha$-harmonicizeable case further permits convenient Time-Frequency (TF) processing. Still, the question of how to handle multichannel signals in this context remains unclear. Indeed, $\alpha$-stable distributions have the noticeable disadvantage of not having finite second-order moments, prohibiting algorithms such as MUSIC or ESPRIT. Some research around this powerful framework has focused on beamforming under $\alpha$-stable noise [20], but this does not translate to the case of $\alpha$-stable target sources. More recent studies such as [21] considered Independent Component Analysis (ICA) for $\alpha$-stable sources and successfully applied the method for the processing of heavy-tailed biomedical data in [22]. This is relevant in our setting because ICA and localization are related problems [23], [24] in the sense that estimating a mixing matrix is related to estimating DoAs. However, the approach in [21] suffers from several intrinsic limitations. Firstly, it considers sources that are independent and identically
distributed (i.i.d.) in the time domain, which is a poor assumption in audio. Secondly, it relies on instantaneous mixing scenarios, for which the mixing matrix boils down to just scalar gains applied in audio. Secondly, it relies on instantaneous mixing scenarios, for distributed (i.i.d.) in the time domain, which is a poor assumption also called mixing filters [27]:

Theory leads us to take the mixtures as a linear combination of the complex entries. Hence, the study becomes to estimate the magnitude of all signals received by the microphone array along with physical assumptions about sound propagation to allow for exact inference even in massively multiplexed scenarios. A very interestingly, the resulting algorithm of the source signals are here taken as independent of both time and frequency. The scale parameters for any source signals are taken as isotropic. Let $s_\alpha$ be the characteristic function defined on $\mathbb{C}$ as [18, p58]:

$$\forall \theta \in \mathbb{C}, \varphi_\alpha (\theta) \triangleq \mathbb{E}\left[ \exp \left( i \Re (\theta^* v) \right) \right] = \exp \left( - |\theta|^\alpha \right),$$

where $\triangleq$ denotes a definition, $|$ denotes the modulus, $i$ denotes complex conjugation, $\Re(z)$ is the real part of $z \in \mathbb{C}$ and $\gamma \geq 0$ is a scale parameter that basically characterizes the amplitude of $v \sim S_\alpha S_c(Y)$. This is a useful distribution for modeling impulsive data in signal processing [29], [30], [31]; the closer the characteristic exponent $\alpha$ is to 0, the heavier the tail is.

In this paper, we assume that each source $s_l$ is an isotropic process [19]. This boils down to taking the random variables $s_l(f, t)$ for all $l, f, t$ as mutually independent, and distributed according to:

$$s_l(f, t) \sim S_\alpha S_c(Y_l).$$

The scale parameters $Y_l$ can be viewed as the amplitudes of the latent sources and are equal to 0 when there is no source in the direction considered. As can be seen, the scale parameters $Y_l$ of the source signals are here taken as independent of both time and frequency. This strongly contrasts with the LGM model [14] for which they are time and frequency dependent. The fact that is picking a heavy-tail distribution such as $S_\alpha S_c$ precisely allows for such a simple marginal model. Let $\mathbf{Y} \triangleq [Y_1, \ldots, Y_L]^T$ be the main $L \times 1$ quantity of interest to estimate, coined in [32] as the discrete spatial measure (DSM).

II-B. Isotropic $\alpha$-stable distributions and source models

Let $v$ be a complex random variable and $\varphi_\alpha$ its characteristic function (cf.f.). We say that $v$ follows a symmetric isotropic $\alpha$-stable distribution with $\alpha \in [0, 2]$, denoted $S_\alpha S_c$, if and only if $\varphi_\alpha$ has the following form:

$$\forall \theta \in \mathbb{C}, \varphi_\alpha (\theta) \triangleq \mathbb{E}\left[ \exp \left( i \Re (\theta^* v) \right) \right] = \exp \left( - |\theta|^\alpha \right),$$

where $\triangleq$ denotes a definition, $|$ denotes the modulus, $i$ denotes complex conjugation, $\Re(z)$ is the real part of $z \in \mathbb{C}$ and $\gamma \geq 0$ is a scale parameter that basically characterizes the amplitude of $v \sim S_\alpha S_c(Y)$. This is a useful distribution for modeling impulsive data in signal processing [29], [30], [31]; the closer the characteristic exponent $\alpha$ is to 0, the heavier the tail is.

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II-C. Spatial measure for multichannel $\alpha$-stable convolutive mixtures

According to (1) and (4), all vectors $x(f, 1), \ldots, x(f, T)$ share the same distribution for a given frequency $f$. Moreover, it can be shown that they are symmetric $\alpha$-stable random vectors but they are not isotropic. Let $\varphi_\alpha$ be the characteristic function of the distribution of $x(f, t)$ for any $t$:

$$\forall \theta \in \mathbb{C}, \varphi_\alpha (\theta) = \mathbb{E}\left[ \exp \left( - \int_{a \in \mathcal{R}_C} |\theta, a|^\alpha \, d\Gamma_f (a) \right) \right].$$

where $\{., .\}$ is the inner product on $\mathbb{C}^K$. It can be uniquely expressed for $\alpha \in [0, 2]$ as [18, p58]:

$$\forall \theta \in \mathbb{C}, \varphi_\alpha (\theta) = \exp \left( - \int_{a \in \mathcal{R}_C} |\theta, a|^\alpha \, d\Gamma_f (a) \right).$$

where $\mathcal{R}_C$ is the complex $K$-dimensional sphere $\mathcal{R}_C^K \triangleq \left\{ (z_1, \ldots, z_K) \in \mathbb{C}^K; \sum_{k=1}^K |z_k|^2 = 1 \right\}$, and $\Gamma_f$ is called the spectral measure, and is symmetric in the sense that for any continuous function $\varphi$ defined on $\mathcal{R}_C^K$ and for any $z \in \mathcal{R}_C^K$, we have $\int_{a \in \mathcal{R}_C} \varphi(a) d\Gamma_f (z) = \int_{a \in \mathcal{R}_C} \varphi(a) d\Gamma_f (a)$. We now introduce the quantity:

$$I_f (\theta) \triangleq - \ln (\varphi_\alpha (\theta)),$$

called the Levy exponent [33] of the distribution of $x(f, t)$. Combining (6) and (7), we get:

$$I_f (\theta) = \int_{a \in \mathcal{R}_C} |\theta, a|^\alpha \, d\Gamma_f (a).$$

In this paper, we will exploit the Levy exponent model estimation. To do this, it is desirable for computational reasons to approximate the integral in (8) as a finite sum. In practice, the sampling of a high dimensional sphere such as $\mathcal{R}_C^K$ would raise the tricky issue of the curse of dimensionality. This is basically what
is done in [21] in the real case. To avoid it, the particular approach we propose in this paper is to exploit the additional information provided by the physical model (2).

Proposition 1. In our case (1), $d\Gamma_f(a)$ is a sum of point masses:

$$d\Gamma_f = \sum_{l=1}^{L} T_l \| A_l(f) \|^2 \delta_{a_l(f)},$$  

(9)

where $\| \cdot \|$ stands for the Hermitian norm and $\delta_{a_l(f)}$ denotes the Dirac measure centered at $a_l(f) \triangleq A_l(f) / \| A_l(f) \|$ and associated to the equivalence relation $a \sim b \iff \exists z \in \mathcal{C}_l, \ a = zb$.

Proof: In short, we generalize [18, p70] to the complex case by substituting (1), (3) and (4) into (5) and by considering the independence of the sources, which yields:

$$\forall \theta \in \mathbb{C}^K, I_f(\theta) = -\ln \left( \prod_{l=1}^{L} \phi_{a_l(f)}(\langle A_l(f), \theta \rangle) \right)$$

$$= \sum_{l=1}^{L} T_l |\langle \theta, A_l(f) \rangle|^2.$$  

(10)

Since the same expression of $\phi_f(\theta)$ can be obtained by substituting (9) into (6), and since the spectral measure $\Gamma_f$ introduced in (6) is unique, (10) proves (9).

In this study, for each frequency $f$, we estimate $\Upsilon$ by exploiting relation (10) for $L$ values $\theta_1(f), \ldots, \theta_L(f)$ of its operand. Assuming we have chosen those $\theta_l(f)$, let:

$$I_f \triangleq [I_f(\theta_1(f)), \ldots, I_f(\theta_L(f))]^T$$  

(11)

be the (nonnegative) $L \times 1$ vector gathering the Levy exponents of the mixture at frequency $f$ for these $\theta_l(f)$. Defining now the matrix $\Psi_f$ of size $L \times L$ by:

$$[\Psi_f]_{l,i} \triangleq |\langle \theta_l, A_i(f) \rangle|^2,$$  

(12)

equation (10) leads to $I_f = \Psi_f \Upsilon$. Since this equality holds for all $f$, we can concatenate all $I_f$ into the $FL \times 1$ vector $I$ and all $\Psi_f$ into the known $FL \times L$ matrix $\Psi$. We obtain:

$$I = \Psi \Upsilon,$$  

(13)

which is our starting point for estimating $\Upsilon$. In practice, we pick $\theta_l(f) = A_l(f)$, because this choice proved effective and leads to a symmetric matrix $\Psi_f$. Note however that other options may be considered.

III. PARAMETER ESTIMATION

We now present a way to estimate $\Upsilon$ using (13). This method is divided into two steps: firstly, we estimate $\hat{I}$ from the data and secondly, we estimate $\hat{\Upsilon}$ based on the knowledge of this estimate, the knowledge of $\Psi$ and the equality $I = \Psi \Upsilon$.

III-A. Estimation of $I$: empirical levy exponent

The classical empirical characteristic function (ECF) of the observations $x(f, 1), \ldots, x(f, T)$, denoted $\tilde{\phi}_f$, is defined as [34]:

$$\forall \theta \in \mathbb{C}^K, \tilde{\phi}_f(\theta) = \frac{1}{T} \sum_{t=1}^{T} \exp \left( i \Re \langle \theta, x(f, t) \rangle \right).$$  

(14)

It would be logical to apply (14) to estimate $I_f$ as $\hat{I}_f = -\ln (\tilde{\phi}_f(\theta))$. However, taking the logarithm of (14) may raise an issue because $\tilde{\phi}_f$ could be complex-valued numerically. Fortunately, in the symmetric $\alpha$-stable case, we can address this issue in the following way:

Proposition 2. In the SoS$_c$ case, we can define an unbiased estimator of the chf. as:

$$\forall \theta \in \mathbb{C}^K, \hat{\phi}_f(\theta) = \left| \frac{T}{T-1} \sum_{t=1}^{T} \exp \left( i \Re \langle \theta, x(f, t) \rangle \right) \right|^2.$$  

(15)

Proof: From the $\alpha$-stability assumption, we have that the two sets of observations: $\{x(f, t)\}^T_{t_1}$ and $\{x(f, t_1) - x(f, t_2)\}^T_{t_1, t_2}$ share the same probability distribution. Replacing the first one by the second one in (14), and factorizing the result, we finally get (15).

Since $\tilde{\phi}_f$ is nonnegative, we can define the empirical Levy exponent: $\hat{I}_f(\theta) = -\ln \hat{\phi}_f(\theta)$.

The strategy we adopt is simply to compute the empirical Levy exponent $\hat{I}_f(\theta)$ of the data for all $\theta_l(f) = A_l(f)$. As in section II-A, we put all those $\hat{I}_f$ estimates together and note them $\hat{I}$. Following (13), they obey:

$$\hat{I} \approx \Psi \Upsilon.$$  

(16)

It is fundamental to note here that the estimate $\hat{I}$ of the Levy exponent is obtained after only one pass over the data through a very simple procedure (15). Parameter estimation is then achieved from this compressed $L F \times 1$ vector $\hat{I}$ only, through $\hat{I} \approx \Psi \Upsilon$, and not using the much larger $F \times T \times K$ observation dataset $x$. We see here we have an example of the sketching framework recently discussed in [26], that naturally arises when considering $\alpha$-stable random variables. This fact means that the proposed estimation procedure is not only of linear complexity, it is massively parallelizable over both time and frequency.

III-B. Estimation of $\Upsilon$ with nonnegative optimization

Our objective is the estimation of the nonnegative quantity $\Upsilon$, so that (16) is verified. A natural idea for this purpose is to estimate it so as to minimize the discrepancies between the left and right-hand sides of (16), by picking:

$$\hat{\Upsilon} \leftarrow \arg\min_{\Upsilon \geq 0} \sum_{f,l} d_\beta \left( \hat{I} \mid \Psi \Upsilon \right),$$  

(17)

where $d_\beta$ is any data-fit cost function such as the $\beta$-divergence [35]:

$$d_\beta(x|y) = \frac{1}{\beta-1} (x^\beta + (\beta - 1)y^\beta - \beta xy^{\beta-1}) \forall x > 0, y > 0, \beta \in \mathbb{R} \setminus \{0, 1\}.$$  

To solve this optimization problem, we note that all quantities involved in (17) are nonnegative, so that it is natural to adopt a now-classical multiplicative update strategy as in nonnegative matrix factorization (NMF, [36]) and to iterate the following update:

$$\hat{\Upsilon} \leftarrow \hat{\Upsilon} \cdot \frac{G_+ \left( \hat{\Upsilon} \right)}{G_- \left( \hat{\Upsilon} \right)}.$$  

(18)

where $a \cdot b$ and $a \div b$ stand for element-wise multiplication and division of $a$ and $b$, respectively. $G_+ \left( \hat{\Upsilon} \right)$ and $G_- \left( \hat{\Upsilon} \right)$ are obtained by the following calculation:

$$\nabla_\Upsilon d_\beta \left( \hat{I} \mid \Psi \Upsilon \right) = \Psi \left( \left( \Psi \Upsilon \right)^{\beta-1} - \Psi \left( \left( \Psi \Upsilon \right)^{\beta-2} \cdot \hat{I} \right) \right) \cdot G_+ \left( \hat{\Upsilon} \right) \div G_- \left( \hat{\Upsilon} \right).$$  

(19)
The algorithm box below summarizes this approach.

**Algorithm 1** Estimation of the spatial measure $\hat{Y}$

1) **Input**
   - Number $L$ of possible positions.
   - Steering vectors $A_l(f)$ as in (2) or directly using RIR.
   - STFTs $x$ of the mixture, of size $F \times T \times K$.
   - Number of iterations.
   - Parameters $\alpha$ and $\beta$ divergence to be used.

2) **Sketching:**
   - for all $f$, compute $\hat{I}_f(A_l(f)) = -\ln \hat{\varphi}_f(A_l(f))$ using (15)

3) **Parameter estimation**
   - Compute $\Psi_f$ according to (12) with $\theta_l(f) = A_l(f)$.
   - Compute $Y$ by iterating over (18).

**IV. EVALUATION**

In this section, we investigate the performance of the proposed method with room impulse responses RIRs, simulating a room of dimensions $5 \times 4 \times 3$ m with 0.4 s of reverberation time [37]. The sources are taken as voice samples from the CMU dataset with sample rates of 16 kHz and distributed randomly on a $5 \times 4$ plane, 1.5 m above the floor. The candidate locations for analysis are positioned on a grid of 10 cm step size, corresponding to $L = 2091$, and 5 cm shifted with respect to the true sources’ locations to ensure realistic experiments. Then, the observations $x(f,t)$ are obtained by propagating the sources the RIR (2). The excerpts considered last 15 seconds ($T = 391$) and we use only the $F = 128$ frequency bands corresponding to the interval [1000, 3000] Hz.

We pick $\alpha = 1.2$ after [19] and we set the beta divergence parameter to $\beta = 0$ (Itakura-Saito divergence). We run 70 iterations of the NMF algorithm in step 3) of Algorithm 1.

The baseline method we use here for comparison is the steered response power (SRP, [38]). It consists in backpropagating the observations towards the sources locations using the steering vectors $A_l(f)$. Basically, it computes simple inner products between $x(f,t)$ and $A_l(f)$.

We define the SRP measure $\hat{B} = [\hat{B}_1, \ldots, \hat{B}_L]^T$ as:

$$\forall l, \hat{B}_l = \frac{1}{FT} \sum_{f,t} |A_l^*(f)x(f,t)|. \tag{20}$$

To compare the performance of both approaches, we first define the ground truth as the support of the true DSM $Y$, that is: 1 in the active directions and 0 otherwise, followed by a smoothing using a Gaussian kernel with a 10cm length-scale. We compute a total number of 200 simulations, i.e. 20 independent trials for each case of $J = 1, \ldots, 4$ sources and $K = 3, 4, 10, 20, 30$ randomly positioned microphones.

We first note in Fig. 1 that the score obtained with SRP is much lower than the correlation obtained with the proposed method. After investigation, it turns out that this bad performance is mostly due to the fact that much of the energy of the SRP is spread out across many directions. We highlight that a large number of microphones delivers better results ($\approx 65\%$ for $K = 30$ and $\approx 21\%$ for $K = 3$).

2https://www.audiolabs-erlangen.de/fau/professor/habets/software/rir-generator
3Carnegie Mellon University dataset: http://www.festvox.org/cmu_faf/
VI. REFERENCES


