Application of the Weibull distribution for the optimization of maintenance policies of an electronic railway signaling system

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ABSTRACT: This paper presents the advantages of using Weibull distribution to describe reliability figures of an electronic railway signaling system, respect to the commonly used exponential distribution. After presenting the context of reliability estimations within the railway domain, this work introduces the approach to use field-return data to build up reliability parameters instead of predictive methodologies, and it is applied to an existing electronic signaling system. A comparison between the two methodologies is also provided, as well as the introduction of further reliability indicators.
A model to improve preventive maintenance tasks defining the optimum time interval is then presented and an example is provided. Some suggestions to improve the process of collecting field-return data are presented impacting both the after-sales logbook and the design of the circuit boards.

1 INTRODUCTION

The adequate choice of maintenance tasks type and periodicity for an electronic system is one of the most common problems when reliability and safety targets have to be achieved. Although corrective maintenance procedures are often taken into account since the design phase of an electronic system, preventive maintenance, intended as the planned or conditioned tasks aiming at preventing the occurrence of a failure or reducing its frequency, seems to be more complicated to be put in place.
This could be caused by the belief that electronic behavior is described by reliability functions based on constant failure rates throughout the entire life. This belief, mixed with reliability models aiming at describing the useful life of electronic components, determines a misinterpretation of reliability performances of complex systems as depicted by Jones & Hayes (1999), Cushing & Mortin (1993), Gray and Paschkewitz (2016).

Although CENELEC EN50129 (2003) standard addresses reliability demonstration as a mandatory step along the whole lifecycle of a railway project, the process to collect these evidences is not provided nor imposed. Moreover, reliability requirements are formalized since the bidding phase in terms of availability/reliability parameters. All these elements could contribute to an abuse of exponential distribution, determining a complete misalignment between predictive and operational reliability figures (Cheng et al. 2006, Roberts et al. 2006, Umiliacchi et al. 2011).
The economic disadvantage related to that kind of approach might deal with non-optimized maintenance policy. The lack of preventive maintenance tasks is justified by wrongly assuming a constant failure rate λ along the whole project life (Boucly, 1998). Moreover, maintenance plans might be defined at the beginning of the project and wrongly never updated if the reliability demonstration is based on unverified hypothesis of constant failure rate (Deloux, 2008, NF EN 60300-3-11, 2010). The final consequence is that the whole lifecycle cost of the system is dramatically underestimated and reliability targets might be not satisfied (Schenkelberg, 2015 and 2016).
The use of more realistic distributions, like the Weibull distribution, is widely adopted in several industrial applications and it has been demonstrated in several works pertaining railway infrastructure (Macchi et al. 2013). Aim of this work is to extend the use of Weibull distribution to railway electronic signaling systems, in which the shortcut of constant failure rate seems to be still preferred.

In order to prove the advantages of Weibull distribution instead of exponential one, field-return data of different projects and boards have been used to build up Weibull distribution in order to compare MBTF values. The use of datasets coming from after-sales database that are not designed for reliability purposes presents some limits that have to be managed by additional efforts before building reliability distributions. In particular, data are affected by right-censoring because not all the boards of the projects have failed. This point does not impact the determination of reliability parameters, but it has to be handled by means of adaptive approach and Weibull parameters have to be corrected when new failures will occur. Moreover, due to the heterogeneity of projects, some propositions to enhance the reliability of collected data are proposed. These propositions deal with implementations of new functions of the board, allowing monitoring several parameters.

In the meanwhile, these data represent a good basis to determine reliability figures, compared to the use of exponential distributions. Once datasets have been opportunely manipulated, several reliability figures have been determined, allowing evaluating the impact over the time of using exponential or Weibull distribution. In particular, a parametric approach for the optimization of preventive maintenance has been proposed. It is based on the estimation of costs of preventive and corrective maintenance tasks, as well as the reliability parameters of the analyzed distribution.

2 WEIBULL CHARACTERIZATION OF AN ELECTRONIC RAILWAY SIGNALING SYSTEM

2.1 *Use-case definition*

In order to shift from constant predictive reliability parameters to time-dependent field-based figures, the Weibull distribution has been chosen to describe the behavior of an electronic railway signaling system, identified as use-case (Lyonnet 1992, Birolini 2007). The Weibull distribution, widely used in life data analysis, is mathematically defined by its *pdf* (Probability Density Function) equation:

$$f(T) = \frac{\beta}{\eta}\left(\frac{T-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{T-\gamma}{\eta}\right)^{\beta}} \tag{1}$$

Where:

β is the shape parameter, also known as the Weibull slope;

η is the scale parameter;

γ is the location parameter.

One of the most important aspects of this distribution is the impact of β value on the whole Weibull distribution. If $\beta < 1$, the failure rate decreases with time, and this condition is known as *infantile* or *early-life failures*. Weibull distributions with β close to or equal to 1 have a fairly constant failure rate, indicative of useful life or random failures.

When $\beta > 1$, the distribution presents a failure rate that increases with time, and the failures are known as *wear-out failures*. These comprise the three sections of the classic "bathtub curve". A mixed Weibull distribution with one subpopulation with $\beta < 1$, another with $\beta = 1$ and a third one with $\beta > 1$ would have a failure rate plot identical to the bathtub curve (Lyonnet 1992, Birolini 2007).

Since the number of field-return data needs to be relevant to determine significant Weibull parameters, the reference use-case is a signaling system based on electronic boards already deployed and in revenue service since years. Both wayside and on-board equipment have been considered for this application, because of the possible impact on the whole maintenance costs.

The reference system is composed by eleven types of circuit boards that are opportunely combined to implement the complete signaling system. All these circuit boards are the line replaceable units (LRU) of the system; a LRU is the element that can be replaced on site and with standard techniques and tools to restore the operating status following a failure.

2.2 *Field-return data preparation*

The database used to collect field-return data is the after-sales service logbook. This department collects all the boards brought back to the maintenance workshops for repairing when they are out of service or have experienced supposed failures after the deployment. A huge amount of information is available within this logbook:

- *Name and serial number of the board*: it allows to uniquely identifying the circuit board.
- *The customer and the project data*: although this information seems not related to reliability, it provides a useful link to the relevant people involved in the project to gather critical information, not included within the logbook. This information is also useful to perform project-specific reliability demonstration tests and evaluations.
- *The part number and revision number*: allowing to clearly associating a specific bill of material (BOM). A single circuit board could have different BOMs during his life, in case of obsolescence phenomena or retrofit.
- *The date* at which the board is brought back to the after sales service. This information is crucial to determine the TTF (*Time To Failure*), but it has to

be completed with further details such the deployment date. Moreover, depending on the customer's policy, this could potentially not be the date at which the failure has occurred. This phenomenon affects the accuracy of TTF calculation and the subsequent reliability calculation.

-*Customer analysis*: typically, it describes customer observations about the failure event.

-*After-Sales analysis*: based on the knowledge of the circuit board and on the analysis of the customer, it describes the real failure that affected the board. In some cases, no failures are detected and the logbook reports the "No Fault Found" mention.

Despite the number and the quality of information included into the after-sales logbook is impressive, the structure of that database has not been designed with the purpose of facilitating reliability evaluations: the number of boards deployed within a given project as well as the deployment date are not included into the logbook. For that reason, key resources of the targeted projects shall be systematically involved.

2.3 *Field-return data analysis: the case of censored data*

After-sales logbook has been deeply analyzed by selecting the eleven types of boards mentioned before. The number of logs and project is huge: several thousands of logs and dozens of projects are indicated, covering a period from 90's until nowadays.

Investigations have been made to gather all the possible data from the maximum number of projects.

Some old projects were affected by censored data, meaning that observations of failures and TTF are only partial known. It is the case of projects for which the complete deployment of the signaling system has lasted months or years, so that for the boards brought back to the after-sales it is not possible to properly calculate the TTF.

In other cases, the start of revenue service phase has been delayed although the signaling system was already deployed. It is then not possible to estimate the TTF due to the fact that the time in which the system has been powered on and used is not known.

Another source of data censoring is the supply policy of specific customers: in some cases, customers buy big lots of boards that are stocked in a dispatching center during an unknown period. The customer decides when and which project has to be provided with a given number of boards, impacting the reliability analysis.

Censoring might also affect the moment at which the failure is accounted:

- On-field failure are not immediately recorded, but they might depend on the frequency of inspections of the system;
- The customer might wait for collecting several circuit boards before bring them back to the provider's maintenance workshop, so that the moment at which they are recorded in after-sales logbook might hide relevant information in terms of reliability.

Even some already known chart formats designed for reliability purposes, such as the *Nevada charts* for warranty data analysis, might not be sufficient to cope with that kind of problems.

2.4 *Field-return data analysis: calculation of Weibull parameters*

Some projects already in revenue service have been chosen because of the accuracy of information provided within the logbook; these data have been mixed with interviews to relevant people involved in each project. This allowed gathering more complete information for the purpose of the calculation of Weibull parameters.

Mainly three projects have been selected and the field data of one wayside (hereafter called "Board_WS", where *WS* means wayside) and one on-board circuit board (hereafter called "Board_OB", where *OB* means onboard) have been collected. Table 1 shows synthetic information about each project, called *Prj1*, *Prj2* and *Prj3*: the revenue service date and the number of boards installed. These data are essential to evaluate reliability figures. TTF information for Wayside board have been gathered both on Prj1 and Prj2.

Table 1. Boards vs. projects information matrix

Board	Project	Revenue Service	Number of boards
Board_WS	Prj1	2003	122
Board_WS	Prj2	2007	172
Board_OB	Prj3	2009	336

Cumulated failures and TTF information have been reported for each [board, project] couple on a Weibull chart, as depicted in Figure1, Figure2 and Figure3.

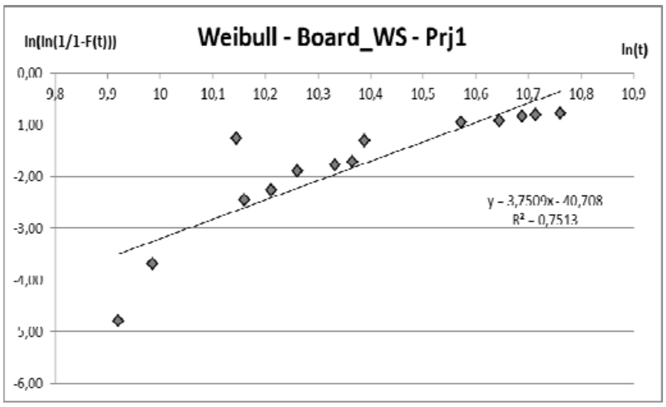

Figure 1. Weibull chart of Board_WS within Prj1 project

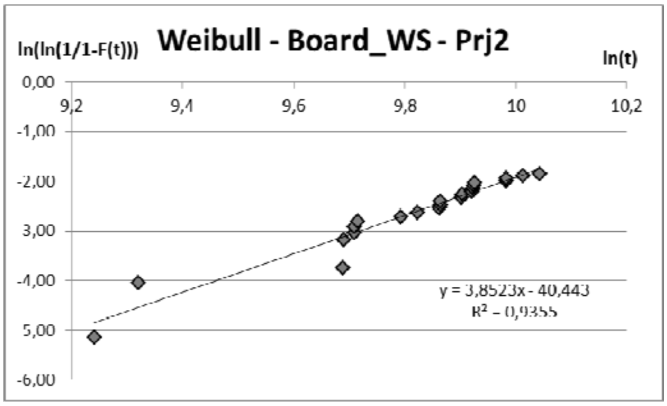

Figure 2. Weibull chart of Board_WS within Prj2 project

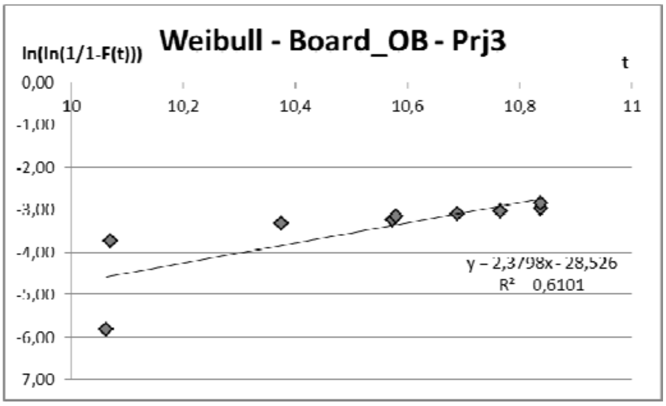

Figure 3. Weibull chart of Board_OB within Prj3 project

A rough graphic approach quickly helps understanding that the β factor is far from assuming '1' value in all the three cases. Consequently, the hypothesis of constant failure rate, proper of exponential distribution, can't be retained. Datasets have been tested to verify if data follow Weibull distribution. The coefficient of determination "R^2" has been used, and it has values 0,7513 for Prj1, 0,9355 for Prj2 and 0,6101 for Prj3. We can conclude that data follow Weibull distribution. Numeric iterative techniques, by using the "least squares" method, have been implemented allowing calculating a straight line that best fits available data. The equation of this straight line is presented in each Weibull chart, and the slope represents the Weibull parameter β.

Datasets used for the determination of Weibull distribution are mainly right-censored: this is not limiting the determination of the parameters, but it is necessary to correct them when new data will be available through an adaptive approach along the whole project life.

Table 2 recaps for each card and project what are the exact β and η values. The β value >1 confirms that the distribution is in its wear-out phase. Considering that Board_WS is the same for both Prj1 and Prj2, we can conclude that the behavior of the distribution is not exactly the same within the two projects.

Table 2. Weibull Parameters β and η for use-case boards

Board	Project	β	η
Board_WS	Prj1	3.75	51,683
Board_WS	Prj2	3.85	36,264
Board_OB	Prj3	2.37	160,667

Once β and η have been calculated, it is then possible to determine and plot the failure rate over the time, comparing the behavior in case of exponential distribution versus the case of Weibull one.

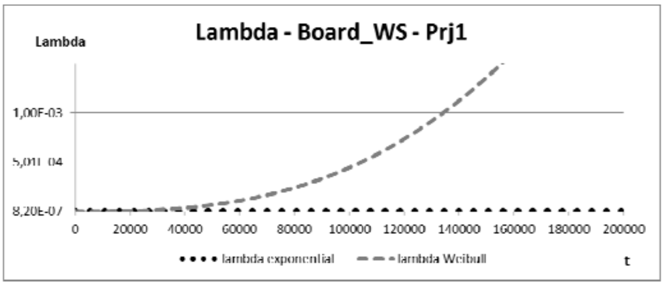

Figure 4. Comparison of Failure Rates for Prj1

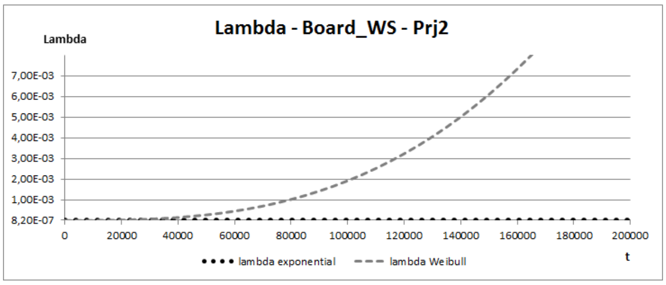

Figure 5. Comparison of Failure Rates for Prj2

Figure 4 and 5 compare the constant failure rate, according to exponential distribution, and the time-dependent failure rate, according to Weibull distribution, over the time. It can be observed that the two values quickly diverge and that after 10 years (100,000 hours) they have almost four orders of magnitude of difference.

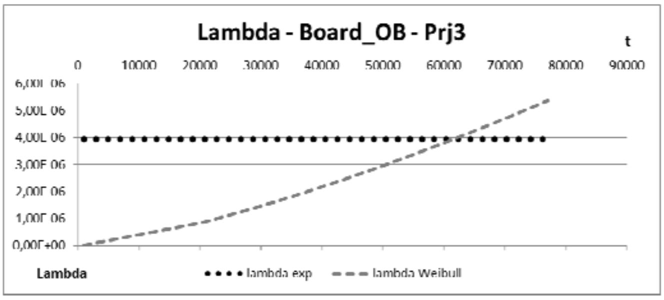

Figure 6. Comparison of Failure Rates for Prj3

Figure 6 presents the same comparison for Board_OB and Prj3. In this case the divergence between the two values is restrained due to the fact that β value is lower than the two previous cases,

Table 3 presents a comparison between the predictive MTBF (*Mean Time Between Failures*) figure calculated according to the exponential distribution and the MTBF figure calculated according to Weibull distribution. Exponential MTBF is calculated by using "*RDF2000*" reliability prediction models and mission profile and stress parameters have been chosen according to project requirements. RDF2000 is a reliability prediction standard providing empirical formula to deduce MTBF for several families of components by taking into account environmental and qualitative parameters (Institut de la Maitrise des risques 2009). Limits of constant failure rate approach appear evident in Table 3, since Board_WS presents the same exponential MTBF value for Prj1 and Prj2, instead of Weibull based MTBF that is calculated by using β and η values previously calculated. This application underlines also the fact that Weibull MTBF is more limiting compared with the exponential one.

Table 3. Exponential vs. Weibull MTBF

Board	Project	Exp. MTBF [h]	Weibull MTBF [h]
Board_WS	Prj1	$1.22\ 10^{6}$	$4.69\ 10^{4}$
Board_WS	Prj2	$1.22\ 10^{6}$	$3.28\ 10^{4}$
Board_OB	Prj3	$2.54\ 10^{5}$	$1.42\ 10^{5}$

Finally, the reliability function for each board within each given project is plotted for both exponential

$$R(t) = \exp^{\left(-\lambda t\right)} \tag{2}$$

and Weibull distribution.

$$R(t) = \exp^{\left(-\frac{t}{\eta}\right)^{\beta}} \tag{3}$$

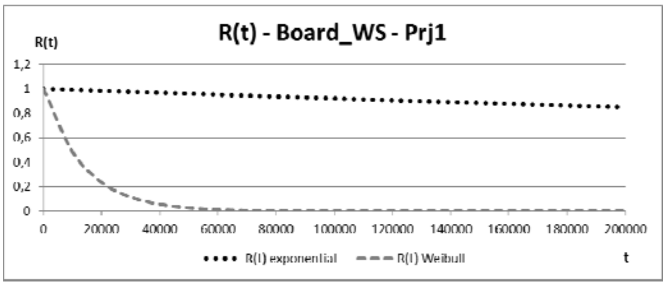

Figure 7. Comparison of reliability functions for Prj1.

Figure 7 and 8 compare reliability behavior over the time: the value of reliability function for Weibull distribution quickly tends to zero due to the wear-out described by the value assumed by β.

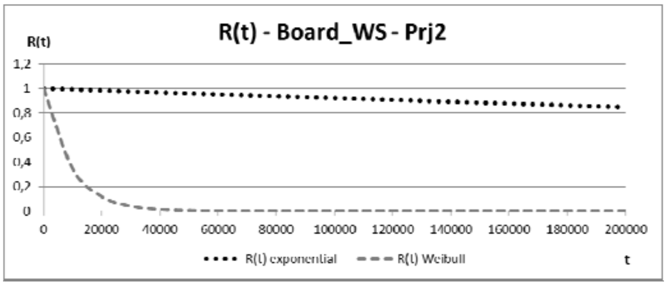

Figure 8. Comparison of reliability functions for Prj2.

Figure 9 presents the same evidences, but the slope of reliability function for Weibull distribution is lower compared to the previous cases, coherently with the values of β.

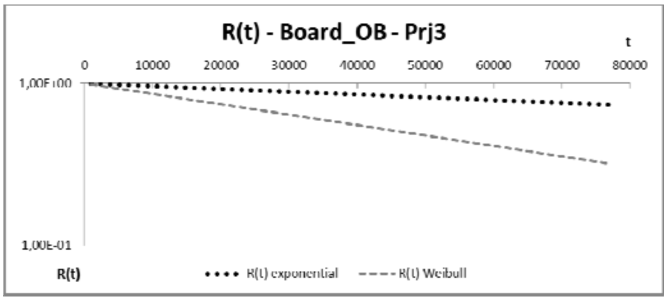

Figure 9. Comparison of reliability functions for Prj3.

These results highlight the possible bias of the use of exponential distribution in case of reliability centered maintenance policies.

It is possible to provide further reliability indicators that allow perceiving the differences between exponential and Weibull distributions. One of them is the half-life time. It is defined at the time required for a quantity to reduce to half its initial value. For exponential distributions (equation 4), the half-time figure only depends on λ.

$$t_{1/2_{\exp}} = \frac{\ln(\lambda)}{2} \tag{4}$$

For Weibull distributions, the half-life figure depends on more parameters of the specific distribution (equation 5).

$$t_{1/2_{Weib}} = \eta \ln(2)^{1/\beta} \tag{5}$$

.

Table 4 shows the application of half-life time parameter to all the boards and projects: the paradox of constant failure rate results in having the same half-life value for Board_WS in projects Prj1 and Prj2.

Table 4. Half-life time exponential vs. Weibull

Board	Project	half-life Exponential [days]	half-life Weibull [days]
Board_WS	Prj1	35,220	1953
Board_WS	Prj2	35,220	1373
Board_OB	Prj3	7330	5738

This parameter calculated according to Weibull distribution presents values of two orders of magnitude less favorable then the exponential distribution. For the Board_OB the order of magnitude is the same, even if the exponential distribution is again more optimistic.

3 OPTIMIZATION OF PREVENTIVE MAINTENANCE

The presence of wearing-out circuit boards within the use-case signaling system is the condition allowing the introduction of systematic preventive *as-good-as-new* maintenance tasks. This consists in replacing a board before a possible failure, due to the wear-out, occurs. The choice of the time at which the maintenance is scheduled depends on the β factor of the board, that is the measure of the wear-out degree, and on the ratio between the average cost of systematic preventive maintenance task and the average cost of corrective maintenance task.
Let's consider the following parameters:
p: the cost of a corrective maintenance task (replacement after a failure); it is assumed that this cost is equivalent to the cost of preventive maintenance task (preventive replacement);
P: the indirect cost caused by the consequences of a failure (i.e. the unavailability cost, like transfer of penalties from a railway operator to the supplier of the signaling equipment).
It is possible defining the average cost of a corrective maintenance task as:

$$C_1 = \frac{p + P}{MTBF} \tag{6}$$

With the same approach, it is possible defining the average cost of systematic preventive maintenance task as:

$$C_2(\theta) = \frac{p + P(1 - R(\theta))}{m(\theta)} \tag{7}$$

Where:
p is the cost of a corrective maintenance task;
θ is the time when the preventive replacement is performed;
$P(1 - R(\theta))$ is the residual cost linked to the risk of a failure before θ and evaluated through the distribution probability F(θ);
$m(\theta)$ is the average lifespan of components not exceeding θ, because they have been replaced at that time.

$$m(\theta) = \int_0^\theta R(t)dt \tag{8}$$

If the ratio between (2) and (1) is lower than '1', it is then convenient introducing systematic preventive maintenance tasks.

$$\frac{C_2(\theta)}{C_1} = \frac{p + P(1 - R(t))}{m(\theta)} \times \frac{MTBF}{p + P} \tag{9}$$

Where R(t) can be modeled through 2-parameters Weibull distribution:

$$R(t) = e^{-\left(\frac{\theta}{\eta}\right)^\beta} \tag{10}$$

And

$$MTBF = \eta \times \Gamma\left(1 + \frac{1}{\beta}\right) \tag{11}$$

Where Γ is the Gamma function calculated in (1+1/β) value

If we set and define:

$$x = \frac{\theta}{\eta} \tag{12}$$

and

$$r = \frac{P}{p} \tag{13}$$

the equation, dependent on β and *r* is:

$$\frac{C(x)_2}{C_1} = \frac{1 + r(1 - e^{-x^\beta})}{m(x)} \times \frac{\Gamma\left(1 + \frac{1}{\beta}\right)}{1 + r} \tag{14}$$

Solutions of this equation can be studied by varying the ratio between (7) and (6).
An abacus of solutions, depending on different values of β and *r* has been calculated and depicted in figure 10.

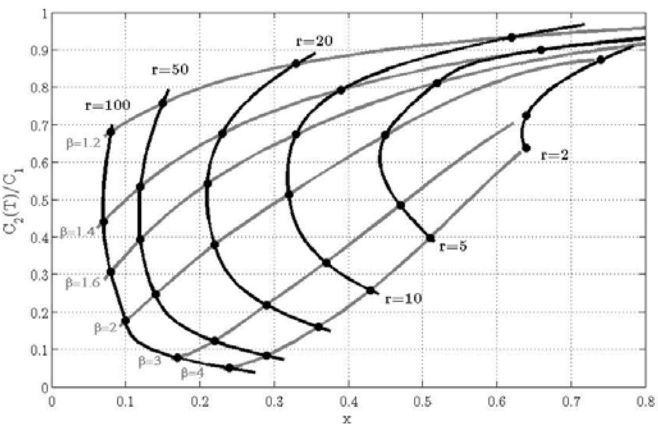

Figure 10. Solution for Abacus for optimization of systematic preventive maintenance interval.

It allows to roughly identify *x* (the systematic preventive maintenance interval) and the ratio between the cost of systematic preventive maintenance task and the cost of corrective maintenance.

Equation (14) has been coded so that exact solution can be provided for each β and *r* values: in order to determine the systematic preventive maintenance interval, and evaluate the associated economic ratio, it is necessary to know the β of the distribution and the *r* of the project.

If we consider a case of β=3, η=50,000 and *r*=50, it is possible to exactly determine the preventive maintenance interval *t*=10,950 and $C_2(x)/C_1$ =0.1220, as depicted in Figure 11.

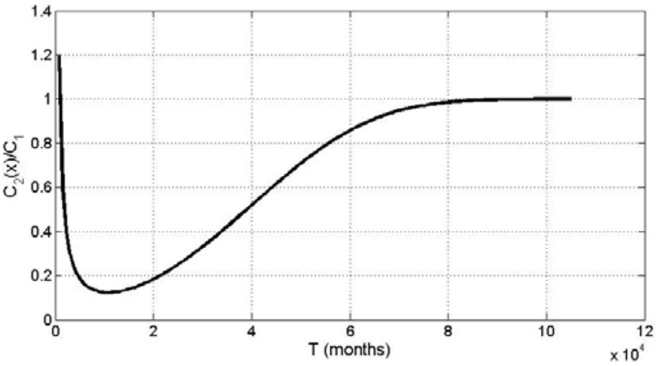

Figure 11. Solution for β=3, η=50,000 and r=50

4 INTRODUCTION TO RELIABILITY ORIENTED IMPROVEMENTS

As demonstrated in previous chapters, some elements strongly impact the reliable collection of field-return data. This might determine data censoring phenomena, with the consequent impossibility of calculating reliability figures and Weibull parameters.

The first element to be enhanced is the after-sales logbook: it should contain all the information required for a quick and relevant calculation of reliability figures. The number of boards deployed in a specific project as well as the revenue service date should be systematically included. Precise information about the time at which failures occur should also be required to the customers.

Another element of improvement can be done at the level of the design of the circuit boards. Usually, the efforts are made to reduce the MTTR in case of failure: the diagnostic system is implemented to allow the quick identification of the failed LRU, so that the maintainer is able to shrink the unavailability time.

The design of the boards could be enhanced by adding functions allowing counting the number of operating hours. For circuit boards already equipped with logic blocs like CPU (Central Processing Unit) boards it is possible by adding a real-time clock (RTC) with a backup battery, and a non-volatile memory, usually already present in CPU circuit boards. For “non-intelligent” circuit boards like power supply, front-end and power controller boards the improvement could be implemented by redesigning the circuit in order to add the same components required for the CPU boards. The feasibility of this improvement is justified since the cost of the components to add is much lower than the whole cost of the board.

A further improvement could be the possibility of monitoring some environmental parameters that could affect the reliability of the boards. Considering that temperature, physical solicitations and low quality power supply are some of the most influencing parameters, the design of the boards could be revised to add some temperature sensors (eg. very low cost negative temperature coefficient NTC devices), integrated accelerometers like cheap MEMS (Micro Electro-Mechanical Sensors) that are monitored through microprocessor. The collection over the time of all these information can increase the knowledge of the system in terms of reliability, and it allows defining correlations between stresses and reliability reduction. These improvements allow gathering several mission profiles that can be used to predict the behavior in case of new projects.

5 CONCLUSIONS

This work has presented the advantages of using Weibull distribution to calculate reliability figures of an electronic railway signaling system in operation, in spite of the commonly used exponential distribution. Although reliability prediction models have historically adopted exponential distribution for modeling the behavior of electronic components, the hypothesis of constant failure rate needs to be verified by using field-return data as deeply done in several further industrial domains. This information have been used to build-up Weibull charts and evaluate the β value for different circuit boards and different projects of the electronic railway signaling system used as reference.

Reliability figures have been calculated following Weibull distribution and compared with already existing predictive figures, demonstrating that the hypothesis of constant failure rate is not respected, since β is >1. In order to increase the knowledge in terms of reliability of the use-case system, the half-life figure has been introduced as reliability parameter and calculated according to exponential and Weibull distribution. Both failure rate and reliability functions have been plotted and the results highlight non-negligible differences.

The exploitation of the field-return data has not been possible for all the circuit boards of the use-case due to censoring phenomena. Some key information was missing and it has not been possible to build-up the Weibull chart for all the boards. The censoring phenomena are partially due to the structure of the logbook, which has not been designed for reliability purposes. Censoring is caused by several factors. They mainly deal with the structure of the logbook, which has not been designed for reliability purposes, with supply policy of specific customers and with procedures to gather on-field failures. Another underlined aspect is that data suffers from right-censoring, because not all the elements have failed within the different projects. This is not an issue for the first estimation of reliability figures, but these figures need to be corrected when new failures will occur within the different projects. This approach is strongly linked to the use of tunable preventive maintenance tasks, whose periodicity has to be updated along the whole life of the project.

Based on the evidences provided, it has been proposed a model to optimize preventive systematic maintenance tasks, allowing reducing the impact of the wear-out of the electronic system. The model proposed allows to define precise systematic preventive maintenance intervals and the economic benefits respect to corrective maintenance. As a conclusion, some suggestions have been proposed in order to enhance the whole process for collecting field-return data, by improving both the after-sales logbook and the design of the boards composing the signaling system, in order to enhance the accuracy of field-return data gathering. A particular accent has been put on the need of monitoring and gathering environmental parameters that could affect the reliability of the boards: temperature, physical solicitations and low quality power supply are some of the parameters to be taken into account, and design improvements have been suggested. The improved design, combined with the proposed methodology to calculate reliability figures based on Weibull distribution, allows building up a portfolio of mission profiles that can be used to predict the behavior in case of new projects based on the reference signaling system.

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