t-résilient snapshot immédiat
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Dans un système de $n$ processus communiquant par mémoire partagée, un snapshot immédiat est un état de la mémoire assurant que si l’écriture réalisée par $q$ est dans le snapshot de $p$ alors le snapshot de $p$ contient le snapshot obtenu par $q$. Un snapshot immédiat peut être réalisé dans un système où au plus $n - 1$ processus peuvent tomber en panne. Dans un système où $t$ processus peuvent tomber en panne, un processus peut obtenir un snapshot contenant les valeurs écrites par jusqu’à $n - t$ processus. On définit ainsi le $t$-résilient snapshot immédiat et on établit des liens entre ce problème et les problèmes d’accord comme le consensus et le $k$-accord.

**Mots-clés :** Mémoire partagé, snapshot immédiat, tolérance aux pannes, consensus, $k$-accord.

1 Introduction

We consider a distributed computing model of $n$ asynchronous processes among which any subset of up to $t$ processes may crash communicating by shared memory.

The snapshot object was first proposed over a decade ago [1, 2] and has since been intensively studied by the distributed algorithms community. A snapshot object can be seen as a data structure initially empty, which can then contain at most $n$ pairs (one per process), each made up of a process index and a value. This object provides the processes with two operations denoted update and snap. The invocation $\text{update}(v)$ by a process $p_i$ adds the pair $\langle i, v \rangle$ to the data structure and the operation snap returns all the pairs already written in the data structure.

The immediate snapshot (IS) object has been introduced in [3, 11], and later investigated in [5, 15]. It is a variant of the snapshot object. An immediate snapshot provides the processes with a single operation $\text{write\_snapshot}()$ that a process may invoke at most once. The invocation $\text{write\_snapshot}(v)$ by a process $p_i$ adds the pair $\langle i, v \rangle$ to the object and returns a set of pairs $\text{view}_i$ belonging to the object such that if $\langle j, w \rangle$ is in $\text{view}_i$, then $\text{view}_j \subseteq \text{view}_i$.

The noteworthy feature of the iterated immediate snapshot model is the following. It has been shown by Borowsky and Gafni in [5], that this model is equivalent to the usual read/write wait-free model ($(n - 1)$-crash model) for task solvability with the wait-freedom progress condition (any non-faulty process obtains a result). Its advantage lies in the fact that its runs are more structured and easier to analyze than the runs in the basic read/write shared memory model [14]. It is also the basis of the combinatorial topology approach for distributed computing (e.g., [10]). Hence, IS objects constitute the algorithmic foundation of distributed iterated computing models.

When considering the $t$-crash $n$-process model where $t < n$, and assuming that each correct process writes a value, a process may wait for values written by $(n - t)$ processes without risking being blocked forever.

This naturally leads to the notion of a $t$-immediate snapshot object, which generalizes the basic $(n - 1)$-immediate snapshot object. More precisely, when considering a $t$-immediate snapshot object in a $t$-crash $n$-process model, an invocation of $\text{write\_snapshot}()$ by a process returns a set including at least $(n - t)$ pairs

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1 This paper is an extended abstract of [7]; the original title is *t-resilient snapshot immédiat*.

2 Carole Delporte-Gallet, Hugues Fauconnier and Michel Raynal are supported by ANR DESCARTES
(while it would return a set of \(x\) pairs with \(1 \leq x \leq n\) if the object was an immediate snapshot object). Hence, a \(t\)-immediate snapshot object allows processes to obtain as much information as possible from the other processes while guaranteeing progress.

The obvious question is then the implementability of a \(t\)-immediate snapshot object in the \(t\)-crash \(n\)-process model.

Implementations of an \((n-1)\)-immediate snapshot object is described in [3]. For the other values of \(t\) \((0 < t < n - 1)\), this question is answered in this paper, which shows that it is impossible to implement a \(t\)-IS object in a \(t\)-crash \(n\)-process model when \(0 < t < n - 1\). More precisely we prove that implementing a \(t\)-IS object is equivalent \(^3\) to implementing consensus when \(t < n/2\) and enables to implement \((2t - n + 2)\)-Set agreement when \(n/2 \leq t < n - 1\).

At first glance, this impossibility result may seem surprising. First, an IS object is a snapshot object (a) whose operations update and snap are glued together in a single operation \texttt{write\_snapshot()}\(^{\textit{1}}\), and (b) satisfying an additional property linking the sets of pairs returned by concurrent invocations. Then, as already indicated, a \(t\)-IS object is an IS object such that the sets returned by \texttt{write\_snapshot()} contain at least \((n - t)\) pairs. This property on the sets returned by a snapshot object can be trivially implemented in a \(t\)-crash \(n\)-process model. Hence, while a \(t\)-snapshot object can be implemented in the \(t\)-crash \(n\)-process model, a \(t\)-IS object cannot when \(0 < t < n - 1\). Second, because in general smallest is the number of possible faults easiest is the implementation. But in the \(t\)-IS, a small number of failure, induces a harder problem each output set must have the size at least \(n - t\).

2 Model

We consider a distributed computing model of \(n \geq 3\) asynchronous sequential processes denoted \(p_1, \ldots, p_n\) among which any subset of up to \(t\) \((0 < t < n)\) processes may crash. The processes cooperate by reading and writing Single-Writer Multi-Reader atomic read/write registers [13].

**One-shot immediate snapshot object.** An immediate snapshot object (IS) is a set, initially empty, that will contain pairs made up of a process index and a value. Let us consider a process \(p_i\) that invokes \texttt{write\_snapshot}(\(v\)). This invocation adds the pair \((i, v)\) to the object, and returns to \(p_i\) a set, called view and denoted \(\text{view}_i\), such that the sets returned to the processes collectively satisfy the following properties.

- Termination. The invocation of \texttt{write\_snapshot()} by a correct process terminates.
- Self-inclusion. \(\forall i: (i, v) \in \text{view}_i\).
- Validity. \(\forall i: (j, v) \in \text{view}_i \Rightarrow p_j\) invoked \texttt{write\_snapshot}(\(v\)).
- Containment. \(\forall i, j: (\text{view}_i \subseteq \text{view}_j) \lor (\text{view}_j \subseteq \text{view}_i)\).
- Immediacy. \(\forall i, j: (i, v) \in \text{view}_j \Rightarrow (\text{view}_i \subseteq \text{view}_j)\).

**K-Set agreement.** \(k\)-Set agreement was introduced by S. Chaudhuri [6], it generalizes consensus which corresponds to the case \(k = 1\). A \(k\)-Set agreement object is a one-shot object that provides the processes with a single operation \texttt{propose\_k}(\(v\)). This operation allows the invoking process \(p_i\) to propose a value it passes as an input parameter (called \texttt{proposed}\(v\)) and obtain a value (called \texttt{decided}\(v\)). The object is defined by the following set of properties.

- Termination. The invocation of \texttt{propose\_k()} by a correct process terminates.
- Validity. A decided value is a proposed value.
- Agreement. No more than \(k\) different values are decided.

It is shown in [8, 4, 11, 16] that the problem is impossible to solve if \(k \leq t\).

**\(t\)-Immediate Snapshot.** A \(t\)-immediate snapshot object (denoted by \(t\)-IS) is an immediate snapshot object with the following additional property.

- Output size. The set \(\text{view}\) obtained by a process is such that \(|\text{view}| \geq n - t\).

3 Results

**\(t\)-Immediate Snapshot is Impossible if \(0 < t < n - 1\).** This section presents an algorithm Figure 1 to achieve (1) consensus from \(t\)-IS for \(0 < t < n/2\), and (2) \(k\)-Set agreement (in short \(k\)-SA) from \(t\)-IS for

\(^3\). \(A\) is equivalent to \(B\) if \(A\) can be (computationally) reduced to \(B\) and reciprocally.
**t-resilient immediate snapshot**

1. **PROPOSE(v)**
   
   begin
   
   view ← I.write_snapshot(v); /* I shared immediate snapshot*/
   VIEW[i] ← view; /* VIEW is a shared array*/
   wait(∥{ j such that VIEW[j] ≠ ⊥}∥ = t + 1);
   let view be the smallest of the previous (t + 1) views;
   return(smallest proposed value in view)
   
   end PROPOSE

**Figure 1:** Solving consensus from t-IS if 0 < t < n/2 (code for p1)

write_snapshot(vi)

begin

S.update((i, vi)); /* S shared snapshot*/
view ← ∅; dec ← ∅; k ← 1; launch the tasks T1 and T2.

Task T1:

repeat k ← k + 1
    do aux ← S.snapshot()
    until (dec ⊆ aux ∧ |aux| ≥ n − t)
    dec ← CONS[k], propose1(aux)
    if ((i, vi) ∈ dec) ∧ (view = ∅) then view ← dec end if
    until |aux| = n

end task T1

Task T2: wait(view ≠ ∅) : return(view) end task T2.

end write_snapshot

**Figure 2:** Implementing t-IS with consensus (code for p0)

k = 2t − n + 2 (e.g., (n − 2)-SA agreement from (n − 2)-IS if t = n − 2). As these problems are impossible to solve [4, 11, 16], we show that it is impossible to implement a t-IS object when 0 < t < n − 1.

Intuitively this algorithm works because there is a set of at least ℓ ≥ n − t processes, that obtained the same view min_view (or crashed before returning from write_snapshot()), and this view is the smallest view obtained by a process and its size is |min_view| = ℓ. If 0 < t < n/2, as ℓ ≥ n − t and (n − t) + (t + 1) > n, it follows from the waiting predicate, that, any process that executes line 5, obtains a copy of min_view, and consequently we have view = min_view at line 5. It follows that no two processes can decide different values. If n/2 ≤ t < n − 1, we have n − t ≤ t. The m = (n − t) − 1 biggest views will never be selected by the processes, and consequently these processes obtain at most t − m = t − ((n − t) − 1) = 2t − n + 1 different smallest views. Hence, these processes may decide at most 2t − n + 1 different values.

**Theorem 1** A t-IS object cannot be implemented if 0 < t < n − 1.

**From Consensus to t-IS** if 0 < t ≤ n − 1. While a snapshot object is atomic (operations on a snapshot object can be linearized [12]), an IS object and a k-immediate snapshot objects are not atomic (its operations cannot always be linearized).

So we cannot apply the universality result of Herlihy [9], and we have to write a specific algorithm given Figure 2.

**t-Immediate Snapshot and k-Set agreement.** With the algorithms Figures 1 and 2, we get:

**Theorem 2** Consensus and t-IS are equivalent if 0 < t < n/2.

We have shown that from t-IS when n/2 ≤ t < n − 2 we can implement (2t − n + 2)-Set agreement. Can we do consensus as in the case 0 < t < n/2? By a simulation argument, we show that consensus is not solvable with t-immediate snapshot when n/2 ≤ t < n proving that the computational power of t-immediate snapshot when 0 < t < n/2 is strictly stronger than the computational power of t-immediate snapshot when n/2 ≤ t < n.
Theorem 3 If $0 < t < n/2$ then $t$-IS can implement $(2t - n + 2)$-Set agreement and cannot implement consensus. Consensus implements $t$-IS.

<table>
<thead>
<tr>
<th>$1 \leq t &lt; n/2$</th>
<th>$n/2 \leq t &lt; n - 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$-IS and consensus are equivalent</td>
<td>$t$-IS implements $(2t - n + 2)$-Set agreement</td>
</tr>
<tr>
<td>$t$-IS does not implement consensus</td>
<td>consensus implements $t$-IS</td>
</tr>
</tbody>
</table>

Table 1: Summary of results presented in the paper

4 Conclusion

The paper has shown that, while it is possible to build an $(n − 1)$-IS object in the asynchronous read/write $(n − 1)$-crash model, it is impossible to build a $t$-IS object in an asynchronous read/write $t$-crash model when $0 < t < n − 1$. It follows that the notion of an Iterative immediate snapshot distributed model seems inoperative for these values of $t$. The results of the paper are summarized in Table 1.

Interestingly, this study shows that there are two contrasting impossibility results in asynchronous read/write $t$-crash $n$-process systems. Consensus is impossible as soon as $t > 0$, while $t$-immediate snapshot is impossible as soon as $t < n − 1$.

As a final remark, some computability problems remain open. As an example, is it possible to implement a $t$-IS object from $(2t − n + 2)$-Set agreement?

Références