t-résilient snapshot immédiat
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Dans un système de \( n \) processus communiquant par mémoire partagée, un snapshot immédiat est un état de la mémoire assurant que si l’écriture réalisée par \( q \) est dans le snapshot de \( p \) alors le snapshot de \( p \) contient le snapshot obtenu par \( q \). Un snapshot immédiat peut être réalisé dans un système où au plus \( n-1 \) processus peuvent tomber en panne. Dans un système où \( t \) processus peuvent tomber en panne, un processus peut obtenir un snapshot contenant les valeurs écrites par jusqu’à \( n-t \) processus. On définit ainsi le \( t \)-résilient snapshot immédiat et on établit des liens entre ce problème et les problèmes d’accord comme le consensus et le \( k \)-accord.

Mots-clés : Mémoire partagé, snapshot immédiat, tolérance aux pannes, consensus, \( k \)-accord.

1 Introduction

We consider a distributed computing model of \( n \) asynchronous processes among which any subset of up to \( t \) processes may crash communicating by shared memory.

The snapshot object was first proposed over a decade ago [1, 2] and has since been intensively studied by the distributed algorithms community. A snapshot object can be seen as a data structure initially empty, which can then contain at most \( n \) pairs (one per process), each made up of a process index and a value. This object provides the processes with two operations denoted update and snap. The invocation update\( (v) \) by a process \( p_i \) adds the pair \( \langle i, v \rangle \) to the data structure and the operation snap returns all the pairs already written in the data structure.

The immediate snapshot (IS) object has been introduced in [3, 11], and later investigated in [5, 15]. It is a variant of the snapshot object. An immediate snapshot provides the processes with a single operation write\( \text{snapshot}(v) \) that a process may invoke at most once. The invocation write\( \text{snapshot}(v) \) by a process \( p_i \) adds the pair \( \langle i, v \rangle \) to the object and returns a set of pairs \( \text{view}_i \) belonging to the object such that if \( \langle j, w \rangle \) is in \( \text{view}_j \) then \( \text{view}_j \subseteq \text{view}_i \).

The noteworthy feature of the iterated immediate snapshot model is the following. It has been shown by Borowsky and Gafni in [5], that this model is equivalent to the usual read/write wait-free model ((\( n-1 \))-crash model) for task solvability with the wait-freedom progress condition (any non-faulty process obtains a result). Its advantage lies in the fact that its runs are more structured and easier to analyze than the runs in the basic read/write shared memory model [14]. It is also the basis of the combinatorial topology approach for distributed computing (e.g., [10]). Hence, IS objects constitute the algorithmic foundation of distributed iterated computing models.

When considering the \( t \)-crash \( n \)-process model where \( t < n \), and assuming that each correct process writes a value, a process may wait for values written by \( (n-t) \) processes without risking being blocked forever.

This naturally leads to the notion of a \( t \)-immediate snapshot object, which generalizes the basic \( (n-1) \)-immediate snapshot object. More precisely, when considering a \( t \)-immediate snapshot object in a \( t \)-crash \( n \)-process model, an invocation of write\( \text{snapshot}(v) \) by a process returns a set including at least \( (n-t) \) pairs.
(while it would return a set of \( x \) pairs with \( 1 \leq x \leq n \) if the object was an immediate snapshot object). Hence, a \( t \)-immediate snapshot object allows processes to obtain as much information as possible from the other processes while guaranteeing progress.

The obvious question is then the implementability of a \( t \)-immediate snapshot object in the \( t \)-crash \( n \)-process model.

Implementations of an \((n - 1)\)-immediate snapshot object is described in [3]. For the other values of \( t \) (\( 0 < t < n - 1 \)), this question is answered in this paper, which shows that it is impossible to implement a \( t \)-IS object in a \( t \)-crash \( n \)-process model when \( 0 < t < n - 1 \). More precisely we prove that implementing a \( t \)-IS object is equivalent\(^3\) to implementing consensus when \( t < n / 2 \) and enables to implement \((2t - n + 2)\)-Set agreement when \( n / 2 \leq t < n - 1 \).

At first glance, this impossibility result may seem surprising. First, an IS object is a snapshot object (a) whose operations update and snap are glued together in a single operation \( \text{write} \_\text{snapshot}() \), and (b) satisfying an additional property linking the sets of pairs returned by concurrent invocations. Then, as already indicated, a \( t \)-IS object is an IS object such that the sets returned by \( \text{write} \_\text{snapshot}() \) contain at least \((n - t)\) pairs. This property on the sets returned by a snapshot object can be trivially implemented in a \( t \)-crash \( n \)-process model. Hence, while a \( t \)-snapshot object can be implemented in the \( t \)-crash \( n \)-process model, a \( t \)-IS object cannot when \( 0 < t < n - 1 \). Second, because in general smallest is the number of possible faults easiest is the implementation. But in the \( t \)-IS, a small number of failure, induces a harder problem each output set must have the size at least \( n - t \).

2 Model

We consider a distributed computing model of \( n \geq 3 \) asynchronous sequential processes denoted \( p_1, ..., \ p_n \) among which any subset of up to \( t \) (\( 0 < t < n \)) processes may crash. The processes cooperate by reading and writing Single-Writer Multi-Reader atomic read/write registers [13].

One-shot immediate snapshot object. An immediate snapshot object (IS) is a set, initially empty, that will contain pairs made up of a process index and a value. Let us consider a process \( p_i \) that invokes \( \text{write} \_\text{snapshot}(v) \). This invocation adds the pair \((i, v)\) to the object, and returns to \( p_i \) a set, called view and denoted \( \text{view}_i \), such that the sets returned to the processes collectively satisfy the following properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Termination</td>
<td>The invocation of ( \text{write} _\text{snapshot}() ) by a correct process terminates.</td>
</tr>
<tr>
<td>Self-inclusion</td>
<td>( \forall i : (i, v) \in \text{view}_i )</td>
</tr>
<tr>
<td>Validity</td>
<td>( \forall i : ((j, v) \in \text{view}_i) \Rightarrow p_j \text{ invoked } \text{write} _\text{snapshot}(v) ).</td>
</tr>
<tr>
<td>Containment</td>
<td>( \forall i, j : (\text{view}_i \subseteq \text{view}_j) \lor (\text{view}_j \subseteq \text{view}_i) ).</td>
</tr>
<tr>
<td>Immediacy</td>
<td>( \forall i, j : (i, v) \in \text{view}_i \Rightarrow (\text{view}_i \subseteq \text{view}_j) ).</td>
</tr>
</tbody>
</table>

\( k \)-Set agreement. \( k \)-Set agreement was introduced by S. Chaudhuri [6], it generalizes consensus which corresponds to the case \( k = 1 \). A \( k \)-Set agreement object is a one-shot object that provides the processes with a single operation \( \text{propose}_k() \). This operation allows the invoking process \( p_i \) to propose a value it passes as an input parameter (called \( \text{proposed} \) value), and obtain a value (called \( \text{decided} \) value). The object is defined by the following set of properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Termination</td>
<td>The invocation of ( \text{propose}_k() ) by a correct process terminates.</td>
</tr>
<tr>
<td>Validity</td>
<td>A decided value is a proposed value.</td>
</tr>
<tr>
<td>Agreement</td>
<td>No more than ( k ) different values are decided.</td>
</tr>
</tbody>
</table>

It is shown in [8, 4, 11, 16] that the problem is impossible to solve if \( k \leq t \).

\( t \)-Immediate Snapshot. A \( t \)-immediate snapshot object (denoted by \( t \)-IS) is an immediate snapshot object with the following additional property.

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output size</td>
<td>The set ( \text{view} ) obtained by a process is such that (</td>
</tr>
</tbody>
</table>

3 Results

\( t \)-Immediate Snapshot is Impossible if \( 0 < t < n - 1 \). This section presents an algorithm Figure 1 to achieve (1) consensus from \( t \)-IS for \( 0 < t < n / 2 \), and (2) \( k \)-Set agreement (in short \( k \)-SA) from \( t \)-IS for

\( ^3 \) A is equivalent to B if A can be (computationally) reduced to B and reciprocally.
**t-resilient immediate snapshot**

```plaintext
PROPOSE(v)
begin
  view ← 1.write_snapshot(v); /* I shared immediate snapshot*/
  VIEW[i] ← view; /* VIEW is a shared array*/
  wait(|{j such that VIEW[j] ≠ ⊥}| = t + 1);
  let view be the smallest of the previous (t + 1) views;
  return(smallest proposed value in view)
end PROPOSE
```

**Figure 1:** Solving consensus from t-IS if 0 < t < n/2 (code for p₁)

```plaintext
write_snapshot(v₁)
begin
  S.update((i, v₁)); /* S shared snapshot*/
  view ← ∅; dec ← ∅; k ← −1; launch the tasks T₁ and T₂.
  Task T₁:
  repeat k ← k + 1
    do aux ← S.snapshot()
    until (dec ⊂ aux ∧ |aux| ≥ n − t)
    dec ← CONS[k], propose₂(aux)
    if ((i, v₁) ∈ dec) ∧ (view = ∅) then view ← dec end if
    until |aux| = n
  end task T₁
  Task T₂: wait(view ≠ ∅); return(view) end task T₂.
end write_snapshot
```

**Figure 2:** Implementing t-IS with consensus (code for p₂)

k = 2t − n + 2 (e.g., (n − 2)-SA agreement from (n − 2)-IS if t = n − 2). As these problems are impossible to solve [4, 11, 16], we show that it is impossible to implement a t-IS object when 0 < t < n − 1.

Intuitively this algorithm works because there is a set of at least ℓ ≥ n − t processes, that obtained the same view min_view (or crashed before returning from write_snapshot()), and this view is the smallest view obtained by a process and its size is |min_view| = ℓ. If 0 < t < n/2, as ℓ ≥ n − t and (n − t) + (t + 1) > n, it follows from the waiting predicate, that, any process that executes line 5, obtains a copy of min_view, and consequently we have view = min_view at line 5. It follows that no two processes can decide different values. If n/2 ≤ t < n − 1, we have n − t ≤ t. The m = (n − t) − 1 biggest views will never be selected by the processes, and consequently these processes obtain at most t − m = t − ((n − t) − 1) = 2t − n + 1 different smallest views. Hence, these processes may decide at most 2t − n + 1 different values.

**Theorem 1** A t-IS object cannot be implemented if 0 < t < n − 1.

**From Consensus to t-IS if 0 < t ≤ n − 1.** While a snapshot object is atomic (operations on a snapshot object can be linearized [12]), an IS object and a k-immediate snapshot objects are not atomic (its operations cannot always be linearized).

So we cannot apply the universality result of Herlihy [9], and we have to write a specific algorithm given Figure 2.

**t-Immediate Snapshot and k-Set agreement.** With the algorithms Figures 1 and 2, we get:

**Theorem 2** Consensus and t-IS are equivalent if 0 < t < n/2.

We have shown that from t-IS when n/2 ≤ t < n − 2 we can implement (2t − n + 2)-Set agreement. Can we do consensus as in the case 0 < t < n/2? By a simulation argument, we show that consensus is not solvable with t-immediate snapshot when n/2 ≤ t < n proving that the computational power of t-immediate snapshot when 0 < t < n/2 is strictly stronger than the computational power of t-immediate snapshot when n/2 ≤ t < n.
Theorem 3 If $0 < t < n/2$ then $t$-IS can implement $(2t - n + 2)$-Set agreement and cannot implement consensus. Consensus implements $t$-IS.

<table>
<thead>
<tr>
<th>$1 \leq t &lt; n/2$</th>
<th>$n/2 \leq t &lt; n - 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$-IS and consensus are equivalent</td>
<td>$t$-IS implements $(2t - n + 2)$-Set agreement</td>
</tr>
<tr>
<td>$t$-IS does not implement consensus</td>
<td>consensus implements $t$-IS</td>
</tr>
</tbody>
</table>

**Table 1:** Summary of results presented in the paper

4 Conclusion

The paper has shown that, while it is possible to build an $(n - 1)$-IS object in the asynchronous read/write $(n - 1)$-crash model, it is impossible to build a $t$-IS object in an asynchronous read/write $t$-crash model when $0 < t < n - 1$. It follows that the notion of an Iterative immediate snapshot distributed model seems inoperative for these values of $t$. The results of the paper are summarized in Table 1.

Interestingly, this study shows that there are two contrasting impossibility results in asynchronous read/write $t$-crash $n$-process systems. Consensus is impossible as soon as $t > 0$, while $t$-immediate snapshot is impossible as soon as $t < n - 1$. As a final remark, some computability problems remain open. As an example, is it possible to implement a $t$-IS object from $(2t - n + 2)$-Set agreement?

Références


