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ABSTRACT

Data envelopment analysis (DEA) has proven to be a useful technique for evaluating the relative performance of comparable and homogeneous decision-making units (DMUs). In recent years, DEA-based resource allocation and target setting approaches have gained more and more attention from both practitioners and academic researchers. In this paper, we propose a new mechanism to simultaneously adopt the principles of common weights and efficiency invariance in allocating multiple resources and setting multiple targets among DMUs. To obtain the final plan, we minimize the deviation between the possible plan based on common weights and another feasible plan emphasizing efficiency invariance. If the minimum deviation equals zero, one optimal plan will be determined. In general situations, however, the proposed approach will present two plans that have a non-zero deviation. One is generated by using a common set of weights for all DMUs in such a way that the change of efficiencies are minimized, while the other is generated by strictly keeping efficiency scores unchanged yet having similar or even identical weights on input-output measures for each DMU to the utmost extent. The efficacy and usefulness of the proposed approach are demonstrated using a numerical example

from previous literature and an empirical application to an urban bus company in China.
Keywords: Data Envelopment Analysis (DEA), Resource allocation, Target setting, Common weights, Efficiency invariance

1 Introduction

In many managerial applications, there exists a set of comparable and homogeneous decision-making units (DMUs) that are operating under a central decision maker (Korhonen and Syrjänen, 2004; Fang and Zhang, 2008). The decision maker has sufficient power and responsibility to control the production process, determine decision parameters, manage resources, plan outcomes, etc. An essential problem frequently faced by the decision maker is the requirement to manage the production effectively to achieve particular goals and/or satisfy pre-specified regulations. The production naturally involves the usage of input resources and production of output targets. Based on this background, many entities will face problems of resource allocation and target setting. For example, a university hires several dozen scholars for its secondary departments and logically the board of directors will set goals on achievements such as publications and research funds. Each department will have to introduce a certain number of talents and accordingly undertake some responsibilities. Within a country, the central government may offer a fixed financial subsidy across provinces to realize a national reduction commitment on carbon emissions. To fully cover the total allowance and reduction goal, all provinces will be assigned a share of resources and quotas at the same time. In such situations, how the input resources and output targets should be distributed among peer DMUs in an equitable and fair way is a problem of considerable importance from both practical applications and academic interest.

Our approach to deal with the resource allocation and target setting problem is based on the data envelopment analysis (DEA) methodology, which has been studied for a long time. DEA, first introduced by Charnes et al. (1978) and further developed by Banker et al. (1984), has proved to be a preferable method for organization performance evaluations and has been applied to many disciplines since its inception (Emrouznejad et al., 2008; Jeang and Liang, 2012). Although DEA was initially proposed for the purpose of performance evaluation, the empirical analysis of operating status by DEA can provide some valuable insights for resource allocation (Lu and Hung, 2010). Also, it is now widely accepted that an important application of DEA-based models is resource allocation and target setting (Amirteimoori and Kordrostami, 2005; Amirteimoori and Tabar, 2010; Lotfi et al., 2013; Hatami-Marbini et al., 2015). As Beasley (2003) indicated, *resource allocation* is the setting of input (output) levels for DMUs when the organization has limited input resources (production possibility) and the input (output) levels should be simultaneously determined for all DMUs, whereas *target setting* is the setting of input (output) levels for DMUs when these can be set for each DMU individually without reference to organizational restrictions. Here in this paper, we adopt the first resource allocation concept. In other words, the purpose of this study is to dispatch multiple fixed input resources and multiple pre-specified total output targets across different DMUs.

The resource allocation problem has become one of the most important application areas of DEA-based approaches. Mandell (1991) formulated bi-criteria mathematical programming problems for allocating service resources among different public service delivery sites. Oral et al. (1991) considered DEA-based performance evaluation for research programs and accordingly awarded research funds. Golany et al. (1993) took the impacts of allocation schemes on the efficiency score into account and used additive DEA models to implement the resource allocation. Cook and Kress (1999) made the first attempt at fixed cost allocation (a special resource), and the authors explored the efficiency invariance and input Pareto optimality aspects of resource allocation. The authors suggested that the allocation mechanism should reflect the performance of current measures, and any change in efficiency scores would be unfair. Beasley (2003) proposed another perspective to allocating the fixed cost, one which maximizes the overall post-allocation efficiency scores with a set of common weights. The underlying logic of common weights is that the traditional

DEA methods have used extreme points to compare one DMU to the rest of the sample set (Jeang and Liang, 2012), with weights being chosen to favor the evaluated DMU, while the common weights concept approaches the evaluation process by favoring all DMUs equally. Cook and Zhu (2005) further extended the Cook and Kress (1999) approach to the output orientation and proposed an executable method in the multi-input and multi-output case. However, Lin (2011a) argued that the Cook and Zhu (2005) method would be infeasible when some additional constraints are taken into account. Amirteimoori and Shafiei (2006) and Amirteimoori and Emrouznejad (2011) considered the problem of allocating a fixed reduction quota of resources to all DMUs, with the efficiency scores again being required to remain unchanged. Pachkova (2009) introduced transfer costs of resources into resource allocation, which is realized by a price matrix. The author traded off the maximum allowed reallocation cost and the highest possible total efficiency for all DMUs. Fang and Zhang (2008) allocated the resources by maximizing both the total efficiency score and individual efficiency scores. Milioni et al. (2011) proposed an ellipsoidal frontier model for resource allocation. Li et al. (2013) proved that all DMUs could be efficient with a common set of weights after the allocation, and they defined a satisfactory degree concept to obtain the final unique allocation. Wu et al. (2013) adopted a bargaining game to make DMUs compete for a common set of weights according to their ecological efficiency and current emission levels. By viewing all DMUs as competitors negotiating with the others, Du et al. (2014) used the game cross-efficiency concept to develop an iterative method to allocate input resources.

In addition, studies on target setting are also abundant in the existing DEA literature. Golany (1988) proposed a multi-objective linear programming procedure to set up goals for desired outputs based on DEA. Thanassoulis and Dyson (1992) presented target setting as being concerned with the solution of various mathematical programs. In their paper, the authors incorporated a preference structure to attach different importances to potential changes in input/output levels. Golany and Tamir (1995) formulated a resource allocation model which simultaneously determines input and output targets based on maximizing total outputs. Yang et al. (2009) combined performance evaluation with target setting in a programming model, in which the preferences of decision makers are taken into account in an interactive fashion. Lozano and Villa (2009) proposed two target setting DEA models; the first one is an interactive multi-objective method, while the other uses a lexicographic multi-objective approach. Matin and Azizi (2011) addressed the target setting problem with negative data. In recent years some articles have studied the context of environmental factors such as CO_2 emission allowance (Wang et al., 2013; Sun et al., 2014; Ji et al., 2017) and carbon emissions abatement quota (Feng et al., 2015; Wu et al., 2016).

Apart from the above literature, there have been studies that address the input resource allocation and output target setting in a unified framework. For example, Amirteimoori and Kordrostami (2005) presented a DEA-based method for allocating fixed inputs and setting pre-specified outputs. In that paper, the authors adopted a method similar to that of Beasley (2003) to use common weights to maximize the overall average efficiency score. Amirteimoori and Tabar (2010) assumed that after the allocation of input resources and output targets all DMUs should be efficient under a set of common weights, and each DMU should be allocated a share of resources and targets proportional to its current input consumption and output production. Towards this end, Amirteimoori and Tabar (2010) introduced goals achievement variables for the efficiency level, allocated resources, and set targets to obtain a unique allocation plan. Lotfi et al. (2013) and Hatami-Marbini et al. (2015) also applied a common-weights DEA approach and Goal Programming (GP) concept to allocate resources and set targets, and the post-allocation efficiency scores were maximized. Lin (2011a) proposed a DEA-based model to allocating input resources while setting output targets, and the efficiency invariance principle was reformulated in that paper. Further, based on a parallel production system Bi et al. (2011) addressed resource allocation and target setting under a network DEA framework. They generated the final plan using three criteria, namely, common weights, efficiency maximization, and improving the worst performing subunit as much as possible.

By surveying the relevant literature, the authors have identified two pairs of most relevant features that are incorporated in DEA-based resource allocation and/or target setting approaches: common weights and variable weights, efficiency invariance and efficiency maximization.

Table 1

Features of selected resource allocation approaches based on DEA.

Methods	Weights		Efficiency	
	Common	Variable	Invariance	Maximization
Cook and Kress (1999)		x	x	
Beasley (2003)	x			x
Jahanshahloo et al. (2004)		x	x	
Lozano and Villa (2004)		x		x
Amirteimoori and Kordrostami (2005)	x		x	
Cook and Zhu (2005)		x	x	
Fang and Zhang (2008)		x		x
Asmild et al. (2009)		x		x
Li et al. (2009)		x		x
Amirteimoori and Tabar (2010)	x			x
Bi et al. (2011)	x			x
Lin (2011a)		x	x	
Lin (2011b)		x	x	
Nasrabadi et al. (2012)		x	x	
Fang (2013)		x		x
Li et al. (2013)	x			x
Lotfi et al. (2013)	x			x
Mostafaee (2013)		x	x	
Si et al. (2013)	x			x
Du et al. (2014)		x		x
Fang (2015)		x		x
Lin and Chen (2016)		x	x	
Lin et al. (2016)		x	x	

Table 1 summarizes the main features of selected resource allocation approaches in terms of weights and efficiency, which implies possible research gaps. Amirteimoori and Kordrostami (2005) is the only paper using both common weights and the efficiency invariance principle, but that paper used a common set of weights to conduct the performance evaluation, and the models developed were nonlinear, which makes their solution more difficult. In that work, the authors obtained a unique allocation by minimizing the difference between the maximum and minimum deviation of the allocated resource and the set target across the DMUs.

To address the issue of resource allocation and target setting, we believe that a common set of weights should be used in such a way as to maintain efficiencies unchanged after the allocated resources and set targets are added as additional inputs and outputs. The common set of weights implies that all DMUs make equal evaluations of these input-output measures in the reference set, hence the resulting allocation and setting scheme can be accepted as fair by all DMUs. It is not far-fetched to suggest that it would be more acceptable by considering common weights for the resource allocation and target setting problem. On the other hand, note that all DMUs' relative performances are only dependent on the existing inputs and outputs measures, which is beyond the control of individual DMUs, hence the resource allocation and target setting should be implemented according to its current efficiency status. If efficiency variances are allowed, the generated plan is not acceptable to the decision maker whose DMUs have no control on the allocated resources and set targets (Lin and Chen, 2016). This is due to the fact that some inefficient DMUs could improve their efficiency scores by utilizing these allocated resources and set targets. Based on these observations, such an arrangement of using common weights and the efficiency invariance principle for resource allocation and target setting can be judged more fair and equitable, and the generated plan can be more acceptable.

In this paper, we reconsider the issue of resource allocation and target setting. Compared with previous studies, the current paper takes into account common weights and the efficiency invariance principle, simultaneously. Common weights imply less difficulty and resistance to implementing the resulted allocation and setting plan, while the efficiency invariance principle reflects the current performance and also the desire for fairness. Both the two principles have been studied in many articles, however, to the best of our knowledge, few studies have combined the two principles in one method. Our purpose is to address the resource allocation and target setting in such a way that common weights are determined and at the same time the efficiency scores remain unchanged for each of the DMUs. Ideally, the resources and targets would be allocated and set in such a way that a common set of weights is used and the efficiency scores remain unchanged. However, sometimes it may be infeasible to strictly satisfy the two principles simultaneously, and so in general cases, we will try to satisfy common weights and efficiency invariance principle as much as possible. Consequently, the proposed approach may generate two possible plans, with one using common weights to avoid efficiency change to the utmost extent, whereas the other emphasizes unchanged efficiency scores and maintains similar evaluations of these measures for all DMUs. The two possible plans in general cases allow the central decision maker to make a trade-off between equal evaluations (i.e., common weights) and efficiency invariance considerations. As compared with Amirteimoori and Kordrostami (2005), we will use the classical CCR model to calculate the efficiency scores, and common weights are used to restrict the efficiency scores for all DMUs when generating the resource allocation and target setting plan. Besides, the gap between the maximal value and the minimal value in our method is smaller than that of Amirteimoori and Kordrostami (2005). In addition, we consider the case in which multiple resources and multiple targets are determined simultaneously for all DMUs, which can be solved only by a linear model, therefore, the proposed approach is more general.

The remainder of this paper is organized as follows: In Section 2, we introduce the resource allocation and target setting problem and propose a DEA-based approach based on common weights and the efficiency invariance principle. Afterward, a numerical example from previous literature and a real application to an urban bus company are used to demonstrate the efficacy of the proposed approach in Section 3. Finally, Section 4 concludes this paper and provides directions for future research.

2 Problem Description and Mathematical Models

This section addresses the input resource allocation and output target setting problem based on common weights and the efficiency invariance principle. To this end, a preliminary is provided in Section 2.1. Afterward, the two principles, common weights and efficiency invariance, are modeled in Section 2.2. Next, a mathematical model is proposed to generate the resource allocation and target setting plan in Section 2.3. The proposed approach is supposed to seek common weights and keep efficiency scores unchanged simultaneously.

2.1 Preliminary

Following a traditional framework in DEA literature, let us consider a case of n homogeneous DMUs. The jth $\mathrm{DMU}\left(j=1,\dots,n\right)$ consumes a column vector of input $X_j=\left(x_{1j},\dots,x_{mj}\right)^T$ to produce a column vector of output $Y_j=\left(y_{1j},\dots,y_{sj}\right)^T$. Here the superscript T represents vector transposition. Supposing that $DMU_k\left(k=1,\dots,n\right)$ is under evaluation, Charnes et al. (1978) proposed model (1), the first DEA model and known as the classical CCR model, to calculate its relative efficiency score.

$$
\begin{aligned}
& e_k^* = max\sum\nolimits_{r=1}^{s} u_r y_{rk} \\
& s.t.\sum\nolimits_{r=1}^{s} u_r y_{rj} - \sum\nolimits_{i=1}^{m} v_i x_{ij} \le 0, \forall j \\
& \quad \sum\nolimits_{i=1}^{m} v_i x_{ik} = 1 \\
& \quad u_r, v_i \ge \varepsilon, \forall r, i.
\end{aligned}
\tag{1}
$$

In model (1), u_r and v_i are unknown relative weights attached to the output r and input i, respectively. Here, ε is a sufficiently small positive value used to avoid zero weights. The efficiency score $e_k^*\left(k=1,\dots,n\right)$ ranges from zero to one, with DMU_k being identified as efficient if $e_k^*=1$. Otherwise, the evaluated DMU_k is considered as inefficient.

The following envelopment model is a dual of model (1).

$$
\begin{aligned}
& \theta_k^* = min\,\theta \\
& s.t.\sum\nolimits_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{ik}, \forall i \\
& \quad \sum\nolimits_{j=1}^{n} \lambda_j y_{rj} \ge y_{rk}, \forall r \\
& \quad \lambda_j \ge 0, \forall j
\end{aligned}
\tag{2}
$$

where λ_j are intensity variables used to construct the efficiency frontier. It is clear that we have $e_k^* = \theta_k^*$.

Now suppose that there exists a central decision maker who has a series of input resources $R_f\left(f = 1,...,g\right)$ to be allocated across the DMUs. Accordingly, by investing these additional inputs the decision maker would logically expect a series of pre-specified output targets $T_p\left(p = 1,...,q\right)$ to be met in future periods. Therefore, output targets should be set for different DMUs at the same time as decisions are made about allocating input resources. Then, the problem emerges of how to allocate these input resources and set output targets among these DMUs in an appropriate fashion (Amirteimoori and Kordrostami, 2005; Amirteimoori and Tabar, 2010; Lotfi et al., 2013; Hatami-Marbini et al., 2015).

Denote the resources allocated to and targets set for $DMU_j\left(j = 1,...,n\right)$ as $r_{fj} \geq 0$ and $t_{pj} \geq 0$, respectively, such that

$$\sum\nolimits_{j=1}^{n} r_{fj} = R_f, r_{fj} \geq 0, \forall f, \tag{3}$$

$$\sum\nolimits_{j=1}^{n} t_{pj} = T_p, t_{pj} \geq 0, \forall p. \tag{4}$$

The above equations (3) and (4) ensure that both the allocated resources $r_{fj}\left(j = 1,...,n\right)$ and set targets $t_{pj}\left(j = 1,...,n\right)$ sum precisely to $R_f\left(f = 1,...,g\right)$ and $T_p\left(p = 1,...,q\right)$, respectively.

Without loss of generality, here we take r_{fj} and t_{pj} as additional inputs and outputs different from current measures. This means that there will be some other weights $u_{s+p}\left(p = 1,...,q\right)$ and $v_{m+f}\left(f = 1,...,g\right)$ attached to p^{th} target and f^{th} resource, respectively. The readers can refer to Li et al. (2009) for a case in which the allocated fixed cost is taken as a supplement of other inputs. As a result, the post-allocation evaluation model can be reformulated as the follows.

$$
\begin{aligned}
&\delta_k^* = max \frac{\sum_{r=1}^{s} u_r y_{rk} + \sum_{p=1}^{q} u_{s+p} t_{pk}}{\sum_{i=1}^{m} v_i x_{ik} + \sum_{f=1}^{g} v_{m+f} r_{fk}} \\
&s.t. \frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{p=1}^{q} u_{s+p} t_{pj}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{f=1}^{g} v_{m+f} r_{fj}} \le 1, \forall j \\
&\quad \sum_{j=1}^{n} t_{pj} = T_p, \forall p \\
&\quad \sum_{j=1}^{n} r_{fj} = R_f, \forall f \\
&\quad t_{pj} \ge 0, r_{fj} \ge 0, \forall p, f, j \\
&\quad u_r, u_{s+p}, v_i, v_{m+f} \ge \varepsilon, \forall r, p, i, f.
\end{aligned} \tag{5}
$$

By inserting $u_{s+p} t_{pk} = \tau_{pk} \left(p = 1,...,q \right)$ and $v_{m+f} r_{fk} = \gamma_{fk} \left(f = 1,...,g \right)$, model (5) can be equivalently changed into model (6).

$$
\begin{aligned}
&\delta_k^* = max \frac{\sum_{r=1}^{s} u_r y_{rk} + \sum_{p=1}^{q} \tau_{pk}}{\sum_{i=1}^{m} v_i x_{ik} + \sum_{f=1}^{g} \gamma_{fk}} \\
&s.t. \frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{p=1}^{q} \tau_{pj}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{f=1}^{g} \gamma_{fj}} \le 1, \forall j \\
&\quad \sum_{j=1}^{n} \tau_{pj} = u_{s+p} T_p, \forall p \\
&\quad \sum_{j=1}^{n} \gamma_{fj} = v_{m+f} R_f, \forall f \\
&\quad \tau_{pj} \ge 0, \gamma_{fj} \ge 0, \forall p, f, j \\
&\quad u_r, u_{s+p}, v_i, v_{m+f} \ge \varepsilon, \forall r, p, i, f.
\end{aligned} \tag{6}
$$

2.2 Common weights and efficiency invariance

To approach the issue of resource allocation and target setting, many previous researchers applied various ideas or principles to obtain their perspectives and insights. At any rate, the authors believe that there exist two essential principles which should be satisfied for all DEA-based resource allocation and/or target setting methods: common weights and efficiency invariance.

1) *Common weights*: The concept of common weights in DEA literature was first introduced by Roll et al. (1991). In this paper, the principle of common weights indicates that the input resource and output target must be allocated and set in such a way that all unknown relative weights should be determined simultaneously for all DMUs, and all DMUs' efficiencies are unchanged based on a common set of weights. A common set of weights means that all DMUs allow equal endogenous evaluations

on inputs and outputs in the reference set (Lotfi et al., 2013). This point is of considerable importance, since the central decision maker has the power to control and plan resources and outcomes, whereas individual DMUs have no control over resources and targets.

2) *Efficiency invariance*: This principle means that the relative efficiency scores before and after allocation (and also target setting) should remain unchanged. The rationale, initiated by Cook and Kress (1999), is that any improvement or deterioration of the relative efficiency scores is unreasonable and unfair. If efficiency variances are allowed, some inefficient DMUs would improve their efficiency scores by utilizing these allocated resources and setting targets accordingly, which is unacceptable to the decision makers whose DMUs have no control over the allocated resources and set targets (Lin and Chen, 2016). Besides, any unbalanced improvement of efficiency scores for efficient DMUs (which actually cannot improve) and inefficient DMUs would bring about some difficulties and organizational resistances in implementing the generated plan.

Both the two principles should be taken as necessary conditions for resource allocation and target setting. As a result, by using common weights the resource allocation and target setting should be addressed with concerning the current relative efficiency status.

Naturally, the following system (7) is required for satisfying common weights and the efficiency invariance principle simultaneously.

$$\begin{aligned}
&\frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{p=1}^{q} \tau_{pj}}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{f=1}^{g} \gamma_{fj}} = e_j^*, \forall j \\
&\sum_{j=1}^{n} \tau_{pj} = u_{s+p} T_p, \forall p \\
&\sum_{j=1}^{n} \gamma_{fj} = v_{m+f} R_f, \forall f.
\end{aligned} \tag{7}$$

Here, u_r, u_{s+p}, v_i and v_{m+f} are unknown common weights which represent equal evaluations of inputs and outputs, and e_j^* is the traditional CCR efficiency solved from model (1). It is clear that system (7) can always guarantee the usage of common weights, but in many real applications it does not necessarily satisfy the efficiency invariance principle, since e_j^* is a possible efficiency score but may not be the optimal. To consider the efficiency invariance principle more seriously, we must step further. To this end, model (8) is used to recalculate the relative efficiency score for $DMU_k\left(k=1,\ldots,n\right)$.

$$\begin{aligned}
& min\,\rho \\
& s.t.\sum\nolimits_{j=1}^{n}\lambda_j x_{ij} \le \rho x_{ik}, \forall i && (8a)\\
& \quad \sum\nolimits_{j=1}^{n}\lambda_j r_{fj} \le \rho r_{fk}, \forall f && (8b)\\
& \quad \sum\nolimits_{j=1}^{n}\lambda_j y_{rj} \ge y_{rk}, \forall r && (8c)\\
& \quad \sum\nolimits_{j=1}^{n}\lambda_j t_{pj} \ge t_{pk}, \forall p && (8d)\\
& \quad \lambda_j \ge 0, \forall j. && (8e)
\end{aligned}$$

To satisfy the efficiency invariance principle, we find that the optimal objective function of model (8) must be equal to that of model (2) for each DMU, basing on the comparison between model (8) and model (2). In addition, the optimal solutions of model (2) should also be optimal solutions to model (8). That is, to meet the efficiency invariance condition constraints (8b) and (8d) in model (8) must be redundant. As a result, the optimal solutions of model (2) will be determined such that constraints (8b) and (8d) automatically hold. Denote the optimal solution of model (2) as λ_j^{k*} when $DMU_k\left(k=1,\ldots,n\right)$ is under evaluation, and then the efficiency invariance condition is satisfied if system (9) is held.

$$\begin{aligned}
& \sum\nolimits_{j=1}^{n}\lambda_j^{k*} r_{fj} \le \theta_j^* r_{fk}, \forall f, \\
& \sum\nolimits_{j=1}^{n}\lambda_j^{k*} t_{pj} \ge t_{pk}, \forall p.
\end{aligned} \qquad (9)$$

This condition was firstly introduced by Cook and Zhu (2005), using a form in strict equality. Later, Lin (2011a) extended it to inequalities. Here the problem we are considering is resource allocation and target setting, therefore, both allocated input resources and set output targets are involved. If we separate the set of all original DMUs into two subsets based on model (2), with *E* for efficient DMUs and *N* for inefficient DMUs, system (9) reduces to system (10).

$$\begin{aligned}
& \sum\nolimits_{j\in E}\lambda_j^{k*} r_{fj} \le \theta_j^* r_{fk}, \forall f, k\in N, \\
& \sum\nolimits_{j\in E}\lambda_j^{k*} t_{pj} \ge t_{pk}, \forall p, k\in N.
\end{aligned} \qquad (10)$$

To sum up, the following system (11) is required if we want to approach the resource allocation and target setting problem based on the efficiency invariance principle.

$$\begin{aligned}
&\sum\nolimits_{j\in E}\lambda_j^{k*} r_{fj} \le \theta_j^* r_{fk}, \forall f, k \in N\\
&\sum\nolimits_{j\in E}\lambda_j^{k*} t_{pj} \ge t_{pk}, \forall p, k \in N\\
&\sum\nolimits_{j=1}^{n} t_{pj} = T_p, \forall p\\
&\sum\nolimits_{j=1}^{n} r_{fj} = R_f, \forall f.
\end{aligned} \tag{11}$$

2.3 Proposed model for resource allocation and target setting

As discussed in Section 2.2, systems (7) and (11) can be considered as formulations of the common weights and the efficiency invariance principle, respectively. If both systems (7) and (11) are satisfied by the same solution, then the common weights and efficiency invariance principles are simultaneously satisfied in the resource allocation and target setting plan. Without loss of generality, we introduce the resource allocation and target setting plan $\left(\left\{r_{fj}^{cw}\right\},\left\{t_{pj}^{cw}\right\}\right)$ that strictly adopts common weights based on system (7), and plan $\left(\left\{r_{fj}^{ei}\right\},\left\{t_{pj}^{ei}\right\}\right)$ that always satisfies the efficiency invariance principle based on system (11). To simultaneously seek common weights and keep efficiency unchanged as much as possible, we should minimize the distance from $\left(\left\{r_{fj}^{cw}\right\},\left\{t_{pj}^{cw}\right\}\right)$ to $\left(\left\{r_{fj}^{ei}\right\},\left\{t_{pj}^{ei}\right\}\right)$. Further, by taking the importance of each input-output measure into account, this can be implemented by minimizing the deviation of $\left(\left\{\gamma_{fj}^{cw}\right\},\left\{\tau_{pj}^{cw}\right\}\right)$ and $\left(\left\{\gamma_{fj}^{ei}\right\},\left\{\tau_{pj}^{ei}\right\}\right)$. The above idea can be modeled as model (12).

$$
\begin{aligned}
& d^* = min\sum\nolimits_{j=1}^{n}\left(\sum\nolimits_{f=1}^{g}\left|\gamma_{fj}^{cw}-\gamma_{fj}^{ei}\right|+\sum\nolimits_{p=1}^{q}\left|\tau_{pj}^{cw}-\tau_{pj}^{ei}\right|\right) \\
& s.t.\sum\nolimits_{r=1}^{s}u_r y_{rj}+\sum\nolimits_{p=1}^{q}\tau_{pj}^{cw}=e_j^*\left(\sum\nolimits_{i=1}^{m}v_i x_{ij}+\sum\nolimits_{f=1}^{g}\gamma_{fj}^{cw}\right),\forall j \\
& \quad \sum\nolimits_{j=1}^{n}\tau_{pj}^{cw}=u_{s+p}R_p,\forall p \\
& \quad \sum\nolimits_{j=1}^{n}\gamma_{fj}^{cw}=v_{m+f}R_f,\forall f \\
& \quad \sum\nolimits_{j\in E}\lambda_j^{k*}\gamma_{fj}^{ei}\le\theta_j^*\gamma_{fk}^{ei},\forall f,k\in N \\
& \quad \sum\nolimits_{j\in E}\lambda_j^{k*}\tau_{pj}^{ei}\ge\tau_{pk}^{ei},\forall p,k\in N \\
& \quad \sum\nolimits_{j=1}^{n}\tau_{pj}^{ei}=u_{s+p}R_p,\forall p \\
& \quad \sum\nolimits_{j=1}^{n}\gamma_{fj}^{ei}=v_{m+f}R_f,\forall f \\
& \quad \tau_{pj}^{cw},\tau_{pj}^{ei}\ge 0,\gamma_{fj}^{cw},\gamma_{fj}^{ei}\ge 0,\forall p,f,j \\
& \quad u_r,u_{s+p},v_i,v_{m+f}\ge\varepsilon,\forall r,p,i,f.
\end{aligned} \tag{12}
$$

Here the constraints in model (12) are mixture of systems (7) and (11). Note that u_{s+p} and v_{m+f} are also used to convert system (11) into its current formulation in model (12).

Further, by substituting $\left|\gamma_{fj}^{cw}-\gamma_{fj}^{ei}\right|+\gamma_{fj}^{cw}-\gamma_{fj}^{ei}=2a_{fj},\left|\gamma_{fj}^{cw}-\gamma_{fj}^{ei}\right|-\gamma_{fj}^{cw}+\gamma_{fj}^{ei}=2b_{fj}$ and $\left|\tau_{pj}^{cw}-\tau_{pj}^{ei}\right|+\tau_{pj}^{cw}-\tau_{pj}^{ei}=2\alpha_{pj},\left|\tau_{pj}^{cw}-\tau_{pj}^{ei}\right|-\tau_{pj}^{cw}+\tau_{pj}^{ei}=2\beta_{pj}$, model (12) is equivalently converted into the following linear model (13).

$$
\begin{aligned}
& d^* = min\sum\nolimits_{j=1}^{n}\left[\sum\nolimits_{f=1}^{g}\left(a_{fj}+b_{fj}\right)+\sum\nolimits_{p=1}^{q}\left(\alpha_{pj}+\beta_{pj}\right)\right] \\
& s.t.\sum\nolimits_{r=1}^{s}u_r y_{rj}+\sum\nolimits_{p=1}^{q}\tau_{pj}^{cw}=e_j^*\left(\sum\nolimits_{i=1}^{m}v_i x_{ij}+\sum\nolimits_{f=1}^{g}\gamma_{fj}^{cw}\right),\forall j \\
& \quad \sum\nolimits_{j=1}^{n}\tau_{pj}^{cw}=u_{s+p}T_p,\forall p \\
& \quad \sum\nolimits_{j=1}^{n}\gamma_{fj}^{cw}=v_{m+f}R_f,\forall f \\
& \quad \sum\nolimits_{j\in E}\lambda_j^{k*}\gamma_{fj}^{ei}\le\theta_j^*\gamma_{fk}^{ei},\forall f,k\in N \\
& \quad \sum\nolimits_{j\in E}\lambda_j^{k*}\tau_{pj}^{ei}\ge\tau_{pk}^{ei},\forall p,k\in N \\
& \quad \sum\nolimits_{j=1}^{n}\tau_{pj}^{ei}=u_{s+p}T_p,\forall p \\
& \quad \sum\nolimits_{j=1}^{n}\gamma_{fj}^{ei}=v_{m+f}R_f,\forall f \\
& \quad \gamma_{fj}^{cw}-\gamma_{fj}^{ei}=a_{fj}-b_{fj},\forall f,j \\
& \quad \tau_{pj}^{cw}-\tau_{pj}^{ei}=\alpha_{pj}-\beta_{pj},\forall p,j \\
& \quad \tau_{pj}^{cw},\tau_{pj}^{ei}\ge 0,\gamma_{fj}^{cw},\gamma_{fj}^{ei}\ge 0,\forall p,f,j \\
& \quad u_r,u_{s+p},v_i,v_{m+f}\ge\varepsilon,\forall r,p,i,f \\
& \quad a_{fj},b_{fj},\alpha_{pj},\beta_{pj}\ge 0,\forall f,p,j.
\end{aligned} \tag{13}
$$

Using simple linear algebra, it is easy to verify that model (13) is always feasible.

Based on the optimal objective function of model (13), we have:

Theorem 1: *The resource allocation and target setting plan satisfying common weights and the efficiency invariance principle simultaneously can be obtained if the optimal objective function of model (13) reaches zero, i.e., d*=0.*

Proof: Given $d^*=0$, it holds that $a_{fj}^* = b_{fj}^* = \alpha_{pj}^* = \beta_{pj}^* = 0, \forall f, p, j.$

Again, note that $\gamma_{fj}^{cw} - \gamma_{fj}^{ei} = a_{fj} - b_{fj}, \forall f, j$ and $\tau_{pj}^{cw} - \tau_{pj}^{ei} = \alpha_{pj} - \beta_{pj}, \forall p, j$, so we have $\gamma_{fj}^{cw*} - \gamma_{fj}^{ei*} = 0, \forall f, j$ and $\tau_{pj}^{cw*} - \tau_{pj}^{ei*} = 0, \forall p, j$.

Therefore, it must be true that $\gamma_{fj}^{cw} = \gamma_{fj}^{ei}\left(\forall f, j\right)$ and $\tau_{pj}^{cw} = \tau_{pj}^{ei}\left(\forall p, j\right)$. This implies that the resource allocation and target setting plan based on common weights also satisfies the efficiency invariance principle. □

If the optimal objective function of model (13) is greater than zero, then it is impossible to strictly satisfy the common weights and efficiency invariance principles simultaneously. In that case, there exist two optimal plans for resource allocation and target setting, namely, $\left(r_j^{cw*}, t_j^{cw*}\right) = \left(\left\{r_{fj}^{cw*}\right\}, \left\{t_{pj}^{cw*}\right\}\right)$ and $\left(r_j^{ei*}, t_j^{ei*}\right) = \left(\left\{r_{fj}^{ei*}\right\}, \left\{t_{pj}^{ei*}\right\}\right)$. The former strictly uses common weights to approach the resource allocation and target setting, and it has the least distance to the optimal plan based on the efficiency invariance principle. In other words, the plan $\left(r_j^{cw*}, t_j^{cw*}\right)$ is generated in such a way that a common set of weights are imposed on input-output measures for all DMUs and the change of efficiency scores is reduced as much as possible. On the contrary, to address the resource allocation and target setting the second plan always satisfies the efficiency invariance principle, and it also has the least distance to the optimal plan with common weights. Then, the plan $\left(r_j^{ei*}, t_j^{ei*}\right)$ is generated in such a way that the efficiency scores remain unchanged for all DMUs and the difference in relative weights of input-output measures made by different DMUs is minimized.

3 Numerical Applications

In this section we will use both a numerical example from previous literature and a real case in China to illustrate the proposed approach. Firstly in Section 3.1, we consider the dataset from Cook and Kress (1999) and compare our results with some other methods. Afterwards, in Section 3.2, the proposed approach is applied to an urban bus company in China.

3.1 The Cook and Kress (1999) case

For the sake of a comprehensive comparison with some similar methods in the literature, here we use the fixed cost allocation problem in Cook and Kress (1999) as a numerical example to illustrate the proposed resource allocation and target setting approach. This example has been studied by many papers in the DEA literature and can be taken as a special case for simultaneous resource allocation and target setting, in which there are no output targets to be set, hence we have T=0. As shown in Table 2, there exist twelve DMUs, with each consuming three inputs to produce two outputs (m=3, s=2), and there exists a common resource R=100 (here meaning the shared cost) to be allocated across all DMUs.

Table 2
A simple example from Cook and Kress (1999).

DMU	Input 1	Input 2	Input 3	Output 1	Output 2
1	350	39	9	67	751
2	298	26	8	73	611
3	422	31	7	75	584
4	281	16	9	70	665
5	301	16	6	75	445
6	360	29	17	83	1070
7	540	18	10	72	457
8	276	33	5	78	590
9	323	25	5	75	1074
10	444	64	6	74	1072
11	323	25	5	25	350
12	444	64	6	104	1199

Solving model (1) n times determines a set of relative efficiencies for all DMUs, which is given in the second column in Table 3. Through running such an input-oriented CCR-DEA model on these data in Table 2, we obtain five efficient DMUs (DMU_4, DMU_5, DMU_8, DMU_9 and DMU_{12}) and seven inefficient DMUs (DMU_1, DMU_2, DMU_3, DMU_6, DMU_7, DMU_{10} and DMU_{11}). Solving model (13) determines an equitable allocation for the common resource. The optimal objective function will be 3.5222e-9, implying that for this case the two principles, common weights and efficiency invariance, cannot be strictly satisfied simultaneously. As a result, two possible allocation plans will be generated, with one using common weights to minimize the efficiency change as much as possible, while the other keeping the efficiencies unchanged and making the evaluations on inputs and outputs for individual DMUs be similar or even identical as much as possible. The allocation results and post-allocation efficiencies are given in last four columns of Table 3. It is notable that the values of the common resource allocated to each DMU are different for these two plans, but the largest difference emerges for DMU_9 and reaches only 0.1125. Therefore, we can conclude that although the two principles cannot be simultaneously satisfied for this numerical example, the two possible allocation plan can be very nearly the same in our method. For the first plan emphasizing common weights, five inefficient DMUs (DMU_1, DMU_3, DMU_6, DMU_7 and DMU_{10}) are found to have different efficiency scores relative to the original pre-allocation efficiencies, while the remaining seven DMUs have no changes in efficiency scores. For the second plan emphasizing efficiency invariance, the post-efficiencies are absolutely the same as the original CCR efficiency for all DMUs and no variations of efficiency scores would emerge.

Table 3
Results.

DMU	CCR efficiency	Common weights		Efficiency invariance	
		Allocation	Efficiency	Allocation	Efficiency
1	0.75670	8.7611	0.76295	8.7703	0.75670
2	0.92300	7.8689	0.92300	7.8771	0.92300
3	0.74702	9.9663	0.75235	9.9731	0.74702
4	1.00000	6.9661	1.00000	6.9705	1.00000
5	1.00000	7.4589	1.00000	7.4605	1.00000
6	0.96123	8.6296	0.96351	8.6342	0.96123
7	0.86041	8.3224	0.86046	8.3320	0.86041
8	1.00000	7.7280	1.00000	7.7326	1.00000
9	1.00000	7.5359	1.00000	7.4234	1.00000
10	0.83178	8.8650	0.84849	8.9192	0.83178
11	0.33333	7.5656	0.33333	7.5700	0.33333
12	1.00000	10.3291	1.00000	10.3351	1.00000

Besides this, extreme zeroes are avoided for both of the allocation plans. Accepting the suggestion of Lin (2011a) that an equitable and fair allocation should assign a positive value to each DMU, our allocations are reasonable and acceptable.

To demonstrate some significant features of the proposed approach, we will compare our results with some other methods. It is noteworthy that very few articles are found to use both the common weights and efficiency invariance principles simultaneously, hence we will address the comparison from these two principles separately, as given in Table 4 and Table 5 respectively.

Table 4

Different allocations based on common weights.

DMU	Our approach	Beasley (2003)	Amirteimoori and Kordrostami (2005)	Si et al. (2013)	Li et al. (2013)	Lotfi et al. (2013)
1	8.7611	6.78	8.2196	7.6475	6.3839	8.199
2	7.8689	7.21	6.8582	8.4118	7.4219	7.462
3	9.9663	6.83	9.4972	8.6216	6.6827	4.284
4	6.9661	8.47	6.3242	8.1091	8.8327	9.301
5	7.4589	7.08	6.6768	8.6938	7.6335	4.807
6	8.6296	10.06	8.3817	9.5669	9.6989	15.370
7	8.3224	5.09	11.7389	8.3333	4.2765	0
8	7.7280	7.74	6.4879	9.9628	8.3526	7.339
9	7.5359	15.11	7.2912	8.6505	15.8710	16.330
10	8.8650	10.08	10.6125	8.3457	9.7510	11.598
11	7.5656	1.58	7.2885	2.8032	0.4550	0
12	10.3291	13.97	10.6233	11.854	14.6404	15.310

For the resource allocation plan based on common weights, several results from Beasley (2003), Amirteimoori and Kordrostami (2005), Si et al. (2013), Li et al. (2013) and Hosseinzadeh Lotfi et al. (2013) are also provided here in Table 4. All these methods address the resource allocation problem in such a way that a common set of weights is attached to each DMU, but our proposed approach tries to keep the relative efficiency unchanged as much as possible, while the other methods are developed for the purpose of efficiency-maximization. Consequently, the post-allocation efficiencies in our method change very little as compared with the original pre-allocation efficiency scores, while the allocation plans generated by the other methods improve the efficiency scores for some DMUs. Moreover, as the efficiency principle used in our proposed approach reflects the current relative efficiency status based on given inputs-outputs, the gap of the value of the allocated resource among these twelve DMUs is smaller than the other five methods in Table 4 (3.3631<12.39, 5.4147, 9.0508, 14.1854, 15.310). Since a smaller gap implies less difficulty and organizational resistance to implement the generated allocation (Li et al., 2009; Fang, 2015), from this perspective our allocation would be more acceptable to all DMUs.

Table 5

Different allocations based on efficiency invariance.

DMU	Our approach	Cook and Kress (1999)	Cook and Zhu (2005)	Lin (2011a)	Mostafaee (2013)	Lin and Chen (2016)
1	8.7703	14.52	11.22	5.69555	8.89296	9.83
2	7.8771	6.74	0	9.24432	8.89296	7.53
3	9.9731	9.32	16.95	5.47825	8.89296	9.93
4	6.9705	5.60	0	10.1637	8.89296	5.20
5	7.4605	5.79	0	7.08163	6.65446	5.20
6	8.6342	8.15	15.43	4.93396	8.89296	9.10
7	8.3320	8.86	0	8.39439	8.89296	5.85
8	7.7326	6.26	0	7.33435	6.65446	8.96
9	7.4234	7.31	17.62	2.92289	6.65446	8.07
10	8.9192	10.08	21.15	3.50746	8.89296	9.69
11	7.5700	7.31	17.62	2.92288	8.89296	8.07
12	10.3351	10.08	0	32.32062	8.89296	12.56

For the resource allocation plan based on the efficiency invariance principle, several results from Cook and Kress (1999), Cook and Zhu (2005), Lin (2011a), Mostafaee (2013), and Lin and Chen (2016) are also provided here in Table 5. All these methods address the resource allocation problem in such way that the post-allocation efficiency scores are the same as that of the pre-allocation efficiency scores, but our proposed approach tries to perform similar or even identical evaluations on inputs and outputs for all DMUs as much as possible, while the other methods have no constraints on weights and different sets of weights are attached to these inputs and outputs by different DMUs. As a result, the allocation in our proposed approach is supposed to be more acceptable as common evaluations of these input-output measures can be easily accepted by all DMUs.

Among these methods in Table 5, Cook and Kress (1999) allocates the same fixed cost to some DMUs when they have identical inputs but different outputs, which is the case for two pairs: DMU_9 and DMU_{11}, DMU_{10} and DMU_{12}. Therefore, Cook and

Kress (1999) determine the cost allocation entirely from the input side, as argued by Beasley (2003). It is notable that all these efficiency-invariance-based methods, including our proposed approach, will allocate the same or approximately the same resources to DMUs with identical inputs. For these two pairs, however, only our proposed approach will allocate different values of resources to these four DMUs, while the other methods in Table 5 will allocate an identical amount to one or two pairs. Note that DEA is a non-parametric methodology depending on both inputs and outputs, and from this perspective our allocation emphasizing the efficiency invariance principle is better than others as it fits the characteristics of the DEA framework (Lin and Chen, 2016). In addition, with the exception of the Mostafaee (2013) allocation, our allocation determines a minimum gap among these DMUs, which is important from the perspective of a smaller gap implying less difficulty and organizational resistance to implementation of the generated allocation (Li et al., 2009; Fang, 2015). Although the Mostafee (2013) allocation obtains the minimum gap among these six methods, its allocation plan allocates an identical amount to many different DMUs, and so it is thought to lack sufficient discrimination power for allocating different resources to DMUs.

3.2 A real application to the urban bus company

In this subsection we will apply the proposed approach to an empirical example of urban bus company activities. The dataset consists of a bus company with 24 public transportation lines located in Sichuan Province, China. The data for this paper is obtained from operations in 2014 and here each bus line is considered to be an independent DMU. We use four variables from the dataset as inputs and two variables as outputs. Inputs consist number of platforms (x_1), fixed assets (x_2), staff costs (x_3), and lastly, the operations costs except for the staff costs (x_4). Outputs include the punctuality rate relative to the time schedule (y_1) and number of passengers (y_2). The pre-specified inputs and outputs used in this paper are accordingly summarized in Table 6, and Table 7 lists the original data.

In late 2014, the bus company made an attempt to delegate the right of advertisement sales to each bus line. Since the advertisement values derived from each line are different, such an attempt is supposed to make full use of its individual advertisement values. Based on this background, the headquarters of this bus company would like to arrange all its available administrative staffs to sell the advertising boards for these 24 bus lines. Now this company has 76 administrative employees responsible for the advertisement sales, and a total sales target in the next period is set to nine million Yuan. Naturally, the problem emerges of how to dispatch these 76 administrative employees to these 24 bus lines and accordingly set sales targets for each bus line in an equitable and fair way. As a result, here we have R=76 and T=900. For the simplification of research and without loss of generality, here we assume the allocated resources (i.e., administrative employees) and set targets (sales of advertisements) are continuous variables.

Since this is a new attempt for this bus company to sell advertisements, the attitudes of all bus lines are very important. To guarantee the successful implementation of this reforming attempt, the bus company needs to communicate with all of its bus lines. Also, these bus lines should reach a consensus on the resource allocation and target setting plan. Specifically, all bus lines expect to bargain for an equal evaluation (i.e., common weights) of these input-output measures, which promises a common evaluation for their relative efficiencies when generating the plan. Besides, as the current relative performance is a main indicator for the bus company to evaluate its bus lines, the allocated resources and set targets should not be intentionally used to change the efficiency scores. From this perspective the efficiency invariance principle is also a preliminary condition. To sum up, both common weights and efficiency invariance are desired by this bus company and its bus lines, thus our proposed approach can be of important significance.

Based on the traditional CCR model (1), we obtain the relative efficiency scores for all 24 bus lines, as given in the second column in Table 8.

Table 6
Input and output variables.

Input/output	Variable	Unit
Input	Platform	Number count
	Fixed assets	10-thousand Yuan
	Staff costs	10-thousand Yuan
	Operations costs (except for the staff costs)	10-thousand Yuan
Output	Punctuality rate	Percentage
	Passenger	10-thousand person time

Table 7
Input-output data for 24 bus lines.

DMU	x_1	x_2	x_3	x_4	y_1	y_1
1	19	2436.41	297.08	2074.65	96.97	1647.67
2	27	4966.40	631.59	2933.43	87.13	1277.1
3	19	2342.40	354.07	3088.38	88.41	1470.86
4	19	3412.10	336.40	2813.00	74.75	1437.71
5	33	3287.51	490.37	3320.14	84.14	1331.88
6	24	2873.10	389.97	1461.32	98.51	994.68
7	22	2353.91	701.17	1363.37	98.77	855.67
8	23	2468.50	635.04	1986.59	70.52	1141.15
9	22	4102.80	585.93	1990.38	98.97	1130.23
10	28	4237.10	767.45	1095.78	98.50	981.38
11	23	2963.51	714.76	1660.33	81.99	862.33
12	18	2412.91	466.44	1158.30	93.01	921.93
13	21	2068.90	642.36	1825.54	69.97	806.02
14	23	3237.20	703.35	2051.99	79.76	1049.16
15	19	2763.11	611.42	1186.76	97.05	862.25
16	24	3972.40	537.24	2523.40	92.73	1173.32
17	16	3141.61	406.52	676.44	98.58	613.66
18	19	2373.70	431.42	1751.26	87.95	1084.97
19	17	3352.31	557.25	1235.25	76.25	1202.87
20	23	3614.00	762.99	1906.61	94.72	1010.29
21	19	3292.61	643.79	1076.34	97.69	951.3
22	26	4700.81	733.14	2725.06	88.76	1296.05
23	23	4386.70	575.44	1533.30	91.52	1211.16
24	22	2871.11	689.71	2738.35	91.01	1603.09

It can be seen that five units are efficient with an efficiency score of one (DMUs 1, 7, 12, 17, and 19), while the remaining nineteen DMUs are inefficient with a score strictly less than one. The relative efficiency scores vary greatly for different DMUs, from 0.6088 to 1.0000. Also, there are many efficiency improvement potentials for these bus lines. Solving model (13) determines an optimal objective function which is equal to zero, implying that the two principles of common weights and efficiency invariance can be satisfied simultaneously for this application. Consequently, our method determines the resource allocation and target setting scheme, as given in the third to eighth columns in Table 8. Since the two principles are satisfied simultaneously, here the two schemes are identical.

Table 8
Results of resource allocation and target setting.

DMU	Efficiency	Common weights			Efficiency invariance			T/R
		Resource	target	Efficiency	Resource	Target	Efficiency	
1	1.0000	2.7819	39.5295	1.0000	2.7819	39.5295	1.0000	14.2096
2	0.6088	4.0212	34.7866	0.6088	4.0212	34.7866	0.6088	8.6508
3	0.9463	2.7015	36.3262	0.9463	2.7015	36.3262	0.9463	13.4466
4	0.8726	2.7818	34.4923	0.8726	2.7818	34.4923	0.8726	12.3993
5	0.6385	3.8788	35.1916	0.6385	3.8788	35.1916	0.6385	9.0729
6	0.9579	2.8880	39.3093	0.9579	2.8880	39.3093	0.9579	13.6114
7	1.0000	3.3617	47.7686	1.0000	3.3617	47.7686	1.0000	14.2096
8	0.7226	2.8357	29.1166	0.7226	2.8357	29.1166	0.7226	10.2679
9	0.8032	3.3611	38.3611	0.8032	3.3611	38.3611	0.8032	11.4132
10	0.9454	3.0387	40.8210	0.9454	3.0387	40.8210	0.9454	13.4338
11	0.7056	3.6584	36.6805	0.7056	3.6584	36.6805	0.7056	10.0263
12	1.0000	2.9516	41.9412	1.0000	2.9516	41.9412	1.0000	14.2096
13	0.8210	2.7444	32.0164	0.8210	2.7444	32.0164	0.8210	11.6661
14	0.6552	3.4989	32.5751	0.6552	3.4989	32.5751	0.6552	9.3102
15	0.9547	3.0846	41.8450	0.9547	3.0846	41.8450	0.9547	13.5659
16	0.7047	3.6305	36.3538	0.7047	3.6305	36.3538	0.7047	10.0135
17	1.0000	2.5594	36.3676	1.0000	2.5594	36.3676	1.0000	14.2096
18	0.9275	2.8623	37.7233	0.9275	2.8623	37.7233	0.9275	13.1794
19	1.0000	2.6035	36.9955	1.0000	2.6035	36.9955	1.0000	14.2096
20	0.7417	3.4998	36.8853	0.7417	3.4998	36.8853	0.7417	10.5393
21	0.9867	2.9432	41.2657	0.9867	2.9432	41.2657	0.9867	14.0207
22	0.6434	3.8742	35.4196	0.6434	3.8742	35.4196	0.6434	9.1425
23	0.8697	3.2180	39.7682	0.8697	3.2180	39.7682	0.8697	12.3581
24	0.8403	3.2210	38.4600	0.8403	3.2210	38.4600	0.8403	11.9404

Based on the optimal resource allocation and target setting results in Table 8, it is easy to verify that the post-allocation efficiency scores will be identical to the pre-allocation efficiencies after the allocated resources and set targets are taken as additional inputs and outputs. It is notable that Bus Line 2 will receive the most resources among these 24 lines, and on the contrary, Line 17 will receive less resources compared with the other lines. On the other hand, Bus Line 7 will be set a larger target than all other lines, whereas the smallest target is set for Line 8. These findings highlight the connection between the current efficiencies and the possible target-to-resource ratios in our proposed approach. It shows that the bus line with lower efficiency score would like to obtain a smaller target-to-resource ratio, as demonstrated by the results in the ninth column in Table 8. Besides, it can be seen that 13 of the DMUs receive a share of the total administrative employees less than the average value (76/24=3.1667), and 11 of the DMUs are responsible for a larger advertisement sales than the average quota (900/24=37.5000). We can conclude that the allocation and target plan is equitable enough. Also, 21.47% of the resources are allocated to these five efficient bus lines, which is consistent with the fact that 20.06% of the targets are set to efficient DMUs. This demonstrates again that we set the shares of output targets intentionally being consistent with the allocation shares of input resources.

In addition, we find that the additional inputs and outputs (i.e., allocated resource and set targets) are consistent with the current inputs and outputs. In other words, the DMUs with less inputs and more outputs are more likely to receive less resources and be set larger targets, as compared with the others. For example, DMU_1's inputs are less than DMU_2, DMU_5, DMU_{14}, DMU_{16}, DMU_{22} and DMU_{24}, and its outputs are more than these DMUs. The results show that the resource allocated to DMU_1 is less than these six other DMUs, and also, the target set for DMU_1 is the largest among these seven DMUs.

All in all, from the above comparison and analysis for the numerical example and empirical application, we can see that our proposed method for the resource allocation and target setting reduces the gap among all DMUs and has enough discrimination power to generate a plan allocating different shares of resources and targets to different DMUs. Besides these advantages, the current efficiency scores and input-output values are well considered in our proposed approach. Therefore, using the common weights and efficiency invariance principle simultaneously for resource allocation and target setting is very reasonable and attractive.

4 Conclusions and perspectives

The resource allocation and target setting problem is of vital importance in many managerial applications. In this paper, we have established a new general framework to allocate multiple resources and set multiple targets across DMUs. Our work is based on DEA-based models, which help a central decision maker to allocate multiple input resources and set multiple output targets across the DMUs by taking into account common weights and the efficiency invariance principle, simultaneously. Since the two criteria may not always be satisfied by a single plan, the proposed approach can produce two possible plans in general cases. One plan is generated in the sense of insisting on common weights, and the change of efficiency scores is avoided for each DMU as much as possible. On the contrary, the other plan is generated by insisting on keeping the efficiency scores unchanged for all DMUs, and the evaluations on inputs and outputs determined by each DMU are made to be similar or even identical as much as possible. Both of the plans are obtained at the same time by minimizing the distance of the two plans. Ideally, one optimal plan will be determined if the minimum deviation equals zero. Finally, the proposed approach was applied to a numerical example from previous literature and a real case of an urban bus company to demonstrate its efficacy and usefulness. The proposed approach would be of significant importance in circumstances where any individual DMU has no control of its efficiency but all DMUs try to reach a consensus on weighting these inputs and outputs. The two possible allocation schemes in general cases allow the central decision maker to make a trade-off between equal evaluations (i.e., common weights) and efficiency invariance consideration.

A basic feature of this paper is that all models in our approach are based on the constant returns to scale property, however, the proposed approach may be infeasible in some situations if the variable returns to scale assumption is incorporated. Actually, the BCC version of model (13) is feasible for the numerical example in Section 3.1, but infeasible for the empirical application in Section 3.2. Besides, if the two principles are strictly satisfied, only one optimal resource allocation and target setting plan will be determined. However, the non-uniqueness problem would be a main concern if two different plans are obtained. The effect is reduced in our method because there exist multiple plans based on emphasizing each principle, and for each principle multiple plans are generated so as to have the least distance to the set of plans emphasizing the other principle.

Note that this paper uses two essential principles (i.e., common weights and efficiency invariance) to approach the resource allocation and target setting problem, but we are not implying that other criteria such as equality and effectiveness are not important. Actually, many articles have worked on various criteria. Therefore, future research efforts may attempt to consider multiple principles comprehensively and focus on the according trade-offs to develop multiple-objective approaches. Besides, this paper adopts a radial efficiency concept, and similar approaches can also be addressed with non-radial models. In addition, this paper considers only conventional inputs and outputs. A possible research avenue may be the idea of taking undesirable outputs into account under natural and managerial disposability, as introduced by Sueyoshi and Goto (2010). The managerial disposability implies that a firm can reduce its undesirable outputs to a certain amount through increasing its input resources. We believe that the resource allocation and target setting problem under such situations can be of vital significance in real applications, especially when the environmental issue increasingly becomes a hot topic.

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