t-résilient snapshot immédiat
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Dans un système de $n$ processus communiquant par mémoire partagée, un *snapshot immédiat* est un état de la mémoire assurant que si l’écriture réalisée par $q$ est dans le snapshot de $p$ alors le snapshot de $p$ contient le snapshot obtenu par $q$. Un snapshot immédiat peut être réalisé dans un système où au plus $n-1$ processus peuvent tomber en panne. Dans un système où $t$ processus peuvent tomber en panne, un processus peut obtenir un snapshot contenant les valeurs écrites par jusqu’à $n-t$ processus. On définit ainsi le $t$-résilient snapshot immédiat et on établit des liens entre ce problème et les problèmes d’accord comme le consensus et le $k$-accord.

**Mots-clés :** Mémoire partagé, snapshot immédiat, tolérance aux pannes, consensus, $k$-accord.

## 1 Introduction

We consider a distributed computing model of $n$ asynchronous processes among which any subset of up to $t$ processes may crash communicating by shared memory.

The snapshot object was first proposed over a decade ago [1, 2] and has since been intensively studied by the distributed algorithms community. A snapshot object can be seen as a data structure initially empty, which can then contain at most $n$ pairs (one per process), each made up of a process index and a value. This object provides the processes with two operations denoted update and snap. The invocation `update(v)` by a process $p_i$ adds the pair $(i, v)$ to the data structure and the operation `snap` returns all the pairs already written in the data structure.

The immediate snapshot (IS) object has been introduced in [3, 11], and later investigated in [5, 15]. It is a variant of the snapshot object. An immediate snapshot provides the processes with a single operation `write_snapshot(v)` that a process may invoke at most once. The invocation `write_snapshot(v)` by a process $p_i$ adds the pair $(i, v)$ to the object and returns a set of pairs `view_i` belonging to the object such that if $(j, w)$ is in `view_i` then `view_j` ⊆ `view_i`.

The noteworthy feature of the iterated immediate snapshot model is the following. It has been shown by Borowsky and Gafni in [5], that this model is equivalent to the usual read/write wait-free model ($(n-1)$-crash model) for task solvability with the wait-freedom progress condition (any non-faulty process obtains a result). Its advantage lies in the fact that its runs are more structured and easier to analyze than the runs in the basic read/write shared memory model [14]. It is also the basis of the combinatorial topology approach for distributed computing (e.g., [10]). Hence, IS objects constitute the algorithmic foundation of distributed iterated computing models.

When considering the $t$-crash $n$-process model where $t < n$, and assuming that each correct process writes a value, a process may wait for values written by $(n-t)$ processes without risking being blocked forever.

This naturally leads to the notion of a $t$-immediate snapshot object, which generalizes the basic $(n-1)$-immediate snapshot object. More precisely, when considering a $t$-immediate snapshot object in a $t$-crash $n$-process model, an invocation of `write_snapshot(v)` by a process returns a set including at least $(n-t)$ pairs.
(while it would return a set of \( r \) pairs with \( 1 \leq r \leq n \) if the object was an immediate snapshot object). Hence, a \( t \)-immediate snapshot object allows processes to obtain as much information as possible from the other processes while guaranteeing progress.

The obvious question is then the implementability of a \( t \)-immediate snapshot object in the \( t \)-crash \( n \)-process model.

Implementations of an \((n - 1)\)-immediate snapshot object is described in [3]. For the other values of \( t \) (\( 0 < t < n - 1 \)), this question is answered in this paper, which shows that it is impossible to implement a \( t \)-IS object in a \( t \)-crash \( n \)-process model when \( 0 < t < n - 1 \). More precisely we prove that implementing a \( t \)-IS object is equivalent\(^\S\) to implementing consensus when \( t < n/2 \) and enables to implement \((2t - n + 2)\)-Set agreement when \( n/2 \leq t < n - 1 \).

At first glance, this impossibility result may seem surprising. First, an IS object is a snapshot object (a) whose operations update and snap are glued together in a single operation \texttt{write\_snapshot()}, and (b) satisfying an additional property linking the sets of pairs returned by concurrent invocations. Then, as already indicated, a \( t \)-IS object is an IS object such that the sets returned by \texttt{write\_snapshot()} contain at least \((n - t)\) pairs. This property on the sets returned by a snapshot object can be trivially implemented in a \( t \)-crash \( n \)-process model. Hence, while a \( t \)-snapshot object can be implemented in the \( t \)-crash \( n \)-process model, a \( t \)-IS object cannot when \( 0 < t < n - 1 \). Second, because in general smallest is the number of possible faults easiest is the implementation. But in the \( t \)-IS, a small number of failure, induces a harder problem each output set must have the size at least \( n - t \).

## 2 Model

We consider a distributed computing model of \( n \geq 3 \) asynchronous sequential processes denoted \( p_1, \ldots, p_n \) among which any subset of up to \( t \) (\( 0 < t < n \)) processes may crash. The processes cooperate by reading and writing Single-Writer Multi-Reader atomic read/write registers [13].

**One-shot immediate snapshot object.** An immediate snapshot object (IS) is a set, initially empty, that will contain pairs made up of a process index and a value. Let us consider a process \( p_i \) that invokes \texttt{write\_snapshot(v)}. This invocation adds the pair \((i, v)\) to the object, and returns to \( p_i \) a set, called view and denoted \( \text{view}_i \), such that the sets returned to the processes collectively satisfy the following properties.

- **Termination.** The invocation of \texttt{write\_snapshot()} by a correct process terminates.
- **Self-inclusion.** \( \forall i : \langle i, v \rangle \in \text{view}_i \).
- **Validity.** \( \forall i : (\langle j, v \rangle \in \text{view}_i) \Rightarrow p_j \text{ invoked } \text{write\_snapshot}(v) \).
- **Containment.** \( \forall i, j : (\text{view}_i \subseteq \text{view}_j) \lor (\text{view}_j \subseteq \text{view}_i) \).
- **Immediacy.** \( \forall i, j : (\langle i, v \rangle \in \text{view}_j) \Rightarrow (\text{view}_i \subseteq \text{view}_j) \).

**\( k \)-Set agreement.** \( k \)-Set agreement was introduced by S. Chaudhuri [6], it generalizes consensus which corresponds to the case \( k = 1 \). A \( k \)-Set agreement object is a one-shot object that provides the processes with a single operation denoted \texttt{propose\_k()}. This operation allows the invoking process \( p_i \) to propose a value it passes as an input parameter (called \texttt{proposed value}), and obtain a value (called \texttt{decided value}). The object is defined by the following set of properties.

- **Termination.** The invocation of \texttt{propose\_k()} by a correct process terminates.
- **Validity.** A decided value is a proposed value.
- **Agreement.** No more than \( k \) different values are decided.

It is shown in [8, 4, 11, 16] that the problem is impossible to solve if \( k \leq t \).

**\( t \)-Immediate Snapshot.** A \( t \)-immediate snapshot object (denoted by \( t \)-IS) is an immediate snapshot object with the following additional property.

- **Output size.** The set \( \text{view} \) obtained by a process is such that \( |\text{view}| \geq n - t \).

## 3 Results

**\( t \)-Immediate Snapshot is Impossible if \( 0 < t < n - 1 \).** This section presents an algorithm Figure 1 to achieve (1) consensus from \( t \)-IS for \( 0 < t < n/2 \), and (2) \( k \)-Set agreement (in short \( k \)-SA) from \( t \)-IS for

\(^\S\) A is equivalent to B if A can be (computationally) reduced to B and reciprocally.
t-resilient immediate snapshot

1  **PROP**ose**(v)**
2  begin
3    view ← 1.write snapshot**(v)** /* I shared immediate snapshot*/
4  VIEW[i] ← view; /* VIEW is a shared array*/
5  wait(|{ j such that VIEW[j] ≠ ⊥}| = t + 1);
6  let view be the smallest of the previous (t + 1) views;
7  return(smallest proposed value in view)
8  end **PROP**ose

**Figure 1:** Solving consensus from t-IS if 0 < t < n/2 (code for \( p_0 \))

1  **write**.snapshot**(v)**
2  begin
3    S.update((i, v_i); /* S shared snapshot*/
4  view ← ⌀; dec ← ⌀; k ← -1; launch the tasks T1 and T2.
5  Task T1 :
6    repeat k ← k + 1
7      do aux ← S.snapshot()
8        until (dec ∋ aux ∨ |aux| ≥ n - t)
9        dec ← CONS[k], prop**ose**(aux)
10       if (\( (i, v_i) ∉ dec \) ∧ (view = ⌀)) then view ← dec end if
11      until |aux| = n
12  end task T1
13  Task T2 : wait(view ≠ ⌀) ; return(view) end task T2.
14  end **write**.snapshot

**Figure 2:** Implementing t-IS with consensus (code for \( p_0 \))

\( k = 2t - n + 2 \) (e.g., \( (n - 2) \)-SA agreement from \( (n - 2) \)-IS if \( t = n - 2 \)). As these problems are impossible to solve [4, 11, 16], we show that it is impossible to implement a t-IS object when \( 0 < t < n - 1 \).

Intuitively this algorithm works because there is a set of at least \( \ell ≥ n - t \) processes, that obtained the same view \( \text{min\_view} \) (or crashed before returning from **write**.snapshot()), and this view is the smallest view obtained by a process and its size is |\( \text{min\_view} \)| = \( \ell \). If \( 0 < t < n/2 \), as \( \ell ≥ n - t \) and \( (n - t) + (t + 1) > n \), it follows from the waiting predicate, that, any process that executes line 5, obtains a copy of \( \text{min\_view} \), and consequently we have view = \( \text{min\_view} \) at line 5. It follows that no two processes can decide different values. If \( n/2 ≤ t < n - 1 \), we have \( n - t ≤ t \). The \( m = (n - t) - 1 \) biggest views will never be selected by the processes, and consequently these processes obtain at most \( t - m = t - ((n - t) - 1) = 2t - n + 1 \) different smallest views. Hence, these processes may decide at most \( 2t - n + 1 \) different values.

**Theorem 1** A t-IS object cannot be implemented if \( 0 < t < n - 1 \).

**From Consensus to t-IS if** \( 0 < t ≤ n - 1 \). While a snapshot object is atomic (operations on a snapshot object can be linearized [12]), an IS object and a \( k \)-immediate snapshot objects are not atomic (its operations cannot always be linearized).

So we cannot apply the universality result of Herlihy [9], and we have to write a specific algorithm given Figure 2.

**t-Immediate Snapshot and k-Set agreement.** With the algorithms Figures 1 and 2, we get :

**Theorem 2** Consensus and t-IS are equivalent if \( 0 < t < n/2 \).

We have shown that from t-IS when \( n/2 ≤ t < n - 2 \) we can implement (\( 2t - n + 2 \))-Set agreement. Can we do consensus as in the case \( 0 < t < n/2 \)? By a simulation argument, we show that consensus is not solvable with t-immediate snapshot when \( n/2 ≤ t < n \) proving that the computational power of t-immediate snapshot when \( 0 < t < n/2 \) is strictly stronger than the computational power of t-immediate snapshot when \( n/2 ≤ t < n \).
Theorem 3 If $0 < t < n/2$ then $t$-IS can implement $(2t - n + 2)$-Set agreement and cannot implement consensus. Consensus implements $t$-IS.

<table>
<thead>
<tr>
<th>$1 \leq t &lt; n/2$</th>
<th>$n/2 \leq t &lt; n-1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$-IS and consensus are equivalent</td>
<td>$t$-IS implements $(2t - n + 2)$-Set agreement $t$-IS does not implement consensus consensus implements $t$-IS</td>
</tr>
</tbody>
</table>

Table 1: Summary of results presented in the paper.

4 Conclusion

The paper has shown that, while it is possible to build an $(n-1)$-IS object in the asynchronous read/write $(n-1)$-crash model, it is impossible to build a $t$-IS object in an asynchronous read/write $t$-crash model when $0 < t < n-1$. It follows that the notion of an Iterative immediate snapshot distributed model seems inoperative for these values of $t$. The results of the paper are summarized in Table 1.

Interestingly, this study shows that there are two contrasting impossibility results in asynchronous read/write $t$-crash $n$-process systems. Consensus is impossible as soon as $t > 0$, while $t$-immediate snapshot is impossible as soon as $t < n-1$.

As a final remark, some computability problems remain open. As an example, is it possible to implement a $t$-IS object from $(2t - n + 2)$-Set agreement?

Références