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Small positive loops on overtwisted manifolds

Roger Casals, Francisco Presas, and Sheila Sandon

Given an overtwisted contact manifold, we prove that there can be no positive loops of contactomorphisms that are generated by a $\mathcal{C}^0$–small Hamiltonian function.

1. Introduction

In 2000 Eliashberg and Polterovich [EP00] noticed that the natural notion of positive contact isotopies, i.e. contact isotopies that move every point in a direction positively transverse to the contact distribution, induces for certain contact manifolds a partial order on the universal cover of the contactomorphism group. Such contact manifolds are called *orderable*. Since the work of Eliashberg and Polterovich orderability has become an important subject in the study of contact topology. In particular it has been discovered to be deeply related to the contact non–squeezing phenomenon [EKP06] (see also the exposition [Gi09]) and, more recently, to the non–degeneracy of a natural bi–invariant metric that is defined on the universal cover of the contactomorphism group [CS15].

As Eliashberg and Polterovich explained, orderability of a contact manifold is equivalent to the non–existence of a positive contractible loop of contactomorphisms. By now many contact manifolds are known to be orderable and many are known not to be, but it is still not well–understood where the boundary between the orderable and non–orderable world lies. In particular it is not known whether there is a relation between overtwistedness and orderability, since not a single overtwisted contact manifold is known to be orderable or not to be. In this article we prove the following result.

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Key words and phrases: overtwisted contact structures, positive loops of contactomorphisms, orderability.

Theorem 1. *Let $\big(M, \xi = \ker \alpha\big)$ be a closed overtwisted contact 3–manifold. Then there exists a real positive constant $C(\alpha)$ such that any positive loop $\{\phi_\theta\}$ of contactomorphisms which is generated by a contact Hamiltonian $H : M \times \mathbb{S}^1 \longrightarrow \mathbb{R}^+$ satisfies*

$$\|H\|_{\mathcal{C}^0} \geq C(\alpha)\,.$$

In other words, on closed overtwisted contact 3–manifolds there are no positive loops of contactomorphisms that are generated by a $\mathcal{C}^0$–small contact Hamiltonian. Note that there is no loss of generality in assuming that the contact Hamiltonian is 1–periodic, see Lemma 3.1.A in [EP00]. It is important to notice that our result does not imply that overtwisted contact manifolds are orderable, because the contraction of a positive contractible loop of contactomorphisms is not necessarily performed via positive loops. For instance, it was even proved in [EKP06, Theorem 1.11] that for the standard tight contact sphere any contraction of a positive contractible loop must be sufficiently negative somewhere. Theorem 1 states though that there exists a lower bound for a Hamiltonian function that generates a positive loop of contactomorphisms. Intuitively, in the presence of an overtwisted disc a positive isotopy returning to the identity requires a minimal amount of energy.

The specificity of our result is that we deal with $\mathcal{C}^0$*–small* contact Hamiltonians. Indeed, let us prove that the non–existence of a positive loop of contactomorphisms that is generated by a $\mathcal{C}^1$*–small* Hamiltonian holds on any contact manifold. Consider first the $\mathcal{C}^2$–small case. If the Hamiltonian $H_\theta : M \longrightarrow \mathbb{R}$ is $\mathcal{C}^2$–small then the generated loop $\{\phi_\theta\}$ is $\mathcal{C}^1$–small and so the contact graphs[1] $\mathrm{gr}(\phi_\theta)$ are Legendrian sections in a Weinstein neighborhood of the diagonal Δ in the contact product $M \times M \times \mathbb{R}$. Since a Weinstein neighborhood is contactomorphic to a neighborhood of the zero section of $J^1(\Delta) = J^1(M)$, the graphs $\mathrm{gr}(\phi_\theta)$ are of the form $\{j^1 f_\theta\}$ for a family of smooth functions f_θ on M. Because of the Hamilton–Jacobi equation (see [Ar, Section 46]), positivity of the loop $\{\phi_\theta\}$ implies that the family f_θ is strictly increasing, yielding a contradiction. If the Hamiltonian function is only $\mathcal{C}^1$–small, and thus the loop $\{\phi_\theta\}$ is $\mathcal{C}^0$–small, then the graphs $\mathrm{gr}(\phi_\theta)$ are still contained in a Weinstein neighborhood of the diagonal in the contact product but they are not necessarily sections anymore, and so they cannot be written as 1-jet of functions. However it follows from Chekanov theorem [Che96, Cha95] that they have generating functions quadratic at infinity and

[1]See for example [Sa13, CS15] for the definition of contact graphs, contact products and more details on arguments similar to the one that follows.

so an argument similar to the one above (or the results in [CFP10, CN10]) allows to conclude also in this case. As far as we know, Theorem 1 is the first result in the literature that shows the non–existence of a positive loop in the case when the Hamiltonian is $\mathcal{C}^0$–small. Our proof strongly uses overtwistedness in several points, and does not give an intuition of whether or not the result should also be true for tight contact manifolds. However it seems plausible to us that this might be the case.

Although Theorem 1 only applies to overtwisted contact 3–manifolds, a higher–dimensional analogue can also be stated. The careful reader can try to generalize the result to non–fillable contact manifolds containing a PS–structure [Ni06, Pr07], a GPS–structure [NP10] or a blob [MNW13] with the appropriate hypotheses on the Chern class of the contact distributions. The precise statement is not part of this article due to its technicality and to the fact that no new geometric ideas are required for the argument to work. We would also like to remark that the constant $C(\alpha)$ appearing in the statement of Theorem 1 can be explicitly computed with an algorithm. This procedure is however quite involved and requires an explicit symbolic solution for a system of differential equations as well as the expression of the flow of a vector field. In particular, a numerical estimate for $C(\alpha)$ is hardly attainable and hence we do not discuss this point further in the article.

As a consequence of Theorem 1, we can bound from below not only the supremum norm of the Hamiltonian of a positive loop but also its L^1–norm, in the following sense.

Corollary 2. *Let $\big(M, \xi = \ker \alpha\big)$ be a closed overtwisted contact 3–manifold. Then there exists a real positive constant $C(\alpha)$ such that any positive loop of contactomorphisms $\{\phi_\theta\}$ which is generated by a contact Hamiltonian H_θ, $\theta \in \mathbb{S}^1$, satisfies*

$$\int_0^1 \|H_\theta\|_{\mathcal{C}^0}\, d\theta \geq C(\alpha)\,.$$

Corollary 2 can be deduced from Theorem 1 as follows. Suppose that there is a positive loop $\{\phi_\theta\}$ which is generated by a contact Hamiltonian H_θ that satisfies

$$\int_0^1 \|H_\theta\|_{\mathcal{C}^0}\, d\theta \leq C(\alpha)\,.$$

Define a reparametrization $\beta : [0,1] \longrightarrow [0,1]$ of the time–coordinate by requiring $\dot\beta(\theta) = \frac{\|H_\theta\|_{\mathcal{C}^0}}{\int_0^1 \|H_\theta\|_{\mathcal{C}^0}\, d\theta}$ and write $\psi_{\beta(\theta)} = \phi_\theta$. Then we have $H_\theta = \dot\beta(\theta) G_{\beta(\theta)}$, where $G_{\beta(\theta)}$ is the Hamiltonian of the reparametrized loop $\psi_{\beta(\theta)}$.

For all $\theta \in \mathbb{S}^1$ we have

$$\max_x G_{\beta(\theta)}(x) = \frac{\int_0^1 \|H_\theta\|_{C^0}\, d\theta}{\|H_\theta\|_{C^0}} \|H_\theta\|_{C^0} = \int_0^1 \|H_\theta\|_{C^0}\, d\theta \leq C(\alpha)$$

contradicting Theorem 1.

The geometric core of the proof of Theorem 1 can be shortly described in two parts. First, any overtwisted contact manifold (M, ξ) can be embedded with trivial symplectic normal bundle in an exact symplectically fillable contact 5–manifold (X, ξ_X). Second, the existence of a small positive loop of contactomorphisms on (M, ξ) implies the existence of a PS–structure on (X, ξ_X). This yields a contradiction, according to the main result of [Ni06]. The construction of a PS–structure on X is based on techniques similar to those used by Niederkrüger and the second author [NP10] to study the size of tubular neighborhoods of contact submanifolds.

The paper is organized as follows. Section 2 recalls basic definitions and facts about overtwisted contact manifolds. In Section 3 we explain how to construct a PS–structure in the total space of the contact fibration $M \times \mathbb{D}^2$, where M is an overtwisted contact 3–manifold, starting from a small positive loop of contactomorphisms of M. Theorem 1 is proved in Section 4 assuming an embedding result that will be proved in Section 5.

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2. Preliminaries on overtwisted contact manifolds

We refer to the book of Geiges [Ge] for the basics about contact structures, and recall here only the definitions and facts about overtwisted contact manifolds that will be needed in the rest of the article. A 3–dimensional contact manifold (M, ξ) is said to be *overtwisted* if it contains an overtwisted disc, i.e. an embedded 2–disc Δ such that the characteristic foliation $T\Delta \cap \xi$ contains a unique singular point in the interior of Δ and $\partial\Delta$ is the only closed leaf of this foliation. A contact manifold is said to be tight if it is not overtwisted.

As follows from the results of Lutz and Martinet [Lu70, Ma71], there exists an overtwisted contact structure in any homotopy class of 2–plane fields. Moreover, by the classification of overtwisted contact structures achieved by Eliashberg [El89], we also know that on a given homotopy class of 2–plane fields there exists exactly one overtwisted contact structure. More precisely we have the following result.

Theorem 3 ([El89]). *Let ξ and ξ' be overtwisted contact structures on a 3–dimensional manifold M, and suppose that they are homotopic as 2–plane fields. Then ξ and ξ' are isotopic contact structures.*

The notion of an overtwisted contact structure does not readily generalize to higher–dimensional contact manifolds. The following geometric model was proposed by Niederkrüger [Ni06].

Definition 4. Let (M, ξ) be a contact 5–manifold. A *plastikstufe* $\mathrm{PS}(\mathbb{S}^1)$ in M with singular set $\mathbb{S}^1$ is an embedding of a solid torus

$$\iota : \mathbb{D}^2 \times \mathbb{S}^1 \longrightarrow M$$

with the following properties:

a. The boundary $\partial\mathbb{D}^2 \times \mathbb{S}^1$ is the unique closed leaf of the foliation $\ker(\iota^*\alpha)$ on $\mathbb{D}^2 \times \mathbb{S}^1$.

b. The interior of $\mathbb{D}^2 \times \mathbb{S}^1$ is foliated by an $\mathbb{S}^1$–family of stripes $(0,1) \times \mathbb{S}^1$ spanned between $\mathbb{S}^1 \times \{0\}$ and asymptotically approaching $\partial\mathbb{D}^2 \times \mathbb{S}^1$ on the other side.

In particular, Property a. implies that the boundary of the solid torus is a Legendrian torus and the core $\{0\} \times \mathbb{S}^1$ is transverse to the contact distribution ξ.

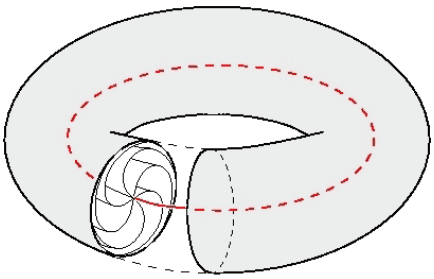

Figure 1: An embedded PS–structure in a contact 5–fold.

A plastikstufe is also referred to in the literature as a PS–structure.

By results of Gromov and Eliashberg [Gr85, El89], in dimension 3 the presence of an overtwisted disc obstructs the existence of symplectic fillings. The higher–dimensional analogue of this fact is the following theorem by Niederkrüger.

Theorem 5 ([Ni06]). *Let (M, ξ) be a contact 5–manifold with a PS–structure. Then M does not admit an exact symplectic filling.*

As we will explain, the argument used to prove Theorem 1 is based on the insertion of a PS–structure in an exact symplectically fillable manifold, thus yielding a contradiction with Theorem 5. The techniques that provide the embedding of the PS–structure are based on the study of certain contact structures on the manifold $M \times \mathbb{D}^2$. This will be explained in the next section.

3. PS–structures and contact fibrations

In the first part of this section we will recall, following Lerman [Le04] and [Pr07], the notion of a contact fibration and its relation to the group of contactomorphisms via the monodromy diffeomorphism. We will then show in Proposition 9 how to apply these concepts in order to construct a PS–structure on the total space of the contact fibration $M \times \mathbb{D}^2$, starting from a sufficiently small positive loop of contactomorphisms of M.

A smooth fiber bundle $\pi : X \longrightarrow B$ is said to be a *contact fibration* if there exists a hyperplane distribution $\xi_X = \ker \alpha_X$ on X such that its restriction $\xi = \ker(\pi) \cap \xi_X$ defines a contact structure in each fiber. In particular $\left(\xi, d\alpha_X|_{\ker(\pi)}\right)$ is a subbundle of the not necessarily symplectic bundle ξ_X. Note that ξ is itself a symplectic bundle. This data leads to a natural choice of connection.

Definition 6. Let $\pi : \big(X, \xi_X = \ker \alpha_X\big) \longrightarrow B$ be a contact fibration. Then the distribution $\xi^{\perp d\alpha_X} \subset \xi_X$ is called the *contact connection* associated to the contact fibration.

In other words, for a point p of B and a tangent vector $v \in T_pB$, the horizontal lift of v at some $\tilde{p} \in \pi^{-1}(p)$ with respect to the contact connection is the unique vector $\tilde{v} \in T_{\tilde{p}}X$ such that $\pi_*\tilde{v} = v$, $\tilde{v} \in \ker(\xi_X)$ and $\iota_{\tilde{v}} d\alpha_X = 0$ on ξ. Note that the contact connection only depends on ξ_X, not on the choice of the 1–form α_X with $\xi_X = \ker \alpha_X$. The parallel transport along a segment joining two points $q, p \in B$ is defined as in the smooth case, but in the contact framework it is enhanced from a diffeomorphism to a contactomorphism between the fibers of q and p. Moreover, the definition of the contact connection implies that the trace by parallel transport of a submanifold that is tangent to the contact structure on the fibers is also tangent to the distribution on the total space. A precise statement of these properties is the content of the following proposition.

Proposition 7. *([Le04, Pr07]) Let $\pi : \big(X, \xi_X = \ker \alpha_X\big) \longrightarrow B$ be a contact fibration with closed fibers. Consider a point $p \in B$ and an immersed path $\gamma : [0,1] \longrightarrow B$ with $\gamma(0) = p$. Then parallel transport along γ with respect to the contact connection defines a path of diffeomorphisms*

$$\widetilde{\gamma}_t : \pi^{-1}(p) \longrightarrow \pi^{-1}(\gamma(t))$$

with the following properties:

a. *The diffeomorphisms $\widetilde{\gamma}_t$ are contactomorphisms.*

b. *Let L be an isotropic submanifold of $\pi^{-1}(p)$ and consider the map*

$$\mathfrak{t} : L \times [0,1] \longrightarrow X, \quad (p,t) \longmapsto \widetilde{\gamma}_t(p),$$

then $im(\mathfrak{t})$ is an immersed isotropic submanifold of (X, ξ_X). It is an embedded isotropic submanifold if γ is an embedded path.

Note that the closedness condition for the fibers is technical and only used to ensure that the vector fields implicitly appearing in the statement are complete.

There are instances in which the contactomorphisms generated via parallel transport have a simple description. The following example will be used in the proof of our results.

Let $\big(M, \xi = \ker \alpha\big)$ be a contact manifold. A time–dependent function H_θ on M induces a path of contactomorphisms $\{\phi_\theta\}$, which is defined to be the flow of the time–dependent vector field X_θ satisfying

$$\begin{aligned} \iota_{X_\theta}\alpha &= H_\theta, \\ \iota_{X_\theta} d\alpha &= -dH_\theta + dH_\theta(R_\alpha)\,\alpha \end{aligned} \tag{1}$$

where R_α is the Reeb vector field associated to α. The function H_θ is called the *contact Hamiltonian* with respect to the contact form α of the contact isotopy $\{\phi_\theta\}$. In contrast to the symplectic case, any contact isotopy can be written as the flow of a contact Hamiltonian, see [Ge, Section 2.3].

Consider the manifold $M \times \mathbb{D}^2$, where $\mathbb{D}^2$ denotes the 2–disc with polar coordinates (r, θ). Let $H : M \times \mathbb{D}^2 \longrightarrow \mathbb{R}$ be a function such that $H \in O(r^2)$ at the origin and $\partial_r H > 0$. Then the 1–form

$$\alpha_H = \alpha + H(p, r, \theta) d\theta$$

defines a contact structure ξ_H on the manifold $M \times \mathbb{D}^2$. In particular, suppose that $H : M \times \mathbb{S}^1 \longrightarrow \mathbb{R}$ is a positive function. Then $\alpha_H = \alpha + H(p, \theta) \cdot r^2 d\theta$ is a contact form in $M \times \mathbb{D}^2$.

Lemma 8. *Let $\big(M, \xi = \ker \alpha\big)$ be a contact manifold, and $H_\theta : M \longrightarrow \mathbb{R}$ an $\mathbb{S}^1$–family of positive smooth functions. Consider the contact fibration*

$$\pi : \big(M \times \mathbb{D}^2, \ker \alpha_H\big) \longrightarrow \mathbb{D}^2.$$

Then parallel transport along $\gamma(\theta) = (1, -\theta)$ is the contact flow of the Hamiltonian H_θ.

Proof. The horizontal lift with respect to the contact connection of the vector field ∂_θ at a point $(1, \theta)$ is of the form

$$\widetilde{X} = \partial_\theta - X_\theta,$$

where X_θ satisfies the equations $\iota_{X_\theta}\alpha = H_\theta$ and $\iota_{X_\theta} d\alpha = -dH_\theta + dH_\theta(R_\alpha)\,\alpha$. Indeed, the lift is unique and $\widetilde{X}$ satisfies both $\alpha_H(\widetilde{X}) = 0$ and $\iota_{\widetilde{X}} d\alpha_H = 0$ on ξ. The statement then follows from equations (1). □

Let us explain how to use Lemma 8 to construct a PS–structure in $\big(M \times \mathbb{D}^2(\delta), \ker(\alpha + r^2 d\theta)\big)$, where $\mathbb{D}^2(\delta)$ denotes the 2–disc of radius δ, using a positive loop of contactomorphisms in M.

Proposition 9. *Assume that $\{\phi_\theta\}$ is a positive loop of contactomorphisms of an overtwisted contact manifold $\big(M, \xi = \ker \alpha\big)$ which is generated by a contact Hamiltonian H_θ, $\theta \in \mathbb{S}^1$, with $H_\theta < \delta^2$ for some $\delta \in \mathbb{R}^+$. Then there is a PS–structure on $\big(M \times \mathbb{D}^2(\delta), \ker(\alpha + r^2 d\theta)\big)$.*

Proof. Note first the following general fact. Suppose that that $\pi : (X, \xi_X) \longrightarrow \Sigma$ is a contact fibration over a smooth compact surface Σ, such that the fibers are closed overtwisted contact manifolds. Suppose also that there exists an embedded loop $\gamma : \mathbb{S}^1 \longrightarrow \Sigma$ whose time–1 parallel transport $\widetilde{\gamma}_1$ is the identity. Then there exists a PS–structure in the pre–image $\pi^{-1}(\gamma(\mathbb{S}^1))$. Indeed, since the fiber $\pi^{-1}(\gamma(0))$ is overtwisted we can consider an embedded overtwisted disc Δ in it and define the map

$$\begin{array}{rcl} \rho : \Delta \times \mathbb{S}^1 & \longrightarrow & X \\ (p, \theta) & \longmapsto & \rho(r, \theta) = \widetilde{\gamma}_\theta(p). \end{array}$$

Then property b. in Proposition 7 implies that $im(\rho)$ is a PS–structure. By combining this fact with Lemma 8 we see that if $\{\phi_\theta\}$ is a positive loop of contactomorphisms on a contact manifold $\big(M, \xi = \ker \alpha\big)$ then there is a PS–structure on $\big(M \times \mathbb{D}^2(1), \ker(\alpha + H_\theta r^2 d\theta)\big)$ where H_θ is the Hamiltonian function of $\{\phi_\theta\}$. The PS–structure lies in the boundary defined by the equation $\{r = 1\}$. Note that if $H_\theta < \delta^2$ for some $\delta \in \mathbb{R}^+$ then there exists a strict contact embedding

$$\big(M \times \mathbb{D}^2(1), \ker(\alpha + H_\theta r^2 d\theta)\big) \longrightarrow \big(M \times \mathbb{D}^2(\delta), \ker(\alpha + r^2 d\theta)\big)$$

given by the map $(p, r, \theta) \longmapsto (p, \sqrt{H_\theta(p)}r, \theta)$. A PS–structure in $\big(M \times \mathbb{D}^2(1), \ker(\alpha + H_\theta r^2 d\theta)\big)$ contained in the hypersurface defined as $\{r = 1\}$ is sent to a PS–structure in $\big(M \times \mathbb{D}^2(\delta), \ker(\alpha + r^2 d\theta)\big)$ contained in the hypersurface defined as $\{r = \sqrt{H_\theta}\}$. We have thus obtained the required PS–structure in $\big(M \times \mathbb{D}^2(\delta), \ker(\alpha + r^2 d\theta)\big)$. □

4. Proof of Theorem 1

In this section we prove Theorem 1 in the case when $c_1(\xi) = 0$. As we will explain, the general case also follows from the same argument modulo Proposition 11 that will be proved in the last section.

Let (M, ξ) be a 3–dimensional overtwisted contact manifold and assume that $\{\phi_\theta\}$ is a positive loop of contactomorphisms, generated by a contact Hamiltonian H_θ, $\theta \in \mathbb{S}^1$. We want to show that if H_θ is small in $\mathcal{C}^0$–norm then the existence of $\{\phi_\theta\}$ gives a contradiction with Theorem 5.

Recall from the previous section that if $\{\phi_t\}$ is a positive loop of contactomorphisms of M which is generated by a sufficiently small contact Hamiltonian H_θ ($\theta \in \mathbb{S}^1$) then there is a PS–structure on $\big(M \times \mathbb{D}^2(\delta), \ker(\alpha + r^2 d\theta)\big)$ for some small $\delta \in \mathbb{R}^+$. Note that the manifold $\big(M \times \mathbb{D}^2(\delta), \ker(\alpha + r^2 d\theta)\big)$ is the standard contact neighborhood of a codimension–2 contact submanifold with trivial symplectic normal bundle, see [Ge, Section 2.5.3]. The result of the previous section implies thus the following proposition.

Proposition 10. *Let (X, ξ_X) be a contact 5–manifold and (M, ξ) a codimension–2 overtwisted contact 3–manifold with trivial symplectic normal bundle. Suppose that $\{\phi_\theta\}$ is a positive loop of contactomorphisms which is generated by a sufficiently small contact Hamiltonian. Then there exists a PS–structure in a neighborhood of M in X.*

In the case when $c_1(\xi) = 0$, Theorem 1 follows from Proposition 10. Indeed any contact manifold (M, ξ) can be embedded as a contact submanifold into its unit cotangent bundle $\mathbb{S}T^*M$, by using the map $e_\alpha : M \longrightarrow \mathbb{S}T^*M$ defined by $e_\alpha(p) = \big(p, \alpha_p\big)$ where α is a contact form for ξ. Note that if $c_1(\xi) = 0$ then $e_\alpha(M) \subset \mathbb{S}T^*M$ has trivial symplectic normal bundle. Certainly, the symplectic normal bundle of $e_\alpha(M)$ inside $\mathbb{S}T^*M$ is isomorphic to ξ, and thus it is trivial if its Euler class $c_1(\xi)$ vanishes. If there was a small positive contact Hamiltonian H_θ that generated a loop of contactomorphisms then Proposition 10 would give a PS–structure inside $\mathbb{S}T^*M$. But the existence of a PS–structure inside $\mathbb{S}T^*M$ is impossible by Theorem 5 because $\mathbb{S}T^*M$ is an exact symplectically fillable manifold, a filling being given by $\mathbb{D}T^*M$. More precisely, a tubular neighborhood of $e_\alpha(M)$ inside $\mathbb{S}T^*M$ is contactomorphic to $\big(M \times \mathbb{D}^2(\delta), \ker(\alpha + r^2 d\theta)\big)$ for some $\delta > 0$. If the $\mathcal{C}^0$–norm of H_θ is smaller than δ^2 then we would obtain a PS–structure inside $\mathbb{S}T^*M$. The square of the maximal size δ of a tubular neighborhood $M \times \mathbb{D}^2(\delta)$ of M inside $\mathbb{S}T^*M$ gives in this case the constant $C(\alpha)$ that appears in the statement of Theorem 1.

In the general case, i.e. when $c_1(\xi)$ does not necessarily vanish, the proof of Theorem 1 follows from the same argument, combined with the following proposition.

Proposition 11. *Every 3–dimensional overtwisted contact manifold can be embedded as a contact submanifold with trivial symplectic normal bundle into an exact symplectically fillable contact 5–manifold.*

The proof of this result will be given in the next section. Assuming it, Theorem 1 is proved as follows. Given an overtwisted contact 3–manifold

(M,ξ), by Proposition 11 it can be embedded with trivial symplectic normal bundle into an exact symplectically fillable contact 5–manifold (X,ξ_X). If there was a sufficiently small contact Hamiltonian H_θ generating a positive loop of contactomorphisms then Proposition 10 would give a PS–structure in X, contradicting Theorem 5.

5. Contact embeddings with trivial normal bundle

In this section we will prove Proposition 11, i.e. that every overtwisted contact 3–manifold (M,ξ) can be embedded with trivial symplectic normal bundle into an exact symplectically fillable contact 5–manifold (X,ξ_X). The idea of the proof is to start with a contact embedding of (M,ξ) into its unit cotangent bundle $\mathbb{S}T^*M$ and then perform contact surgeries in an appropriate way in order to make the symplectic normal bundle trivial while keeping the symplectic fillability of the resulting 5–manifold. As we will see the process will also modify the contact structure on the initial overtwisted 3–manifold M. One of the crucial points of the proof will be to make sure that the modified contact structure on the 3–manifold will still be overtwisted and moreover in the same homotopy class as cooriented 2–plane fields as the initial one. Then it will be isotopic to it according to Eliashberg's classification theorem, confer Theorem 3.

We start by briefly recalling the notion of a Lutz twist, its effect on the homotopy class of the contact structure and its relation to contact surgery and symplectic cobordism. See [Ge] for more details on these notions.

Let K be a positive transverse knot in (M,ξ). A *Lutz twist*[2] along K is an operation that changes (see [Ge, Section 4.3]) the contact structure in a neighborhood of K. The resulting contact structure ξ^K on M is always overtwisted. The effect of a Lutz twist on the homotopy class of the contact structure can be described as follows. Given two 2–plane fields ξ_0 and ξ_1 there are two cohomology classes $d^2(\xi_0,\xi_1)\in H^2(M,\mathbb{Z})$ and $d^3(\xi_0,\xi_1)\in H^3(M,\mathbb{Z})$ that measure the obstruction for ξ_0 and ξ_1 to belong to the same homotopy class of plane fields. We refer to [Ge, Section 4.3] for details and for a proof of the following results.

Proposition 12. *Let $K\subset M$ be a positive transverse knot on ξ. Then $d^2(\xi,\xi^K)=-pd([K])$.*

[2]We consider a Lutz twist what is called a half Lutz twist in other places of the literature.

Proposition 13. *Let $K \subset M$ be a null–homologous positive transverse knot on ξ with self–linking number $sl(K)$. Then $d^2(\xi, \xi^K) = 0$ and $d^3(\xi, \xi^K) = sl(K)$.*

Following Eliashberg [El90] and Weinstein [We91], fix a Legendrian knot L on a contact manifold 3–manifold M and fix the relative (-1)–framing with respect to the canonical contact framing associated to the knot. If we perform on M a handle attachment along L, then the resulting cobordism has a natural symplectic structure. The bottom boundary of this cobordism, i.e. the initial contact manifold, is a concave boundary of the symplectic structure. The upper boundary is convex and therefore it has an induced contact structure, which is said to be obtained from the initial one by contact (-1)–surgery. The inverse operation is called a contact $(+1)$–surgery.

As proved by Ding, Geiges and Stipsicz [DGS05], the effect of a Lutz twist on a contact manifold can be described in terms of contact surgery as follows. Given a Legendrian knot $L \subset (M, \xi)$, denote by $t(L)$ a positive transverse push–off of L and by $\sigma(L)$ a Legendrian push–off of L with two added zig–zags. Then we have the following result.

Proposition 14 ([DGS05]). *Let (M, ξ) be a contact 3–manifold and L a Legendrian knot for ξ. The contact structure obtained by a Lutz twist along $t(L)$ is isotopic to the contact structure resulting from a contact $(+1)$–surgery along L and $\sigma(L)$.*

Being the inverse of a (-1)–surgery, the contact $(+1)$–surgery in a contact 3–fold corresponds to a symplectic 2–handle attachment to the concave boundary of a bounded part of the symplectization, i.e. we obtain a symplectic cobordism in which the new boundary is concave. Consider the transverse knot $K = t(L) \subset (M, \xi^K)$ and the belt spheres $\lambda_K, \lambda_K^\sigma \subset (M, \xi^K)$ corresponding to the contact $(+1)$–surgeries along L and $\sigma(L)$ in (M, ξ) described in Proposition 14. Then λ_K and λ_K^σ are two Legendrian knots in (M, ξ^K). Since Proposition 14 is a local result, both Legendrian knots can be assumed arbitrarily close to K. The following observation will be used in our argument.

Lemma 15. $[K] = [L] = [\lambda_K]$.

Proof. By definition $[K] = [t(L)] = [L]$. The equality $[L] = [\lambda_K]$ follows from the fact that the surgery in Proposition 14 is smoothly trivial. This implies the statement. See Proposition 6.4.5 in [Ge] for further details. □

A consequence of the description in Proposition 14 is the existence of an exact symplectic cobordism realizing a Lutz twist. More precisely we have the following result.

Corollary 16. *Let (M,ξ) be a contact 3–manifold and $K = t(L)$ a positive transverse knot which is a positive transverse push–off of a Legendrian knot L. Then there exists an exact symplectic cobordism (W,ω) from (M,ξ^K) to (M,ξ), which is realized by a 2–handle attachment along the Legendrian link $\lambda_K \cup \lambda_K^\sigma$.*

The convex end of (W,ω) is the contact boundary (M,ξ), the concave end is (M,ξ^K). A Lutz untwist is thus tantamount to an exact symplectic cobordism. It is central to note that the convex end of an exact symplectic cobordism is exact symplectically fillable if the concave end is. This fact will be crucial in our proof of Proposition 11, because it will ensure that the 5–manifold X into which we will embed (M,ξ) will still be fillable. Indeed, as we will see, X will be obtained by constructing an exact symplectic cobordism between contact 5–manifolds with an exact symplectically fillable concave end. This cobordism will restrict to a cobordism between contact 3–manifolds as the one described in Corollary 16.

In our argument we will also use the following result.

Lemma 17. *Let (M,ξ) be an overtwisted contact 3–manifold and Δ a fixed overtwisted disc. Consider a Legendrian link L in M disjoint from Δ. Then there exists a Legendrian link Λ disjoint from $L \cup \Delta$ such that ξ is isotopic to $\xi^{t(L\cup\Lambda)}$.*

Proof. Consider a Legendrian link $\widetilde{L}$ disjoint from $L \cup \Delta$ and with homology class $[\widetilde{L}] = -[L]$. Then Proposition 12 implies that $d^2(\xi, \xi^{t(L\cup\widetilde{L})}) = 0$. Let K be a null–homologous knot contained in a Darboux ball with self–linking number $-d^3(\xi, \xi^{t(L\cup\widetilde{L})})$. Propositions 12 and 13 imply that $\Lambda = \widetilde{L} \cup K$ satisfies $d^2(\xi, \xi^{t(L\cup\Lambda)}) = 0$ and $d^3(\xi, \xi^{t(L\cup\Lambda)}) = 0$. Theorem 3 concludes the statement of the Lemma. □

We are now almost ready to state and prove two results, Propositions 19 and 20, that will be the two main steps in the proof of Proposition 11. Proposition 19 will be an adaptation to higher dimensions of Proposition 14. We first discuss the smooth model for it.

Denote by $M_L(\tau)$ the manifold obtained by surgery along a knot $L \subset M$ with framing τ. In case this is a contact surgery along a Legendrian knot,

the notation stands for a contact (-1)–surgery. The following observation is a strictly differential topological statement.

Lemma 18. *Let X be a smooth 5–manifold and M a codimension–2 submanifold. Consider a knot L in M and a framing τ of L in X. Suppose that τ restricts to a framing τ_s of L in M. Then a surgery on X along L with framing τ induces a surgery on M along L with framing τ_s.*

Proof. The statement can be seen as a consequence of the description of a surgery as a handle attachment. The gradient flow used to glue a 6–dimensional 2–handle $H^6 \cong \mathbb{D}^2 \times \mathbb{D}^4$ along the attaching sphere L in $X \times \{1\} \subset X \times [0,1]$ restricts to a gradient flow in the submanifold $M \times \{1\}$. This describes the attachment of a 4–dimensional 2–handle $H^4 \cong \mathbb{D}^2 \times \mathbb{D}^2$ along L in $M \times \{1\} \subset M \times [0,1]$. Note that the belt 3–sphere in the handle H^6 intersects the surgered submanifold $M_L(\tau_s)$ along the belt 1–sphere of the handle H^4. □

Lemma 18 provides the smooth model for the symplectic cobordism we shall construct to prove Proposition 11. Proposition 14 concerns contact 3–manifolds and a 4–dimensional symplectic cobordism. In view of Lemma 18 we can adapt Proposition 14 to the context of a codimension–2 contact submanifold in a contact 5–manifold. The result is as follows.

Proposition 19. *Let (X, ξ_X) be a contact 5–manifold and (M, ξ) a codimension–2 overtwisted contact submanifold. Consider a transverse knot $K = t(L)$ in M which is the core of a Lutz tube and such that $c_1(\nu_M) = pd([\lambda_K])$. Denote $\lambda = \lambda_K \cup \lambda_K^\sigma$. Then there exists a framing τ of λ in (X, ξ_X) restricting to the Legendrian framing τ_s of λ in (M, ξ) such that $M_\lambda(\tau_s)$ is contactomorphic to M with a Lutz untwist along K, and the symplectic normal bundle of $M_\lambda(\tau_s)$ in $X_\lambda(\tau)$ is trivial.*

Proof. The contact (-1)–surgery that occurs on the contact 3–manifold (M, ξ) is the procedure described in Proposition 14 and Corollary 16. It suffices to explain the choice of framing τ for the link λ in X. The Legendrian framing τ_s for λ in (M, ξ) is extended to a framing τ for λ in X. This extension is obtained as follows.

Consider a section $\mathfrak{s} : M \longrightarrow \nu_M$ transverse to the 0–section and such that

$$\lambda_K = Z(\mathfrak{s}), \quad \text{where } Z(\mathfrak{s}) = \{p \in M : \mathfrak{s}(p) = 0\}.$$

This section exists since $c_1(\nu_M) = pd([\lambda_K])$. It is used to define the extension of the Legendrian framing τ_s to τ for λ_K. Let us discuss this in detail and

the effect of the surgery. It can be considered in two stages. First, surgery along the Legendrian link λ_K. The required framing along λ_K is defined to be $\tau = (\tau_s, \mathfrak{s}_*\tau_s)$. Thus τ is constructed using the differential $\mathfrak{s}_*$ of the section $\mathfrak{s}$. The section $\mathfrak{s}$ cannot be used since it vanishes along λ_K. Consider polar coordinates $(r, w_1, w_2) \in \mathbb{D}^4 \subset \mathbb{C}^2$ with $(w_1, w_2) \in \mathbb{S}^3$. The framing τ provides a diffeomorphism

$$f_\tau : \mathbb{S}^1 \times \mathbb{D}^2 \times \mathbb{D}^2 \longrightarrow \mathcal{U}(\lambda_K) \subset X, \quad (\theta; r, w_1, w_2) \longmapsto f_\tau(\theta; r, w_1, w_2)$$

and we can suppose that $f_\tau(\mathbb{S}^1 \times \mathbb{D}^2 \times \{0\}) = \mathcal{U}(\lambda_K) \cap M$, for a neighborhood $\mathcal{U}(\lambda_K)$ of $\lambda_K \subset X$. The differential $\mathfrak{s}_*$ identifies the pull–back $f_\tau^*(\nu_M)$ of the normal bundle with the trivial bundle $\mathbb{C} \longrightarrow \mathbb{S}^1 \times \mathbb{D}^2 \times \{0\}$ over a neighborhood of $\lambda_K \subset M$. We can also suppose that the section $\mathfrak{s}$ in these local coordinates is $((f_\tau)^*\mathfrak{s})(\theta; r, w_1) = rw_1$. The function rw_1 is well–defined although the coordinate w_1 is not well–defined at $r = 0$.

The surgery substitutes the core $\lambda_K \cong \mathbb{S}^1 \times \{0\} \times \{0\} \subset \mathbb{S}^1 \times \mathbb{D}^2 \times \mathbb{D}^2$ with coordinates $(\theta; r, w_1, w_2)$ by $\{0\} \times \mathbb{S}^3 \subset \mathbb{D}^2 \times \mathbb{S}^3$ with coordinates $(r, \theta; w_1, w_2)$ along the common boundary $\mathbb{S}^1 \times \mathbb{S}^3 = \{(r, \theta; w_1, w_2) : r = 1\}$. The section $((f_\tau)^*\mathfrak{s})(\theta; r, w_1) = rw_1$ can be substituted by a section of the form

$$g : \mathbb{D}^2 \times \mathbb{S}^3 \longrightarrow \mathbb{C}, \quad (r, \theta; w_1, w_2) \longmapsto g(r, \theta; w_1, w_2) = \rho(r)w_1$$

where $\rho : \mathbb{R} \longrightarrow \mathbb{R}^+$ is a positive smooth function. In particular it is non–vanishing and provides a trivialization of the normal bundle of the surgered submanifold $M_{\lambda_K}(\tau_s)$ in the surgered manifold $X_{\lambda_K}(\tau)$.

Second, surgery along the Legendrian link λ_K^σ. The manifold $M_{\lambda_K}(\tau_s)$ has trivial normal bundle in $X_{\lambda_K}(\tau)$. Thus there exists a global framing τ_ν of this normal bundle. Denote the restriction of this global framing τ_ν to the Legendrian knot λ_K^σ by $\tau_\nu|_{\lambda_K^\sigma}$. Then the framing $\{\tau_s, \tau_\nu|_{\lambda_K^\sigma}\}$ is a framing of the normal bundle of λ_K^σ inside $X_{\lambda_K}(\tau)$. Thus, once the surgery along λ_K^σ is performed with the framing $\{\tau_s, \tau|_{\lambda_K^\sigma}\}$, the resulting normal bundle is still trivial. Denote by τ the extended framing as described above for both λ_K and λ_K^σ. Then the normal bundle of $M_\lambda(\tau_s)$ in $X_\lambda(\tau)$ is trivial. □

A minor modification of the argument for Proposition 19 yields the following result.

Proposition 20. *Let (X, ξ_X) be a contact 5–manifold and (M, ξ) a codimension–2 overtwisted contact submanifold with trivial normal bundle. Consider a transverse knot $K = t(L)$ in M which is the core of a Lutz tube*

and denote $\lambda = \lambda_K \cup \lambda_K^\sigma$. *Then there exists a framing* τ *of* λ *in* (X, ξ_X) *restricting to the Legendrian framing* τ_s *of* λ *in* (M, ξ) *such that* $M_\lambda(\tau_s)$ *is contactomorphic to* M *with a Lutz untwist along* K *and the symplectic normal bundle of* $M_\lambda(\tau_s)$ *in* $X_\lambda(\tau)$ *is trivial.*

Proof. In this case there is no need to use $\mathfrak{s}_*$ since the section $\mathfrak{s}$ can be chosen to be non–vanishing. Thus we choose the framing described in the second part of the surgery in Proposition 19. Id est, the framing induced by $\mathfrak{s}$. The surgery along λ with this framing preserves the triviality of the normal bundle. □

We are now ready to prove Proposition 11.

Let (M, ξ) be an overtwisted contact 3–manifold. We want to show that there is a contact embedding with trivial symplectic normal bundle of (M, ξ) into an exact symplectically fillable contact 5–manifold.

Fix an overtwisted disc Δ in (M, ξ) and take a Legendrian link L in (M, ξ) which is disjoint from Δ and such that $pd([L]) = c_1(\xi)$. By Lemma 17 we know that there exists a Legendrian link Λ in (M, ξ) disjoint from L and Δ and such that ξ is isotopic to $\overline{\xi} := \xi^{t(L\cup\Lambda)}$. Consider a contact embedding $(M, \overline{\xi}) \longrightarrow \mathbb{S}T^*M$ defined by some contact form $\overline{\alpha}$ for $\overline{\xi}$. The symplectic normal bundle of this embedding is isomorphic to $\overline{\xi}$ and hence to ξ. Note that L and Λ are still Legendrian in $(M, \overline{\xi})$. Consider the transverse push–offs, with respect to $\overline{\xi}$, $K = t(L)$ and $\kappa = t(\Lambda)$.

First, we apply Proposition 19 to $(M, \overline{\xi})$ inside $(X, \xi_X) := \mathbb{S}T^*M$, and $K = t(L)$. We can apply it because the symplectic normal bundle of $(M, \overline{\xi})$ inside (X, ξ_X) is $\overline{\xi}$ and we know that

$$c_1(\overline{\xi}) = c_1(\xi) = pd([L]) = pd([K]) = pd([\lambda_K]).$$

The last equality holds by Lemma 15. After applying Proposition 19 we get contact structures $\overline{\xi}'$ on M and ξ'_X on X such that $(M, \overline{\xi}')$ embeds into (X, ξ'_X) with trivial symplectic normal bundle, and $\overline{\xi}'$, ξ'_X are obtained from $\overline{\xi}$, ξ_X by performing a Lutz untwist along K.

Second, consider $\kappa = t(\Lambda)$ as a transverse link in $(M, \overline{\xi}')$ and apply Proposition 20 to $(M, \overline{\xi}')$ inside (X, ξ'_X) and $\kappa = t(\Lambda)$. We obtain contact structures $\overline{\xi}''$ on M and ξ''_X on X such that $(M, \overline{\xi}'')$ embeds into (X, ξ''_X) with trivial symplectic normal bundle and $\overline{\xi}''$, ξ''_X are obtained from $\overline{\xi}'$, ξ'_X by performing a Lutz untwist along κ.

Recall that $\overline{\xi}$ was obtained from ξ by performing a Lutz twist along $K \cup \kappa = t(L \cup \Lambda)$. We have thus that $\overline{\xi}''$ and ξ are in the same homotopy class. Since the overtwisted disc has not been affected by the previous operations,

Theorem 3 implies that the two contact structures $\overline{\xi}''$ and ξ are actually isomorphic. We have thus obtained an embedding

$$(M, \xi) \longrightarrow (X, \xi''_X)$$

with trivial symplectic normal bundle. The contact manifold (X, ξ_X) is exact symplectically fillable, it follows from Corollary 16 and the discussion after it that (X, ξ''_X) is also exact symplectically fillable. This finishes the proof of Proposition 11 and hence the proof of Theorem 1 in the general case.

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