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GRAVITY INDUCED MIXING IN A VERTICAL TUBE

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Introduction

Mixing between two miscible fluids of different densities ($\rho_1$ and $\rho_2$) and same viscosity ($\eta$) superimposed in unstable configuration, is related to the development of Rayleigh-Taylor instabilities\textsuperscript{1,2}. We shall be concerned here with the case of miscible fluids which is relevant to many practical situations\textsuperscript{3,4,5} such as laser induced nuclear fusion, extraction columns or fire propagation in vertical shafts. The growth of the mixing zone is often observed to be quadratic with time\textsuperscript{6,7,8} and then linear\textsuperscript{6,9}. Front displacement laws close to $t^{-0.5}$ have been reported in vertical tubes\textsuperscript{4,5}.

The aim of this work is to study this mixing in a long vertical tube. Previous studies have analyzed gravity induced mixing in tubes\textsuperscript{4,5}. However, in both cases, the initial interface is located at one of the ends of the tube and the diffusive growth of the concentration profiles can not be observed simply. In our experiments, the interface is initially at mid-height and the concentration profiles are measured before the mixing zone reaches either end of the tube. Instabilities develop then in two semi-infinite media. Above a threshold density contrast $At_m$, these profiles display a self-similar dependence on the ratio of distance and of the square root of time.

Experimental procedure

In a vertical perspex tube (4 m high and 20 mm in diameter), an heavy fluid is placed over a lighter miscible one. The light fluid of density $\rho_1$ and viscosity $\eta$, is a water-glycerol mixture between 0 and 60 % mass of glycerol, dyed with nigrosine 40 mg/l. The heavy fluid of density $\rho_2$ and same viscosity $\eta$, is the same solution of water-glycerol with a CaCl$_2$ concentration between 0.05 and 300 g/l. Density contrasts are characterized by the Atwood number $At = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ that varies from $2 \times 10^{-5}$ to $9 \times 10^{-2}$, and viscosity by the quotient $\eta/\eta_0$ ($\eta_0$ being the viscosity of pure water) between 1 and 12.

Mixing is initiated by opening a slot valve located at mid-height. Then we study mixing along the tube during approximately 20 minutes, by recording with a digital camera images of a 2.6 m long central section of the tube (1300 x 20 pixels) at 2 s intervals. Profiles of the mean concentration in the tube section are then obtained using an independent calibration. Normalized profiles $C(x,t)$ are finally computed using reference images corresponding to the two pure solutions. Successive profiles are juxtaposed into spatiotemporal diagrams : grey levels correspond to the value of $C(x,t)$ (black for the dyed lighter solution, white for the transparent heavy one).

Experimental results for water ($\eta=\eta_0$\textsuperscript{10})

At high density contrasts, fluid volumes of characteristic scale about 1 cm move randomly at velocities of the order of a few mm/s over distances of the order of the tube diameter : this flow is weakly turbulent and induces an efficient mixing similar to eddy diffusion. (Fig.1-a)

Only a continuously varying greyshade is visible on the spatiotemporal diagram (Fig.1-b) implying that the amplitude and size of relative concentration fluctuations are small (Fig.3-a). The S shape of the $C(x,t)$ curves and the lack of clear-cut boundary (Fig.1-b) of the mixing zone, suggest a diffusive process ; this is confirmed by the fact that, when concentration profiles obtained at different times in the same experiment are plotted as a function of the scaling variable $x/\sqrt{t}$ (Fig.1-c), all curves collapse onto a single one $C(x/\sqrt{t})$, which is moreover perfectly fitted by an error function solution of a 1D diffusion equation. The fit provides an effective diffusivity $D$, which has
thus a purely macroscopic meaning. Such fully self-similar sets of profiles are observed in the range of Atwood numbers: $4 \times 10^{-3} \leq At \leq 9 \times 10^{-2}$.

![Figure 1](image1.png)

**Figure 1.** Mixing regime obtained for $At = 1.5 \times 10^{-2}$
(a) video images, 300 mm above the valve; (b) spatiotemporal diagram; (c) $C$ vs $x/\sqrt{t}$ at 3 successive times.

At lower density contrasts, the initial instability finger (Fig.2-a) displays a mushroom shape characteristic of finite Reynolds numbers ($10$ or more). Transverse waves develop in the wake of the finger which takes an helical shape. They reflect instabilities which are due to the strong shear gradient at the interface between the ascending and descending fluids, and induce transverse mixing across the pipe. Fluid patches of higher density contrast (darker colour on Fig.2-a) propagate in the wake of the leading tip (dashed lines on Fig.2-a) and move faster than it. Their successive arrival at the front prevents its speed from decreasing through mixing with the surrounding fluid.

![Figure 2](image2.png)

**Figure 2.** Mixing regime obtained for $At = 8 \times 10^{-4}$
(a) video images, 300 mm above the valve; (b) spatiotemporal diagram; (c) $C$ vs $x/\sqrt{t}$ at 3 successive times.

The mixing region has a sharp boundary (Fig.2-b) corresponding to the tip of the finger and materialized by concentration steps on the concentration profiles (arrow on Fig.2-b). The local slope of the boundary (continuous line on Fig.2-b) represents the instantaneous velocity $V_t$ of the tip of the displacement front. At low $At$ values, $V_t$ is roughly constant with time. At higher density contrasts, the coordinate $x_t$ of the tip varies approximately as $t^{0.5}$ ($4.5$). The typical front velocity $V_t$ (measured at short times) increases, as expected physically, from 1 to 5 mm/s over the range of $At$ values investigated. However, inspite of these important differences between this regime and the previous one, all profiles still overlay when plotted as a function of the reduced variable $x/\sqrt{t}$, and can be fitted by an error function. Oblique streaks visible on the spatiotemporal diagram (dotted line on Fig.2-b) correspond to the internal motions observed previously (velocity $V_f$). The self-similar trend is however only followed between the concentration steps (arrows on Fig.3-b). This latter evolution towards a fully macroscopically diffusive regime occurs later as $At$ decreases.
Figure 3. Concentration profiles obtained at $t = 480\ s$ for 3 different density contrasts
(a) $At = 3.5\ 10^{-2}$; (b) $At = 4\ 10^{-4}$; (c) $At = 5\ 10^{-5}$.

At still lower Atwood numbers ($At < At_m = 1.5\ 10^{-4}$), a region of average concentration about 0.5, develops in the central part of the tube (Fig.3-c), which corresponds to a stable parallel counterflow of the two fluids: the light fluid ascending and the heavy one going down, without mixing. The length of this region increases with time and helical instabilities persist only near the ends of the tube. In this regime, the global concentration profiles cannot be fitted by error functions and mixing is not diffusive any more.

**Influence of viscosity on the mixing process**

The influence of viscosity was studied using water-glycerol mixtures in the same proportions in the two fluids. Viscosities $\eta$ up to $12\eta_0$ can be reached, while retaining the Newtonian behaviour of the fluids.

The spatiotemporal diagrams display a similar sequence of variations as for water when $At$ decreases: starting from a purely diffusive regime, sharp boundaries appear, and then a stable counterflow region. One observes qualitatively that, in order to obtain a similar spatiotemporal diagram (and thus a similar mixing regime), the density contrast between the two fluids must be higher when their common viscosity is increased.

**Macroscopic diffusivity variations**

For $\eta = \eta_0$ (water) (Fig.4-a), at lower $At$ values ($At_m = 1.5\ 10^{-4} < At < 10^{-2}$), $D$ is about constant and of the order of $2.5\ 10^{-4}\ m^2\ s^{-1}$, despite the different flow configurations observed in the same range (Fig.1-b and 2-b). At large $At$ values ($10^{-2} < At < 9\ 10^{-2}$), $D$ increases roughly linearly with $At$ from $2.5\ 10^{-4}$ to $6\ 10^{-4}\ m^2\ s^{-1}$.

Since mixing is associated to internal fluid motions in the mixing zone, $D$ may be considered as the product of their characteristic velocity $V_f$ and a length $l$. $V_f$ determined from the slope of the streaks (dotted line on Fig.2-b), increase with $At$ from 2 to 10 mm/s over the range studied, while the characteristic length $l$ decreases and saturates with $At$ increasing (Fig.4-a). From this point of view, the fact that $D$ is roughly constant in the range $1.5\ 10^{-4} < At < 10^{-2}$ would reflect a compensation between variations of $V_f$ and $l$. For $10^{-2} < At < 9\ 10^{-2}$, $l$ reaches a lower limit of the order of 45 mm, probably related to the tube diameter: the increase of $D$ with $At$, would then reflect that of $V_f$. 
Figure 4. Diffusivity variations with (a) density contrast (also $V_f$ and $l$ vs $At$) and (b) viscosity.

At a given $At$ value, $D$ increases with viscosity. This result, at first surprising, can be explained by the experimental fact that the wakes are less destabilized in the case of viscous fluids. Consequently, the velocity of the front $V_f$ and that of internal fluid motions $V_l$ both increase with $\eta$.

At a given viscosity $\eta$, the diffusivity is a linear function of Atwood number: $D=D_0(\eta)\cdot(1+aAt)$, with $a$ independent of $\eta$. The variation of $D_0(\eta)$ is plotted on Fig.4b for different $At$ values. There results a phenomenological expression giving the diffusivity $D$ as a function of Atwood number $At$ and viscosity $\eta$: $D= c \cdot \eta \cdot (1-b \cdot \eta/\eta_0) \cdot (1+aAt)$, where $a$ is of the order of 0.012, $b$ is about 0.05 and $c = 200$.

**Conclusion**

We have shown the existence of a macroscopic diffusive regime in gravity induced mixing in vertical tubes, and found a simple dependence of the corresponding diffusivity on the density contrast and the viscosity. In subsequent work, the influence of the diameter of the tube and of its inclination on the mixing will be studied, as well as that of the fluid rheology.

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1 Lord Rayleigh, "Investigation of the character of the equilibrium of an incompressible heavy fluid of variable density", *Scientific Papers, ii, 200-7*, Cambridge, GB (1900).