Expectile prediction through asymmetric kriging
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Kriging (see Krige (1951)) aims at predicting the conditional mean of a random field \((Z_t)_{t \in T}\) given the values \(Z_{t_1}, \ldots, Z_{t_n}\) of the field at some points \(t_1, \ldots, t_n \in T\), where typically \(T \subset \mathbb{R}^d\). It seems natural to predict, in the same spirit as Kriging, other functional fields. In our study, we focus on expectiles for elliptical random fields. We also did a similar work for quantiles (see Maume-Deschamps et al. (2016b)).

### Elliptical Distributions

Cambanis et al. (1981) give the representation : the random vector \(X \in \mathbb{R}^d\) is elliptical with parameters \(\mu \in \mathbb{R}^d\) and \(\Sigma \in \mathbb{R}^{d \times d}\), if and only if

\[
X = \mu + R \Lambda U, 
\]

where \(\Lambda \Lambda^T = \Sigma,\) \(U \in \mathbb{R}^d\) is a \(d\)-dimensional random vector uniformly distributed on \(S^{d-1}\) (the unit disk of dimension \(d\)), and \(R\) is a non-negative random variable independent of \(U \in \mathbb{R}^d\). Furthermore, \(X\) is said consistent if:

\[
\mathbb{R} \ni X \overset{c}{\sim} \chi^2 (d). 
\]

### Example Regression

From now on, we consider the following context: \((X(t))_{t \in T}\) is an \(R\)-elliptical random field. We consider \(N\) observations at locations \(t_1, \ldots, t_n \in T\), called \((X(t_1), \ldots, X(t_n))\). In order to predict the value of \(X_0 = X(t)\) given \(X_0 = X(t_1), \ldots, X(t_n)\), we approximate \(X(t)\) by:

\[
\hat{e}_\alpha(X(t)|X_0) = \beta_0 + \beta_1 x_0, 
\]

where \(\beta_0\) and \(\beta_1\) are solutions of the following minimization problem:

\[
(\beta_0, \beta_1) = \arg \min \sum_{i=1}^n (y_i - \hat{y}_i)^2. 
\]

\[
\hat{y}_i = \beta_0 + \beta_1 x_i. 
\]

In the general case, the term \(\frac{2}{2\alpha} \hat{y}_i^2 \alpha X_0\) is difficult to compute. This is why we propose some other predictors.

### Expectile Regression

In our context of elliptical random fields, we are able to solve this minimization problem, and then define the Expectile Regression Predictor:

\[
\hat{e}_\alpha(X(t)|X_0) = \mu + \Sigma V_1^\alpha + \frac{1}{\alpha} \xi_{11} \Sigma_1 \Sigma_{12} R \Lambda U. 
\]

Furthermore, its distribution

\[
\hat{e}_\alpha(X(t)|X_0) = \hat{e}_\alpha(X(t)|X_0) + \Sigma V_1^\alpha + \frac{1}{\alpha} \xi_{11} \Sigma_1 \Sigma_{12} R \Lambda U. 
\]

### Extremal expectiles

In this section, the aim is to establish a relation between \(V_\alpha^1\) and \(V_\alpha^2\) for extremal values of \(\alpha\). For that, we do an assumption : their exist \(0 < c < +\infty\) and \(\gamma \in \mathbb{R}\) such as:

\[
\lim_{\alpha \to 1} \frac{1 - V_\alpha^1(x)}{V_\alpha^2(x)} = \gamma 
\]

Under this assumption, we can define Extreme Conditional Expectiles Predictors:

\[
\hat{e}_\alpha(X(t)|X_0) = \mu + \Sigma V_1^\alpha + \frac{1}{\alpha} \xi_{11} \Sigma_1 \Sigma_{12} R \Lambda U. 
\]