t-résilient snapshot immédiat
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Dans un système de $n$ processus communiquant par mémoire partagée, un snapshot immédiat est un état de la mémoire assurant que si l’écriture réalisée par $q$ est dans le snapshot de $p$ alors le snapshot de $p$ contient le snapshot obtenu par $q$. Un snapshot immédiat peut être réalisé dans un système où au plus $n-1$ processus peuvent tomber en panne. Dans un système où $t$ processus peuvent tomber en panne, un processus peut obtenir un snapshot contenant les valeurs écrites par jusqu’à $n-t$ processus. On définit ainsi le t-résilient snapshot immédiat et on établit des liens entre ce problème et les problèmes d’accord comme le consensus et le $k$-accord.

Mots-clés : Mémoire partagé, snapshot immédiat, tolérance aux pannes, consensus, $k$-accord.

1 Introduction

We consider a distributed computing model of $n$ asynchronous processes among which any subset of up to $t$ processes may crash communicating by shared memory.

The snapshot object was first proposed over a decade ago [1, 2] and has since been intensively studied by the distributed algorithms community. A snapshot object can be seen as a data structure initially empty, which can then contain at most $n$ pairs (one per process), each made up of a process index and a value. This object provides the processes with two operations denoted update and snap. The invocation $\text{update}(v)$ by a process $p_i$ adds the pair $(i, v)$ to the data structure and the operation $\text{snap}$ returns all the pairs already written in the data structure.

The immediate snapshot (IS) object has been introduced in [3, 11], and later investigated in [5, 15]. It is a variant of the snapshot object. An immediate snapshot provides the processes with a single operation $\text{write}\_\text{snapshot}()$ that a process may invoke at most once. The invocation $\text{write}\_\text{snapshot}(v)$ by a process $p_i$ adds the pair $(i, v)$ to the object and returns a set of pairs $\text{view}_i$ belonging to the object such that if $(j, w)$ is in $\text{view}_j$, then $\text{view}_j \subseteq \text{view}_i$.

The noteworthy feature of the iterated immediate snapshot model is the following. It has been shown by Borowsky and Gafni in [5], that this model is equivalent to the usual read/write wait-free model ($(n-1)$-crash model) for task solvability with the wait-freedom progress condition (any non-faulty process obtains a result). Its advantage lies in the fact that its runs are more structured and easier to analyze than the runs in the basic read/write shared memory model [14]. It is also the basis of the combinatorial topology approach for distributed computing (e.g., [10]). Hence, IS objects constitute the algorithmic foundation of distributed iterated computing models.

When considering the $t$-crash $n$-process model where $t < n$, and assuming that each correct process writes a value, a process may wait for values written by $(n-t)$ processes without risking being blocked forever.

This naturally leads to the notion of a $t$-immediate snapshot object, which generalizes the basic $(n-1)$-immediate snapshot object. More precisely, when considering a $t$-immediate snapshot object in a $t$-crash $n$-process model, an invocation of $\text{write}\_\text{snapshot}()$ by a process returns a set including at least $(n-t)$ pairs.

\footnote{This paper is an extended abstract of [7]; the original title is \textit{t-resilient snapshot immédiat}.}
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(while it would return a set of \(x\) pairs with \(1 \leq x \leq n\) if the object was an immediate snapshot object). Hence, a \(t\)-immediate snapshot object allows processes to obtain as much information as possible from the other processes while guaranteeing progress.

The obvious question is then the implementability of a \(t\)-immediate snapshot object in the \(t\)-crash \(n\)-process model.

Implementations of an \((n-1)\)-immediate snapshot object is described in [3]. For the other values of \(t\) (\(0 < t < n-1\)), this question is answered in this paper, which shows that it is impossible to implement a \(t\)-IS object in a \(t\)-crash \(n\)-process model when \(0 < t < n-1\). More precisely we prove that implementing a \(t\)-IS object is equivalent\(^\S\) to implementing consensus when \(t < n/2\) and enables to implement \((2t - n + 2)\)-Set agreement when \(n/2 \leq t < n-1\).

At first glance, this impossibility result may seem surprising. First, an IS object is a snapshot object (a) whose operations update and snap are glued together in a single operation \texttt{write\_snapshot()}, and (b) satisfying an additional property linking the sets of pairs returned by concurrent invocations. Then, as already indicated, a \(t\)-IS object is an IS object such that the sets returned by \texttt{write\_snapshot()} contain at least \((n-t)\) pairs. This property on the sets returned by a snapshot object can be trivially implemented in a \(t\)-crash \(n\)-process model. Hence, while a \(t\)-snapshot object can be implemented in the \(t\)-crash \(n\)-process model, a \(t\)-IS object cannot when \(0 < t < n-1\). Second, because in general smallest is the number of possible faults easiest is the implementation. But in the \(t\)-IS, a small number of failure, induces a harder problem each output set must have the size at least \(n-t\).

2 Model

We consider a distributed computing model of \(n \geq 3\) asynchronous sequential processes denoted \(p_1, \ldots, p_n\) among which any subset of up to \(t\) (\(0 < t < n\)) processes may crash. The processes cooperate by reading and writing Single-Writer Multi-Reader atomic read/write registers [13].

One-shot immediate snapshot object. An immediate snapshot object (IS) is a set, initially empty, that will contain pairs made up of a process index and a value. Let us consider a process \(p_i\) that invokes \texttt{write\_snapshot}(\(v\)). This invocation adds the pair \((i, v)\) to the object, and returns to \(p_i\) a set, called view and denoted \texttt{view}_i, such that the sets returned to the processes collectively satisfy the following properties.

- Termination. The invocation of \texttt{write\_snapshot()} by a correct process terminates.
- Self-inclusion. \(\forall i: (i, v) \in \texttt{view}_i\).
- Validity. \(\forall i: ((j, v) \in \texttt{view}_i) \Rightarrow p_j \text{ invoked } \texttt{write\_snapshot}(v)\).
- Containment. \(\forall i, j: (\texttt{view}_i \subseteq \texttt{view}_j) \lor (\texttt{view}_j \subseteq \texttt{view}_i)\).
- Immediacy. \(\forall i, j: (i, v) \in \texttt{view}_j \Rightarrow (\texttt{view}_i \subseteq \texttt{view}_j)\).

\(k\)-Set agreement. \(k\)-Set agreement was introduced by S. Chaudhuri [6], it generalizes consensus which corresponds to the case \(k = 1\). A \(k\)-Set agreement object is a one-shot object that provides the processes with a single operation \texttt{propose()}\(k\). This operation allows the invoking process \(p_i\) to propose a value it passes as an input parameter (called \texttt{proposed value}), and obtain a value (called \texttt{decided value}). The object is defined by the following set of properties.

- Termination. The invocation of \texttt{propose()}\(k\) by a correct process terminates.
- Validity. A decided value is a proposed value.
- Agreement. No more than \(k\) different values are decided.

It is shown in [8, 4, 11, 16] that the problem is impossible to solve if \(k \leq t\).

\(t\)-Immediate Snapshot. A \(t\)-immediate snapshot object (denoted by \(t\)-IS) is an immediate snapshot object with the following additional property.

- Output size. The set \texttt{view} obtained by a process is such that \(|\texttt{view}| \geq n-t\).

3 Results

\(t\)-Immediate Snapshot is Impossible if \(0 < t < n-1\). This section presents an algorithm Figure 1 to achieve (1) consensus from \(t\)-IS for \(0 < t < n/2\), and (2) \(k\)-Set agreement (in short \(k\)-SA) from \(t\)-IS for

\(\S\). A is equivalent to B if A can be (computationally) reduced to B and reciprocally.
t-resilient immediate snapshot

$$\text{PROPOSE}(v)$$

1. $$\text{begin}$$
2. $$\text{view} \leftarrow \text{I.write\_snapshot}(v) ; /* I shared immediate snapshot*/$$
3. $$\text{VIEW}[i] \leftarrow \text{view} ; /* \text{VIEW} \text{is a shared array*/}$$
4. $$\text{wait}([\{ j \text{ such that } \text{VIEW}[j] \neq \bot \}] = t + 1) ;$$
5. $$\text{let view be the smallest of the previous } (t + 1) \text{ views} ;$$
6. $$\text{return(smallest proposed value in view)}$$
7. $$\text{end PROPOSE}$$

**Figure 1:** Solving consensus from t-IS if $$0 < t < n/2$$ (code for $$p_0$$)

$$\text{write\_snapshot}(v_i)$$

1. $$\text{begin}$$
2. $$S.\text{update}(i, v_i) ; /* S shared snapshot*/$$
3. $$\text{view} \leftarrow \emptyset ; \text{dec} \leftarrow \emptyset ; k \leftarrow -1 ; \text{launch the tasks } T_1 \text{ and } T_2.$$
4. $$\text{Task } T_1 :$$
5. $$\text{repeat } k \leftarrow k + 1$$
6. $$\text{do aux } \leftarrow \text{S.snapshot}()$$
7. $$\text{until } (\text{dec} \subset \text{aux} \land \text{aux} \geq n - t)$$
8. $$\text{dec} \leftarrow \text{CONS}[k], \text{propose}_2(\text{aux})$$
9. $$\text{if } ((i, v_i) \in \text{dec}) \land (\text{view} = \emptyset) \text{ then view } \leftarrow \text{dec} \text{ end if}$$
10. $$\text{until } |\text{aux}| = n$$
11. $$\text{end task } T_1$$
12. $$\text{Task } T_2 : \text{wait}(\text{view} \neq \emptyset) ; \text{return(view)} \text{ end task } T_2.$$  
13. $$\text{end write\_snapshot}$$

**Figure 2:** Implementing t-IS with consensus (code for $$p_0$$)

$$k = 2t - n + 2$$ (e.g., $$(n - 2)$$-SA agreement from $$(n - 2)$$-IS if $$t = n - 2$$). As these problems are impossible to solve [4, 11, 16], we show that it is impossible to implement a t-IS object when $$0 < t < n - 1$$.

Intuitively this algorithm works because there is a set of at least $$\ell \geq n - t$$ processes, that obtained the same view $$\text{min\_view}$$ (or crashed before returning from write\_snapshot()), and this view is the smallest view obtained by a process and its size is $$|\text{min\_view}| = \ell$$. If $$0 < t < n/2$$, as $$\ell \geq n - t$$ and $$(n - t) + (t + 1) > n$$, it follows from the waiting predicate, that, any process that executes line 5, obtains a copy of $$\text{min\_view}$$, and consequently we have $$\text{view} = \text{min\_view}$$ at line 5. It follows that no two processes can decide different values. If $$n/2 \leq t < n - 1$$, we have $$n - t \leq t$$. The $$m = (n - t) - 1$$ biggest views will never be selected by the processes, and consequently these processes obtain at most $$t - m = t - ((n - t) - 1) = 2t - n + 1$$ different smallest views. Hence, these processes may decide at most $$2t - n + 1$$ different values.

**Theorem 1** A t-IS object cannot be implemented if $$0 < t < n - 1$$.

**From Consensus to t-IS if $$0 < t \leq n - 1$$.** While a snapshot object is atomic (operations on a snapshot object can be linearized [12]), an IS object and a k-immediate snapshot objects are not atomic (its operations cannot always be linearized).

So we cannot apply the universality result of Herlihy [9], and we have to write a specific algorithm given Figure 2.

**t-Immediate Snapshot and k-Set agreement.** With the algorithms Figures 1 and 2, we get:

**Theorem 2** Consensus and t-IS are equivalent if $$0 < t < n/2$$.

We have shown that from t-IS when $$n/2 \leq t < n - 2$$ we can implement $$(2t - n + 2)$$-Set agreement. Can we do consensus as in the case $$0 < t < n/2$$? By a simulation argument, we show that consensus is not solvable with t-immediate snapshot when $$n/2 \leq t < n$$ proving that the computational power of t-immediate snapshot when $$0 < t < n/2$$ is strictly stronger than the computational power of t-immediate snapshot when $$n/2 \leq t < n$$. 


Theorem 3 If \( 0 < t < n/2 \) then \( t\)-IS can implement \((2t - n + 2)\)-Set agreement and cannot implement consensus. Consensus implements \( t\)-IS.

<table>
<thead>
<tr>
<th>( 1 \leq t &lt; n/2 )</th>
<th>( n/2 \leq t &lt; n - 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t)-IS and consensus are equivalent</td>
<td>( t)-IS implements ((2t - n + 2))-Set agreement ( t)-IS does not implement consensus consensus implements ( t)-IS</td>
</tr>
</tbody>
</table>

Table 1: Summary of results presented in the paper

4 Conclusion

The paper has shown that, while it is possible to build an \((n - 1)\)-IS object in the asynchronous read/write \((n - 1)\)-crash model, it is impossible to build a \( t\)-IS object in an asynchronous read/write \( t\)-crash model when \( 0 < t < n - 1 \). It follows that the notion of an Iterative immediate snapshot distributed model seems inoperative for these values of \( t \). The results of the paper are summarized in Table 1.

Interestingly, this study shows that there are two contrasting impossibility results in asynchronous read/write \( t\)-crash \( n\)-process systems. Consensus is impossible as soon as \( t > 0 \), while \( t\)-immediate snapshot is impossible as soon as \( t < n - 1 \).

As a final remark, some computability problems remain open. As an example, is it possible to implement a \( t\)-IS object from \((2t - n + 2)\)-Set agreement?

Références


