Discretization of space and time: mass-energy relation, accelerating expansion of the Universe, Hubble constant

Luca Roatta

Discretization of space and time: mass-energy relation, accelerating expansion of the Universe, Hubble constant

Luca Roatta

E-mail: lucaroa@gmail.com

Abstract

Assuming that space and time can only have discrete values, we obtain the expression of the gravitational potential energy that at large distance coincides with the Newtonian. In very precise circumstances it coincides with the relativistic mass-energy relation: this shows that the Universe is a black hole in which all bodies are subjected to an acceleration toward the border of the Universe itself. Since the Universe is a black hole with a fixed radius, we can obtain the density of the Universe and the value of the Hubble constant.

1. Introduction

Let's assume, as work hypothesis, the existence of both discrete space and discrete time, namely spatial and temporal intervals not further divisible; this assumption leads to some interesting consequences. Here we find that the discrete gravitational potential energy is very similar to the Newtonian, with which coincides at large distances.

It is also shown that the expression of the gravitational potential energy of a body inside a black hole, far from its center and far from its border, corresponds to the relativistic mass-energy relation: it means that the Universe is a black hole. In the same circumstances the body undergoes a constant acceleration toward the border of the Universe itself.

So, if we suppose that neither space nor time are continuous, but that instead both are discrete, and following the terminology used in a previous document[1], we call l_0 the fundamental length and t_0 the fundamental time.

2. Gravitational potential energy

In a previous document[2] we obtained that in a discrete context the modified expression of the gravitational force exerted by a body A having mass M on a body B having mass m at the distance $r>d$ from A is:

$$f(r) = -mc^2 \frac{r}{(r-l_0)} \frac{d}{(r-d)^2} \tag{1}$$

where

$$d = \frac{GM}{c^2} \tag{2}$$

is the radius of the black hole of mass M.

If $r \gg l_0$ Eq. (1) becomes:

$$f(r) = -mc^2 \frac{d}{(r-d)^2} \tag{3}$$

In a continuous context, if we want to calculate the potential energy, we have to perform an integration; in a discrete context, however, the integral should be replaced by a summation.

But, if $r \gg l_0$, we can perform the integration without problems, because the effects of the discretization are negligible. To give an idea of when this might be valid, it must be noted that, by virtue of the value obtained[3] for l_0, already at the level of the atomic nucleus this approximation is certainly valid, being the size of the latter about 20 orders of magnitude greater than l_0. This makes perfectly permissible, even in the discrete context, the use of integrals and derivatives at astronomical scale.

From the mathematical point of view, the structure of Eq. (3) is equivalent to the Newtonian; from the physical point of view, the approximation introduced allows us to use the same reasonings that lead to determine the traditional potential energy, and then we have:

$$U(r) = -mc^2 \frac{d}{r-d} \tag{4}$$

We can immediately verify that at very large distances ($r \gg d$), Eq. (4) coincides with the Newtonian $U(r)=-GMm/r$.

Eq. (4) shows that, for $r=d=GM/c^2$, $U(r)$ is infinite and negative; this means that we must provide infinite energy to move the body away from this position: that is, it can not be removed.

So far we have considered the case where $r>d$.

If $d>r$, the reasoning have to change, because in the first case, in analogy with Newtonian physics, it has been imposed $U(\infty)=0$ because the two bodies do not interact at infinite distance. Here this condition can no longer be imposed: r can not extend indefinitely, because of the limitation $d>r$. When $d>r$ Eq. (1) becomes[2]:

$$f(r) = -mc^2 \frac{r}{(r+l_0)} \frac{d}{(d-r)^2} \tag{5}$$

and Eq. (3) becomes:

$$f(r) = -mc^2 \frac{d}{(d-r)^2} \tag{6}$$

Examining Eq. (5), we can note that the two bodies do not interact for $r=0$: in this case the condition to impose is $U(0)=0$.

We must make a clarification: strictly speaking we could not consider $r=0$, because we have imposed the condition $r \gg l_0$; however, we have seen that this approximation is already valid if the order of magnitude of r is the atomic nucleus, a very little distance that at astronomical scale can be surely approximated to zero. Under these conditions, we can again perform the integration instead of the summation, and the result for $U(r)$ is an expression that coincides exactly with Eq. (4), which therefore can be used either for the case $r>d$ either for the case $d>r$.

3. Relativistic mass-energy relation

If we consider the case *d>r* and we also impose the condition *d»r*, Eq. (4) becomes:

$$U(r)=mc^2 \tag{7}$$

We can obtain this result only imposing strict conditions: $r»l_0$ and *d»r*; that is, if the body is far away from the center of a black hole and far away from its edge, but anyway *inside*.

We got a result apparently strange that tells us that the gravitational potential energy of a body can be reduced to its energy mc^2; but as well it tells us that this can happen *only* if the body is inside a black hole.

This is a different interpretation of mc^2 with respect to the relativistic one.

Since the relation $E=mc^2$, that in a discrete context coincides with the gravitational potential energy $U(r)=mc^2$, has been widely demonstrated both theoretically and experimentally[4], the conditions in which we live are the ones, *the only ones*, that allow the correctness of this relation; then we must necessarily conclude that we live in a black hole, albeit far away from the edge.

So we also have to necessarily conclude that *the Universe is a black hole*.

This is not a completely new concept: some papers[5][6] investigate more or less deeply this possibility. However, in this discrete theory the fact that the Universe is a black hole emerges as a consequence of the correctness of the mass-energy relation.

Let's now calculate the radius that the universe should have if it were a black hole.

There are very different estimations of the mass of the Universe[7]: if we consider a mass of about $\alpha \cdot 10^{53}\ kg$, where α is of the order of some units, we obtain that the radius of the black hole, given by Eq. (2), is approximately $0.7 \cdot \alpha \cdot 10^{26}\ m$, that is about a few tens of billion light years. This is a very rough assessment, but it is compatible with the estimate of the size of the observable Universe. This strengthens the assertion that we are really inside a black hole and that this black hole is the whole Universe.

4. Accelerating expansion of the Universe

If we consider the case *d>r* and we also impose the condition *d»r*, the same conditions imposed to obtain $U(r)=mc^2$, Eq. (6) becomes:

$$f(r)=-\frac{mc^2}{d} \tag{8}$$

and the acceleration of the body is:

$$a(r)=-\frac{c^2}{d} \tag{9}$$

Again, we can obtain this result only imposing strict conditions: $r»l_0$ and *d»r*; that is, if the body is far away from the center of a black hole and far away from its edge, but anyway *inside*.

We have found[2] that, when *d>r*, the acceleration a body undergoes is toward the edge of the black hole, that in this case is the Universe.

Substituting the value of *d* shown by Eq. (2) in Eq. (9) we obtain:

$$a(r)=-\frac{c^4}{GM} \tag{10}$$

where *M* is the mass of the Universe.

So a body, in the same conditions that allow the correctness of the mass-energy relation, undergoes a constant acceleration toward the edge of the Universe whose numeric value is $a(r) \sim 1.2/\alpha \cdot 10^{-9}\ m/s^2$, where α is of the order of some units.

It is a very small amount, but not null, which tells us that, with the conditions $r \gg l_0$ and $d \gg r$, all bodies are moving towards the edge of the Universe with motion uniformly accelerated. The acceleration, however, greatly increases in proximity of the universe edge, as indicated by Eq. (6).

This means that the Universe is undergoing an accelerated expansion? It is not so: in this discrete theory, the Universe is a black hole with a well defined radius depending only on its mass and thus constant. Instead, the objects inside the black hole (the Universe) are attracted toward the border with accelerated motion.

This is not the first time that the term c^4/GM appears in a document[8], but in this discrete theory it indicates the constant acceleration a body undergoes when it is inside a black hole, far away from its edge.

5. Where are we?

Experimental verifications of the relation $E=mc^2$ indicate a very precise correspondence[8]; from the point of view of this discrete theory it is equivalent to say that our position in the Universe is very close to its center: according to Eq. (4) the higher is the precision of the mass-energy relation, the more we are close to the center of the Universe.

The difference between Eq. (4) and mc^2 when $d \gg r$ is:

$$\Delta E = U(r) - mc^2 = mc^2 \frac{1}{1-\frac{r}{d}} - mc^2 \sim mc^2 \left(1+\frac{r}{d}\right) - mc^2 \sim mc^2 \frac{r}{d} \tag{11}$$

So the relative difference is

$$\frac{\Delta E}{mc^2} = \frac{U(r)-mc^2}{mc^2} = \frac{r}{d} \tag{12}$$

In a document[9], a group of physicists at MIT reported that the mass-energy relation has been verified with an accuracy of about 10^{-6}. If we could assume that this very high accuracy coincides with the relative difference in Eq. (12), we should deduce that our actual position in the Universe is at $r=10^{-6}\, d$ from the center, that is at about $0.7 \cdot \alpha \cdot 10^4$ light years, where α is of the order of some units; it's a very little distance if compared with the radius of the Universe. It seems that the center of the Universe could be inside the Milky Way.

Measuring the mass-energy relation, we can determine our position in the Universe in terms of the ratio r/d. Measuring also the acceleration evidenced in the expansion of the Universe, and using both Eq. (4) and Eq. (6), we can obtain the values for r and d.

6. Density of the Universe

We have seen that in this discrete theory the universe is a black hole and its radius, given by Eq. (2), is constant. So also its density is constant:

$$\rho = \frac{M}{\frac{4}{3}\pi d^3} = \frac{3c^6}{4\pi G^3 M^2} \tag{13}$$

In a continuous context, from the equations of General Relativity has been found that

$$\rho_c = \frac{3H_0^2}{8\pi G} \tag{14}$$

where H_0 is the Hubble constant and ρ_c is the critical density: if the universe has an higher density, its expansion will stop and it will collapse back on itself (closed universe); if the universe has a lower density, its expansion will never stop (open universe); if the universe has exactly the critical density, its expansion will stop after an infinite time (flat universe).

Recent measurements[7] using different methods have shown with high accuracy that the density of the universe is very close to the critical density, so the universe should be flat.

In this discrete theory there is no *critical density*, but only *density*, because the universe in not expanding nor contracting. The model of the continuous context that is the closest to the discrete one is the *flat universe*, in which the density of the universe is exactly the critical density.

Equating Eq. (13) and Eq. (14) we obtain:

$$H_0=\sqrt{2}\frac{c^3}{GM} \tag{15}$$

that, for Eq. (2), can be written as

$$H_0=\sqrt{2}\frac{c}{d} \tag{16}$$

There is still a quite high uncertainty on the measured value of H_0, but if we use the widely accepted value of $H_0 \sim 70\ km\ s^{-1}\ Mpc^{-1}$, we obtain $d \sim 6\ Gpc \sim 18\ Gly$.

From Eq. (15) we can also find the mass of the universe in terms of G and H_0:

$$M=\sqrt{2}\frac{c^3}{GH_0} \tag{17}$$

that indicates a value of $M \sim 3 \cdot 10^{53}\ kg$.

Eq. (17) is very close to the Hoyle-Carvalho formula, obtained[7] in a different way.

7. Conclusion

Assuming that space and time can only have discrete values, we obtained the expression of the gravitational potential energy, that under very strict conditions coincides with mc^2. Under the same conditions a body undergoes a constant acceleration that tends to move it away from the center of the Universe. These conditions show that the Universe is a black hole and that our position is close to its center. We also obtained the constant density of the universe and the value of the Hubble constant.

References

[1] Luca Roatta. Discretization of space and time in wave mechanics: the validity limit. 2017. <hal-01476650>. Available at https://hal.archives-ouvertes.fr/hal-01476650

[2] Luca Roatta. Discretization of space and time: a slight modification to the Newtonian gravitation which implies the existence of black holes. 2017. <hal-01493148>. Available at https://hal.archives-ouvertes.fr/hal-01493148

[3] Luca Roatta. Discretization of space and time: determining the values of minimum length and minimum time. 2017. Available at https://hal.archives-ouvertes.fr/hal-01498171

[4] Ying-Qiu Gu. A Sensitive Test of Mass-Energy Relation. 2011. Available at https://arxiv.org/pdf/hep-th/0610189.pdf

[5] R. K. Pathria, "The Universe as a Black Hole," Nature, vol. 240, 1972.

[6] Ludovic Joxe. Our black hole. 2013. Available at https://hal.archives-ouvertes.fr/hal-01326681

[7] Dimitar Valev. Estimations of total mass and energy of the universe. 2010. Available at https://arxiv.org/pdf/1004.1035.pdf

[8] Jean-Philippe Bruneton. Notes on several phenomenological laws of quantum gravity. 2013. Available at https://arxiv.org/pdf/1308.4044.pdf

[9] Pritchard and C. Nature issue 22, 2005. Cited at http://news.mit.edu/2005/emc2