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## Taylor-Couette instability in thixotropic yield stress fluids

Mathieu Jenny,\* Sébastien Kiesgen de Richter, Nicolas Louvet,  
Salahedine Skali-Lami, and Yvan Dossmann

*LEMETA, UMR No. 7563, Université de Lorraine, CNRS, F-54500 Vandoeuvre-lès-Nancy, France*

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We consider the flow of thixotropic yield stress fluids between two concentric cylinders. To account for the fluid thixotropy, we use Houška's model [Houška, Ph.D. thesis, Czech Technical University, Prague, 1981] with a single structural parameter driven by a kinetic equation. Because of the yield stress and the geometric inhomogeneity of the stress, only a part of the material in the gap may flow. Depending on the breakdown rate of the structural parameter, the constitutive relation can lead to a nonmonotonic flow curve. This nonmonotonic behavior is known to induce a discontinuity in the slope of the velocity profile within the flowing material, called shear banding. Thus, for fragile structures, a shear-banded flow characterized by a very sharp transition between the flowing and the static regions may be observed. For stronger structures, the discontinuity disappears and a smooth transition between the flowing and the static regions is observed. The consequences of the thixotropy on the linear stability of the azimuthal flow are studied in a large range of parameters. Although the thixotropy allows shear banding in the base flow, it does not modify fundamentally the linear stability of the Couette flow compared to a simple yield stress fluid. The apparent shear-thinning behavior depends on the thixotropic parameters of the fluid and the results about the onset of the Taylor vortices in shear-thinning fluids are retrieved. Nevertheless, the shear banding modifies the stratification of the viscosity in the flowing zone such that the critical conditions are mainly driven by the width of the flowing region.

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### I. INTRODUCTION

Yield stress fluids, such as emulsions, foams, mud, and gels, are of industrial interest. Because of their high number of applications, they have been intensively studied over the few past decades. Many of them have an inner microstructure, responsible for the yield stress when an external load is applied, that resists large-scale rearrangement. The destruction of this microstructure by the flow is responsible for a complex phenomenon, named thixotropy. The competition between the internal reorganization and the macroscopic flow induces a complex time dependence of the rheological parameters, such as the apparent viscosity. The complex behavior of such fluids raises the question of their flow stability in industrial conditions (melting, mass transfers, etc.).

The Taylor-Couette flow is often considered as a paradigm to study the stability and the transition to turbulence of complex fluids [1–4]. While the Taylor-Couette flow of Newtonian fluids has been extensively studied since the historical work of Taylor [5], much attention has been paid to complex fluids during the past decade. According to the studies of the hydrodynamic stability of shear thinning [1,3,6] and Bingham fluids [2–4], it is observed that when the viscosity is scaled with the inner-wall shear viscosity, shear thinning has a stabilizing effect, i.e., the appearance of the Taylor vortices is delayed [3]. For simple yield stress fluids, two regions of the flow coexist, a yielded zone close to the rotating cylinder and a static region close to the fixed cylinder. Landry *et al.* [2] have shown that the vortices are localized in the yield zone. Naimi *et al.* [7] have reported that the yield stress appears to stabilize the flow. Few studies have focused on the influence of the thixotropy on the stability of Taylor-Couette flow in thixotropic shear-thinning fluids [8]. Questions remain about the consequences of a microstructural-dependent yield stress on the stability of the flow.

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\*mathieu.jenny@univ-lorraine.fr

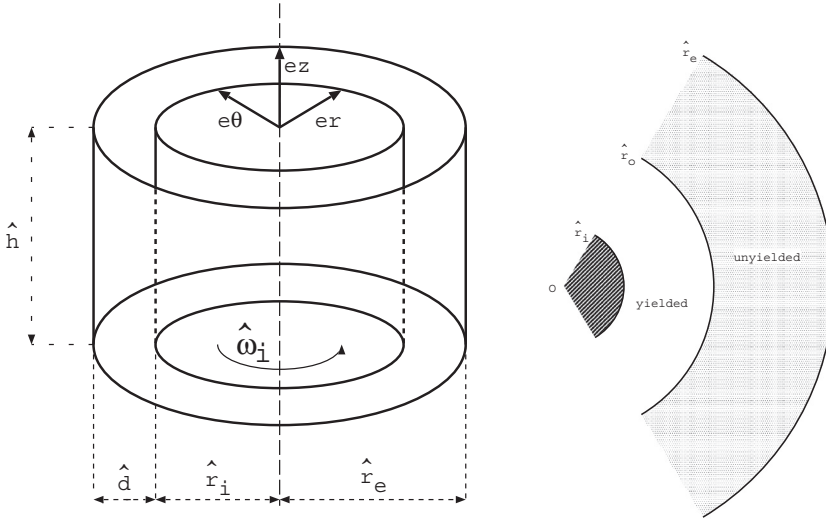


FIG. 1. Taylor-Couette geometry. Here  $\mathbf{e}_r$ ,  $\mathbf{e}_\theta$ , and  $\mathbf{e}_z$  are the unit vectors of the cylindrical coordinate system  $(r, \theta, z)$ .

47 In that context, we study of the stability of a thixotropic yield stress fluid in a Taylor-Couette  
 48 configuration. To model the base flow of such fluids, structural parameter models allow us to take  
 49 into account the inner dynamic, coupled with the surrounding flow. The inner structure is then  
 50 entirely described by the structural parameter. In this article we use Houška's model [9,10]. It is built  
 51 from the Hershel-Bulkley model, commonly used for nonelastic yield stress fluids, considering that  
 52 the consistency  $K$  and the yield stress  $\tau_0$  depend linearly on the structural parameter  $\lambda$ . This robust  
 53 thixotropic fluid model was originally developed to characterize liquid foods such as ketchup or  
 54 yogurt [9–11]. The existence and unicity of a steady solution of this model was recently established  
 55 in pipe flows [12]. Houška's model has been successfully used by Wachs *et al.* [13] to model  
 56 start-up pipe flows of waxy crude oils despite the limitations of the model. In particular, irreversible  
 57 effects are not accounted for, as stressed by Mendes *et al.* [14]. Billingham and Ferguson [15] also  
 58 investigated also steady pipe flow of bentonite mud using Houška's model. The simplicity of the  
 59 model allows us to conclude about the consequences of the thixotropy upon the linear stability of  
 60 the flow. The linear stability analysis of the flow shows that the nature of the linear unstable mode  
 61 is steady and axisymmetric in the large range of the explored parameters and does not depend on  
 62 the thixotropic character of the flow.

## 63 II. CONSTITUTIVE EQUATIONS AND NUMERICAL APPROACH

### 64 A. Flow geometry

65 In the present work we consider the case of an inner cylinder rotating at a given angular velocity  
 66  $\hat{\omega}_i$  so that the velocity at the inner radius  $\hat{r}_i$  is  $\hat{v}_i = \hat{\omega}_i \hat{r}_i$  (Fig. 1). Here the caret denotes a dimensional  
 67 variable. The outer cylinder is static. In this configuration, when the velocity of the inner cylinder is  
 68 sufficiently low, the purely azimuthal steady flow is stable for viscous fluids [3,5,6,16].

### 69 B. Houška's model

70 For a nonzero strain rate, i.e.,  $\hat{\gamma} \neq 0$ , the constitutive law of Houška's model [9,10] is expressed  
 71 as

$$\hat{\boldsymbol{\tau}} = [(K + \Delta K \lambda) \hat{\gamma}^{n_c} + \tau_0 + \tau_1 \lambda] \frac{\hat{\boldsymbol{\gamma}}}{\hat{\gamma}}, \quad (1)$$

72 where  $\overline{\hat{\boldsymbol{\tau}}}$  denotes the stress tensor, the parameters  $\Delta K$  and  $\tau_1$  respectively determine the sensitivity  
 73 of the consistency and the yield stress with the structural parameter  $\lambda$ , and  $n_c$  is the shear-thinning  
 74 index. The strain rate tensor is given by  $\overline{\hat{\boldsymbol{\gamma}}} = \overline{\hat{\nabla}}\hat{\mathbf{v}} + \overline{\hat{\nabla}}\hat{\mathbf{v}}^T$ , where  $\overline{\hat{\nabla}}\hat{\mathbf{v}}$  and the index  $T$  denote the  
 75 gradient tensor of the velocity vector  $\hat{\mathbf{v}}$  and the transposition, respectively. The second invariant of  
 76 the strain rate tensor  $\overline{\hat{\boldsymbol{\gamma}}}$  is defined by

$$\hat{\boldsymbol{\gamma}} = \left( \frac{1}{2} \hat{\gamma}_{ij} \hat{\gamma}_{ij} \right)^{1/2}, \quad (2)$$

77 using the Einstein summation convention where the elements of the strain tensor  $\hat{\gamma}_{ij}$  are defined  
 78 with the components of the fluid velocity  $\hat{\mathbf{v}}$ . In Eq. (2), the indices  $i$  and  $j$  stand for the cylindrical  
 79 coordinates  $r$ ,  $\theta$ , and  $z$  (Fig. 1). The structural parameter  $\lambda$  is determined by the kinetic equation

$$\frac{\partial \lambda}{\partial \hat{t}} + \hat{\mathbf{v}} \cdot \nabla \lambda = a(1 - \lambda) - b\lambda \hat{\boldsymbol{\gamma}}^m, \quad (3)$$

80 where  $a$  and  $b$  are, respectively, the building and the breakdown parameters. The thixotropic  
 81 breakdown index  $m$  is taken to be equal to 1 in the following. The values of the structural parameter are  
 82 within the range  $0 \leq \lambda \leq 1$ . The value  $\lambda = 1$  means that the fluid is fully structured and  $\lambda = 0$  means  
 83 that it is fully unstructured. The kinetic equation (3) governs the evolution of the microstructure,  
 84 which influences the flow by modifying the stress tensor.

### 85 C. Nondimensional equations

86 To nondimensionalize the constitutive equations of the flow in a cylindrical Couette geometry,  
 87 we choose the following references for the density, the velocity, and the length, respectively:

$$\rho_{\text{ref}} = \rho, \quad v_{\text{ref}} = \hat{v}_i, \quad l_{\text{ref}} = \hat{d}, \quad (4)$$

88 where  $\rho$  is the density of the fluid and  $\hat{v}_i$  the inner cylinder velocity. One can build a characteristic  
 89 strain rate  $\hat{v}_i/\hat{d}$  using the latter reference dimensional parameters. For the non-Newtonian fluids,  
 90 several choices can be made for the reference viscosity. We choose the plastic viscosity of the fluid  
 91 at the characteristic strain rate  $\hat{v}_i/\hat{d}$  as a reference viscosity

$$\mu_{\text{ref}} = \mu_0(1 + \Delta K^* \lambda_{\text{ref}}). \quad (5)$$

92 The parameter  $\mu_0 = K(\hat{v}_i/\hat{d})^{n_c-1}$  is the standard reference viscosity of a power-law fluid. The  
 93 structural parameter  $\lambda_{\text{ref}}$  is given by Eq. (3) at equilibrium

$$\lambda_{\text{ref}} = \frac{a}{a + b(\hat{v}_i/\hat{d})^m} = \frac{1}{1 + b^*/a^*}, \quad (6)$$

94 where the nondimensional building and breakdown parameters are

$$a^* = \frac{a\hat{d}}{\hat{v}_i}, \quad b^* = b \left( \frac{\hat{v}_i}{\hat{d}} \right)^{m-1}, \quad (7)$$

95 respectively. Here  $\Delta K^* = \Delta K/K$  is the reduced thixotropic consistency factor. This parameter  
 96 characterizes the dependence of the plastic viscosity with the inner structure of the fluid in  
 97 comparison with the intrinsic consistency  $K$ , which itself depends on the solvent. The reference  
 98 viscosity  $\mu_{\text{ref}}$ , depending on the ratio of the breakdown parameter  $b^*$  over the building parameter  $a^*$ ,  
 99 decreases when  $b^*/a^*$  increases, i.e., when the inner structure of the fluid becomes more and more  
 100 fragile.

101 Using the previous reference dimensions, the Navier-Stokes and mass conservation equations for  
 102 flows of incompressible fluids are

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \frac{1}{\text{Re}} \nabla \cdot \bar{\boldsymbol{\tau}}, \quad (8)$$

$$\nabla \cdot \mathbf{v} = 0, \quad (9)$$

103 where  $\mathbf{v} = \hat{\mathbf{v}}/\hat{v}_i$  stands for the reduced velocity and  $p = \hat{p}/\rho\hat{v}_i^2$  for the reduced pressure. One can  
 104 notice that  $\hat{p}$  is the modified pressure including the hydrostatic pressure. The Reynolds number is  
 105 defined using the reference viscosity (5) by

$$\text{Re} = \frac{\text{Re}_0}{1 + \Delta K^* \lambda_{\text{ref}}}, \quad (10)$$

106 where  $\text{Re}_0 = \frac{\rho\hat{v}_i\hat{d}}{\mu_0}$ . Thus, the reduced stress tensor reads

$$\bar{\boldsymbol{\tau}} = \left[ \left( \frac{1 + \Delta K^* \lambda}{1 + \Delta K^* \lambda_{\text{ref}}} \right) \dot{\gamma}^{n_c} + \text{Bn} \left( \frac{1 + \tau_1^* \lambda}{1 + \tau_1^* \lambda_{\text{ref}}} \right) \right] \bar{\boldsymbol{\gamma}}, \quad (11)$$

107 where  $\dot{\gamma}$  and  $\bar{\boldsymbol{\gamma}}$  are the nondimensional strain rate and strain tensor, respectively, and  $\tau_1^* = \tau_1/\tau_0$  is  
 108 the reduced thixotropic yield stress. Equation (11) involves the Bingham number, which is the ratio  
 109 of the yield stress with the plastic viscous stress

$$\text{Bn} = \text{Bn}_0 \frac{1 + \tau_1^* \lambda_{\text{ref}}}{1 + \Delta K^* \lambda_{\text{ref}}}, \quad (12)$$

110 where  $\text{Bn}_0 = \frac{\tau_0}{K(\hat{v}_i/\hat{d})^{n_c}}$  is the standard Bingham number of a Hershel-Bulkley fluid. The Bingham  
 111 number increases with the yield stress and localization stops occurring for sufficiently high values  
 112 of the Bingham number. According to Eq. (11), the reduced yield stress equals

$$\tau_y = \text{Bn} \left( \frac{1 + \tau_1^* \lambda}{1 + \tau_1^* \lambda_{\text{ref}}} \right). \quad (13)$$

113 For the structural parameter, the nondimensional version of Eq. (3) is

$$\frac{\partial \lambda}{\partial t} + \mathbf{v} \cdot \nabla \lambda = a^*(1 - \lambda) - b^* \lambda \dot{\gamma}^m. \quad (14)$$

114 Equations (11) and (14) describe the coupling between the flow properties and the evolution of the  
 115 microstructure.

#### 116 D. Boundary conditions for the flow

117 The inner and outer reduced radii of the Couette setup are defined by

$$r_i = \frac{\eta}{1 - \eta}, \quad (15)$$

$$r_e = \frac{1}{1 - \eta}, \quad (16)$$

118 with  $\eta = \hat{r}_i/\hat{r}_e$  the radii ratio. The velocity vector  $\mathbf{v}$  is written in the cylindrical basis as  $\mathbf{v} =$   
 119  $v_r \mathbf{e}_r + v_\theta \mathbf{e}_\theta + v_z \mathbf{e}_z$ . The boundary conditions are as follows.

- 120 (i) At the inner radius  $r = r_i$ , the velocity components are  $v_\theta = 1$  and  $v_r = v_z = 0$ .  
 121 (ii) At the outer radius of the flowing zone  $r = r_o$ , the velocity components are  $v_r = v_\theta = v_z = 0$ .  
 122 (iii) In our case, there is a material limit at  $r = r_e$ . Thus, the outer radius  $r_o$  is given by the  
 123 following criterion: If  $\tau(r_e) \geq \tau_y$ ,  $r_o = r_e$ ; otherwise  $\tau(r_o) = \tau_y$ . In the following, we assume that

124 the stress at the interface between the flowing and static regions is the yield stress. Other assumptions  
 125 would be beyond the framework of Houška's model and it would demand a model for the solid phase.

126 At  $r = r_o$ , the yield stress  $\tau_y$  given by Eq. (13) becomes

$$\tau_{y_o} = \text{Bn} \left( \frac{1 + \tau_1^* \lambda_o}{1 + \tau_1^* \lambda_{\text{ref}}} \right), \quad (17)$$

127 with  $\lambda_o$  the structural parameter at the interface. Thus, the last boundary condition becomes  $\tau(r_o) =$   
 128  $\tau_{y_o}$  and the stress  $\tau_{y_o}$  at the interface is defined by Eq. (17). Nevertheless, the stress condition at  
 129 the interface between the fluid and solidlike zone is well defined only if the structural parameter  
 130  $\lambda$  is continuous across the interface. When shear banding occurs, we expect a discontinuity of the  
 131 structural parameter  $\lambda$  at the interface when no diffusion of the structural parameter  $\lambda$  is included  
 132 in Eq. (14). According to Olmsted *et al.* [17], a stress diffusion term must be added in that case to  
 133 conserve the unicity of the steady solution by selecting the stress at the interface between the bands  
 134 (see also Lu *et al.* [18]). It was also shown that a spatially local model, i.e., without any diffusive  
 135 gradient of the stress (diffusive term for the structural parameter in our case), will not correctly  
 136 predict a shear banded state. The steady state depends then on the flow or numerical noise history  
 137 by selecting arbitrarily a stress value at the interface. When adding a diffusive term, the continuity  
 138 of the yield stress or the structural parameter across the interface is ensured. This kind of diffusive  
 139 term was recently interpreted as a nonlocal effect at the molecular scale in the flow of micellar  
 140 suspensions [19]. As the value of the stress diffusion coefficient, similar to the structural diffusion  
 141 coefficient, is found to be very small [19,20], we will focus in the following on the cases where the  
 142 coefficient of diffusion is equal to zero. This assumption implies that the stress interface is fixed. For  
 143 Bingham-like fluids, i.e., when  $n_c = 1$  and  $m = 1$ , Eqs. (11) and (14) for one-dimensional steady  
 144 flows give the following equation for  $\lambda$  over the gap:

$$(b^* \tilde{\tau}_1 - a^* \Delta \tilde{K}) \lambda^2 + [a^* (\Delta \tilde{K} - \tilde{K}) + b^* (\tilde{\tau}_0 - \tau)] \lambda + a^* \tilde{K} = 0, \quad (18)$$

145 where

$$\tilde{K} = \frac{1}{1 + \Delta K^* \lambda_{\text{ref}}}, \quad \Delta \tilde{K} = \frac{\Delta K^*}{1 + \Delta K^* \lambda_{\text{ref}}}, \quad (19)$$

$$\tilde{\tau}_0 = \frac{\text{Bn}}{1 + \tau_1^* \lambda_{\text{ref}}}, \quad \tilde{\tau}_1 = \frac{\text{Bn} \tau_1^*}{1 + \tau_1^* \lambda_{\text{ref}}}. \quad (20)$$

146 Assuming that the stress at the interface is the yield stress  $\tau_{y_o}$ , the second-order polynomial (18) is  
 147 rewritten at  $r = r_o$  setting  $\tau = \tau_{y_o}$  and it becomes

$$-a^* \Delta \tilde{K} \lambda_o^2 + a^* (\Delta \tilde{K} - \tilde{K}) \lambda_o + a^* \tilde{K} = 0. \quad (21)$$

148 The only positive root of (21) is  $\lambda_o = 1$ . In the framework of the considered model without any  
 149 diffusive term, the stress at the fluid-solid interface is always the yield stress of the fully structured  
 150 material  $\tau_{y_s}$  in steady flows. Thus, we can either solve the steady equations with or without shear  
 151 banding, setting the stress at the interface

$$\tau(r_o) = \tau_{y_s} = \text{Bn} \left( \frac{1 + \tau_1^*}{1 + \tau_1^* \lambda_{\text{ref}}} \right). \quad (22)$$

152 Note that adding a diffusive term in Eq. (14) would result in slightly different stress values. It might  
 153 be necessary to compare to experimental results, but it would not modify the conclusions of the  
 154 present paper.

### 155 E. Numerical methods for steady flows

156 Only the flowing region needs to be considered to solve the steady flow. To perform the numerical  
 157 resolution, we use a finite-difference method for the spatial discretization. The mesh points are

158 regularly spaced between the inner radius  $r_i$  and the outer radius  $r_o$ . The stress points are taken  
 159 between two successive velocity points to ensure the numerical accuracy of the scheme for the  
 160 velocity. For the derivative operations, the standard second-order centered scheme is used. The  
 161 numerical method used for the spatial discretization is quite well established and is similar to the  
 162 ones used, for instance, in Refs. [8,21,22]. A validation and a convergence test are performed in  
 163 Sec. IV B.

164 To calculate the base flow, we consider the steady axisymmetric solution of Eqs. (8), (9) and (14),  
 165 i.e.,  $\mathbf{v}_b = V_b(r)\mathbf{e}_\theta$  and  $\lambda = \lambda_b(r)$ . In the flowing region, i.e.,  $r_i \leq r \leq r_o$ , the only nonzero element  
 166 of the strain rate tensor is  $\dot{\gamma}_{r\theta}$ . The strain rate is always nonzero, negative in the flowing region. The  
 167 only nonzero element of the stress tensor  $\bar{\boldsymbol{\tau}}$  is then  $\tau_{r\theta}$ . Considering the previous assumptions for  
 168 the flow, the well known result for steady Couette flow applies:

$$\tau_{r\theta,b} = -\frac{C}{r^2}, \quad (23)$$

169 where the positive constant  $C$  is related to the torque imposed by the inner rotating cylinder. The  
 170 radius  $r_o$  can be obtained from the stress condition on the interface between the yielded and unyielded  
 171 regions:

$$r_o = \sqrt{\frac{C}{\tau_{y0}}}. \quad (24)$$

172 If  $r_o \geq r_e$  according to Eq. (24), all the material in the gap flows and  $r_o = r_e$ . In the next  
 173 section, the flow curves show that the minimal value  $\tau_{\min}$  of the stress may be below  $\tau_{y0}$ . Thus, for  
 174  $\tau_{\min}r_e^2 \leq C < \tau_{y0}r_e^2$ , an alternative to Eq. (24) is to set  $r_o = r_e$ . In practice, this means that if there  
 175 is no interface at the initial state, the fluid region fits the whole gap for  $\tau_{\min}r_e^2 \leq C < \tau_{y0}r_e^2$  and if  
 176 there is a solidlike region in the initial state, the flowing region is confined between  $r_i$  and  $r_o < r_e$   
 177 according Eq. (24).

178 To compute the flow velocity in the yielded region, we calculate the strain rate by solving the  
 179 regular setup of equations at each point of the mesh:

$$\lambda_b = \frac{1}{1 + (b^*/a^*)\dot{\gamma}_b^m}, \quad (25)$$

$$\dot{\gamma}_b = \left( \frac{(C/r^2 - \tau_{yb})(1 + \Delta K^* \lambda_{\text{ref}})}{1 + \Delta K^* \lambda_b} \right)^{1/n_c}, \quad (26)$$

180 with  $\tau_{yb}$  the yield stress given by Eq. (13) replacing  $\lambda$  by  $\lambda_b$ . Once we obtain the strain rate  $\dot{\gamma}_b$  and  
 181 the structural parameter  $\lambda_b$  for a given constant  $C$  by solving the setup of Eqs. (25) and (26), the fluid  
 182 velocity is calculated by the integration of the strain rate with  $V_b(r_o) = 0$  as the boundary condition.  
 183 Finally, one has to find the value of  $C$  such that  $V_b(r_i) = 1$  using the algorithm available in MATLAB  
 184 to calculate the zero of a real nonlinear function. If needed, the pressure  $P_b$  of the base flow can be  
 185 obtained by integrating the equation

$$\frac{\partial P_b}{\partial r} = \frac{V_b^2}{r} \quad (27)$$

186 and setting the inner pressure  $P_b(r_i) = 0$  for instance.

### 187 III. EFFECT OF THE THIXOTROPY ON THE BASE FLOW

188 In the following sections, we set the thixotropic index breakdown to  $m = 1$ . It seems reasonable  
 189 to argue that the structural parameter  $\lambda$  modifies the viscous term and the yield stress with the same  
 190 order of magnitude. Thus, we set  $\Delta K^* = \tau_1^*$  and so  $\text{Bn} = \text{Bn}_0$ . As we focus our study on the cases  
 191 where both flowing and solidlike regions exist, i.e.,  $r_o < r_e$ , the Bingham number is set equal to

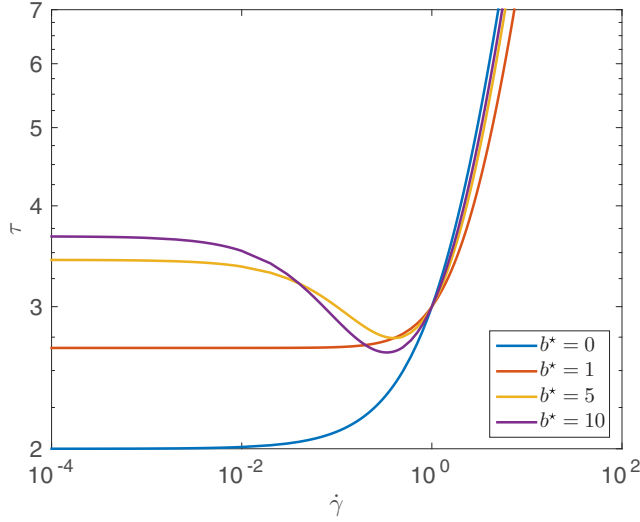


FIG. 2. Composite flow curves, stress  $\tau$  vs the strain rate  $\dot{\gamma}$ , become nonmonotonic depending on the breakdown parameter  $b^*$  ( $Bn = 2$ ,  $n_c = 1$ ,  $\Delta K^* = 1$ ,  $\tau_1^* = 1$ , and  $a^* = 1$ ).

192  $Bn = 2$ . Finally, for steady-state flows only the ratio  $b^*/a^*$  appears and we choose to set  $a^* = 1$   
 193 without loss of generality.

194

#### A. Steady-state flow curves

195

The base flow is computed using 50 grid points in the flowing region of the gap to ensure good  
 196 accuracy for the linear stability analysis as shown in Sec. IV B. We report in Fig. 2 the evolution of the  
 197 composite flow curves under controlled shear rate for different values of the breakdown parameter  
 198  $b^*$ . The composite curves are obtained straightforwardly by replacing the structural parameter  $\lambda$   
 199 by its relation to  $\dot{\gamma}$  (25) in the constitutive law (11). As  $b^*$  increases, the composite curve drops  
 200 from a monotonic to a nonmonotonic behavior, which presents an unstable branch leading to shear  
 201 banding [23–25]. This result indicates that, in the range of parameters studied here, shear banding  
 202 may occur in the base flow. Shear banding is characterized by a discontinuous strain rate  $\dot{\gamma}$  and  
 203 structural parameter  $\lambda$  across the fluid-solid interface [Figs. 3(b) and 3(d)], although in the simple  
 204 localized flow, the transition between the flowing and static regions is smooth [Figs. 3(a) and 3(c)].  
 205 The mechanism underlying shear banding is different from those considering viscoelastic fluids as  
 206 in the major classical works about the stability of shear-banded flows [26]. Structural discontinuity  
 207 is here the underlying cause of shear banding. The localized flow, due to the yield stress, is also  
 208 observed in simple yield stress fluids such as Bingham fluids.

209

Thus, the shear banding may appear if the sign of the derivative of the constitutive relation  
 210  $\tau(\dot{\gamma})$  changes at a critical strain rate  $\dot{\gamma}_0 > 0$ . In other words, the necessary condition to allow the  
 211 shear-banded flow is

$$\exists \dot{\gamma}_0 \geq 0: \frac{\partial \tau}{\partial \dot{\gamma}} = 0. \quad (28)$$

212

In the steady state, the derivative of the stress given by Houška's model is

$$\frac{\partial \tau}{\partial \dot{\gamma}} = \dot{\gamma}^{n_c-1} \left( \frac{n_c (b^*/a^*)^2 \dot{\gamma}^{2m} + (b^*/a^*) [2n_c + (n_c - m) \Delta K^*] \dot{\gamma}^m + n_c (1 + \Delta K^*)}{(1 + \Delta K^* \lambda_{\text{ref}}) [1 + (b^*/a^*) \dot{\gamma}^m]^2} - \frac{m (b^*/a^*) Bn \tau_1^* \dot{\gamma}^{m-n_c}}{(1 + \tau_1^* \lambda_{\text{ref}}) [1 + (b^*/a^*) \dot{\gamma}^m]^2} \right). \quad (29)$$



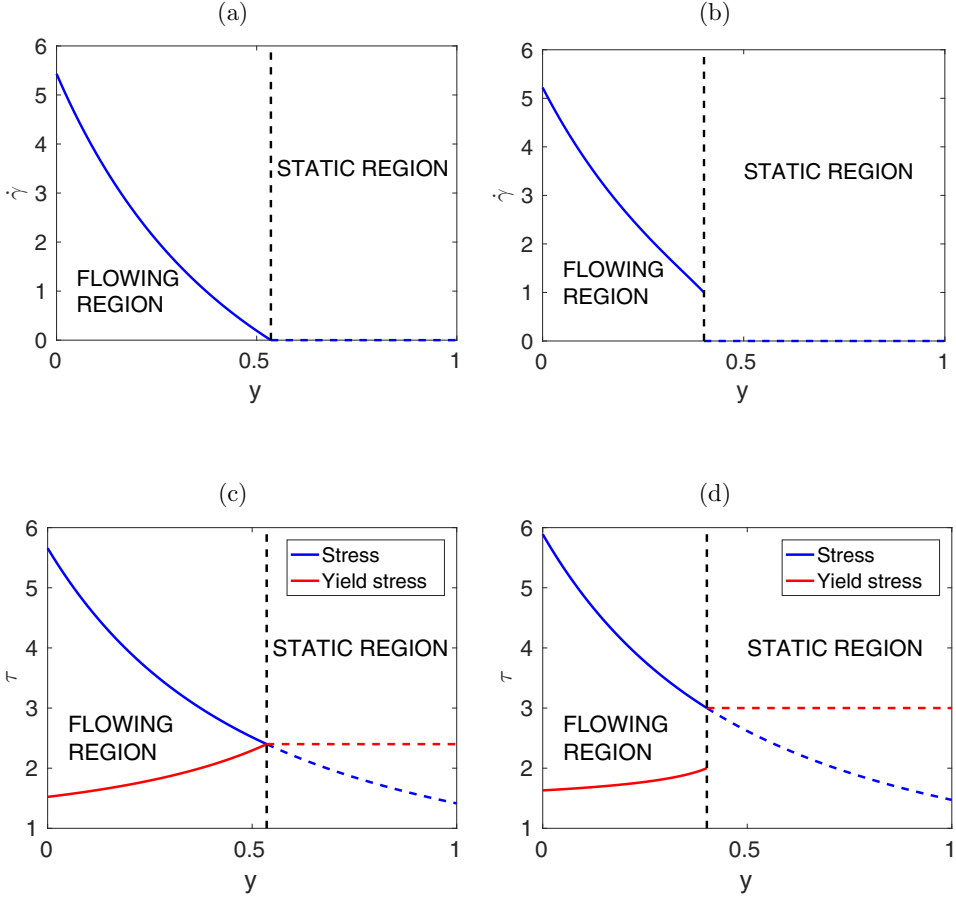


FIG. 3. (a) and (b) Strain rate  $\dot{\gamma}$  in the gap and (c) and (d) stress and yield stress in the gap in (a) and (c) a localized flow at  $b^* = 0.5$  and (b) and (d) a shear-banded flow at  $b^* = 2$ . Other parameters are the Bingham number  $\text{Bn} = 2$ , the shear-thinning index  $n_c = 1$ , the structural dependence of the consistency  $\Delta K^* = 1$ , the structural dependence of the yield stress  $\tau_1^* = 1$ , and the building parameter  $a^* = 1$ .

213 For Bingham-like fluids where  $m = n_c = 1$ , Eq. (28), using the derivative of  $\tau$  given by Eq. (29),  
 214 admits only one positive root  $\dot{\gamma}_0$ :

$$\dot{\gamma}_0 = \frac{a^*}{b^*} \left[ \sqrt{\text{Bn} \tau_1^* \frac{b^*}{a^*} \left( \frac{1 + \Delta K^* \lambda_{\text{ref}}}{1 + \tau_1^* \lambda_{\text{ref}}} \right) - \Delta K^* - 1} \right] \quad (30)$$

215 if

$$1 + \Delta K^* - \text{Bn} \tau_1^* \frac{b^*}{a^*} \left( \frac{1 + \Delta K^* \lambda_{\text{ref}}}{1 + \tau_1^* \lambda_{\text{ref}}} \right) < 0. \quad (31)$$

216 The derivative of  $\tau$  [Eq. (29)] is negative for  $\dot{\gamma} \in [0, \dot{\gamma}_0[$ . As  $\dot{\gamma} = 0$  lies in the forbidden range of  
 217 strain rates, the simple localization is no longer stable. The strain rate at the structural discontinuity  
 218 is the nonzero strain rate

$$\dot{\gamma}_c = \text{Bn} \tau_1^* \left( \frac{1 + \Delta K^* \lambda_{\text{ref}}}{1 + \tau_1^* \lambda_{\text{ref}}} \right) - \frac{1 + \Delta K^*}{b^*/a^*}, \quad (32)$$

219 which ensures the stress condition (22). Figure 2 and Eqs. (30) and (32) show that  $\dot{\gamma}_c \geq \dot{\gamma}_0$ .

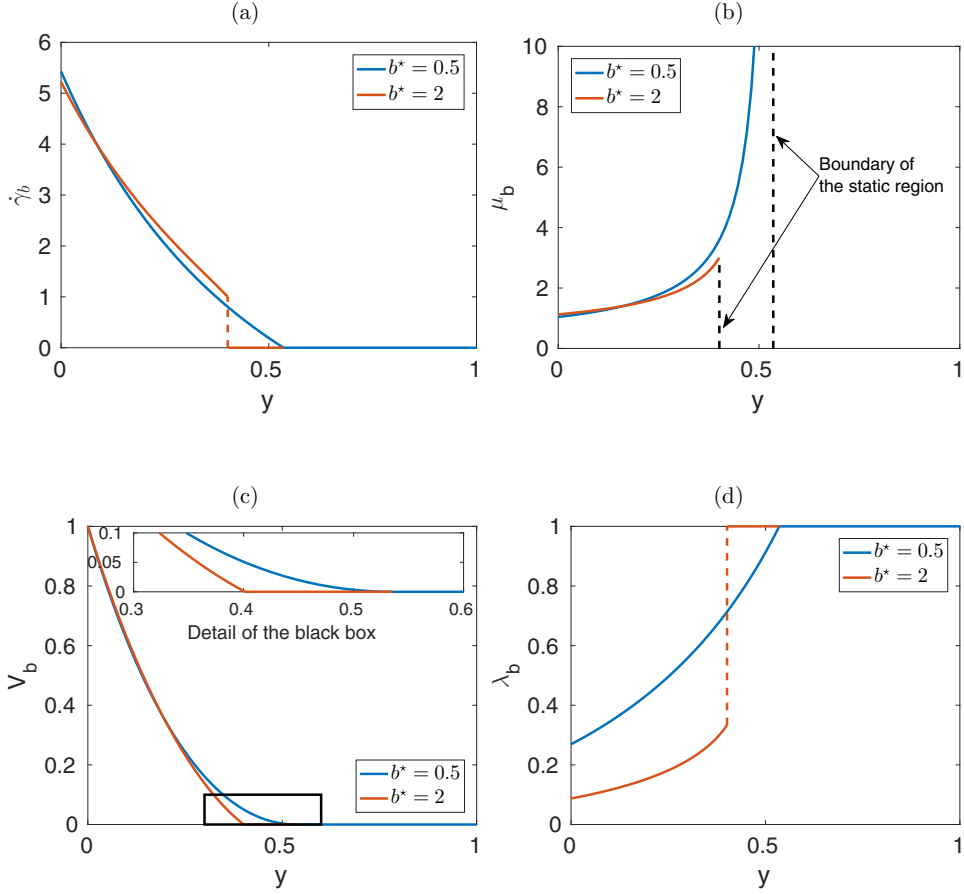


FIG. 4. (a) Strain rate  $\dot{\gamma}_b$ , (b) viscosity  $\mu_b$ , (c) azimuthal velocity  $V_b$ , and (d) structural parameter  $\lambda_b$  of the base flow vs the reduced gap position  $y = (r - r_i)/(r_e - r_i)$  with  $\text{Bn} = 2$ ,  $n_c = 1$ ,  $\Delta K^* = 1$ ,  $\tau_1^* = 1$ , and  $a^* = 1$  for a large gap  $\eta = 0.5$ .

220 Note that if  $n_c < 1$  and  $m \geq 1$ ,

$$\frac{\partial \tau}{\partial \dot{\gamma}} \Big|_{\dot{\gamma} \rightarrow 0^+} \rightarrow +\infty \quad (33)$$

221 and thus there is a range of strain rate values close to zero where  $\tau(\dot{\gamma})$  is a growing function of  $\dot{\gamma}$ .  
 222 The range of positive strain rates where the stress  $\tau$  decreases cannot start at a zero value. This case  
 223 would be similar to the flow curve of a semidilute wormlike micelle solution with a yield stress like  
 224 in Fig. 1(b) of [27]. As there is no diffusion term in our set of equations, sharp discontinuities of the  
 225 strain rate and the structural parameter can appear within the fluid region at a radius  $r_i < r < r_o$  when  
 226  $n_c < 1$ . Our numerical method does not allow such discontinuous fields in the fluid domain except at  
 227 the interface between the fluid and solidlike region, i.e., at  $r = r_o$ . Thus, in the following, we limit our  
 228 parametric study to shear-banded flows with an interface between the flowing and static regions only  
 229 [flow curve corresponding to Fig. 1(c) of [27]], i.e., with  $n_c = 1$  and shear localization with  $n_c \leq 1$ .

230

### B. Velocity profiles and structural parameter

231 The strain rate, the viscosity, the velocity, and the structural parameter profiles are shown in  
 232 Figs. 4(a)–4(d). We see that for all values of  $b^*$ , a flowing and a static region coexist with the  
 233 considered value of Bingham number  $\text{Bn} = 2$ . However, the discontinuity of the strain rate profile

234 depends on the latter parameter and is related to a discontinuity of the structural parameter. For lower  
 235 values of  $b^*$ , the velocity profiles as the structure parameter is continuous as observed, for example,  
 236 in carbopol gels, emulsions, and foams [28,29]. In that case, the shear localization is inherent to the  
 237 existence of the yield stress and no shear banding is observed. By contrast, for  $b^*$  greater than the  
 238 critical value  $b_c^* = 1$ , the shear rate becomes nonzero at the outer boundary of the flowing region  
 239 [Fig. 4(a)]. The underlying cause of this discontinuity of the strain rate is the discontinuity of the  
 240 structural parameter  $\lambda$  across the boundary of the flowing region [Fig. 4(d)]. Such a discontinuous  
 241 strain rate profile between a static and a flowing region has been observed using magnetic resonance  
 242 imaging measurements in cement pastes [30] and bentonite suspensions [29]. It corresponds to a  
 243 steady-state shear-banded velocity profile where the shear rate is equal to a critical shear rate in the  
 244 liquid region and is equal to zero in the solid region.

245 According to Eq. (31), increasing the parameter  $\tau_1^*$  may produce the same effect as increasing  
 246  $b^*/a^*$ . The steady-state flow is controlled by the competition between the restructuring and the  
 247 breakdown effects. The more the structure close to the interface is broken efficiently by the strain  
 248 rate [Fig. 4(d)], the more the viscosity drops significantly and rapidly.

249 Finally, we explore the effect of the shear-thinning index  $n_c$ . When  $n_c < 1$ , the shear-banded  
 250 flows are not observed because the constitutive relation of the material is always a growing function  
 251 for  $\dot{\gamma}$  sufficiently close to zero [Eq. (33)], allowing small values for  $\dot{\gamma}$  in the flow. In that case the  
 252 flow is always simply shear localized and smooth. This contrasts with the previous cases discussed  
 253 above ( $n_c = 1$ ), where small values of  $\dot{\gamma}$  fall in the unstable branch of the flow curve and then lead  
 254 to shear-banded flows. As would be expected from the velocity profiles obtained with Carreau fluids  
 255 by Alibenyahia *et al.* [3], the flow is confined close to the inner cylinder when the shear-thinning  
 256 index  $n_c$  decreases. This confirms that the shear-thinning behavior confines the flow in the inner  
 257 region of the gap where the viscosity is lower.

### 258 C. Interface between the static and the flowing regions

259 Now we focus on the evolution of the width of the flowing region  $y_o = r_o - r_i$  depending on the  
 260 thixotropic parameters and the shear-thinning index  $n_c$ ;  $y_o$  can be obtained in Fig. 4(d) by reading the  
 261 abscissa where  $\lambda$  reaches 1. In Figs. 5(a)–5(c) the curve of  $y_o$  separates the inner flowing region from  
 262 the outer static region. The base flow evolves smoothly from shear localization to shear-banding  
 263 regimes by increasing the value of  $b^*$  or  $\Delta K^*$  and  $\tau_1^*$  [Figs. 5(a) and 5(b)]. Nevertheless, it can be  
 264 observed in Fig. 5(a) that  $y_o$  decreases faster in shear-localized flows than in shear-banded flows  
 265 where  $b^*$  or  $\Delta K^*$  and  $\tau_1^*$  increase. When the breakdown parameter  $b^*$  tends to infinity, the size of  
 266 the flowing region, characterized by the reduced position  $y_o = r_o - r_i$  of the interface between the  
 267 flowing and solidlike regions, decreases to a minimum size corresponding to the one of the fully  
 268 unstructured equivalent Bingham fluid [Fig. 5(a)]. A quite similar remark can be made about  $\Delta K^*$   
 269 and  $\tau_1^*$ . Indeed, increasing the parameters  $b^*$ ,  $\Delta K^*$ , or  $\tau_1^*$  makes the shear-thinning behavior stronger  
 270 in the steady flowing region. Thus, it is not surprising that when the shear-thinning index  $n_c$  decreases,  
 271 the width of the flowing zone also decreases [Fig. 5(c)]. Nevertheless, the model with one structural  
 272 parameter predicts shear banding when the thixotropic parameters grow above some critical values  
 273 given by Eq. (31). The regular Bingham or Hershel-Bulkley laws only describe the shear localization.

274 Now we will study the linear stability of the base flow, whether shear banding is present or not.

## 275 IV. LINEAR STABILITY ANALYSIS

### 276 A. Equation setup

277 To perform a linear analysis of stability, the fluid velocity, the structural parameter, and the  
 278 pressure are decomposed as

$$279 \quad \mathbf{v} = \mathbf{v}_b + \tilde{\mathbf{v}}(r)\exp(\sigma t + in\theta + ikz), \quad (34)$$

$$280 \quad \lambda = \lambda_b + \tilde{\lambda}(r)\exp(\sigma t + in\theta + ikz), \quad (35)$$

$$281 \quad p = P_b + \tilde{p}(r)\exp(\sigma t + in\theta + ikz), \quad (36)$$

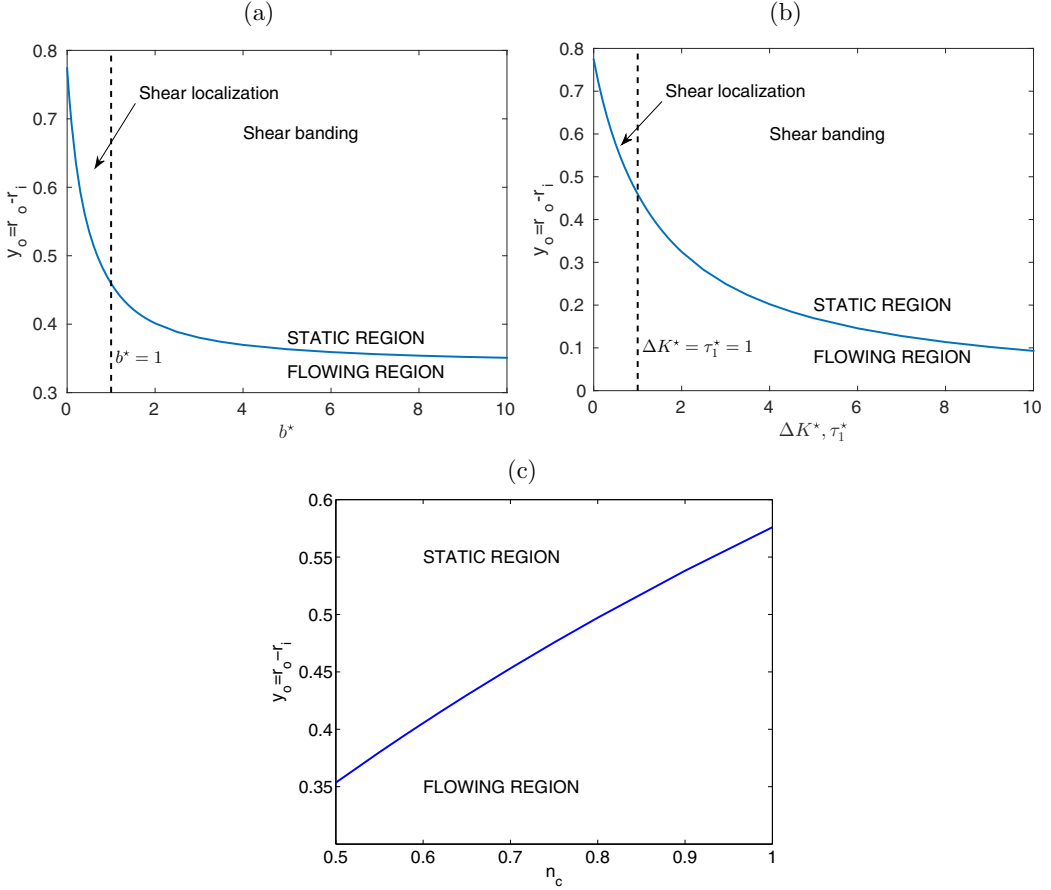


FIG. 5. Reduced position of the interface between flowing and static regions  $y_o = r_o - r_i$  for a large gap  $\eta = 0.5$  with  $Bn = 2$ ,  $n_c = 1$ , and  $a^* = 1$ . (a)  $y_o$  vs  $b^*$  with  $\Delta K^* = 1$  and  $\tau_1^* = 1$ . The vertical dashed line stands for the critical value of  $b^* = 1$  where the strain rate at the interface  $\dot{\gamma}_o$  becomes nonzero. (b)  $y_o$  vs  $\Delta K^*$  or  $\tau_1^*$  with  $b^* = 1$ . The vertical dashed line stands for the critical value of  $\Delta K^* = 1$  (or  $\tau_1^*$ ) where the strain rate at the interface  $\dot{\gamma}_o$  becomes nonzero. (c)  $y_o$  vs  $n_c$  with  $\Delta K^* = 0.5$ ,  $\tau_1^* = 0.5$ , and  $b^* = 1$ . Only shear-localization cases are considered when  $n_c \neq 1$ .

279 where  $\tilde{\mathbf{v}}$ ,  $\tilde{\boldsymbol{\lambda}}$ , and  $\tilde{p}$  are the perturbation of the base flow considering the azimuthal mode  $n$  and the  
 280 axial wave number  $k$ . Injecting Eqs. (34)–(36) in the general setup of Eqs. (8), (14), and (9) and after  
 281 withdrawing the nonlinear terms, the linear setup of equations for the perturbation of the base flow is

$$\sigma \tilde{\mathbf{v}} = -\overline{\nabla} \mathbf{v}_b \cdot \tilde{\mathbf{v}} - \overline{\nabla} \tilde{\mathbf{v}} \cdot \mathbf{v}_b + \frac{1}{\text{Re}} \text{div} \left( \left. \frac{\partial \overline{\boldsymbol{\tau}}}{\partial \dot{\gamma}_{ij}} \right|_b \dot{\gamma}_{ij}(\tilde{\mathbf{v}}) + \left. \frac{\partial \overline{\boldsymbol{\tau}}}{\partial \lambda} \right|_b \tilde{\boldsymbol{\lambda}} \right) - \nabla \tilde{p}, \quad (37)$$

$$\sigma \tilde{\boldsymbol{\lambda}} = -\mathbf{v}_b \cdot \nabla \tilde{\boldsymbol{\lambda}} - \tilde{\mathbf{v}} \cdot \nabla \boldsymbol{\lambda}_b - (a + b \dot{\gamma}_b^m) \tilde{\boldsymbol{\lambda}} - mb \lambda_b \dot{\gamma}_b^{m-1} \left. \frac{\partial \dot{\gamma}}{\partial \dot{\gamma}_{ij}} \right|_b \dot{\gamma}_{ij}(\tilde{\mathbf{v}}), \quad (38)$$

$$0 = \text{div}(\tilde{\mathbf{v}}). \quad (39)$$

282 The indices  $ij$  stand for  $r, \theta$ , or  $z$  and Einstein's convention for summation is used. As the stress is  
 283 always continuous across  $r_o$ , we can use a method similar to those of Frigaard *et al.* [31] and Landry  
 284 *et al.* [2]. It consists in writing the linear setup of Eqs. (37)–(39) in the flowing region of the base  
 285 flow. The displacement of the yield stress boundary is fully driven by the perturbation of the flow

TABLE I. Critical Reynolds number  $Re_c$  and critical axial wave number  $k_c$  for a Newtonian fluid vs the number of nodes  $M$  in the gap with a radii ratio  $\eta = 0.9$ .

$M$	$Re_c$	$k_c$
20	132.492	3.1270
30	131.989	3.1280
40	131.822	3.1283
50	131.746	3.1285
60	131.705	3.1286
100	131.647	3.1287

[see Eqs. (B1) and (B2)] unlike in a viscoelastic context. We consider here rigid boundary conditions for the velocity perturbation, i.e.,  $\tilde{\mathbf{v}} = 0$  at  $r = r_i$  and  $r = r_o$ . Thus, the generalized eigenvalue problem given by the latter setup of Eqs. (37)–(39) can be straightforwardly written in matrix form

$$\sigma \begin{bmatrix} I_v & 0 & 0 \\ 0 & I_\lambda & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ \Lambda \\ P \end{bmatrix} = \begin{bmatrix} L_{vv} & L_{v\lambda} & -G \\ L_{\lambda v} & L_{\lambda\lambda} & 0 \\ D & 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ \Lambda \\ P \end{bmatrix}, \quad (40)$$

where  $V$  is the vertical matrix of the values of the components of velocity  $\tilde{\mathbf{v}}$  at each inner point of the gap,  $\Lambda$  is the vertical matrix of the values of  $\tilde{\lambda}$  at each point of the mesh, including the inner and outer radii, and  $P$  is the vertical matrix of the values of  $\tilde{p}$  taken in the middle points of two successive nodes of the velocity and structural parameter mesh.

The linear problem (40) admits a number of infinite eigenvalues that is two times the number of degrees of freedom of the pressure. The infinite eigenvalues have to be removed because they correspond to nonzero divergence velocity fields.

### B. Convergence test and validation

In order to test the convergence of our numerical scheme and to validate our method, the critical Reynolds number  $Re_c$  and the critical axial wave number  $k_c$  are determined using different number of nodes  $M$  in the gap. The results are given in Tables I and II. For Newtonian fluids, many works allow us to validate our results. In a recent work [8], a similar numerical method gave  $Re_c = 131.66$  and  $k_c = 3.130$  in Newtonian fluids with  $\eta = 0.9$ . Those values are in very good agreement with ours. In addition, for Bingham fluids, Alibenyahia *et al.* [3] found  $Re_c = 127.74943$  and  $k_c = 3.183706$  with a spectral method at  $Bn = 1$  and  $\eta = 0.5$ . Once again, our results in Table II agree with these values within an error below 0.1%. According to Tables I and II and the results of [3,8], we can estimate the relative error for the critical values of the Reynolds number  $Re_c$  and the axial wave number  $k_c$  below 0.1% when  $M \geq 50$ . Thus, we use  $M = 50$  in the following.

TABLE II. Critical Reynolds number  $Re_c$  and critical axial wave number  $k_c$  for a Bingham fluid vs the number of nodes  $M$  in the gap with a radii ratio  $\eta = 0.5$ .

$M$	$Re_c$	$k_c$
20	128.472	3.1695
30	128.057	3.1776
40	127.919	3.1803
50	127.857	3.1816
60	127.823	3.1822
100	127.776	3.1832

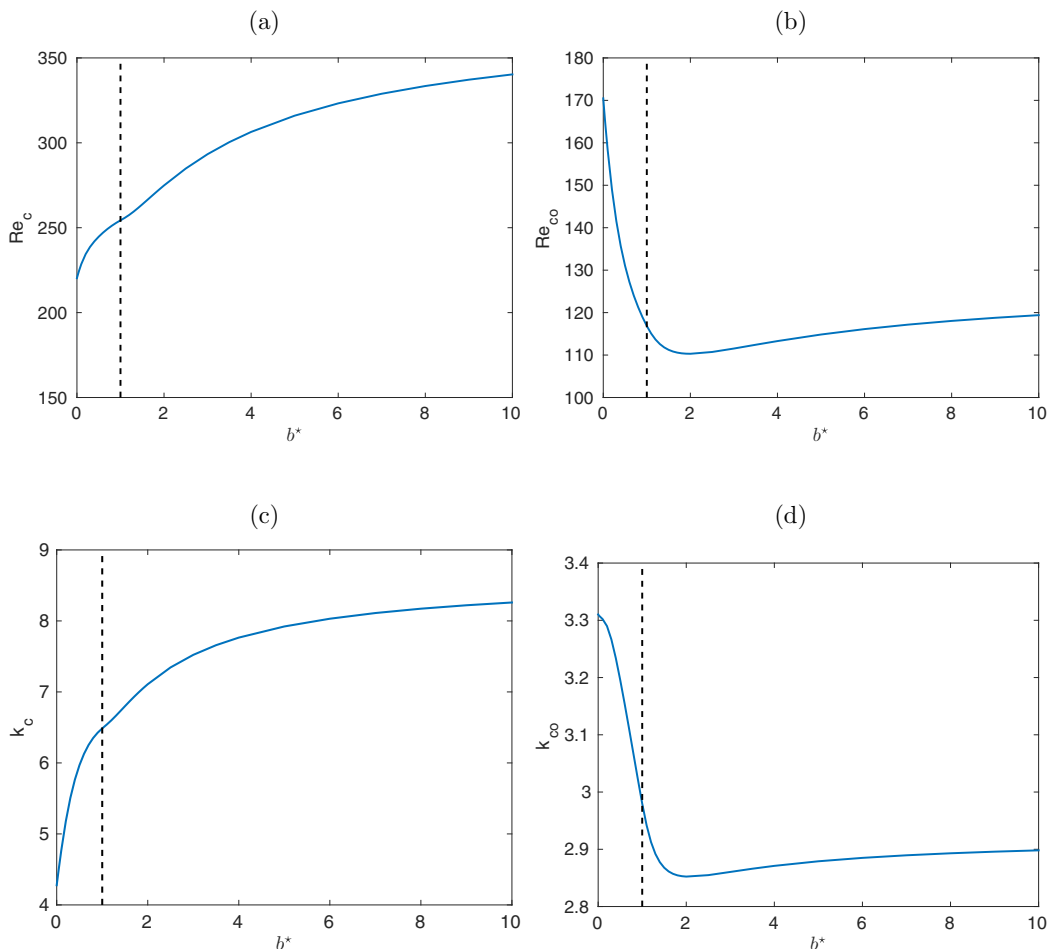


FIG. 6. Critical Reynolds number (a)  $Re_c$  and (b)  $Re_{co} = y_o Re_c$  and critical axial wave number (c)  $k_c$  and (d)  $k_{co} = y_o k_c$  vs  $b^*$ . Large gap  $\eta = 0.5$ ,  $Bn = 2$ ,  $n_c = 1$ ,  $\Delta K^* = \tau_1^* = 1$ , and  $a^* = 1$ . The vertical dashed line stands for the critical value of  $b^* = 1$  where the strain rate at the interface  $\dot{\gamma}_o$  becomes nonzero.

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### C. Stability analysis of Couette flow of thixotropic yield stress fluids

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To determine the critical eigenmode of the linear setup of Eq. (40), the algorithm seeks the minimum of the critical value of the Reynolds number  $Re_c$  depending on the wave number  $k$  for a given azimuthal mode  $n$ . The critical Reynolds number is reached when the real part of the eigenvalue  $\sigma$  is zero. The minimal value of  $Re_c$  is reached at the critical wave number  $k_c$ . We have verified that the critical perturbation is always axisymmetric, i.e.,  $n = 0$ , by computing the critical Reynolds number for the azimuthal modes  $n$  from 0 to 3 within the range of our parameters for the thixotropic yielded fluids. The result is that the Taylor vortices, steady and axisymmetric, correspond always to the most unstable eigenmode of (40). Thus, the critical mode is not oscillating or three dimensional, just as expected for nonthixotropic fluids such as those of Bingham [2]. This suggests that the structural parameter  $\lambda$  plays a passive role. Indeed, the operator  $L_{\lambda\lambda}$  in Eq. (40) is

$$L_{\lambda\lambda} = -\text{diag}\left(a^* + b^* \dot{\gamma}_b^m + \frac{inV_b}{r}\right), \quad (41)$$

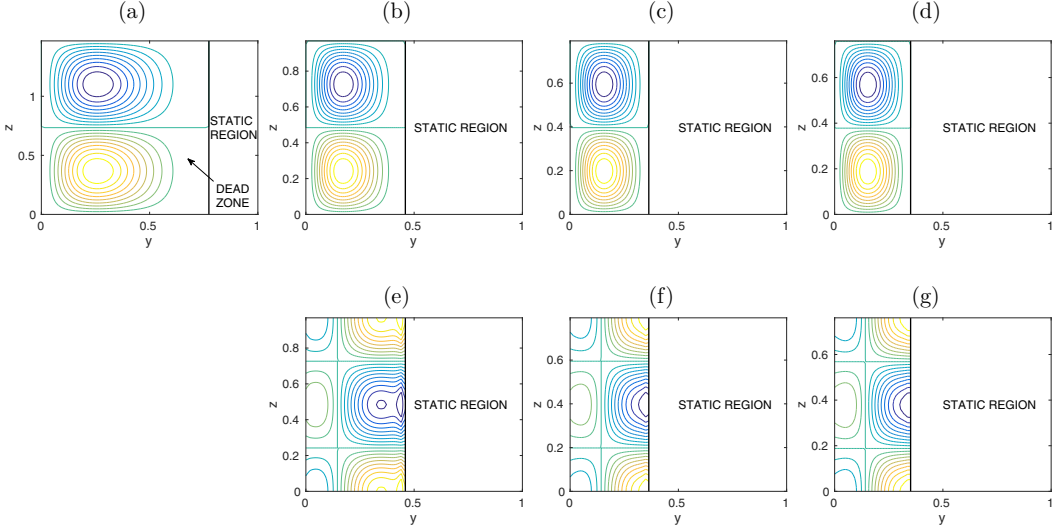


FIG. 7. (a)–(d) Streamlines of the critical velocity field of the perturbation. (e)–(g) Contour plot of the perturbation of the structural parameter  $\lambda$ . The contour lines step is 5% of the normalized amplitude of the perturbation. Blue stands for negative values and red for positive values (counterclockwise and clockwise spin for Taylor vortices). The large gap  $\eta = 0.5$ ,  $Bn = 2$ ,  $n_c = 1$ ,  $\Delta K^* = \tau_1^* = 1$ ,  $a^* = 1$ , and (a)  $b^* = 0$ , (b) and (e)  $b^* = 1$ , (c) and (f)  $b^* = 5$ , and (d) and (g)  $b^* = 10$ .

318 where  $\text{diag}(\cdot)$  stands for the diagonal matrix generated by the value of the argument at each node  
 319 of the mesh. Equation (41) shows that it generates eigenvalues  $\sigma$  such that their real part is always  
 320 negative. This means that the perturbations of  $\lambda$  vanishes without any coupling terms with the  
 321 perturbation of the velocity. This confirms the passive role of Eq. (38). Only the values of the critical  
 322 wave number  $k_c$  and the critical Reynolds number  $\text{Re}_c$  are modified compared to the Newtonian or  
 323 shear-thinning cases. One can notice this result because it means that the unsteady effects of the  
 324 thixotropy in cylindrical Couette flow that might occur above the threshold of the primary instability  
 325 are nonlinear. Nevertheless, shear-banded flows may occur with thixotropic yielded fluids. This is a  
 326 real difference from simple yield stress fluids.

327 The ratio  $b^*/a^*$  denotes the resistance against the strain rate  $\dot{\gamma}$  of the structure described by  
 328  $\lambda$ . The higher  $b^*/a^*$  is, the easier the inner structure of the fluid is broken down by the shear.  
 329 As shown previously in Fig. 5, the flowing region decreases because the yield stress collapses  
 330 with the breakdown of the structure. If the fluid would be a viscous Newtonian fluid, the critical  
 331 Reynolds number would increase because the gap becomes small, stabilizing the flow. In Fig. 6(a)  
 332 the variation of  $\text{Re}_c$  with  $b^*$  suggests that our choice for the reference viscosity  $\mu_{\text{ref}}$  is representative  
 333 of an equivalent Newtonian fluid and thus we retrieve the stabilizing effect of the reduction of the  
 334 gap width. Moreover, as the viscosity of the fluid decreases when the inner structure is broken, the  
 335 fluid is stronger with stronger shear thinning. Thus, it is not surprising that the growth of  $b^*/a^*$   
 336 ends by stabilizing the flow [Fig. 6(a)], as observed experimentally for a large gap by Escudier  
 337 *et al.* [6] and shown by Alibenhahia *et al.* [3] when the shear-thinning index  $n_c < 0.6$  for  $\eta = 0.5$ .  
 338 Nevertheless, it has been known that the yielded flow region width is the relevant length scale since  
 339 the earliest studies on the linear stability of yield stress fluids, e.g., Frigaard *et al.* [31]. The Reynolds  
 340 number  $\text{Re}_o = y_o \text{Re}$  is calculated taking into account the gap width of the yielded region  $y_o$ . The  
 341 collapse of the width  $y_o$  of the flowing zone (Fig. 5) is stronger than the stabilizing effect of shear  
 342 thinning. Thus, the critical Reynolds number  $\text{Re}_{co}$  decreases until  $y_o$  is close to its minimum value  
 343 corresponding to a fully unstructured fluid [Fig. 6(b)]. The critical wave number  $k_c$  mainly follows  
 344 the evolution of the fluid gap width and thus it increases with  $b^*$  [Fig. 6(c)]. By recalculating the wave

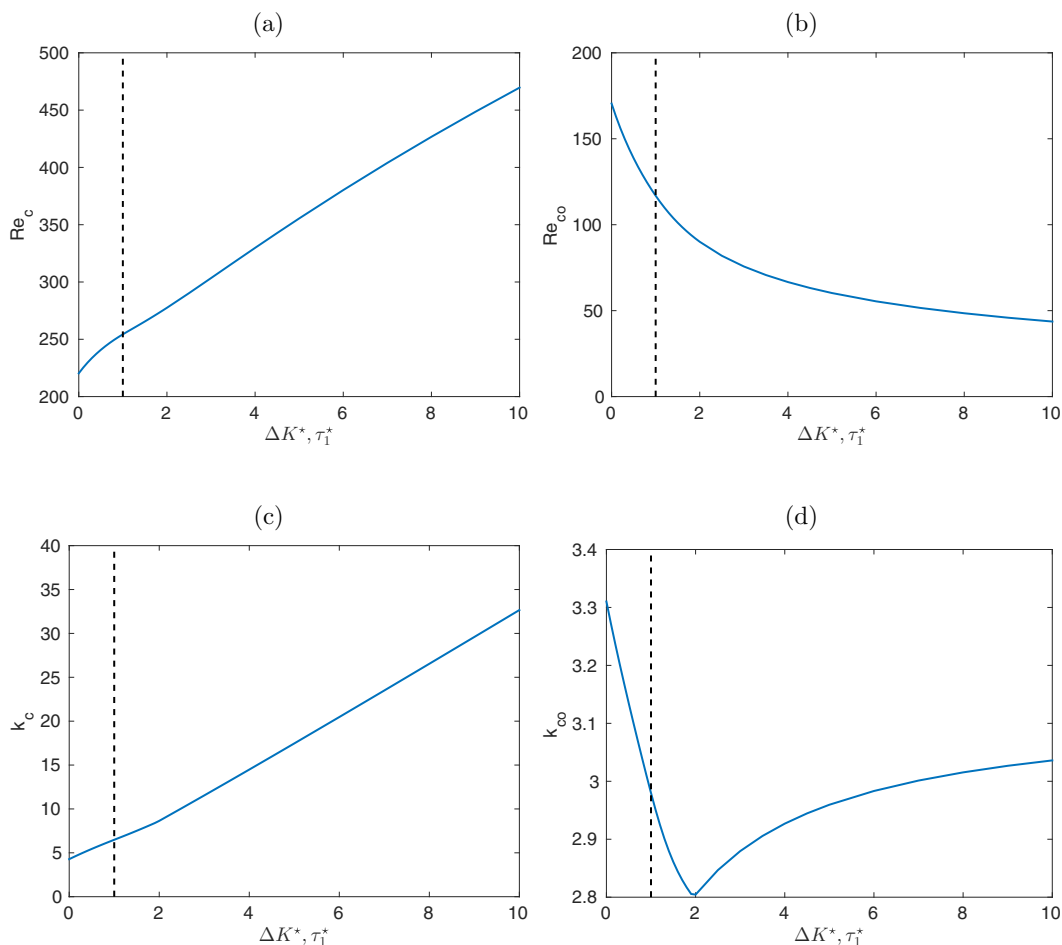


FIG. 8. Critical Reynolds number (a)  $Re_c$  and (b)  $Re_{co} = y_o Re_c$  and critical axial wave number (c)  $k_c$  and (d)  $k_{co} = y_o k_c$ . The large gap  $\eta = 0.5$ ,  $Bn = 2$ ,  $n_c = 1$ ,  $a^* = 1$ , and  $b^* = 1$ . The vertical dashed line stands for the critical value of  $\Delta K^* = 1$  where the strain rate at the interface  $\dot{\gamma}_o$  becomes nonzero.

345 number considering the fluid gap as the effective gap, we show that the wavelength  $L_z = 2\pi/k_{co}$  first  
 346 increases ( $k_{co}$  decreases) [Fig. 6(d)] because the vortices fill better the flowing region [Figs. 7(a)–7(d)]  
 347 when  $b^*$  increases. It can be characterized by the width between the contour line corresponding to the  
 348 5% level of the streamlines and the outer limit of the flowing region [Figs. 7(a)–7(d)]. This zone is the  
 349 so-called dead zone [Fig. 7(a)]. The vortices are squeezed toward the inner wall [Fig. 7(a)] because of  
 350 the stratification of the viscosity. When the shear-banding appears, the stratification of the viscosity  
 351 is weakened and the Taylor vortices thicken [Figs. 7(c) and 7(d)]. The wave number  $k_{co}$  tends to an  
 352 asymptotic value that corresponds to the rolls of an equivalent nonstructured fluid ( $\Delta K^* = \tau_1^* = 0$ ).  
 353 To compute the critical Reynolds number of an equivalent nonthixotropic fluid when the gap width  
 354 corresponds to  $y_o$  for  $b^* = 10$ , we have to set  $\eta = 0.7403$  and  $Bn = 0.7018$ . The critical Reynolds  
 355 number is  $Re_c = 126.9870$  and the critical wave number  $k_c = 2.9177$ . These values have to be  
 356 compared to the  $Re_{co} = 119.4021$  and  $k_{co} = 2.8981$  found when  $b^* = 10$ . As expected, for high  
 357 values of  $b^*$ , this fits with an equivalent nonthixotropic (simple) fluid flowing in a smaller gap.

358 It is worth noticing that the perturbation of the structural parameter  $\lambda$  corresponds to the convection  
 359 of the structure by the Taylor vortices. In Figs. 7(e)–7(g) the negative zone of the perturbation



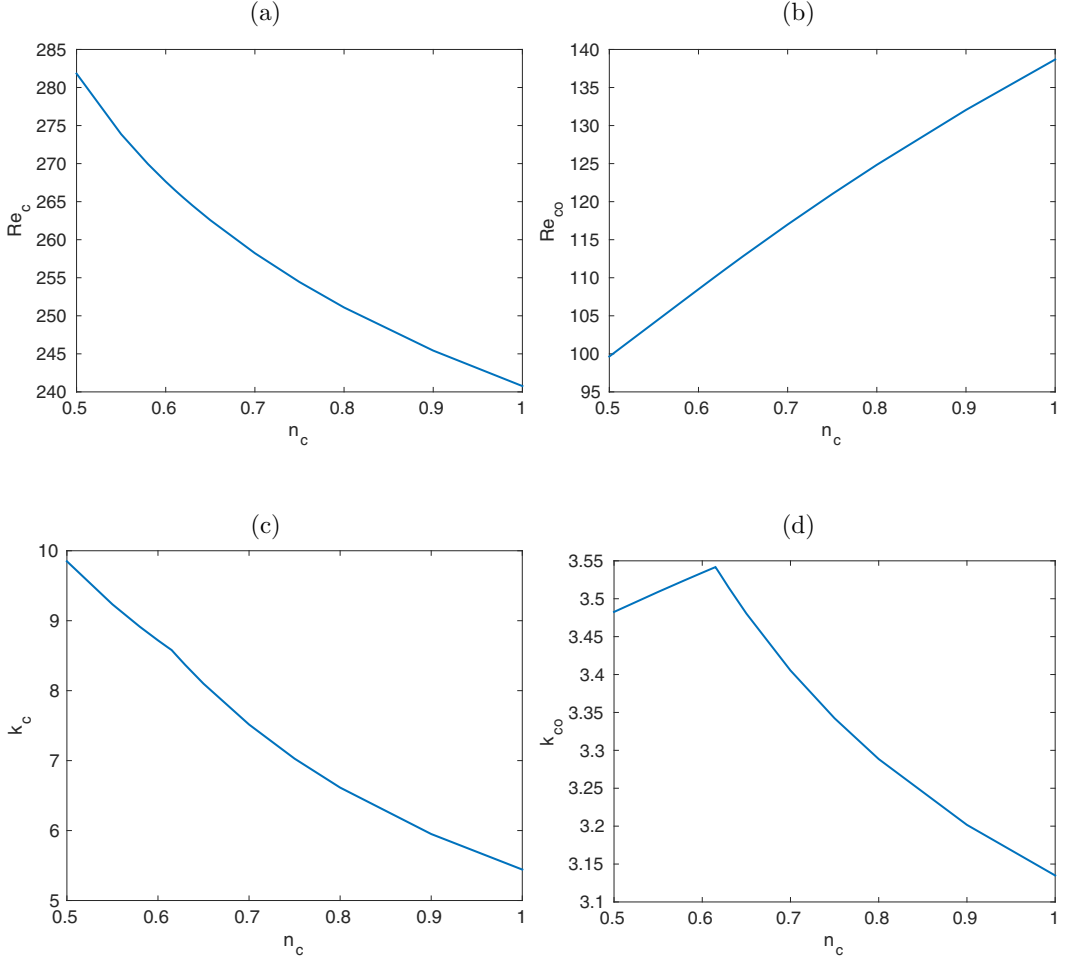


FIG. 9. Critical Reynolds number (a)  $Re_c$  and (b)  $Re_{co} = y_0 Re_c$  and critical axial wave number (c)  $k_c$  and (d)  $k_{co} = y_0 k_c$ . The large gap  $\eta = 0.5$ ,  $Bn = 2$ ,  $\Delta K^* = \tau_1^* = 0.5$ ,  $a^* = 1$ , and  $b^* = 1$ .

360 corresponds to the convection from the inner cylinder where the strain rate destroys the microstructure  
 361 toward the outer cylinder. Thus, in our parameters range, the linear stability is driven by the flow  
 362 that governs the perturbation of the microstructure.

363 The parameters  $\Delta K^*$  and  $\tau_1^*$  stabilize the flow according to Fig. 8(a). Nevertheless, as previously,  
 364 the material gap size is not the most relevant to define the critical Reynolds number. From this point  
 365 of view, the Reynolds number  $Re_{co}$  decreases and the flow is destabilized [Fig. 8(b)]. Indeed, the  
 366 shear-thinning behavior is strengthened when  $\Delta K^*$  and  $\tau_1^*$  increase. The high-viscosity area near the  
 367 interface between the flowing and solidlike regions, corresponding to the dead zone, collapses after  
 368 the onset of the shear banding. This is responsible for the increase of  $k_{co}$  in Fig. 8(d) with  $\Delta K^*$  and  $\tau_1^*$ .

369 As expected, the effect of  $n_c$  seems to be either destabilizing or stabilizing depending on whether  
 370 we track  $Re_c$  [Fig. 9(a)] or  $Re_{co}$  [Fig. 9(b)]. For the shear-localized flows, increasing the shear-  
 371 thinning behavior with the parameters  $b^*$ ,  $\Delta K^*$ , and  $\tau_1^*$  or  $n_c$  has similar effects.

372 To conclude on the effect of the thixotropy on the linear stability of the Couette flow, the critical  
 373 mode corresponds to the axisymmetric Taylor vortices that are also found for simple yield stress  
 374 fluids, such as Bingham fluids [2]. The critical eigenvalue is real, as in simple fluids. The results  
 375 found by Landry *et al.* [2] with Bingham fluids or Alibenyahia *et al.* [3] with shear-thinning fluids

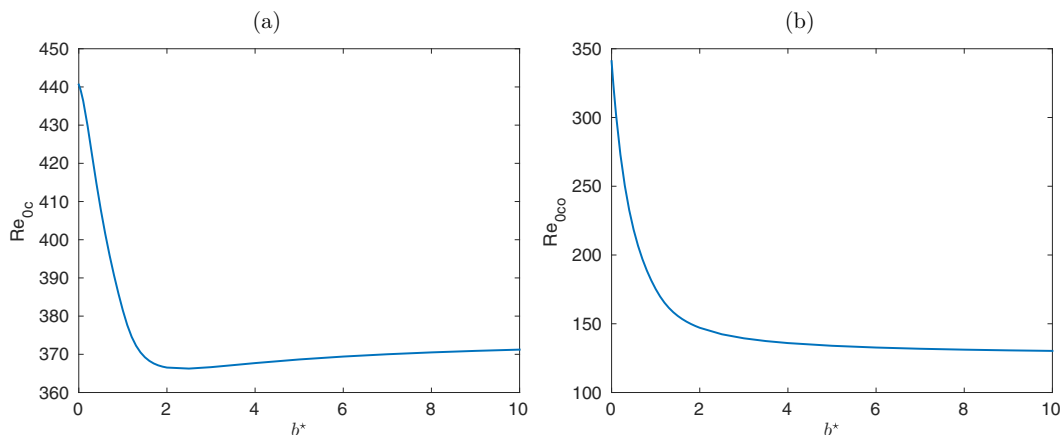


FIG. 10. Critical Reynolds number (a)  $Re_{0c} = (1 + \Delta K^* \lambda_{ref}) Re_c$  and (b)  $Re_{0co} = y_o Re_{0c}$  vs  $b^*$ . The large gap  $\eta = 0.5$ ,  $Bn = Bn_0 = 2$ ,  $\Delta K^* = \tau_1^* = 1$ , and  $a^* = 1$ .

376 are retrieved. The critical perturbation is driven by the inertial term, i.e., by the centrifugal force.  
 377 The yield stress and the shear-thinning behavior confine the rolls toward the inner cylinder in the  
 378 wide-gap case. In a Bingham fluid, Chen *et al.* [4] show that the optimal perturbation is also shifted  
 379 toward the inner cylinder in the wide-gap case. Thus, even during the transient growth preceding  
 380 the onset of the instability, only the inner zone of the gap is perturbed. The perturbation of the  
 381 structural parameter is driven by the convection of the material because of the Taylor vortices.  
 382 The stabilizing or destabilizing effect depends on the reference viscosity used for the definition of  
 383 the Reynolds number. Nevertheless, the key point is that increasing the shear-thinning behavior  
 384 reduces the width of the inner region where the viscosity is low. As would be the case if the material  
 385 gap size would be reduced, it stabilizes the flow. The shear thinning is driven not only by  $n_c$  but also  
 386 by the thixotropic parameters, i.e., the ratio  $b^*/a^*$ ,  $\Delta K^*$ , and  $\tau_1^*$ . Although the thixotropy does not  
 387 produce a qualitative modification of the linear stability of the flow in the case of shear localization,  
 388 it allows for shear banding.

## V. REFERENCE VISCOSITY AND REFERENCE YIELD STRESS

389  
 390 Figures 6(a) and 10(a) show that the characteristic value chosen for the viscosity may dramatically  
 391 change the conclusion about the effect of the parameters on the critical value of the Reynolds  
 392 number. Nevertheless, the asymptotic behavior for large  $b^*$  can also be retrieved from Fig. 10(b):  
 393 The critical Reynolds  $Re_{0co} = 130.2569$  is close to the one of the equivalent case with Bingham fluid,  
 394  $Re_c = 126.9870$ . The viscosity  $\mu_0$  is also a good choice to interpret the results, but our reference  
 395  $\mu_{ref}$  might be more relevant from a practical point of view. Moreover, it reproduces the stabilizing  
 396 effect of thinning the gap, which would be observed with Newtonian fluids. Finally, we defined the  
 397 wall Reynolds number as

$$Re_w = Re/\mu_w, \quad (42)$$

398 where  $\mu_w$  is the shear viscosity of the fluid on the inner cylinder. This viscosity is relevant because  
 399 it fixes the resistive torque on the rotating cylinder, which is measured in classical rheological  
 400 experiments. Moreover, the centrifugal instability at the origin of the onset of the Taylor vortices  
 401 is triggered in the low-viscosity region, close to the inner cylinder. Figure 11(a) shows that the  
 402 inner-wall shear viscosity  $\mu_w$  decreases for  $b^* = 0.6$  just before the onset of the shear banding.  
 403 For the shear-banded flows, the inner-wall shear viscosity increases to reach a limit value when  $b^*$   
 404 becomes high, i.e., when the structure is broken down even when the strain rate is low. The critical

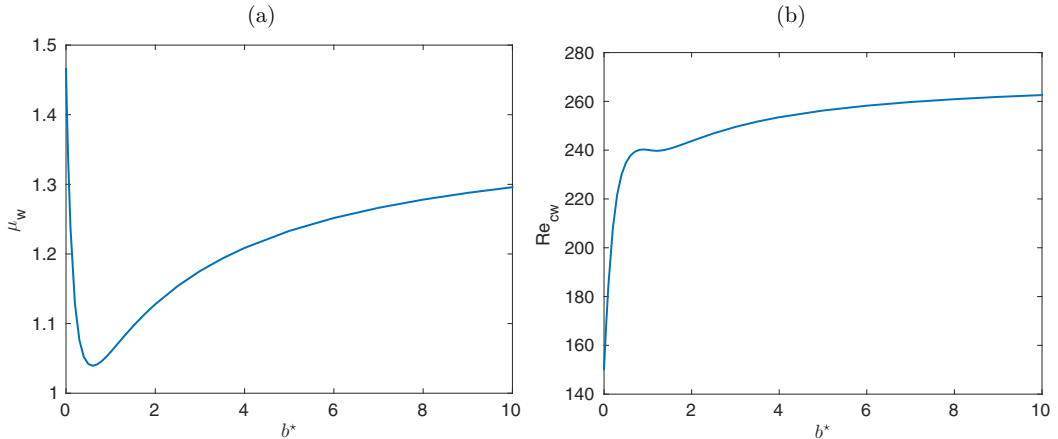


FIG. 11. (a) Inner wall shear-viscosity  $\mu_w$  and (b) critical Reynolds number  $Re_{cw} = Re_c/\mu_w$  scaled with  $\mu_w$  vs  $b^*$ . The large gap  $\eta = 0.5$ ,  $Bn = Bn_0 = 2$ ,  $\Delta K^* = \tau_1^* = 1$ , and  $a^* = 1$ .

405 Reynolds number calculated with the wall viscosity is strongly growing before the onset of the shear  
 406 banding but it is slightly constant ( $\sim 250$ ) for the shear-banded flow [Fig. 11(b)]. This observation  
 407 suggests that the inner-wall shear viscosity is more relevant for the onset of the Taylor vortices when  
 408 the velocity profile of the base flow corresponds to the shear banding, i.e., when the strain rate does  
 409 not approach zero and the viscosity values are finite and moderate.

410 Thus, the thixotropic yield stress fluids behave mainly as a viscous fluid when the structure is  
 411 fragile and the shear banding appears. In this case, the inner-wall viscosity is a good reference to  
 412 predict the onset of the Taylor vortices.

413

## VI. CONCLUSION

414 In this work we have studied the base flow and the linear stability in a Couette cell of a thixotropic  
 415 yield stress material modeled by Houška's model. This model with a single structural parameter  
 416 allows for nonmonotonic composite flow curves depending on the ratio between the building and  
 417 the breakdown parameters  $b^*/a^*$  (Fig. 2). Nonmonotonic composite curves are known to trigger  
 418 shear banding [25]. In shear banding, the structural parameter  $\lambda$  jumps abruptly from a value below  
 419 1 to 1 across the interface between the fluid and the solidlike zones. The shear rate  $\dot{\gamma}$  exhibits a  
 420 discontinuity across the interface and the width of the high-viscosity zone, called the dead zone,  
 421 collapses when shear banding occurs. In shear-banded flows, the selected stress at the interface is the  
 422 yield stress of the solid material where  $\lambda = 1$  if there is no diffusive term for the structural parameter.  
 423 For the flows of simple yield stress fluids, only a smooth transition between the flowing and static  
 424 regions is possible and the so-called dead zone is thicker than for thixotropic yield stress fluids.

425 The primary instability of the Couette flow is studied for a large gap ( $\eta = 0.5$ ) when the Bingham  
 426 number  $Bn$  is sufficiently high to have a solidlike region in the gap. The thixotropy does not modify  
 427 the kind of linear unstable mode. It remains steady and axisymmetric in the large range of parameters  
 428 explored in comparison with simple yield stress fluids [2]. The choice of the reference viscosity  
 429 in the definition of the Reynolds number plays a critical role in determining the stabilizing or the  
 430 destabilizing effect of the thixotropic and the shear-thinning behaviors. By taking our reference  
 431 viscosity  $\mu_{ref}$  or the inner-wall shear-viscosity  $\mu_w$ , the thixotropy stabilizes the Couette flow in both  
 432 cases because it increases the stratification of the viscosity over the gap. This stabilizing effect was  
 433 also found in Ref. [3] with shear-thinning fluids. Nevertheless, rescaling the Reynolds number with  
 434 the flowing gap size  $y_o$  shows that there is a competition between the reduction of the stratification  
 435 of the viscosity that destabilizes the flow and the reduction of the effective gap size because of the

436 breakdown of the fluid structure. The shear banding dramatically reduces the stratification of the  
 437 viscosity, which remains finite and moderate in the fluid region. The flow is squeezed near the inner  
 438 wall, which becomes similar to the one of a circular Couette setup with a smaller gap and without any  
 439 dead zone. Thus, the  $y_o$ -scaled Reynolds number  $Re_{co}$  is only slightly growing and tends towards a  
 440 constant value for high values of the breakdown parameter  $b^*$ . The unsteady effects of the thixotropy  
 441 may thus appear at the secondary instability of the Taylor vortices or it could be due to nonlinear  
 442 effects, which should be studied in future works.

#### 443 APPENDIX A: DETAILS OF THE LINEAR OPERATORS FOR THE VELOCITY PERTURBATION

444 In cylindrical coordinates, the perturbation vector of the velocity is written

$$\tilde{\mathbf{v}} = u\mathbf{e}_r + v\mathbf{e}_\theta + w\mathbf{e}_z. \quad (\text{A1})$$

445 The expressions of the linear operators that appear in Eqs. (37)–(39) in the cylindrical coordinates  
 446 system are

$$-\overline{\overline{\nabla}}_{\mathbf{v}_b} \cdot \tilde{\mathbf{v}} - \overline{\overline{\nabla}} \tilde{\mathbf{v}} \cdot \mathbf{v}_b = \frac{V_b}{r} \left( \frac{2}{r} v - inu \right) \mathbf{e}_r - \left[ \left( \frac{V_b}{r} + \frac{\partial V_b}{\partial r} \right) u + \frac{inV_b}{r} v \right] \mathbf{e}_\theta - \frac{inV_b}{r} w \mathbf{e}_z \quad (\text{A2})$$

447 for the inertial term and

$$-\mathbf{v}_b \cdot \nabla \tilde{\lambda} - \tilde{\mathbf{v}} \cdot \nabla \lambda_b = -\frac{\partial \lambda_b}{\partial r} u - \frac{inV_b}{r} \tilde{\lambda} \quad (\text{A3})$$

448 for the convective term of the structural parameter. The stress terms are

$$\frac{\partial \overline{\overline{\boldsymbol{\tau}}}}{\partial \dot{\gamma}_{ij}} \Big|_b \dot{\gamma}_{ij}(\tilde{\mathbf{v}}) = \left[ \frac{1}{2} (\mu_1 - \mu_b) (\delta_{ir} \delta_{j\theta} + \delta_{i\theta} \delta_{jr}) (\mathbf{e}_r \otimes \mathbf{e}_\theta + \mathbf{e}_\theta \otimes \mathbf{e}_r) + \mu_b (\mathbf{e}_i \otimes \mathbf{e}_j) \right] \dot{\gamma}_{ij}(\tilde{\mathbf{v}}), \quad (\text{A4})$$

$$\frac{\partial \overline{\overline{\boldsymbol{\tau}}}}{\partial \dot{\lambda}} \Big|_b \tilde{\lambda} = \tau_2 \lambda (\mathbf{e}_r \otimes \mathbf{e}_\theta + \mathbf{e}_\theta \otimes \mathbf{e}_r), \quad (\text{A5})$$

449 with

$$\mu_b = \left( \frac{1 + \Delta K^* \lambda_b}{1 + \Delta K^* \lambda_{\text{ref}}} \right) \dot{\gamma}_b^{n_c-1} + \frac{\text{Bn}}{\dot{\gamma}_b} \left( \frac{1 + \tau_1^* \lambda_b}{1 + \tau_1^* \lambda_{\text{ref}}} \right), \quad (\text{A6})$$

$$\mu_1 = n_c \left( \frac{1 + \Delta K^* \lambda_b}{1 + \Delta K^* \lambda_{\text{ref}}} \right) \dot{\gamma}_b^{n_c-1}, \quad (\text{A7})$$

$$\tau_2 = \left( \frac{\Delta K^*}{1 + \Delta K^* \lambda_{\text{ref}}} \dot{\gamma}_b^{n_c-1} + \frac{\text{Bn}}{\dot{\gamma}_b} \frac{\tau_1^*}{1 + \tau_1^* \lambda_{\text{ref}}} \right) \dot{\gamma}_{r\theta, b}. \quad (\text{A8})$$

450 The divergence of the stress tensor in cylindrical coordinates is

$$\begin{aligned} \text{div} \left( \frac{\partial \overline{\overline{\boldsymbol{\tau}}}}{\partial \dot{\gamma}_{ij}} \Big|_b \dot{\gamma}_{ij}(\tilde{\mathbf{v}}) \right) &= \left[ 2\mu_b \frac{\partial^2 u}{\partial r^2} + 2 \left( \frac{\mu_b}{r} + \frac{\partial \mu_b}{\partial r} \right) \frac{\partial u}{\partial r} - \left( n^2 \frac{\mu_1}{r^2} + k^2 \mu_b + \frac{2\mu_b}{r^2} \right) u \right. \\ &\quad \left. + in \frac{\mu_1}{r} \frac{\partial v}{\partial r} - in \left( \frac{2\mu_b + \mu_1}{r^2} \right) v + ik \mu_b \frac{\partial w}{\partial r} \right] \mathbf{e}_r \\ &\quad + \left[ in \frac{\mu_1}{r} \frac{\partial u}{\partial r} + in \left( \frac{2\mu_b + \mu_1}{r^2} + \frac{1}{r} \frac{\partial \mu_1}{\partial r} \right) u \right. \\ &\quad \left. + \mu_1 \frac{\partial^2 v}{\partial r^2} + \left( \frac{\mu_1}{r} + \frac{\partial \mu_1}{\partial r} \right) \frac{\partial v}{\partial r} \right] \mathbf{e}_\theta \end{aligned}$$

$$\begin{aligned}
 & - \left( \frac{2n^2\mu_b + \mu_1}{r^2} + k^2\mu_b + \frac{1}{r} \frac{\partial\mu_1}{\partial r} \right) v - nk \frac{\mu_b}{r} w \Big] \mathbf{e}_\theta \\
 & + \left[ ik\mu_b \frac{\partial u}{\partial r} + ik \left( \frac{\mu_b}{r} + \frac{\partial\mu_b}{\partial r} \right) u - nk \frac{\mu_b}{r} v \right. \\
 & \left. + \mu_b \frac{\partial^2 w}{\partial r^2} + \left( \frac{\mu_b}{r} + \frac{\partial\mu_b}{\partial r} \right) \frac{\partial w}{\partial r} - \left( n^2 \frac{\mu_b}{r^2} + 2k^2\mu_b \right) w \right] \mathbf{e}_z \quad (\text{A9})
 \end{aligned}$$

451 and

$$\text{div} \left( \frac{\partial \bar{\bar{\tau}}}{\partial \lambda} \Big|_b \tilde{\lambda} \right) = in \frac{\tau_2}{r} \tilde{\lambda} \mathbf{e}_r + \left[ \tau_2 \frac{\partial \tilde{\lambda}}{\partial r} + \left( \frac{2\tau_2}{r} + \frac{\partial\tau_2}{\partial r} \right) \tilde{\lambda} \right] \mathbf{e}_\theta. \quad (\text{A10})$$

452 Finally, the coupling term with the velocity in Eq. (38) is

$$\frac{\partial \dot{\gamma}}{\partial \dot{\gamma}_{ij}} \Big|_b \dot{\gamma}_{ij}(\tilde{\mathbf{v}}) = \frac{in}{r} u + \frac{\partial v}{\partial r} - \frac{v}{r}. \quad (\text{A11})$$

453

## APPENDIX B: DEFORMATION OF THE YIELD STRESS BOUNDARY

454 For the Bingham-like problem ( $n_c = 1$ ), the displacement of the interface is fully driven by the  
 455 perturbation of the strain rate by an expression similar to Eq. (3.11) of Landry *et al.* [2]. The details  
 456 of the mathematical proofs are given in Refs. [2,31]. They are based on the continuity of the stress  
 457 across the interface of the plug. The displacement of the interface  $\tilde{r}_o$  for a perturbation of the stress  
 458  $\tilde{\tau}_o$  at the interface is

$$2 \frac{\tilde{r}_o}{r_o} = \frac{\tilde{\tau}_o}{\tau_{ys}}. \quad (\text{B1})$$

459 If  $n_c = m = 1$ , the perturbation of the stress at  $r = r_o$  is linked to the perturbation of the strain rate

$$\tilde{\tau}_o = -\dot{\gamma}_{r,\theta}(\tilde{\mathbf{v}}) \left[ \tilde{K} + \lambda_o \frac{(a^* + \sigma) \Delta \tilde{K} - b^* \tilde{\tau}_1}{a^* + b^* \dot{\gamma}_b + \sigma} \right], \quad (\text{B2})$$

460 where  $\tilde{K}$ ,  $\Delta \tilde{K}$ , and  $\tilde{\tau}_1$  are defined by Eqs. (19) and (20). We recall that  $\sigma$  is an eigenvalue of the  
 461 linear setup of Eq. (40). Near the critical point, the real part  $|\text{Re}(\sigma)| \ll 1$ . As  $\sigma$  is found to be a real  
 462 number, we have  $\sigma = 0$  at the critical point.

463 For  $n_c < 1$ , the linear perturbation of the strain rate  $\delta\dot{\gamma}$  on the interface is negligible because  
 464  $|\delta\dot{\gamma}| \propto |\tilde{\tau}_o|^{1/n_c}$  at  $r = r_o$  [we only consider the case  $\dot{\gamma}_b(r_o) = 0$ ]. This leads to no linear displacement  
 465 of the yield stress boundary. This is compliant with the so-called dead zone, where the rolls vanish  
 466 near the yield stress boundary.

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