Hybrid 3D Mass-Spring System to simulate isotropic materials

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Workshop on Virtual Reality Interaction and Physical Simulation VRIPHYS (2017), pp. 1–2
F. Jaillet and F. Zara (Editors)

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This work was supported by the LABEX PRIMES (ANR-11-LABX-0063) of Université de Lyon,
within the program "Investissements d'Avenir" (ANR-11-IDEX-0007) operated by the French National Research Agency (ANR).

Abstract

Cubic symmetric Mass-Spring Systems (MSS) for simulating isotropic elastic materials are known to be intrinsically limited by the Cauchy constraints (rari-constant theory) involving Poisson ratio of exactly $\nu = 1/4$. To mitigate the impact of this restriction, we propose a new solution based on correction forces, which authorizes to use MSS for materials with $\nu \in]0;0.5[$. The validation of our model is evaluated by comparing its response to various deformations with Finite Element Model (FEM). The results show that the proposed model is relevant to simulate any compressible isotropic elastic material, and in particular nearly incompressible (Poisson ratio $\simeq 0.5$) biological soft tissues for which it is dedicated.

1. Introduction

In computer graphics, the most important aspects of the simulation are computational speed and stability of the system, while in the biomedical field, the most important aspects is the accuracy of the simulation and its mechanical response (like haptic feedback).

The mechanical model called Mass-Spring System (MSS) meets the first two requirements [ChZK98] and it is widely used for gaming purposes enabling to simulate cloths [BFA02] or deformable solids [VO11, THMG04]. They are based on an intuitive approach with the discretization of the object in a set of particles (or masses) connected together by a set of springs. However, the accuracy of their mechanical behavior is difficult to improve and by consequence it is not considered as a suitable model for real tissue simulation. It is due to troubles to correctly adjust the stiffness of springs according to the mechanical properties of the materials [LSH07, SVAT12], and to difficulties to ensure the correct change of volume of compressible bodies. In this context, over the years several studies attempted to reduce the impact of these issues with some solutions. Nevertheless, the main problem is caused by the elasticity theory. Thus, a MSS can modeled only isotropic elastic material with a Poisson's ratio equal to $1/4$ (the Cauchy constraints). This limits its use to only one type of volumetric behavior and prevents from simulating incompressible bodies (like internal organs) which have a Poisson's ratio $\nu \simeq 1/2$.

For these reasons, the finite element method (FEM) seems to be more relevant in the biomedical field than the MSS, with a better description of the mechanical behavior of the material. Nevertheless, this positive aspect is counterbalanced by the advantages of mass-spring models, namely the fast computation and the effortless topological modifications in real-time, that is without additional pre-computation (which is still necessary when using the FEM). Moreover, the demand from biomedical environment is nowadays more and more urgent to have a mechanical model which rejoins the need of real-time accurate simulations, to enable the use of haptic feedbacks for computer-based surgery or training simulator.

In this paper, we propose a mechanical model suitable for real-time and accurate simulations. It ensures a correct volume behavior of the tissue, according to its provided Poisson ratio value, while satisfying the stability conditions. Moreover, to ensure an accurate mechanical behavior, the formulation of our hybrid 3D MSS integrates the mechanical properties of the simulated material.

2. Our Hybrid 3D cubical MSS with correction forces

Our aim is to be able to use the mass-spring model to simulate deformable soft tissues with accuracy. But as previously said, this model is naturally limited to simulate isotropic elastic material with a Poisson ratio fixed to $\nu = 1/4$. Moreover, this model presents some instabilities in compression, when we reach a certain limit of deformation, causing wrinkles on the layers of a sample.

We dealt with these 2 limitations by proposing a hybrid MSS based on 3D cubical MSS model (see Fig. 2).

- To obtain a more remote instability limit, we used 3 types of springs for each element: a spring on each edge, a spring on each diagonal of each face, a spring on each inner diagonal of element.
- The springs are defined by their stiffness constants: k_e, k_f, k_c.
- To enable the simulation of material with Poisson's ratio $\nu \in]0;0.5[$, we add non-linear forces called correction forces.

Stiffness parameters. The formulations of the springs stiffness constants (naturally based on $\nu = 1/4$) have been derived from the energy involved by the deformation of the springs. Their formulation incorporate the Young modulus E of the material and the springs' rest length l with:

$$k_c = \frac{E\ l\ \mathbf{A}}{10/3\ \mathbf{A}+2.5},\ k_f = \frac{E\ l}{10/3\ \mathbf{A}+2.5}, k_e = \left(\frac{8}{3}\mathbf{A}+1\right)\frac{E\ l}{10/3\ \mathbf{A}+2.5}.$$

The parameter $\mathbf{A}$ corresponds to the ratio k_c/k_f introducing in our system a liberty freedom relevant to the considered material.

Correction forces. To go beyond the Cauchy limitations, we add non-linear forces to the linear springs forces applied on each particle during the simulation loop. To ensure a correct volume behavior during the deformation of the material even in compression, their formulation comes from the energy involved by the system due to the variation of the volume.

By focusing on particle P_2 of a considered cubical element, the correction forces applied to P_2 is defined by:

$$\overrightarrow{F_{P_2}} \ = \ \kappa(V - V_0)\ \frac{\partial V}{\partial P_2},$$

where V_0 is the initial volume of the element and V represents an approximation of its current volume which is based on averaged $\vec{x}, \vec{y}, \vec{z}$ vectors defined by:

$$\begin{cases} \vec{x} = \frac{1}{4}(\overrightarrow{P_1P_2} + \overrightarrow{P_4P_3} + \overrightarrow{P_6P_5} + \overrightarrow{P_7P_8}) \\ \vec{y} = \frac{1}{4}(\overrightarrow{P_2P_3} + \overrightarrow{P_1P_4} + \overrightarrow{P_5P_8} + \overrightarrow{P_6P_7}) \\ \vec{z} = \frac{1}{4}(\overrightarrow{P_1P_6} + \overrightarrow{P_3P_8} + \overrightarrow{P_2P_5} + \overrightarrow{P_4P_7}). \end{cases}$$

Moreover, the parameter κ is defined according to the Young modulus E and the Poisson ratio $\nu \in]0;0.5[$ of the material by

$$\kappa = \frac{1}{2}\ \frac{E\ (4\nu - 1)}{(1+\nu)(1-2\nu)}$$

and the gradient $\partial V/\partial B$ is defined by

$$\frac{\partial V}{\partial B} = \left(\frac{1}{4}(\vec{y} \times \vec{z}) - \frac{1}{4}(\vec{z} \times \vec{x}) - \frac{1}{4}(\vec{x} \times \vec{y})\right).$$

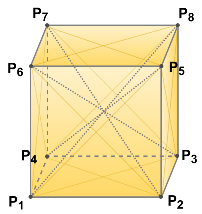

Figure 1: *Basic hexahedral element composed of* 8 *particles.*

Results. The experimental results in comparison with classical MSS show huge improvement in the stability and correct simulation of the incompressible bodies given the mean error of the volume. The Fig. 2 (1-2) shows experimental results of 10% compression on a sample with $E = 20$ kPa and $\nu = 0.4999$. We can see the level of improvement ensured by the use of correction forces and correct formulation of the stiffness parameters in a MSS. We can see the instability of the MSS, which is not occurring when using correction forces and face diagonal spring. Fig. 2 (2-3) on the other hand shows results of a beam sample exposed to gravity force with $E = 400$ kPa and $\nu = 0.3$. Fig. 2 (3) shows how a wrinkle instability can affect a simple deformation of a beam flexion experiment, where the wrinkles at the bottom of a sample change the shape of the deformation.

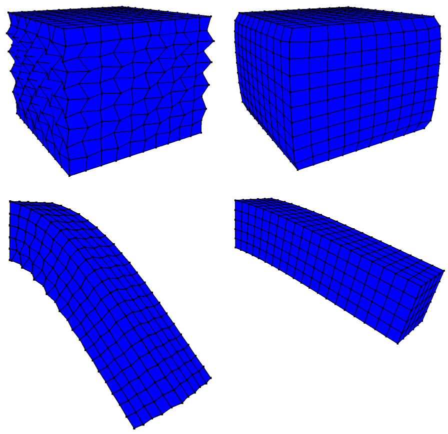

Figure 2: *(From the left) (1) Classical MSS with wrinkle instability,* $V = 115.196$*; (2) MSS with correction forces,* $V = 124.959$*; (3) Classical MSS with wrinkle instability at the bottom; (4) MSS with correction forces, smooth shape without instabilities.*

3. Conclusion

In this paper, we introduced improvements to the classical MSS used to simulate isotropic tissues. Working on cubical-symmetrical meshes, a) we shown the effect of face-diagonal springs on the simulation by avoiding wrinkles in the surface of the object; b) we presented new stiffness formulation, which incorporates the mechanical information about a body into the model, especially Young's modulus and Poisson's ratio; and we shown how to locally correct volumes to keep the simulation physically correct. The solution is suitable for real-time applications, even though the additional computation of the volume and correction forces on each element. The mean performance of 10 s simulation of our hybrid MSS on a sample of 1,000 elements on 8-core processor is 2.88s.

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