Comparison of updating strategies to improve finite element models of multi-axis machine tools

Jesus Maria Hernandez-Vazquez, Iker Garitaonandia, Maria Helena Fernandes, Joseba Albizuri, Jokin Munoa

Comparison of updating strategies to improve finite element models of multi-axis machine tools

J.M. Hernandez-Vazquez[1], I. Garitaonandia[1], M.H. Fernandes[1], J. Albizuri[2], J. Muñoa[3]
[1] Department of Mechanical Engineering, Faculty of Mining and Civil Engineering, University of the Basque Country UPV/EHU, Paseo Rafael Moreno « Pitxitxi » 2, E-48013 Bilbao, Spain
[2]Department of Mechanical Engineering, Faculty of Engineering, University of the Basque Country UPV/EHU, Alameda de Urquijo s/n, E-48013 Bilbao, Spain
[3]Dynamics & Control Research Line, IK4-IDEKO, Pol. Industrial de Arriaga 2, E-20870 Elgoibar, Spain
email: jesusmaria.hernandez@ehu.es, iker.garitaonandia@ehu.es, mariahelena.fernandes@ehu.es, joseba.albizuri@ehu.es, jmunoa@ideko.es

Abstract: In this paper, two alternative strategies to update finite element models of multi-axis machine tools are analyzed. The aim is to obtain improved numerical models that simulate adequately the dynamic behavior of the machine tool over a wide range of operating conditions. First, one- and multiple-configuration strategies are defined according to the number of spatial configurations of the selected machine tool. Then, using experimental modal data and iterative updating techniques, both strategies are tested on a three-axis machining center. Results show that, whatever the strategy selected, relative differences between numerical and experimental natural frequencies are decreased considerably after model updating. Nevertheless, one configuration strategy provides much better results in the configuration selected as reference than in the other, while multiple configuration strategy leads to attractive results for any spatial configuration. Thus, machine tool designers have at their disposal two possible strategies, so that depending on the characteristics of the machine tool and machining processes, may choose the most convenient. In addition, it is shown that multiple-configuration strategy enables to generate efficient finite element models for multi-axis machine tools using experimental data from a few specifically selected spatial configurations. As a result, the improved models will facilitate to establish reliable valuations about the necessity of performing design modifications, and will be the basis to accurately obtain the stability ranges of the machine and consequently diminish chatter vibrations.

Keywords: Updating strategies; Multi-axis machine tools; Finite element method; Experimental modal analysis.

1 INTRODUCTION

Today, machine tool manufacturers devote strong efforts to improve the dynamic behavior of machine tools under different operating conditions and subsequently to ensure the accuracy of the finished workpieces [1,2]. This is a complex task because machine tools are made up of components or substructures connected by guidance systems and drives that allow the relative movement between components, so natural frequencies and mode shapes are changed when different machine configurations are defined.

In the engineering field, and especially concerning on machine tools, it is of great interest to have efficient and flexible design methodologies, like the finite element method (FEM), to guarantee accurate predictions of the real behaviour of the machines. However, there are still some drawbacks that limit the quality and reliability of the results achieved by this method, mainly related to physical uncertainties in material properties and loads, and numerical uncertainties in the modeling and meshing processes.

Another important tool to study the dynamic characteristics of a mechanical system is experimental modal analysis (EMA). Using this technique it is possible to obtain information about the natural frequencies with an error lower than 1 %, and the corresponding mode shapes and modal damping factors. In addition, the last ones can be obtained only experimentally [3,4].

The advantages of EMA make it interesting to validate and improve FE models, so the adapted models may simulate more adequately the dynamic behavior of mechanical systems, such as machine tools. Updating techniques [5, 6] are the most appropriate for achieving this objective, because using them it is possible to modify the finite element model so that its dynamic characteristics resemble those obtained experimentally in the frequency range of interest. Garitaonandia et al. [7,8], Bais et al. [9] and Houming et al.[10] have successfully applied these techniques to machine tools.

The purpose of this work is to analyze different strategies to improve finite element models of multi-axis machine tools using iterative updating techniques. First, one-configuration strategy has been studied; in this option, the improvement is developed over one configuration and updated parameters are directly used for any other configuration. Second option is multiple-configuration strategy, where experimental and numerical data coming from several configurations are used altogether to update parameters of the FE model.

The final aim of this study is to provide elements of judgment that allow to the designer the selection of the adequate strategy, depending on the characteristics of the machine tool and machining processes, and eventually obtain an improved finite element model that simulates the dynamic response of the machine tool over a wide range of working configurations. The ultimate goal would be to optimize the design to eliminate stability problems under variable operating conditions and avoid the pernicious effects of chatter vibrations.

2 UPDATING STRATEGIES TO IMPROVE A FINITE ELEMENT MODEL OF A MACHINE TOOL

2.1 Survey of the updating process

In this work, the starting point is that there will be two models for each machine tool configuration: numerical (finite element) and experimental, which provide different modal analysis results. Updating techniques may be used to take advantages of both models and can be classified into two major groups:

- Direct methods [11,12], where the individual terms of the system FE matrices **K** and **M** are directly adjusted and consequently any physical meaning is lost, and
- Iterative methods [13,14], where changes are made on specific properties of the finite element model, as mass density, modulus of elasticity, etc., providing more flexibility, physical meaning and interpretability. These methods are developed into two different phases: 1) error localization and subsequent parameter selection, and 2) error correction by minimizing an objective function.

Regardless of the selected method, first task must be to assess the degree of correspondence between FE and experimental models, because it is necessary that both models show a considerable degree of correlation, in order to develop the updating procedure successfully.

First, geometrical correlation is developed to match the different coordinate and unit systems used in the models, and then, mode shape correlation is performed to establish a reliable pairing between numerical and experimental modes, usually by means of the Modal Assurance Criterion (MAC)[15].

Anyway, it is important to emphasize that FE model updating is heavily dependent on the quality of test data [16]. Therefore, special attention must be paid to carry out experimental modal analysis in the machine tool adequately, in order to minimize errors during execution.

2.2 Error localization and parameter selection

This is the first phase of the updating process and its aim is to select those physical parameters of the FE model which have been modeled incorrectly or whose values show significant uncertainties. Different error localization techniques may help in this matter [17,18]. In this work, sensitivity analysis has been used to find out which parameters cause main changes in the selected responses (natural frequencies and MAC values).

In the finite element method, a sensitivity analysis provides a sensitivity matrix **S**, whose terms show how a particular response quantity Y changes with respect to a variation of a model parameter P. If there are a set of parameters n, their influence on a set of responses m can be expressed in matrix form as

$$\Delta Y = S \cdot \Delta P \tag{1}$$

Where

$$s_{ij} = \partial Y_i / \partial P_j \tag{2}$$

Due to its lower computational cost, it is convenient to determine s_{ij} using an analytical approach based on the differentiation of the structural undamped eigenvalue equation [19]. Hence, sensitivity expressions for natural frequencies and MAC are as follows:

$$\frac{\partial f_i}{\partial P_j} = \frac{\Phi_i^T \cdot \left(\frac{\partial K}{\partial P_j} - 4\pi^2 f_i^2 \cdot \frac{\partial M}{\partial P_j} \right) \cdot \Phi_i}{8\pi^2 f_i \cdot \Phi_i^T \cdot M \cdot \Phi_i} \tag{3}$$

$$\frac{\partial MAC}{\partial P_j} = 2 \cdot \left[\frac{\left(\Phi_{\exp}^T \cdot \Phi_{FEM}\right) \cdot \left(\Phi_{\exp}^T \cdot \frac{\partial \Phi_{FEM}}{\partial P_j} \right)}{\left(\Phi_{\exp}^T \cdot \Phi_{\exp}\right) \cdot \left(\Phi_{FEM}^T \cdot \Phi_{FEM}\right)} - \frac{\left(\Phi_{\exp}^T \cdot \Phi_{FEM}\right)^2 \cdot \left(\Phi_{FEM}^T \cdot \frac{\partial \Phi_{\exp}}{\partial P_j} \right)}{\left(\Phi_{\exp}^T \cdot \Phi_{\exp}\right) \cdot \left(\Phi_{FEM}^T \cdot \Phi_{FEM}\right)} \right] \tag{4}$$

But, in finite element models, it is common that there are significant differences between the magnitudes of the parameters which are in the denominator of (2). For instance, in cast iron – one of the main materials of the machining center analyzed here – modulus of elasticity and mass density are 175 GPa and 7100 kg/m3 respectively. Hence, the values of the sensitivity coefficients s_{ij} will be different by several orders of magnitude and erroneous conclusions when analyzing **S** matrix could be extracted. Therefore, it is appropriate to use normalized sensitivities instead, defined as

$$\left(s_{ij}\right)_{norm} = \frac{\partial Y_i}{\partial P_j} \cdot P_j \tag{5}$$

Higher sensitivity values in (5) would show parameters whose influence on responses is high, and some of them could be modeled incorrectly. So, at this moment, engineering judgment plays an important role to establish a first selection of parameters.

2.3 Iterative updating

In this step, the previously selected parameters of the FE model will be modified to adjust experimental and numerical responses. An iterative updating procedure based on sensitivity analysis has been used to estimate the optimal parameter changes $\mathbf{\Delta P_{opt}}$ that minimize differences between FEM and EMA frequencies and bring MAC values to 100% [3].

Recalling (1), when applying $\mathbf{\Delta P}$ to the initial FE model, vector of responses $\mathbf{Y_{FEM}}$ will be modified as

$$Y_{MFEM} = Y_{FEM} + \Delta Y = Y_{FEM} + S \cdot \Delta P \tag{6}$$

The objective would be to obtain an ideal vector of parameter changes, $\mathbf{\Delta P_{ideal}}$, so

$$Y_{MFEM} = Y_{FEM} + S \cdot \Delta P_{ideal} = Y_{FEM} + \Delta Y_{exact} = Y_{EMA} \tag{7}$$

But, in general, it will be impossible to achieve the exact change of the model responses, $\Delta \mathbf{Y}_{exact}$. Instead, an optimal vector $\Delta \mathbf{Y}_{opt}$ will be obtained as

$$\Delta Y_{exact} = \Delta Y_{opt} + E = S \cdot \Delta P_{opt} + E \qquad (8)$$

And rearranging (7) and (8) leads to

$$E = (Y_{EMA} - Y_{FEM}) - S \cdot \Delta P_{opt} \qquad (9)$$

Applying a least-squares criterion [20], an optimal solution $\Delta \mathbf{P}_{opt}$ would be obtained minimizing the sum of the squared terms of vector **E**.

But, if different types of responses are selected, for example, natural frequencies and MAC values, sensitivity levels will be quite different between finite element model parameters, as explained in paragraph 2.2, and matrix **S** might be ill-conditioned.

Therefore, it is highly recommended to use relative responses differences and relative parameters instead of absolute ones, and normalized sensitivities as well. Moreover, it is possible to apply weighting coefficients *w* to parameter and response values, expressing the degree of confidence on these terms. For instance, values of natural frequencies are more reliable than MAC ones, which involve mode shapes.

So, any component of the error vector **E** would be expressed as

$$E_i = w_{iY} \cdot \frac{(Y_{iEMA} - Y_{iFEM})}{Y_{iEMA}} - \sum_{j=1}^{n} \frac{\partial Y_i}{\partial P_j} \cdot \left(w_{jP} \cdot \frac{\Delta P_{jopt}}{P_j} \right) \cdot \frac{w_{iY}}{Y_{iEMA}} \cdot \frac{P_j}{w_{jP}} = \qquad (10)$$

$$= w_{iY} \cdot \frac{(Y_{iEMA} - Y_{iFEM})}{Y_{iEMA}} - \sum_{j=1}^{n} b_{ij} \cdot \left(w_{jP} \cdot \frac{\Delta P_{jopt}}{P_j} \right)$$

or, in matrix form

$$E = \Delta Y_{rel}^{w_Y} - B \cdot \{\Delta P_{rel}^{w_P}\}_{opt} \qquad (11)$$

In addition, as vector of response differences contains large values and Taylor's expression in (1) is truncated after the first term, it is advisable to impose upper and lower bounds to parameter changes. Hence, in order to achieve the desired changes, it will be necessary to develop several iterations.

2.4 Updating strategies

At this point, two updating strategies can be selected to improve the FE model. The first one, simplest and classical, is based on one configuration, so that an experimental modal analysis over that configuration is considered; then, physical parameters of the FE model are updated and finally used for other configurations of the machine. In this case matrix B in (11) is derived from matrix **S** of the configuration selected for the strategy (Figure 1).

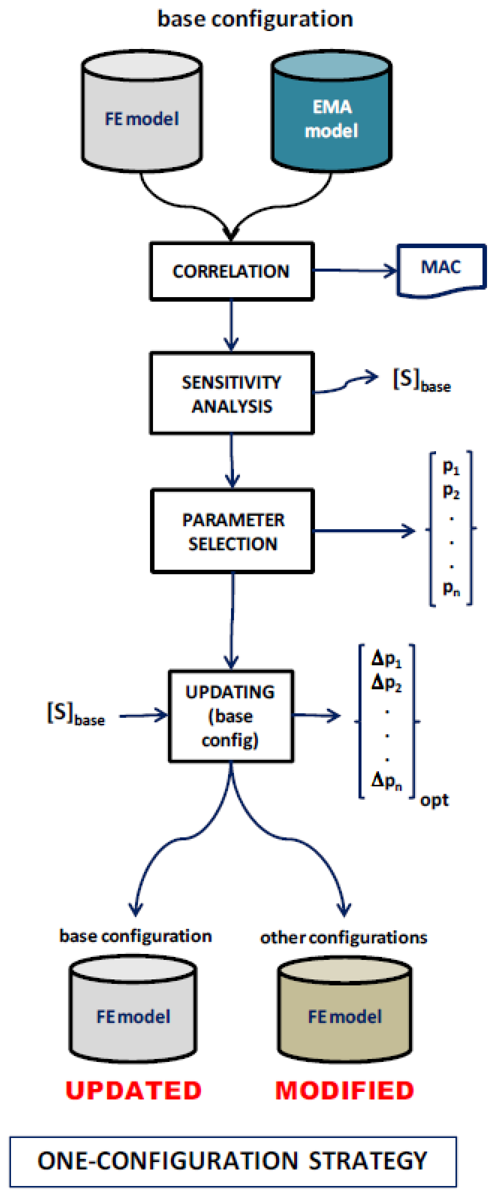

Figure 1. Flowchart of one-configuration strategy.

But, if several experimental modal analyses are available, related to different configurations of the machine tool, it would be possible to update physical parameters of the FE model for all configurations simultaneously (Figure 2).

So, extending expression (1) to take into account multiple configurations, *z*

$$\begin{aligned} \Delta Y_a &= S_a \cdot \Delta P_a \\ \Delta Y_b &= S_b \cdot \Delta P_b \\ &\cdots\cdot \\ \Delta Y_z &= S_z \cdot \Delta P_z \end{aligned} \qquad (12)$$

The previous expression shows *z* parameter vectors, one for each configuration. But, in this case, the differences between FE models are only due to the changing position of the main components. Therefore, it is possible to establish a unique set of parameters for the complete set of configurations, and hence, expression (12) is now

$$\begin{aligned}\Delta Y_a &= S_a \cdot \Delta P \\ \Delta Y_b &= S_b \cdot \Delta P \\ &\dots\dots \\ \Delta Y_z &= S_z \cdot \Delta P\end{aligned} \tag{13}$$

And rearranging (13) leads to

$$\begin{pmatrix}\Delta Y_a \\ \Delta Y_b \\ \dots \\ \Delta Y_z\end{pmatrix} = \begin{pmatrix}S_a \\ S_b \\ \dots \\ S_z\end{pmatrix} \cdot \Delta P \quad \Rightarrow \quad \Delta Y = S \cdot \Delta P \tag{14}$$

which is the base expression for iterative updating (1), but taking into account all the configurations. This technique is known as multi-model updating and has been used by Lauwagie [21] to identify the elastic properties of layered materials,

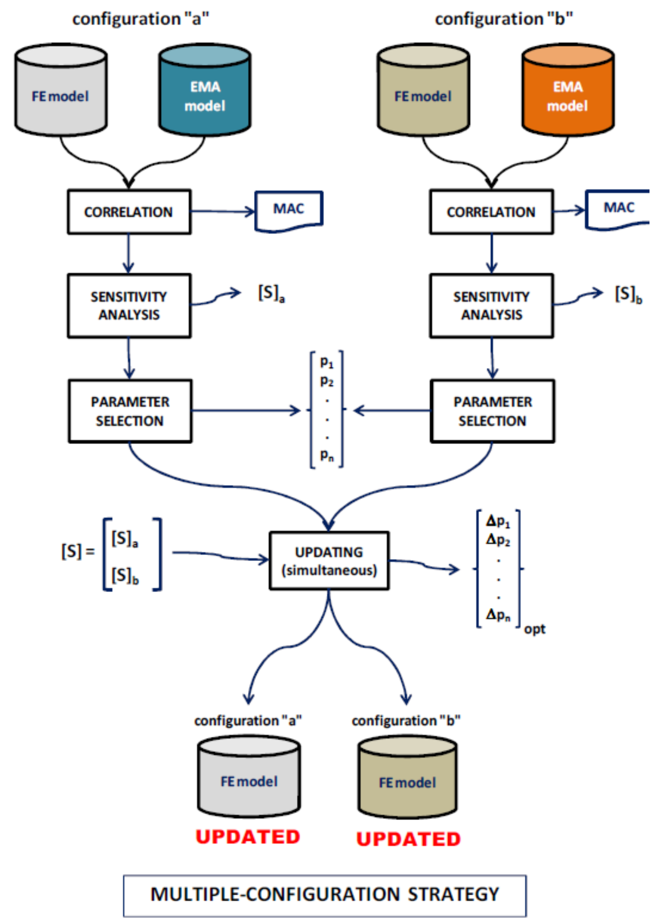

Figure 2. Flowchart of multiple-configuration strategy (two configurations).

3 MULTI-AXIS MACHINE TOOL: MODELING DETAILS AND UPDATING

In this section, the dynamic characteristics of the DANOBAT SISTEMAS DS630 machining center are presented. This three-axis machine tool is made up of four main components – bed frame, column, framework and ram – connected by roller type linear guideways and driven by ball-screws and linear motors. These components can move along X (longitudinal), Y (vertical) and Z (transverse) axes for 1000, 800 and 630 mm, respectively.

First, a finite element model of the machine has been defined. This model, which consists of 12795 nodes and 14980 elements is displayed in figure 3. Mainly shell and solid elements have been used in the modeling procedure.

Special attention has been paid to the modeling of the connections between components. Linear guideways have been modeled using spring elements, assigning high stiffness values in two directions, perpendicular and transverse to the direction of movement, based on stiffness curves provided by the guideway supplier, and very low stiffness values along directions where the movement is developed. A similar modeling has been followed for ball-screws, although in this case high stiffness values have only been established in the direction of movement [22].

Connections to the foundation have been modeled using also spring elements, and motors and the milling head as lumped masses. This initial configuration – Figure 3- has been named RCC (column Right, frame-work Centered and ram Centered).

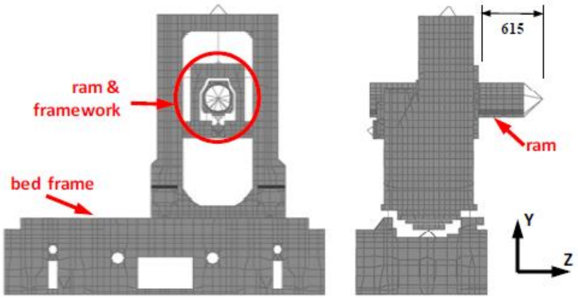

Figure 3. FE model of the machine tool (RCC configu-ration).

A second finite element model of the machine tool has been defined centering the column on the bed frame by sliding it along the X axis, and moving forward the ram along Z axis. This configuration has been named CCF (Figure 4).

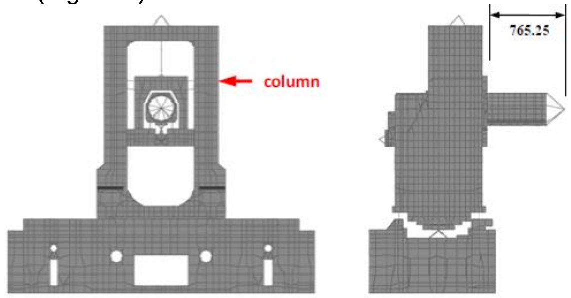

Figure 4. FE model of the machine tool (CCF configu-ration).

Eigenvalues and eigenvectors have been calculated from the assembled mass and stiffness matrices of both numerical models. According to several tests developed under chatter conditions [23,24], the frequency range of interest has been defined as 10 Hz to 150 Hz. Then, experimental modal analyses of the machine tool in RCC and CCF in situ configurations have been carried out. Table 1 shows paired numerical and experimental frequencies and corresponding MAC values for both models [25].

Table 1. Numerical and experimental frequencies.

FEM (RCC)	EMA (RCC)	MAC	FEM (CCF)	EMA (CCF)	MAC
35.75	33.83	91.2	35.78	34.68	97.3
82.96	69.54	57.4	53.19	52.93	77.1
92.15	84.66	77.5	75.7	56.95	80.8
139.61	107.15	70.1	81.77	68.7	74.0
177.8	145.38	92.1	127.3	109.44	62.4
			182.52	152.05	85.6

Looking into RCC configuration, it can be seen that there are two high MAC values (91.2 and 92.1) which express a great degree of correspondence, two intermediate values (77.5 and 70.1) and a lower one of 57.4, which show less confidence. Nevertheless, the last paired ones have been selected due to their great influence on the frequency response function [23].

On the other hand, in CCF configuration there is a very high value (97.3) and a lower value (62.4), whereas the remaining are quite adequate. So, it may be considered that correlation results are satisfactory, and will serve as a basis for an updating phase.

Then, the selection of updating parameters has been done. In this type of machine tools, the main uncertainties in the FE model are concentrated on the stiffness values of the connection and driving elements among main components of the machine tool (Figure 5), on the inertia contribution of the moving concentrated masses (motors and milling head), on the material properties of the cast iron components, and on the stiffness values assigned to the connection elements to the machine foundation. Table 2 shows the selected parameters and their initial values according to the manufacturer.

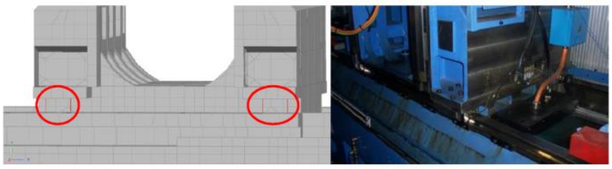

Figure 5. FE model of the machine tool (CCF configuration).

Table 2. Parameters selected for updating.

Connection or component	Parameters	Values	Numbering (Figure 6)
Bed frame - column	Stiffness Y,Z	1350, 1500 N/µm	5, 9
Column - framework	Stiffness X,Z	1000, 950 N/µm	1, 3, 10
Framework - ram	Stiffness X,Y	950, 900 N/µm	2, 6
Y Ball-screw	Stiffness Y	176.7 N/µm	7
Z Ball-screw	Stiffness Z	172.7 N/µm	11
Machine tool-foundation	Stiffness X,Y,Z	140, 1200, 140 N/µm	4, 8, 12
Column	Young's modulus	125 GPa	13
Framework, ram	Young's modulus	175 GPa	14
Bed frame	Young's modulus	125 GPa	15
Milling head	Lumped mass	120 kg	16
Motor Z Direction	Lumped Mass	100 kg	17
Motor Y Direction	Lumped Mass	100 kg	18

Figures 6 and 7 show the normalized sensitivity coefficients for RCC and CCF configurations. Highest values appear in parameters 13 to 18 depending on the responses, while the sensitivities corresponding to stiffness values are smaller. Furthermore, MAC value for the fifth paired mode shapes and second frequency in CCF configuration show a high sensitivity to some stiffness changes.

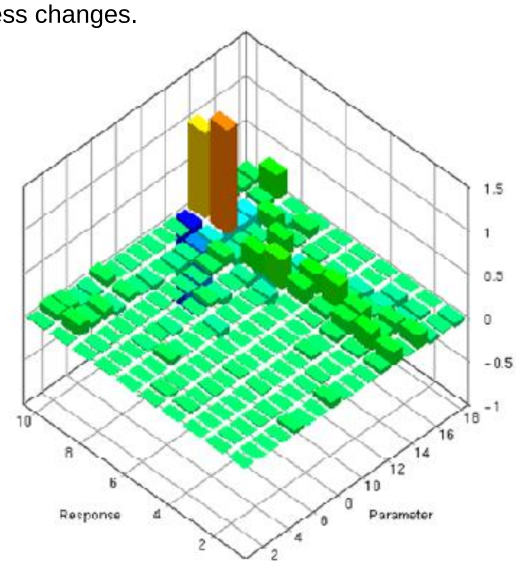

Figure 6. Sensitivity matrix for RCC configuration.

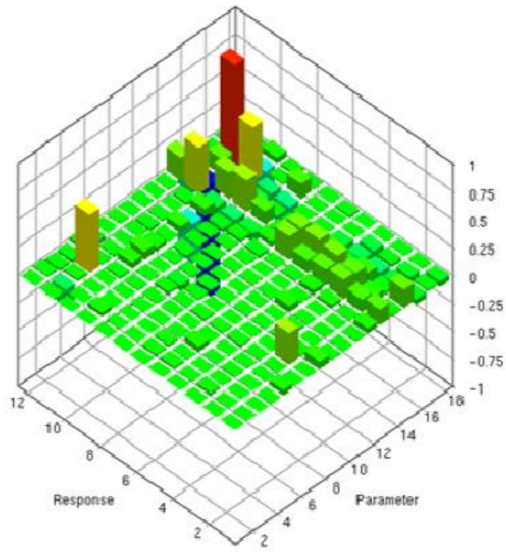

Figure 7. Sensitivity matrix for RCC configuration.

These results have been considered appropriate and selected parameters have been used in the following error correction phase.

Figures 8 and 9 show frequency differences between numerical and experimental models for both configurations after updating using the following three possible strategies:

- RCC first: updating parameters in RCC configuration and then using for CCF configuration (one-configuration strategy).
- CCF first: updating parameters in CCF configuration and then using for RCC configuration (one-configuration strategy).
- M-C: updating parameters for both configurations simultaneously (two-configuration strategy).

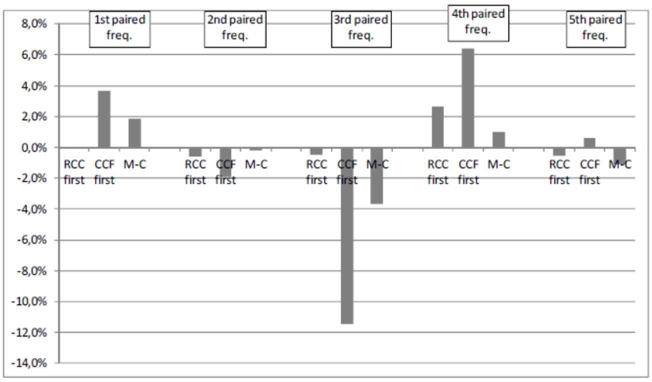

Figure 8. Frequency differences for RCC configuration after updating.

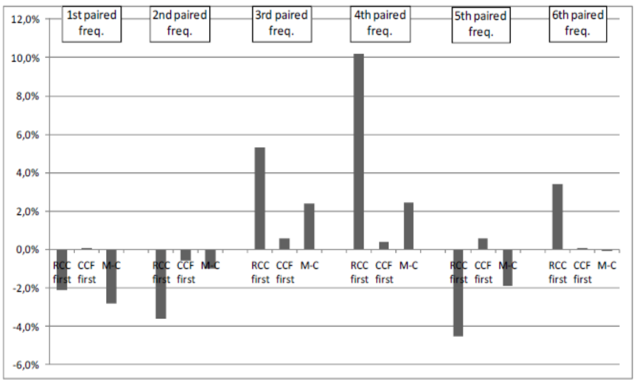

Figure 9. Frequency differences for CCF configuration after updating.

For any configuration, it can be noticed that:

- In nearly all paired mode shapes, it has been managed to reduce the difference between numerical and experimental frequencies – see Table 1 -, so that the updated FE model is closer to the real mechanical system than the initial one. This result is solid whatever the strategy selected.
- When the updating process is based on one configuration, the best improved FE model is that corresponding to the configuration selected and frequency differences are lower than 1 %, in most cases. But, in this strategy, results obtained for the other configuration are deficient and even unacceptable. For instance, a pair of results displayed in figures 8 and 9 show frequency differences greater than 10 %.
- For the two-configuration strategy, frequency differences show intermediate values for both configurations, and most of them lower than 2.5 %. So, they can be considered as acceptable results.

4 CONCLUSIONS

Multi-axis machine tools are complex mechanical systems made up of sliding substructures connected by specific elements, so relative movement is allowed, which causes variable dynamic characteristics.

In this work, two updating strategies to improve finite element models of multi-axis machine tools based on experimental modal data have been studied. Both strategies provide better FE models than initial ones, but substantial differences exist. Thus, in one configuration strategy, the basic configuration is much more improved than the other, which could be unsatisfactorily adapted. This strategy would only be attractive if basic configuration was mostly used under working conditions and other configurations were much less important and used.

Multiple-configuration strategy provides mean errors on the updated FE model for any configuration, so it can be expected that dynamic characteristics are reproduced over a wide range of working conditions.

The methodology presented can be generalized to any multi-axis machine tool and will allow obtaining an improved finite element model by selecting critical configurations in the main working processes of the machine. This model would represent the starting point to optimize the machine design and eliminate stability problems under operating conditions.

ACKNOWLEDGMENTS

This work has been supported by the Ministry of Science and Innovation (DPI2010-18258) and UPV/EHU (GIU 13/10 and UFI 11/29).

REFERENCES

[1] Y. Altintas, C. Brecher, M. Weck, S. Witt, Virtual Machine Tool, CIRP Annals – Manufacturing Technology 54 (2005) 115-138.

[2] G. Quintana, J. Ciurana, Chatter in machining processes: A review, International Journal of Machine Tools & Manufacture 51 (2011) 363-376.

[3] D.J. Ewins, Modal Testing: Theory, Practice and Application, John Wiley & Sons, New York, USA, second edition, 2001.

[4] N.M.M. Maia, J.M.M. Silva, Theoretical and Experimental Modal Analysis, John Wiley & Sons, New York, USA, 1997.

[5] M.I. Friswell, J.E. Mottershead, Finite Element Model Updating in Structural Dynamics, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1995.

[6] T. Janter, Construction oriented updating of dynamic finite element models using experimental modal data, PhD Thesis, Katholieke Universiteit Leuven, Belgium, 1989.

[7] I. Garitaonandia, M.H. Fernandes, J. Albizuri, Dynamic model of a centerless grinding machine based on an updated FE model, International Journal of Machine Tools & Manufacture 48 (2008) 832-840.

[8] I. Garitaonandia, J. Albizuri, J.M. Hernández, M.H. Fernandes, I. Olabarrieta, J. Muñoa, Modeling procedure of a machining center using updating techniques and substructure synthesis, Proceedings of ISMA 2010 – International Conference on Noise and Vibration Engineering, Leuven, Belgium, September 2010, pp. 3815-3827.

[9] R.S. Bais, A.K. Gupta, B.C. Nakra, T.K. Kundra, Studies in dynamic design of drilling machine using updated finite element models, Mechanism and Machine Theory 39 (2004) 1307-1320.
[10] Z. Houming, W. Chengyang, Z. Zhenyu, Dynamic characteristics of conjunction of lengthened shrink-fit holder and cutting tool in highspeed milling, Journal of Material Processing Technology 207 (2008) 154-162.
[11] M. Baruch, I.Y. Bar Itzhack, Optimal Weighted Orthogonalization of Measured Modes, AIAA Journal 16 (1978) 346-351.
[12] E. Fissette, C. Stavrinidis, S.R. Ibrahim, Mathematical Model Update Considerations for Engineering Applications, Proceedings of the 12th International Seminar on Modal Analysis, Leuven, Belgium, September 1987.
[13] R. Fillod, G. Lallement, J.P. Piranda, Q. Zhang, Parametric Correction of Regular Non-Dissipative Finite Element Models, Proceedings of the 11th International Seminar on Modal Analysis, Leuven, Belgium, September 1986.
[14] T. Janter, W. Heylen, P. Sas, QA-Model updating, Proceedings of the 13th International Seminar on Modal Analysis, Leuven, Belgium, September 1988.
[15] R.J. Allemang, Investigation of some multiple input/output frequency response experimental modal analysis techniques, PhD Thesis, University of Cincinnati, Cincinnati, USA, 1980.
[16] L. Eckert, B. Caesar, Influence of Test Data Quality and Completeness on Model Update Results, Proceedings of the 14th International Seminar on Modal Analysis, Leuven, Belgium, September 1989.
[17] E. Fissette, C. Stavrinidis, S.R. Ibrahim, Error Location and Updating of Analytical Dynamic Models using a Force Balance Method, Proceedings of the International Conference "Spacecraft Structures and Mechanical Testing", Noordwijk, The Netherlands, 1988.
[18] J.C. Wei, Q. Zhang, R.J. Allemang, M.L. Wei, Correction of Finite Element Model via Selected Physical Parameters, Proceedings of the 13th International Seminar on Modal Analysis, Leuven, Belgium, September 1988.
[19] R.L. Fox, M.P. Kapoor, Rates of change of eigenvalues and eigenvectors, AIAA Journal 6 (1968) 2426-2429.
[20] A.J. Morris, Foundations of structural optimization: A unified approach, John Wiley & Sons, New York, USA, 1982.
[21] T. Lauwagie, H. Sol, W. Heylen, G. Roebben, Determination of the inplane elasticpropertie s of the different layers of laminated plates by means of vibration testing and model updating, Journal of Sound and Vibration 274 (2004) 529-546.
[22] H. Van Brussel, P. Sas, I. Németh, P. De Fonseca, P. Van den Braembussche, Towards a Mechatronic Compil-er, IEEE/ASME Transactions on Mechatronics 6 (1) (2001) 90-105.
[23] J. Muñoa, M. Zatarain, I. Bediaga, R. Lizarralde, Optimization of Hard Material Roughing by means of a Stability Model, Proceedings of 8th CIRP International Workshop on Modeling of Machining Operations, Chemnitz, Germany, May 10-11, 2005.
[24] J. Muñoa, Z. Dombovari, I. Mancisidor, Y. Yang, M. Zatarain, Interaction between Multiple Modes in Milling Processes, Machining Science and Technology 17 (2013)165-180.
[25] J.M. Hernandez, I. Olabarrieta, I. Garitaonandia, J. Albizuri, M.H. Fernandes, J. Muñoa, Un procedimiento para la mejora de modelos de elementos finitos en máquinas herramienta de múltiples ejes, Actas do X Congresso Ibero-americano em Engenharia Mecânica - CIBEM10, Porto, Portugal, September 4-7, 2011