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Response of a laboratory aquifer to rainfall

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We investigate the response of laboratory aquifer submitted to artificial rainfall, with an emphasis on the early stage of a rain event. In this almost two-dimensional experiment, the infiltrating rainwater forms a groundwater reservoir which exits the aquifer through one side. The resulting outflow resembles a typical stream hydrograph: the water discharge increases rapidly during rainfall, and decays slowly after the rain has stopped. The Dupuit-Boussinesq theory, based on Darcy's law and the shallow-water approximation, quantifies these two asymptotic regimes. At the early stage of a rainfall event, the discharge increases linearly with time, at a rate proportional to the rainfall rate to the power of $3/2$. Long after the rain has stopped, it decreases as the squared inverse of time (Boussinesq 1903). We compare these predictions with our experimental data.

1. Introduction

During a rain event, some of the rainwater flows over the landscape as surface runoff, while the rest of it infiltrates into the porous ground (Horton 1945; McDonnell 1990; Neal & Rosier 1990; Kirchner *et al.* 2000). Drained by gravity, the infiltrating water eventually joins a groundwater reservoir, thus elevating its surface. Within this reservoir, the resulting pressure increase induces a flow towards the drainage network (Sanford *et al.* 1993; Szilagyi & Parlange 1998; Andermann *et al.* 2012; Devauchelle *et al.* 2012).

Through its ability to store water, the aquifer acts as a filter between the rainfall signal and the water output to the drainage network (Sloan 2000; Kirchner *et al.* 2009). A visible consequence of this filtering is that a river still flows long after the rain stopped (Chapman 2003). This drought flow has attracted much attention due to its obvious implications for water resources management, and because it provides information about the groundwater flow (Kirchner *et al.* 2009).

In contrast, little is known about the contribution of groundwater to the streamflow during rainfall. In small catchments, geochemical data based on passive tracers (such as water isotopes or chloride) indicate that storm flows are mostly composed of groundwater (Neal & Rosier 1990; Buttle 1994; Ladouche *et al.* 2001; Kirchner 2003). Still, the groundwater dynamics during a rain event remains unclear (Henderson & Wooding 1964; Beven 1981; Nieber & Walter 1981).

Darcy's law provides a natural representation of the groundwater flow, although networks of large-scale fractures in the aquifer can break this approximation (Long *et al.* 1982; Renard & de Marsily 1997; Erhel *et al.* 2009). Accordingly, in unconfined aquifers, the elevation of the groundwater surface controls the horizontal pressure gradient which drives the flow. As most natural aquifers are far more extended horizontally than vertically, the shallow-water approximation provides a convenient simplification of the problem, referred to as the Dupuit-Boussinesq theory (Dupuit 1863; Boussinesq 1903; Polubarinova Kochina 1962).

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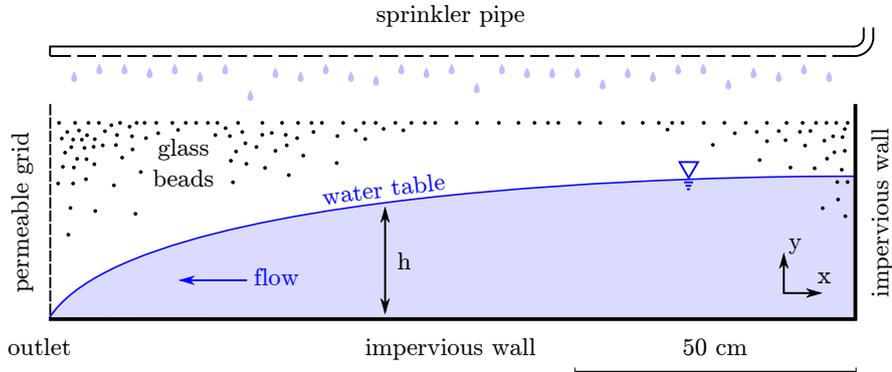


FIGURE 1. Experimental aquifer. The water table (blue line) is submitted to atmospheric pressure. The aspect ratio is preserved.

The Dupuit-Boussinesq theory reproduces accurately the relaxation of the water table observed both in laboratory experiments (Ibrahim & Brutsaert 1965) and in field measurements (Brutsaert & Nieber 1977; Brutsaert & Lopez 1998). To our knowledge, its ability to describe the early stage of a rain event has not been assessed yet.

Here, we investigate the dynamics of a laboratory aquifer, with an emphasis on the early stage of a rain event.

2. Laboratory aquifer

The simplest configuration of a free-surface aquifer consists of a homogeneous porous medium partly filled with water, the flow of which is confined to a vertical plane. We approximate this idealistic representation with a quasi two-dimensional tank filled with glass beads of either 1 mm or 4 mm in diameter (figure 1). Using a Darcy column, we measure a conductivity K of $0.97 \pm 0.06 \text{ cm s}^{-1}$ for the 1 mm glass beads and $K = 5.7 \pm 1 \text{ cm s}^{-1}$ for the 4 mm glass beads. Two vertical glass plates ($143 \times 40 \text{ cm}$) separated by a 5 cm gap hold the glass beads between an impervious vertical wall (right-hand side), an impervious horizontal bottom, and a permeable grid through which water can exit the experiment (right-hand side). The impervious wall corresponds to the drainage divide of a natural aquifer, whereas the outlet of the experiment is a rough representation of the drainage network.

If the grid were in direct contact with air, surface tension would induce a pressure jump at the outlet. To avoid this inconvenience, we spread a soft plastic sheet over the outside of the grid. This device maintains a thin film of water at atmospheric pressure over the grid surface.

To simulate rainfall, a sprinkler pipe is held above the tank. A series of 31 holes spreads regularly along it, and distributes water evenly over the aquifer surface. Rainwater infiltrates vertically through the porous material until it reaches the bottom of the tank, where its accumulation forms a reservoir. Air connects the upper surface of this reservoir to atmospheric pressure through the unsaturated area of the aquifer. This free surface is referred to as the water table.

As the water table expands to accommodate more rainwater, the asymmetry of the boundary conditions causes it to curve towards the aquifer outlet. The resulting pressure field pushes the water out of the aquifer, generating a discharge Q . The flow exiting the

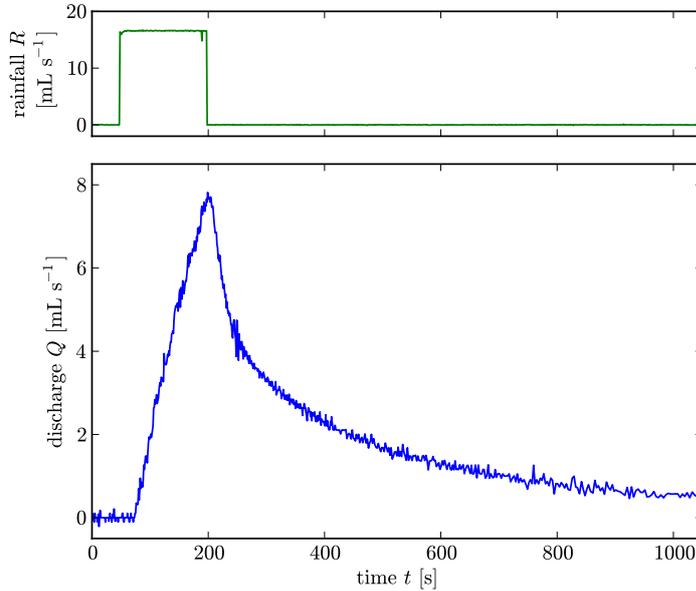


FIGURE 2. Rainfall rate R imposed to our laboratory aquifer (green line, top), and water discharge Q exiting the experiment (blue line, bottom). Measurements are collected at 1 Hz, with a precision of about 0.1 g s^{-1} . In this experimental run, the porous reservoir is made of 1 mm glass beads.

aquifer is collected in a bucket weighted every second with a precision of 0.1 g. The water discharge is then simply the derivative of the bucket weight with respect to time.

A typical experiment begins with an empty aquifer. We then switch on the rainfall and maintain its rate R constant for a few tens of seconds (an electromagnetic flowmeter measures R with a precision of 2%). After a few seconds, the water discharge exiting the aquifer rises quickly, until the rainfall stops (figure 2). At this point, the discharge suddenly decreases, and then relaxes slowly towards zero.

In the experiment of figure 2, the water discharge increases about 40 s after the beginning of the rain. We observe a similar time lag in every experiment. We interpret it as the travel time of rainwater through the 40 cm of unsaturated porous medium which separate the aquifer's surface from its bottom. A vertical velocity of about 1 cm s^{-1} is indeed consistent with the measured conductivity of the saturated aquifer, although one could expect the porous media to be less permeable when unsaturated.

3. Discharge increase

Once the rainwater has reached the bottom of the aquifer, the discharge at the experiment's outlet appears to increase linearly with time (figure 3). This observation holds over the entire range of rainfall rates we were able to investigate (from 2.4×10^{-5} to $5.8 \times 10^{-4} \text{ m s}^{-1}$). Naturally, if we maintain a constant rainfall long enough, the water discharge will eventually saturate to equal the rainfall input. In what follows, we focus on the linear discharge increase which precedes this saturation.

A simple reasoning might explain the linear increase of the discharge during rainfall: far from the outlet, the water table moves upwards in proportion to the water input, that is Rt , and arguably so does the pressure in the reservoir. Assuming discharge is simply

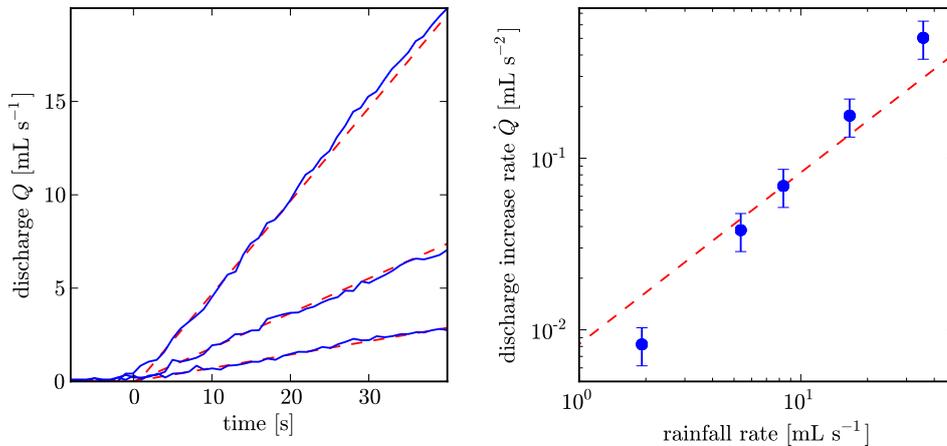


FIGURE 3. *Left*: Increase of the water discharge just after the beginning of rainfall for various rainfall rates (solid blue lines, $R = 36.7$, 16.8 and 8.3 mL s^{-1} from top to bottom). The curves are shifted horizontally so that time is zero at the beginning of the discharge increase. For each run, a linear increase is fitted to the data (dashed red lines). *Right*: Dependence of the discharge increase rate on the rainfall rate (blue dots). A linear relationship is fitted to the data for comparison (dashed red line). In this series of experiments, the porous reservoir is made of 4 mm glass beads.

proportional to the pressure in the reservoir, we conclude that the discharge increase rate should be constant, and proportional to the rainfall rate R .

We can measure the discharge increase rate \dot{Q} by fitting a linear relation to the data, at the beginning of an experimental run (figure 3). Repeating this procedure for various rainfall rates, we find that the discharge increase rate is a growing function of the rainfall rate. This relationship, however, is not linear, and resembles a power law with an exponent larger than one. This finding contradicts the simple reasoning based on the linear pressure increase, thus suggesting that the discharge is not proportional to the elevation of the water table far from the outlet.

The non-linear relation between discharge and pressure is a signature of a free-surface flow. Indeed, when the geometry of the flow is confined, Darcy's law leads to a linear response of the aquifer. To the contrary, when the shape of the water table adjusts to the flow conditions, its dynamics influences the aquifer's response to rainfall.

4. Dupuit-Boussinesq approximation

The Dupuit-Boussinesq approximation combines Darcy's law with the shallow-water approximation to describe the groundwater flow in an unconfined aquifer (Boussinesq 1903; Polubarinova Kochina 1962). In a homogeneous and isotropic porous material, Darcy's law reads

$$\mathbf{v} = -K \nabla \left(\frac{p}{\rho g} + y \right) \quad (4.1)$$

where \mathbf{v} , p , ρ , g and y are the water flux, the pressure, the water density, the acceleration of gravity and the vertical coordinate respectively.

The applicability of the shallow-water approximation to our experiment depends on the aspect ratio of the flow. Based on direct observation through the side panels of the experiment, the elevation h of the water table never exceeds 15 cm above the bottom of

the aquifer, that is, about one tenth of its length. This suggests that the flow is almost horizontal which, through Darcy's law, is equivalent to assuming that the pressure is hydrostatic. Accordingly, the total flux of water through a vertical line halving the aquifer reads

$$q = -K h \frac{\partial h}{\partial x}. \quad (4.2)$$

Mass balance then leads straightforwardly to the one-dimensional Dupuit-Boussinesq equation:

$$\phi \frac{\partial h}{\partial t} = \frac{K}{2} \frac{\partial^2 h^2}{\partial x^2} + R \quad (4.3)$$

where ϕ is the porosity of the aquifer. The porosity is a geometrical quantity which does not depend significantly on the diameter of the glass bead. Based on the difference between the density of glass and the weight of a large collection of beads, we measure $\phi \approx 0.40$ for a random packing of 1 mm beads, and $\phi \approx 0.42$ for 4 mm beads.

Being of second order in space, equation (4.3) must be supplemented with two boundary conditions. The impervious wall bounding the right-hand side of the aquifer imposes that the groundwater flux vanishes there (figure 1):

$$\frac{\partial h}{\partial x} = 0 \quad \text{for } x = L \quad (4.4)$$

where L is the length of the aquifer (we set the origin of the horizontal coordinate at the outlet).

Choosing a boundary condition at the outlet is less straightforward. If we neglect surface tension, we may assume that the water pressure tends towards the atmospheric pressure as the flow approaches the outlet. In the shallow-water framework, this approximation translates into

$$h = 0 \quad \text{for } x = 0. \quad (4.5)$$

Unfortunately, this boundary condition creates a singularity at the outlet. Indeed, if q_o is the flux of groundwater exiting the aquifer, then equation (4.2) implies that the water table elevation behaves like

$$h \sim \sqrt{\frac{2 q_o x}{K}}. \quad (4.6)$$

Strictly speaking, this square root shape is incompatible with the shallow-water approximation, since the slope of the water table diverges near the outlet. Hereafter, we momentarily ignore this inconsistency and use boundary condition (4.5) to solve the Dupuit-Boussinesq equation. We then discuss the possible consequences of this disregard.

5. Early response to rainfall

Our experiments suggest a power-law relationship between the discharge increase rate and the rainfall rate during the early stage of a rain event (figure 3). This is an incentive to look for the asymptotic behaviour of the Dupuit-Boussinesq equation at the beginning of rainfall.

Far from the outlet, we expect the groundwater flow to be insensitive to this boundary, and therefore the water table elevation should increase as Rt/ϕ , where the time t is set to zero when rainwater reaches the bottom of the experiment. In contrast with the simple

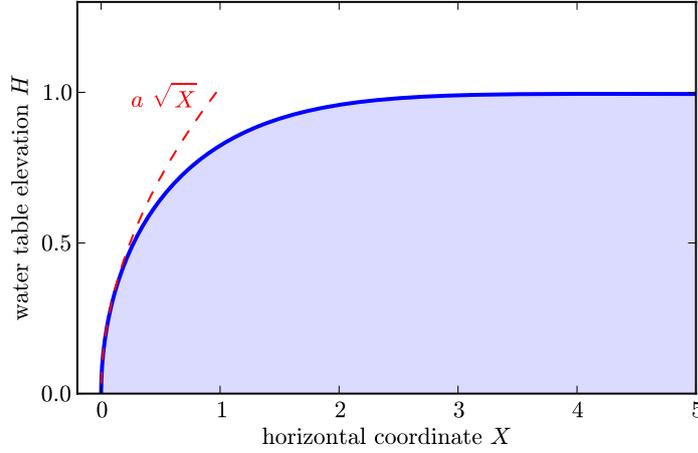


FIGURE 4. Numerical solution of equation (5.2), corresponding to the self-affine shape of the water table during the early stage of a rain event (blue line). Its behaviour near the aquifer outlet is $a\sqrt{X}$ (red dashed line).

reasoning of section 3, we now propose a self-affine shape H for the water table:

$$h(x, t) = \frac{Rt}{\phi} H(X) \quad \text{where} \quad X = \frac{\phi x}{t} \sqrt{\frac{2}{KR}}. \quad (5.1)$$

For the above ansatz to satisfy the Dupuit-Boussinesq equation, the function H must be a solution of the following ordinary differential equation:

$$H H'' + H'^2 + \frac{1}{2} (X H' - H + 1) = 0. \quad (5.2)$$

The boundary condition at the outlet translates into $H(0) = 0$, while the steady rise of the water table far from the outlet formally reads

$$\lim_{X \rightarrow -\infty} H = 1. \quad (5.3)$$

We are not aware of any analytical solution to the above problem, and therefore we approximate H numerically (figure 4). Naturally, the affine transformation defined by equation (5.1) preserves the singularity of the water table near the outlet:

$$H \sim a\sqrt{X} \quad (5.4)$$

where a is a constant adjusted to satisfy the far-field boundary condition (5.3). Using a numerical shooting method to do so, we find $a \approx 1.016$.

To the self-affine regime of the groundwater flow at early time corresponds the following water discharge, in dimensional form:

$$Q(t) \sim a^2 \frac{W}{\phi} \sqrt{\frac{K}{2}} R^{3/2} t \quad (5.5)$$

where W is the width of the aquifer. The above expression is encouraging, as it features the linear time dependence of the discharge we observe in experiments. Furthermore, the prefactor of this relation includes the rainfall rate R to a power larger than one, again in qualitative agreement with observations (figure 3).

To further compare this asymptotic regime to observation, we supplement the data of

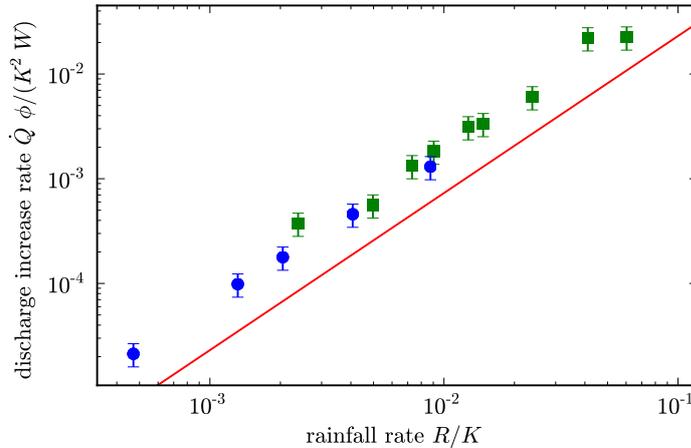


FIGURE 5. Dependence of the discharge increase rate with respect to the rainfall rate, for two series of experiments: 4 mm glass beads (blue dots) and 1 mm glass beads (green squares). The red line represents the asymptotic regime (5.6).

figure 3 with similar experiments involving a less permeable aquifer, made of 1 mm glass beads. Equation (5.5) suggests that the permeability K be scaled out of the data:

$$\frac{\dot{Q} \phi}{K^2 W} \sim \frac{a^2}{\sqrt{2}} \left(\frac{R}{K} \right)^{3/2}. \quad (5.6)$$

Indeed, when rescaled according to the above expression, the discharge increase rate and the rainfall rate from all experiments gather around the same relation, regardless of permeability (figure 5). Fitting a power law through the data yields an exponent of 1.47 ± 0.01 , in reasonable accordance with the $3/2$ exponent of the asymptotic regime. Assuming the asymptotic exponent is correct, the data are best fitted with a prefactor of 2.0 ± 0.1 , that is about three times the theoretical value of 0.73.

The asymptotic regime resulting in equation (5.6) explains the scaling of the discharge increase rate with both the rainfall rate and the conductivity, throughout the range of parameters we were able to explore experimentally. The mismatch of the theoretical prefactor with observation does not depend on the rainfall rate, nor on the conductivity. This observation points to a geometrical effect. The breakdown of the Dupuit-Boussinesq approximation near the outlet suggests that the two-dimensionality of the flow becomes significant there.

The groundwater flow that exits the aquifer is enslaved to the dynamics of the reservoir bulk. Therefore, in the neighbourhood of the outlet, it should reach a quasi-static regime which would facilitate its two-dimensional description. Ideally, such a description could then be matched to the Dupuit-Boussinesq equation to form a more accurate model of the complete reservoir. Unfortunately, none of our attempts to do so have been successful yet.

6. Drought flow

When the rain stops, the water discharge relaxes slowly towards zero. This drainage regime is referred to as “drought flow”. In our experiment, groundwater exits the aquifer near its impervious bottom. In this configuration, the Dupuit-Boussinesq theory predicts

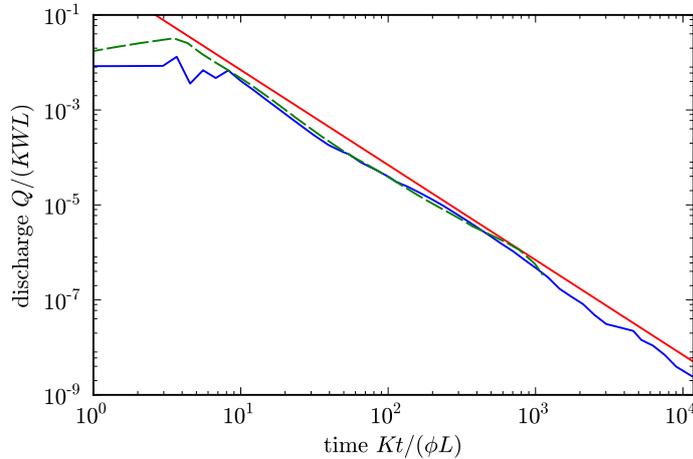


FIGURE 6. Evolution of the water discharge after rainfall has been switched off (blue solid line: 4 mm glass beads, green dashed line: 4 mm glass beads). The red line represents the asymptotic regime (6.3).

that the discharge decreases as $1/t^2$ during the drought flow (Boussinesq 1903; Polubarinova Kochina 1962). We now compare this classical result to our observations.

In the absence of any source term ($R = 0$), the Dupuit-Boussinesq equation (4.3) has a self-similar solution:

$$h(x, t) = \frac{L^2 \phi}{K t} H_d \left(\frac{x}{L} \right) \quad (6.1)$$

where L is the length of the aquifer, and the shape H_d of the water table satisfies

$$H_d H_d'' + H_d'^2 + H_d = 0 \quad (6.2)$$

with two boundary conditions, $H_d'(-1) = 0$ and $H_d(0) = 0$. The discharge associated to this self-similar solution reads

$$Q \sim a_d \frac{\phi^2 W L^3}{K t^2} \quad (6.3)$$

where a_d is a mathematical constant expressed in terms of the Euler gamma function Γ (Brutsaert 2005):

$$a_d = \frac{4}{3} \left(\frac{\Gamma(7/6)}{\sqrt{\pi} \Gamma(2/3)} \right)^3 \approx 0.693. \quad (6.4)$$

In contrast with the early stage of the discharge increase associated to a rain event, the drought flow depends on the length of the aquifer, a property reminiscent of the linear heat equation.

Our experimental aquifer conforms reasonably with the drought flow regime of the Dupuit-Boussinesq approximation (figure 6). After an appropriate rescaling, the relaxation of the water discharge appears to be independent of the aquifer permeability. Fitting a power law to the data yields an exponent of -1.87 ± 0.03 , comparable with the theoretical exponent. Assuming this exponent is exactly -2 , the data is best fitted with a prefactor of 0.48 ± 0.2 , again in reasonable agreement with the theoretical value of a_d .

The slight mismatch between theory and observation is hardly distinguishable from the experimental noise, thus supporting the use of the Dupuit-Boussinesq approximation to interpret the drought flow of our experimental aquifer. We cannot, however, dismiss

any influence of the two dimensionality of the flow on the drought regime, for exactly the same reasons as in section 5.

7. Conclusion

The experiment presented here supports the use of the Dupuit-Boussinesq approximation to describe the response of an aquifer to a rainfall event. In particular, it correctly predicts the increase of the groundwater discharge at the beginning of rainfall, and reveals an unexpected asymptotic behaviour: the discharge increase is proportional to the rainfall rate to the power $3/2$.

If its validity extends to field situations, this strong dependence would induce a fast groundwater contribution to the discharge of rivers, especially under intense rainfall. Therefore its consequences in terms of flood predictions deserve detailed scrutiny. An ideal field site to assess the contribution of this asymptotic regime would be dominated by groundwater hydrology, with negligible surface run-off.

However, even in a simplified laboratory experiment, the fast dynamics of a free-surface aquifer requires more investigations. Indeed, our experiment shows a slight mismatch between the numerical factors yielded by the Dupuit-Boussinesq approximation and measurements. We speculate that the two-dimensionality of the flow might come into play near the outlet, thus changing the effective boundary condition there.

More importantly, the geometry of our aquifer is the most amenable to shallow-water description, because the position of the outlet concentrates the flow near the impervious bottom. In a perhaps more realistic representation of a river, water would exit the aquifer at a finite elevation above the bottom, thus imposing an upwards flow below the outlet. This configuration certainly breaks the Dupuit-Boussinesq approximation, and its asymptotic regimes are still to be identified. It is the subject of present research.

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