Task-layer multiplicity as a measure of community health

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Abstract

A small number of (perhaps only 6) broken-symmetries, marked by the edges of a hierarchical series of physical *subsystem-types*, may underlie the delicate correlation-based complexity of life on our planet's surface. Order-parameters associated with these broken symmetries might in the future help us broaden our definitions of community well-being. For instance we show that a model of metazoan attention-focus, on correlation-layers that look in/out from the 3 boundaries of skin, family & culture, predicts that behaviorally-diverse communities require a characteristic task-layer multiplicity *per individual* of only about $4\frac{1}{4}$ of the six correlation layers that comprise that community. The model may facilitate explorations of task-layer diversity, go beyond GDP & body count in quantifying the impact of policy-changes & disasters, and help manage electronic idea-streams in ways that strengthen community networks. Empirical methods for acquiring task-layer multiplicity data are in their infancy, although for human communities a great deal of potential lies in the analysis of web searches and perhaps other forms of self-reporting.

Keywords: statistical inference, subsystem correlations, broken symmetry, layered complexity, order parameter, evolving codes, community health

1. Introduction

In this paper we examine an empirical way to characterize the extent to which organisms generally, and people in particular, manage to spend time addressing matters that look inward, as well as outward, from *their* boundaries of skin, family, and culture. The approach is inspired by the fact that discussions of both our intelligence and our well-being often center around individual organisms instead of community processes (cf. Sloman and Fernbach (2017)), and that both community and individual measures of well-being face "a prodigious variety of pre-analytic conditions" consistent with commonsense, along with an awareness of scientific insights across disciplines (cf. Bishop (2015)).

The approach here also benefits from the fact that quantitative definitions for order, information, entropy and even available-work have gained sophistication over the past century. Although initially considered properties local to a specific object, such quantities can often be seen as special cases of a more robust "non-local" definition that measures correlations between subsystems (cf. Lloyd (1989)), as does e.g. the binary-logic distinction between true and false which depends on the match between an assertion, and "that in the world around" to which the assertion refers. In fact, the same tools lie at the heart of probability-based (Bayesian) data handling (cf. MacKay (2003); Ghosh et al. (2006)) and model-selection (cf. Burnham and Anderson (2002); Gregory (2005)), which may play a key role implementing and choosing ideas that work in the days ahead.

Modeling the processes by which order emerges, and then fades, is also an area of long-standing interest and increasing activity. Whether we are looking at the evolution of a star from a density fluctuation in an interstellar gas cloud, or emergence of a bubble in the center of a pot of water being heated on the stove, broken symmetries surrounding gradients, boundaries, or pool edges play an important role (cf. Anderson (1972)). Useful concepts sometimes called order-parameters, associated with the spontaneous breaking of symmetries (used here to denote the emergence of newly identifiable asymmetries), are of special interest (e.g. Sethna (2006)). Studies of order-emergence generally have focused on one layer at a time, e.g. on the precipitation of one phase inside another, the development of correlated behaviors between individual cells in an organism, or the formation of special interest groups in a community.

In this paper we specifically focus on the bloom and decline of *layered* complexity. This is a less developed topic, even though studies of "higher-order than pair" correlations in a wide variety of single-layer systems (e.g. Schneidman et al. (2006)) perhaps suggest that the "post-pair" correlations needed, e.g. among internal (base-level) molecules to prepare a one-celled (upper-level) microorganism for survival, may not be coming about by natural selection on the base level alone. Put another way, our microorganism may have to spend resources nurturing (inward-looking) molecular correla-

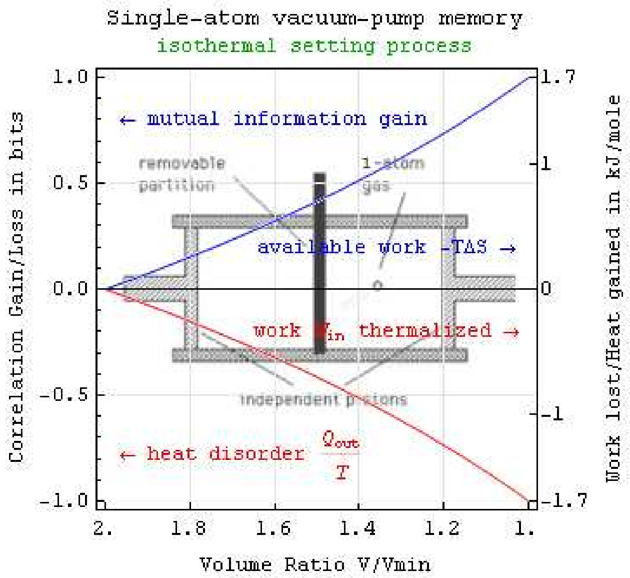

Figure 1: Szilard vacuum-pump memory schematic relating subsystem correlations to reversibly-thermalized work.

tions within, at the same time it is dealing with (outward-looking) challenges from its external environment.

Even if we had robust theoretical underpinnings, however, the selection of order parameters for the upper layers of a complex-system hierarchy is likely to be a matter of field insight, plus trial and error. Lacking much of either at this point, in this paper we propose simply to examine the fractional attention that organisms can give to buffering correlations that look inward and outward from the three highest layers of organization, namely those associated with the boundaries of skin, family and culture.

As we'll see, the approach provides a framework for both characterization and for suprisingly-robust goal formulation (which e.g. works to balance a wide variety of opposing perspectives). However, we will only know what is working if we have ways to obtain data on these matters. That is the next "first step".

2. Symmetry and complexity

We've attempted to outline possible technical connections to order-emergence in simpler systems with an earlier note (Fraundorf (2013)), but these are neither rigorous nor important here. Instead, one might simply consider that in the "natural history of invention", complexity emerges when specific information on broken symmetries, generally associated with gradients, boundaries, or pool edges, becomes available in the outside world. If and when an asymmetry (or external correlation with it, including external awareness of it) fades, the associated complexity fades along with it. Thus for instance liquid water might be seen as isotropic for all practical purposes, even though we know that on the nanoscale it has neither translational nor orientational symmetry.

One of the simplest examples of this is the Szilard vacuum-pump binary memory (Szilard (1929)), in which a symmetric two-piston assembly with removable partition (cf. Fig. 1) contains a single atom at an ambient-stablized temperature T, whose position can be "set" by removing the divider, inserting one piston using available work $W = kT \ln[2]$, followed by return of the partition and removal of the piston. We now know (i.e. have one bit of information about) which side the atom is on. We've added complexity to the world at cost of some thermodynamic availability.

That information can be irreversibly lost if we (i) remove and reinsert the partition, (ii) close our eyes and spin the assembly randomly about an axis through the partition, or (iii) forget which side we put the atom on. Thus at no cost, the world can become less complex. This exercise illustrates the "one-way" nature of spontaneous correlation loss i.e. of entropy increase, the quantitative cost of complexity i.e. of correlation information between subsystems, plus several ways that complexity can spontaneously fade in the absence of effort to keep it in place.

3. Multiple layers, external and internal

Earth life is part of the hierarchy of broken symmetries that began with the collapse of the solar nebula, the accretion of planetesimals to form the planet, and the formation of a surface boundary layer on that planet subjected to the flow of ordered energy (from within and without) to power a layered system of biogeochemical cycles. In these flows shared-electrons first broke the symmetry between in-molecule and extra molecule interactions. In this context many broken symmetries emerged and then faded, but the key symmetry breaks that we focus on here established a hierarchy of correlated subsystems made up of correlated subsystems.

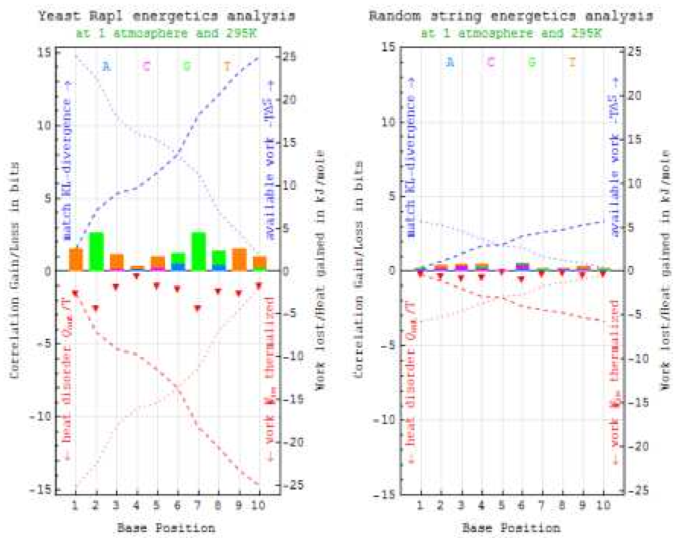

Figure 2: Yeast Rap1 versus equiprobable sequence-energetics, relating correlations to work as in Fig. 1.

Thus one might be tempted to say that life began with the natural invention of bilayer membranes, whose closure allowed the break in symmetry between molecules inside and outside that membrane or cell wall. These single-celled lifeforms can not only tolerate a much wider range of conditions than us multi-celled organisms, but they also invented digital storage of information in molecular codes as illustrated in the Fig. 2 analysis of the information stored in a 10 nucleotide binding sequence (cf. Stormo et al. (1986)), for comparison to the vacuum-pump memory schematic above.

Beyond that, shared resources (like steady-state flows) may have broken the symmetry between in-tissue and external processes, giving rise to our first multi-celled organisms. Beyond this, metazoan skins allowed symmetry between in-organism and out-organism processes to be broken, bias toward family members broke the symmetry between in-family and extra familial processes, and membership-rules (like e.g. tribal xenophobia) broke the symmetry between in-culture and multi-cultural processes.

If we take a closer look at the emergence of order, one might imagine subsystems on a first reference level becoming increasingly correlated through "pair interactions" (cf. Schneidman et al. (2003)) like: (i) argon atoms in a cooling gas or metal atoms in a cooling liquid which form kissing-number 12 clusters, (ii) photosynthetic cells which form a 2D biofilm so that each gets some access to the sunlight, or (iii) children on a playground who fall into a herd when chasing a soccer ball. Next, however, let's imagine the emergence of order one layer up i.e. between clusters of atoms, cells, or individuals. In this case, higher order correlations (we refer to these here as post-pair) between first-level building blocks might be needed, like multi-atom sequences to hold together a polymer, channels between cells in a 3D assembly to provide access to external nutrients, or recognition (by a third child) that when two children are fighting they should probably side with the one which is their sibling.

Although pair correlations between building blocks on one level can of course be key to the survival of higher level assemblies, the rationale behind emergence of higher order correlations (like altruism among individuals) is often easier to see in terms which look inward from the dynamics one level up (like selection in terms of family genetics or group culture cf. Okasha (2008); Nowak et al. (2010); Richerson and Boyd (2004)). This of course might seem at best abstract to researchers used to thinking in terms of lower-level component interactions alone.

There may also be some reason to think about pair correlations (e.g. between molecules inside a cell, neural connections between cells, and individuals in a community) when one is considering order emergence looking outward from the building blocks of one level of organization. In that context, one might tend to think about post-pair correlations between those same building blocks when looking inward from the boundary of a composite entity one level up. In other words, adaptation of a composite entity to the world around often involves pair correlations between similar composite entities, along with post-pair correlations of building blocks internal to that entity. This is the basis for our distinction between outward and inward looking correlations with respect to each boundary in the discussion to follow, even though mapping of these as pair and postpair, respectively, is approximate at best.

In this paper we focus on the perspective of (a) metazoan individuals as both audience and agent, instead of for instance on (b) the perspective of individual micro-organisms, or (c) the perspective of whole family gene-pools even though this is of much recent interest in biology. In that context, therefore, we center our attention on the last three symmetry-break levels (skin, family, culture) and the six subsystem-correlation layers associated therewith.

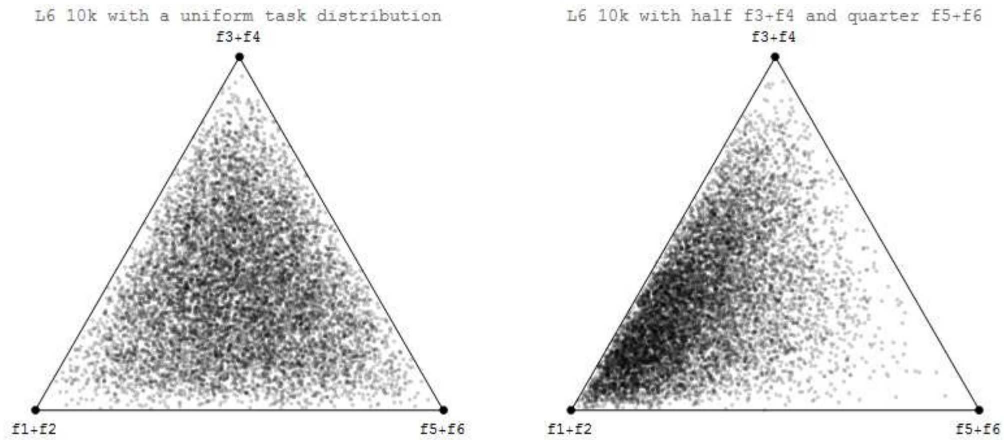

Figure 3: At left is a random simplex-point picked 6-layer population of 10,000 individuals, projected onto a ternary plot with subsystem correlations e.g. in/out from skin in the lower left, in/out from family at top, and in/out from culture at lower right, resulting in $M_{\rm cm} \simeq 6.0$ and $M_{\rm geom} \simeq 4.26$. At right is a similar 6-layer population, in which participation buffering of correlations that look in/out from family has been cut in half, and of correlations that look in/out from culture has been divided by 4, resulting in $M_{\rm cm} \simeq 5.39$ and $M_{\rm geom} \simeq 3.87$. The latter might be expected e.g. for a human population which has limited access to jobs, and even more-limited access to cultural/professional education.

4. A task layer-multiplicity simplex

Selection of order parameters for complex systems is sometimes more of an art than a science. Here as in the selection of order-parameters for simpler (albeit still-complex) thermodynamic systems, we seek a measure based on information available with minimal disruption.

For inputs, we begin with (up to) $L = 6$ normalized positive numbers f_i representing the fraction of an organism's effort allocated to buffering subsystem correlations associated with each of the 6 subsystem correlation-layers i.e. which look in/out from skin, family and culture. In other words, by various means we try to get a sense of the types of tasks that individuals in a given community manage to spend their time on. For vizualization-purposes these six positive f_i values (which add up to 1) allow us to map the layer-focus of organisms to individual points within the *unit 5-simplex* between 6 vertices, just as ternary-diagrams map any three normalized positive-numbers onto an equilateral triangle or 2-simplex in a plane. The latter in this context may be used to project normalized groups of these fractions, as shown in Fig.3, while a hexplot of ternary diagrams might be useful for a more complete view of

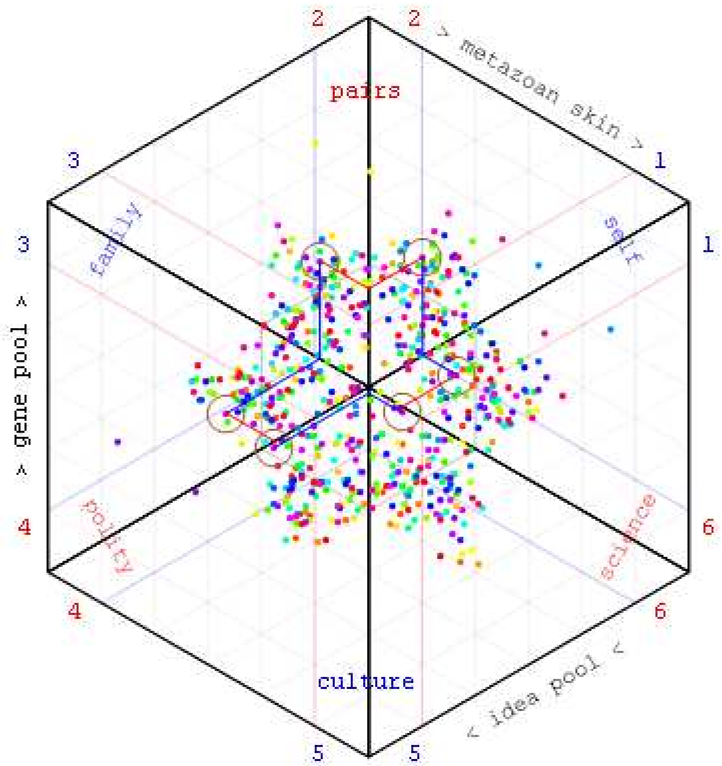

Figure 4: Six-projections of 100-member random simplex point-picked dot-cloud, with projections of one individual organism circled. The attention-fraction associated with the outer-vertices is labeled, while the central vertex in each ternary-plot triangle represents the sum of the remaining fractions.

an $N = 6$ population (cf. Fig. 4).

To inventory order we then define a single metazoan-individual's niche-network layer-multiplicity m as the behavior-defined effective-number of correlation buffering choices, expressed as an entropy-exponential in terms of that organism's set of e.g. $L = 6$ fractional-attention values $\{f\}$:

$$1 \leq \#_{\text{choices}} \equiv m[\{f\}] = \prod_{i=1}^{L} \left(\frac{1}{f_i}\right)^{f_i} = 2^{\#\text{bits}} \leq L \tag{1}$$

where $\Sigma_i f_i = 1$ i.e. sums to one over the level-index $i = 1, L$.

This multiplicity measure can also be expressed in terms of the number of bits of surprisalTribus (1961) or state-uncertainty S in bits about which correlation layer (e.g. self, friends, family, job, culture, profession) they are working on at any given time, i.e. $S = \ln_2[m] = \Sigma_i f_i \ln_2[1/f_i]$. However use of $\#_{\text{choices}}$ instead of $\#_{\text{bits}}$ probably makes more sense here since the numbers are so small.

Population-averages i.e. normalized-sums over all N community members (say using index $j = 1, N$) will be denoted with angle-brackets like $\langle\rangle$. Thus the **population-average individual-multiplicity** is $\langle m \rangle = (1/N)\Sigma_j m_j$. The population-average value for attention-fraction f_i is $\langle f_i \rangle = (1/N)\Sigma_j f_{ij}$ where f_{ij} is the jth individual's layer i attention-fraction.

We'll use $\{\langle f \rangle\}$ to refer to the set of all L attention-fraction population-averages. This allows us to define a **center-of-mass multiplicity** $M_{\text{cm}} = \Pi_i^L (1/\langle f_i \rangle)^{\langle f_i \rangle}$, representing the spread in attention-focus for the community as a whole. In non-social organism communities, for instance, the fraction of time spent on matters of social hierarchy, let alone intra and extra cultural pursuits, may be quite small, pushing the center of mass multiplicity closer to only 3 of the 6 layers that we are considering here.

We may also want to consider **population average-surprisal** or entropy $\langle S \rangle = (1/N)\Sigma_j^N S_j$. This leads simply to the **geometric-average individual-multiplicity**, defined as $M_{\text{geom}} = 2^{\langle S \rangle} = (\Pi_j^N m_j)^{1/N}$ for which it is easy to show that $M_{\text{geom}} \leq M_{\text{cm}}$. Because of this organic relation to the center-of-mass value, we'll use M_{geom} as our indicator of the spread in attention-focus for individual organisms with the community. For instance, a community of individuals might have a center of mass multiplicity of 6 even if half of the individuals only take on nurturing (e.g. inward looking or post-pair correlation) tasks, while the other half only takes on adventuring (i.e. outward-looking) tasks. In that case the geometric average multiplicity would only be about 3.

The inequality above naturally lets us define organism and community **specialization indices**, whose logarithms are KL-divergences, which decrease in value toward 1 only as the spread of individual foci begins to match that of the community as a whole. For the community specialization index R, we use $1 \leq R \equiv M_{\rm cm}/M_{\rm geom} \leq M_{\rm cm}$. The community specialization index R would thus be only about 1 for a community in which all individuals spent equal amounts of time on all six layers, while for a community adopting the "nurture/adventure" (or "yin/yang") dichotomy mentioned above, the specialization index would approach 2.

For use only in Fig. 5, although they are also useful for deriving some inequalities, along with individual multiplicity $m_j \equiv \Pi_i^L f_i^{-f_{ij}}$ one might also define individual specialization indices $r_j = (1/m_j)\Pi_i^L \langle f_i \rangle^{-f_{ij}}$. Like the community specialization index R, r_j will always be between 1 and L.

Finally, we recommend comparison of communities in this context with a "uniform-reference" community, in which all combinations of task assignment are equally probable. In general this will allow researchers to see operating biases toward effort spent buffering sub-system correlations on one layer or another. Comparison of experimental data from real communities, to this reference, might also help explore the possibility that task-layer diversity has a selective advantage, and/or is a useful measure of community well-being. Quantitative aspects of this reference are discussed further in Appendix A.

5. Applications

Describing live communities *quantitatively* in terms of subsystem correlations may be in its infancy. Operational models for describing subsystem correlations in biofilms, within and between species in plant communities, in communities of social insects, as well as in primate communities including our own, can only be done with help from experts with field involvement in each of these areas.

The objective of this section is therefore simply to take a cursory look at some aspects of the potential for such an approach, with a bias toward its application in 6-layer human communities. Moreover we'll focus mainly on uses not for *detailed aspects* of observed distributions, but on center-of-mass task layer-multiplicity $M_{\rm cm}$ as a measure of correlation-layer activity relevant to the survival of living systems, and the perhaps more subtle adaptive-value of task-layer diversity i.e. of a community with specialists and generalists of all sorts. These analyses treat all subsystem-correlation layers equally, in spite of a hierarchical structure which shows they are not i.e. individuals are clearly pre-requisite to family, which in turn may be pre-requisite to culture. By averaging over any given community's population, data in this form is perhaps also by its nature "anonymous" as far as specific individuals in a community are concerned, even though establishing useful protocols for obtaining it in any given community type remain a future challenge to be discussed briefly in the next section.

5.1. task-layer breadth

Imagine that M_{cm} began increasing toward 2 when the metazoan skin of multi-celled organisms predicated the symmetry-break between self-focused behaviors (like hunger & fear) and pair-focused behaviors (like aggression & pair-bonding). When such social organisms began treating their young differently from the young of others, molecular code-pool boundaries facilitated the symmetry-break between family-focused behaviors (like bower-building & child-rearing) and socially-focused behaviors (like status-pursuit & community-service) letting M_{cm} approach 4. M_{cm} was allowed to approach 6 only after communicating organisms began recognizing distinctions between in-group and outsider patterns, allowing idea-pool symmetry-breaks to distinguish behaviors that are culturally-focused (like religion & sports) and extra-cultural (like professional-development & library-building). Astrophysical observations indicate that environments for such multi-layer correlation-structures are short-lived (e.g. Ward and Brownlee (2000)), so quantitative models for M_{cm}'s increase & decrease with time may be worthwhile.

These models might provide integrative measures of social patterns already of interest, like division of responsibility between large and small gamete metazoans (i.e. femaile/male role specialization), and quantitative comparison of the extent and nature of community cultural-correlations from one species to another or from one time to another for a given species. If center-of-mass multiplicity correlates with other measures of health in human communities, it could be especially important for going beyond single-layer measures, like gross domestic product and body count, for taking quantitative account of family and culture when assessing the impact of policy changes and disasters on a given community (cf. Fig. 3).

There are immediate as well as abiding practical possibilities here. Available resources, as well as the preservation of task layer-diversity, means that individual-humans are fallible in that their capabilities will *either* span only a part of the 6-layer correlation-hierarchy that underlies human social-systems today, *or* be spread quite thin across all 6. This is also true, in spite of our evolutionary attraction to social-hierarchies, about the vision of any given leader or demagogue.

Regardless as the ordered-energy available per-capita decreases (with either increasing population or energy-costs), we can expect the 6-layer structure of our social-systems to experience pressure to deconstruct (e.g. Chaisson (2004)). The demagogues of communism and fascism in the last century, as well as the demagogues of religious-fundamentalism today, are evidence of pressure to toss out one layer or another of our social-organization. *Data with which to track, and concepts with which to communicate, about these pressures and their effects will be important* if we want to give human social-systems on earth a chance to do their best.

5.2. task-layer diversity

When diversity of task assignments for individuals, as distinct from the task-layer breadth of attention in the community as a whole, is maximized by random simplex point-picking as outlined in Appendix A, $M^*_{\rm cm} \simeq 6$ but $M^*_{\rm geom} \simeq 4.26$. In other words the opportunity to be equal may not argue that everyone contribute on all layers (specialization index $R \simeq 1$). However we might look for a specialization index closer to 1.4 e.g. significantly less than the $R \simeq 2$ expected for a community with "nurture/adventure" role-specialization. This may help us address the "urgent question" posed in the late 19th century by Emile Durkheim in his dissertation on workplace divisions of labor (Durkheim (1893)), whether to choose roundedness or specialization, by saying "if possible explore roundedness, but specialize when that works better for you". This is consistent with subsequent trends away from rigid divisions of labor (e.g. based on heritage and gender) at home as well as at work.

The physiological division of labor between large and small gamete metazoans in reproductive roles, e.g. in social insect communities, shows that task-layer diversity may not always be an adaptive choice. However communities with higher free-energy per capita and electronic information-flow seem to be moving away from cultural role-divisions. Fig. 5 illustrates by comparing R and $M_{\rm geom}$ of a 6-layer model with task-diversity maximized

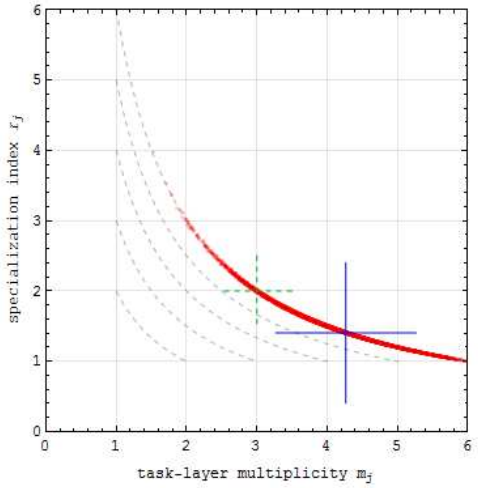

Figure 5: The red dots denote individual specialization indices r_j as a function of individual task-layer multiplicities m_j for organisms in a 6-layer random simplex point-picked population of 10,000 individuals. The blue-cross is the specialization index R for this population, the green dashed-cross for a more specialized "nurture/adventure" population. The dashed lines follow $r_j \simeq L/m_j$ for L of 2 through 6 layers, successively outward from the origin.

by random simplex point-picking (larger plus) with the same quantities for a "yin-yang" community (smaller plus) in which half of the organisms each buffer subsystem correlations directed only inward, or only outward, from skin, family & culture.

6. The data challenge

All of the applications above are predicated on a source of data about attention-focus in a given community. One may attempt to acquire data on some organism communities by direct observation. In human communities, self-reporting and communication-traffic analysis may also be useful particularly for data on short-term changes in attention-focus. A possible self-reporting strategy might involve experience-sampling (Hektner et al. (2007); Killingsworth and Gilbert (2010)) by selecting a layer from 1 to 6 on your phone, when the occasional request comes in. In fact, the community well-being categories in the Gallup-Healthways Well-Being 5 Index (Sears et al. (2014)) might be seen as mapping loosely to correlations that look inward from skin ("physical"), inward from family ("social"), outward from family ("financial"), outward from skin ("community"), and in/out-ward from culture (combined e.g. as belief and profession related "purpose").

7. Conclusion

We describe in this paper a physical "broken-symmetry" approach toward community-structure inventories. It is integrative in that it is inspired by work on broken-symmetries in simpler physical systems, and in that its basics should apply to living systems on other levels of organization and in different astrophysical settings.

Its timing is important because discussions of well-being science have focused on the meaning, measurement, and improvement of individual as distinct from community well-being, and in that context not made explicit connections to the bloom and decline of complexity. As we turn our focus on a finite earth to sustainability, connections of individual well-being to our understanding of the gain and loss of complexity in both physical and biological systems will be important.

By way of example, Cloninger's measures (Cloninger (2004)) of unconscious style or temperament seem largely physiological, but his conscious "idea-mediated" elements of character (namely self-regulation, cooperativeness, and judicial-transcendence as more active elements of our "post-paleolithic" development) might map reasonably well with our interest in one's attention-focus on broken-symmetry subsystem correlations that look in/out, respectively, from skin, family, and culture. Clearly, experts from more than one field are called-upon to acquire and explore data relevant to possible connections like this, and more importantly to put such connections to good use.

Appendix A. The uniform task-layer diversity reference

A nice mathematical feature of simplex models, involving normalized fractions or probabilities, is that they follow the statistics of compositional analysis (cf. Aitchison (1986/2003)). This means that the statistics is already well-explored, and it makes projections from a 5 simplex with 6 vertices into lower dimensional simplex spaces easy as well (cf. Figs. 3 and 4). Hence a wide range of understandable illustrations e.g. of the effect of policy changes and events on a community's focus can be expected as more data on real communities in this format become available.

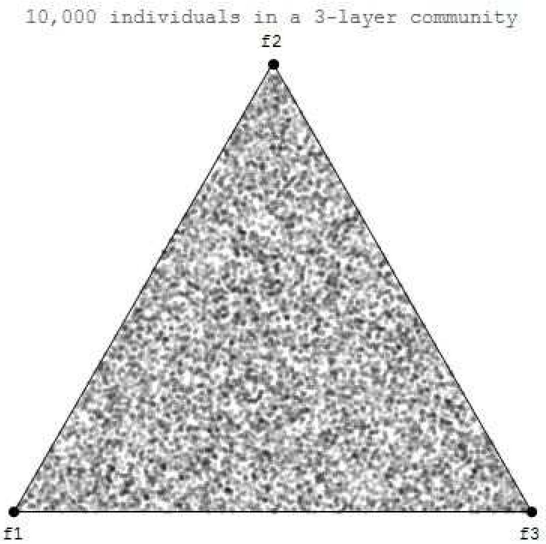

Figure A.6: This is a test of our Dirichlet-based routine for random simplex-point picking, using a unit 2-simplex triangle with 3 vertices, because the uniformity associated with 10, 000 points is easily illustrated on a flat-screen ternary diagram.

For the moment, in order to explore an L-layer community in which all possible mixes of attention-focus for individuals occurs with equal probability, we examined analytical approaches, as wall as algorithms for random simplex-point picking based e.g. on the Dirichlet distribution (cf. Fig. A.6). When running these algorithms on say 100 communites each of a million individuals, they all predict that the center-of-mass multiplicity approaches L, since there is no bias in this random model toward effort directed toward one layer of community organization over another. In other words, we expect the population-average for attention-fraction f_i to equal $1/L$.

This reference value (denoted with an asterisk) for a 6-layer community of $M^*_{\rm cm} \simeq 6$ thus signifies the collective ability of the community to apportion its effort equally toward the buffering of correlations that look in/out from skin, family and culture. Limited historical opportunities, policy changes, disasters, and environmental changes can only reduce this value.

The foregoing quantity, however, says nothing about role-specialization or the lack thereof. For instance, one might think of social-insect communities with extreme amounts of role specialization, but which nonetheless manage to buffer correlations on all the levels needed for their survival. One way to measure this is to look at the breadth of activities for individuals in the community. Rather than measure diversity against a requirement that "alll individuals give equal effort in all layers", however, we propose here that we look for biases in experimental data with respect to a community in which (as above) all possible task-assignments are equally probable. This kind of reference should help examine biases for or against any type of task-layer assignment.

Following rigorous derivation of M^*_{geom} for communities with $L \leq 3$, we infer that a uniform distribution of tasks for arbitrary L will give:

$$M^*_{\text{geom}} = 2^{\int_0^1 df_1 \int_0^{1-f_1} df_2 \cdots \int_0^{1-\Sigma_{i=1}^{L-2} f_i} df_{L-1} (L-1)! S}, \tag{A.1}$$

where as usual $S = \ln_2[\Sigma_{i=1}^{L} f_i^{f_i}]$ and $f_L = 1 - \Sigma_{i=1}^{L-1} f_i$. This implies that for communities of one to eight layers that

$$M^*_{\text{geom}} = \{1, e^{\frac{1}{2}}, e^{\frac{5}{6}}, e^{\frac{13}{12}}, e^{\frac{77}{60}}, e^{\frac{29}{20}}, e^{\frac{223}{140}}, e^{\frac{481}{280}}\} \tag{A.2}$$

This assertion has been checked quantitatively to half dozen significant figures for values through $L = 6$ by simplex-point picking, and suggests that a good rule of thumb (for $L \leq 10$ within 0.5%) is $M^*_{\text{geom}} \simeq 0.65L + 0.35$. Thus unbiased distribution of task assignments in an $L = 6$ community means that individuals on average are buffering subsystem-correlations in only $M^*_{\text{geom}} = e^{29/20} \simeq 4.2631$ layers. This is good news, given that the opportunity to buffer more layers was probably absent during the paleolithic times of our species' evolution. It is also good news for individuals in that, even when the opportunity to "do everything" is available, it may well not be your best choice.

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