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Proper Generalized Decomposition for Smart Grid applications: new routes towards efficient parametric solutions of the load flow problem

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Abstract — The aim of this short communication is to present a new strategy for computing parametric solutions of the power flow problem based on Proper Generalized Decomposition. The robustness of the featured nonlinear solver and its easiness of implementation allow an efficient extension to parametric solutions at affordable computational cost. Targeted potential applications range from long term expansion planning of existing power grids to real time control and operation of smart grids.

Keywords — PGD, smart grid, power flow.

1 Introduction

The nonlinear equations governing the power distribution in an electrical network with given loads and generation units are

\[ \sum_k Y_{jk} V_k = \frac{S_j}{V_j}, \]

where \( Y_{jk} \) are the nodal admittances, \( V_k \) the nodal voltages, and \( S_j = S_{Gj} - S_{Lj} \) is the complex electrical power, whose sign depends on whether the power is generated or demanded. Here and in the rest of the paper the symbol \( * \) is used to denote the complex conjugate. For large power systems iterative methods are often required in order to solve (1).

Fast and accurate numerical solutions are needed in smart grid applications such as real time control, contingency analysis or restoration plan after a failure. Such applications rely on the concepts of probabilistic and optimal load flow [2, 1] in order to take the best possible action out of a set of simulated scenarios. The numerical methods that are classically used in power flow analysis appear as outdated for such applications because they favour either accuracy or fast execution, but never both features at the same time. As pointed out by Trias, the Fast Decoupled Load Flow Method (FDLM) of Stott and Alsac [7], that is nowadays standard of most available software, tend to produce spurious or non-physical solutions when the grid operates far from the design conditions, which is probably the situation where real time control actions are most needed in practical smart grid applications [8]. In the same work the author also proposes new and fast algorithm capable of discerning physical solutions from spurious ones, that is based on the state-of-the-art tools of analytical continuation.

The aim present work is to develop a competitive solver that can be integrated with model order reduction techniques and more specifically with the Proper Generalized Decomposition (PGD). Indeed, when planning control actions on long or short term, several configurations need to be evaluated and uncertainty in the load demand must be taken into account. Our approach consists in considering an equivalent parametric problem where the loads and the generated powers are taken as additional coordinates. By means of PGD, we provide parametric solutions of the power flow equations that can be successively used in optimization problems.

The rest of the paper is structured as follows: in section 2 the basis of the nonlinear solver are briefly recalled while in section 3 how it is possible to integrate PGD through a worked example. Finally conclusions are drawn in section 4.
2 Nonlinear solution of the power flow problem

By reintroducing the nodal currents $I_j$, the power flow problem can be reformulated as follows:

\[ \sum_k Y_{jk} V_k = I_j \]  
\[ I_j V_j^* = S_j^* . \]

Linear system (2) expresses the linear balance of the nodal currents for ohmic transmission lines, as in Kirchhoff’s law, while equations (3) constitute additional constraints imposed by the nodal powers. In this formulation the problem lends itself to Uzawa-based iterative techniques [9], often encountered in constrained optimization. Frequently related to the Uzawa algorithms, the so called Latin method [5] has been expressly developed in the framework of nonlinear mechanics to handle nonlinear constraints and it represents the base from which our solver is inspired. The underlying idea is to seek the solution of the linear and nonlinear constraint equations alternatively, until they coincide within a given tolerance. For a given pair of voltages and currents $V_k^G$, $I_k^G$ verifying the global current balance, the corresponding pair $V_k^L$, $I_k^L$, satisfying the local constraints, is given by the solution of the following system:

\[ S_j = V_j^L I_j^G \]
\[ I_j^G - I_j^L = \beta (V_j^G - V_j^L) . \]

The new pair $V_k^G$, $I_k^G$ is then obtained by solving:

\[ \sum_k Y_{jk} V_k^G = I_j^G \]
\[ I_k^G - I_k^L = \alpha (V_k^G - V_k^L) . \]

The procedure is iterated until $V_k^G \approx V_k^L$. The quantities $\alpha$ and $\beta$ are appropriate search directions that are kept constant through the iterations. In this way, the matrix in linear system (5) only needs to be factorized once, while (4) only requires the solution of a set of uncoupled second order equations. The true strength of this strategy is that the non-linearity and the non-locality are tackled separately, making the overall algorithm extremely efficient. The choice of the search directions allows to accelerate the convergence rate or, as shown in the next section, to simplify the form of (4) and (5).

3 Integrating the Proper Generalized Decomposition: a worked example

To illustrate the procedure we consider the example proposed in reference [6] and shown in figure 1. The goal is to obtain the voltage solution at each node $k$ when an additional capacitor bank is placed at the node $p$ and generates reactive power $q$. The three-dimensional voltage solution can be written in a separated form as:

\[ V_k(p, q) = \sum_i K_i(k) P_i(p) Q_i(q) . \]
The search directions are chosen as $\alpha = 0$ and $\beta = \infty$. In this way, the starting from the initial guess solution $V^G(k,p,q) = V_0$, with $V_0$ the generator voltage, the currents are computed as solution of the local problem

$$V^G_j(p,q)I^L_j(p,q) = S_j^r(p,q).$$

This equation is linear in $I^L_j$ and expresses the problem of finding the separated form of the quotient $I$ of two given functions. The latter can be efficiently solved with PGD (see [4, 3] for details on the implementation). As a second step, the new voltages $V^G_k$ are determined by solving

$$\sum_k Y_{jk}V^G_k(p,q) = I^L_j(p,q). \quad (6)$$

Contrarily to jacobian based iterative methods, the matrix $Y_{jk}$ does not depend on $V$, implying that the linear operator in (6) does not depend on the parameters $p$ and $q$ but only on the coordinate $k$. Because of this consideration, PGD is not even involved in this second step, since the $V^G_k$ can be computed directly from $I^L_j$ by applying the inverse of $Y_{jk}$ to all the functions of $k$. As in the previous section, even in the parametric case the problem only requires the factorization of the matrix once, in the preprocessing stage.

Once the algorithm has converged, the voltage solution can be post-processed according to the requirements of a given application. If for instance, the optimal position and size of the capacitor bank is sought as the one who minimizes the system losses, the loss function $L(p,q)$ can be derived directly from the separated form of $V(k,p,q)$ and the minimum search can be easily performed with simple optimization software, since the objective function is now explicitly available at any point of the parametric space. When the tolerance on the nonlinear solution is set to $10^{-8}$, the loss function can be represented with only 5 terms, as shown in figure 3. The reconstructed function is also shown in figure 3. Note that the optimal position and power the generator are in perfect agreement with the results of reference [6], where the same problem were solved using a Monte Carlo algorithm.

**Figure 2** Separated form of the power loss as a function of the nodal position $p$ and the generated reactive power $q$ of the capacitor bank. Functions of $p$ (a). Functions of $q$ (b).

## 4 Conclusions and Perspectives

The proposed algorithm represents a promising starting point for integrating the PGD in the nonlinear solution of parametric power flow problems. The key feature of the newly proposed approach is a fast nonlinear solver that constructs the solution with a two step procedure. Each step is linear and therefore can easily accommodate PGD. Furthermore, the only linear operator appearing in the equation is constant.
through the iterations and independent on the values of the parameters, hence its factorization can be performed once and stored in memory.

Future work is aimed to study the possibility of integrating the proposed strategy in real time control of power systems, by pre-computing parametric solutions in an off-line phase in order to accelerate the on-line optimization. Simulation of the time-dependent equations is also an ongoing subject. The aim is to provide a real time estimator of the short term system evolution in order to predict and anticipate the onset of dynamic instabilities.

References


