

Accurate and robust vertex placement for edge collapses

Supplemental material of
"Hybrid mesh-volume LoDs for all-scale pre-filtering of complex 3D assets"

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1. Introduction

Mesh simplification algorithms based on edge collapse [LWC*02] are flexible and they are widely used for reducing the resolution of very detailed meshes. The strategy for placing merged vertices after each edge collapse is a key ingredient of such algorithms.

Several vertex placement strategies have been proposed. The most simple approach, known as *half-edge collapse*, consists in placing the vertex v resulting from a collapse at one of the initial vertices (v_1 and v_2 in Fig 1). This leads to inaccurate simplified meshes because objects tend to shrink. This is an important issue for complex objects such as trees because the global appearance may be visibly changed if every leaf is inaccurately simplified. An example is given in Fig 2. Placing v between v_1 and v_2 doesn't perform better.

Vertex placement based on *quadratic error metrics* (or QEM) [GH97, Hop99] has been successfully used for fast simplification while improving the quality of simplified meshes comparing to naive approaches. Unfortunately, QEM methods also tend to shrink the geometry as noted by Hoppe [Hop99].

Several authors addressed the problem of preserving the volume of a mesh within the edge collapse framework [Hop99, LT98, ALSS99]. In fact, volume preservation is not always the right goal: a mesh could have a null or almost null volume locally, for example in the case of thin geometry, and volume-preserving methods are not robust in such case as shown in Fig. 2c. Preserving the area of the surface is not the proper criterion either because the simplification of very rough surfaces would generate incorrect results. Actually, what we want for our LoDs is to minimize some distance between the input mesh and the simplified mesh.

We propose a new vertex placement strategy based on mean square distances between the surface before the collapse and the surface after the collapse. Our algorithm produces results with quality similar to volume-preserving strategies but it is more robust.

2. Accurate vertex placement based on mean square distances

2.1. Background

We propose to place the vertex such that it minimizes two-sided mean square distances in the spirit of Aspert et al. [ASCE02]. Given two vertices v_1 and v_2 (Fig 1), M the set of triangles adjacent to v_1 and v_2 and M' the triangles adjacent to the new vertex v , we want to find a position for v that minimizes

$$d_{sqr}(M, M') = \iint_{p \in M} dist(p, M')^2 dp + \iint_{p' \in M'} dist(p', M)^2 dp' \quad (2.1)$$

with

$$dist(p, M') = \min_{p' \in M'} dist(p, p').$$

This formulation or similar ones have been used for measuring geometric errors in simplified meshes [OVBP11], but have never been used directly for vertex placement to our knowledge. Finding a position for v that minimize Eq. 2.1 is difficult because of non linearities.

2.2. Our approach

We propose to approximate $d_{sqr}(M, M')$ (Eq. 2.1) with a sum of square distances. Our algorithm is iterative and tries to find a better position x for the vertex v at each step.

Let's consider an edge collapse like the one in Fig. 1. Given a current position x , our algorithm works as follow:

1. We choose some points p_i on M , and find for each one the nearest point p'_i on M' . We write points p'_i as functions of x using barycentric coordinates: $p'_i = a'_i x + b'_i$.
2. We choose some points P'_i on M' , and find for each one the nearest point P_i on M . We write points P'_i as functions of x using barycentric coordinates: $P'_i = A'_i x + B'_i$.
3. Then, we minimize square distances between our points:

$$x_{next} = \arg \min_x \left(\sum \|p'_i - p_i\|^2 + \sum \|P'_i - P_i\|^2 \right). \quad (2.2)$$

4. We iterate until convergence or stop after N steps.

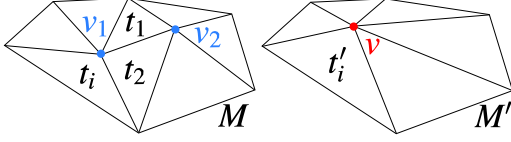


Figure 1: One edge collapse. v_1 and v_2 are merged into a new vertex v . Triangles t_1 and t_2 are collapsed. Every other triangle t_i corresponds to a new triangle t'_i .

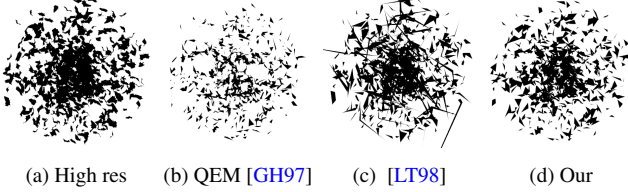


Figure 2: Comparison of vertex placement strategy for accurate mesh simplification with edge collapses. (a): the input high resolution mesh. (b): QEM based simplification tends to shrink the geometry. (c): volume preserving simplification proposed in [LT98]. It leads to instabilities for thin geometries such as leaves. (d): our vertex placement strategy. Unlike volume preserving methods it is robust for thin geometries and almost degenerated cases.

The main idea behind this algorithm is that minimizing Eq. 2.2 also reduces distances between p_i and M' and between points P'_i and M . Computing x_{next} is simple:

$$\begin{aligned} x_{next} &= \arg \min_x \sum \|a'_i x + b'_i - p_i\|^2 + \sum \|A'_i x + B'_i - P_i\|^2 \\ &= \arg \min_x \sum a_i'^2 x^T x + 2a_i'(b'_i - p_i)x \\ &\quad + \sum A_i'^2 x^T x + 2A_i'(B'_i - P_i)x \\ &= \arg \min_x cx^T x + Cx \\ &= -\frac{C}{2c} \end{aligned}$$

with $c = \sum a_i'^2 + \sum A_i'^2$ and $C = \sum 2a_i'(b'_i - p_i) + \sum 2A_i'(B'_i - P_i)$.

2.3. Implementation

We implemented this algorithm and we optimize vertex positions after each collapse. For points p_i , we choose the positions of v_1 and v_2 as well as points at the middle of edges that are adjacent to v_1 and v_2 . For points P'_i , we use the position of vertex v itself (its current position) and points at the middle of edges that are adjacent to v . We artificially stop the optimization after 200 iterations and use

$$\varepsilon = \frac{\text{voxelSize}}{100}$$

as a tolerance error.

We observed that our algorithm converges most of the time with few iterations. For example, during the simplification of the Vessel model (presented in the main paper), the average number of itera-

tions was 7 (measured with 5000 collapses), and the average cost of the optimization was $164 \mu\text{s}$ per edge collapse.

The most time-consuming task in our algorithm is finding the closest points p'_i and P_i . That is why we choose few points p_i and P'_i . We compared our method with the standard QEM strategy and one volume-preserving method (Fig. 2) implemented in CGAL [CGA16]. Our method leads to results that are similar to volume-preserving methods in simple cases, and it is more robust for thin geometry because our minimization has no instabilities.

2.4. Conclusion

We proposed a new vertex placement strategy based on the mean square distance between the input surface and the simplified surface. Our algorithm is more accurate than the standard QEM method and it is more robust than volume-preserving methods. We think that future work on vertex placement strategies based on mean square distances could further improve accuracy and efficiency of mesh simplification algorithms based on edge collapses.

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