On the Discontinuity of ϵ θ -the Dissipation Rate Associated with the Temperature Variance - at the Fluid-Solid Interface for Cases with Conjugate Heat Transfer

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ABSTRACT

Conjugate heat transfer describes the thermal coupling between a fluid and a solid. It is of prime importance in industrial applications where fluctuating thermal stresses are a concern, e.g. in case of a severe emergency cooling (Pressurized Thermal Shock) or long-term ageing of materials (T junctions). For such complex applications, investigations often rely on experiments, high Reynolds RANS (Reynolds-Averaged Navier-Stokes) or wall-modelled LES (Large Eddy Simulation). However, experimental data on conjugate heat transfer are scarce as walls in lab rigs are often made of plexiglas and the transported scalar studied is often a dye. The development of RANS models for conjugate heat transfer is relatively recent, see Craft et al. [1]. In this paper, we establish that the dissipation rate associated with the temperature variance is discontinuous at the fluid-solid interface, in case of conjugate heat transfer. This discontinuity is verified using some DNS (Direct Numerical Simulation) of the turbulent channel flow with conjugate heat transfer. There is currently no RANS model for conjugate heat transfer that takes into account this discontinuity.

1 INTRODUCTION

Conjugate heat transfer simulations are required in industrial applications where fluctuating thermal stresses are a concern. As far as the nuclear industry is concerned, cases like Emergency Core Cooling (Pressurized Thermal Shock [2]) or long-term ageing materials (T junctions [3]) potentially involve thermal coupling between a fluid and a solid.

During the last decades, this thermal coupling has been scrutinized by a number of authors. Following Kirillov [4] : *It is now recognized that the problem of heat transfer between the surface and any medium is a conjugate problem, i.e., the temperature field in the flow and therefore the coefficient of heat transfer also depend on the physical properties of the surface*. Solving such a coupled problem often implies to determine both the fluid and solid temperature, as stated by Khabakhpasheva et al. [5] : *[...] problems of unsteady heat transfer must be considered in combined formulation because of the close connection between the temperature field in a liquid and that in the channel walls*.

However, most of the industrial simulations do not account for this thermal coupling. The main reason being the large discrepancy often found between the characteristic time scales for the temperature evolution in the fluid and solid domain. This discrepancy leads to intractable simulations for the fully coupled problem. As a result, an imposed temperature, heat flux or heat exchange coefficient is often used as a boundary condition for the non-coupled problem. Indeed, such a simplification is not an easy choice, as stated by Kasagi et al. [6] : *it is not straightforward to give a unique thermal boundary condition on the wall surface, which is in contact with a turbulent flow, because of the wall-side unsteady heat conduction associated with the intrinsic unsteadiness of turbulence*. In addition to being a difficult choice, it is an important one: it is often overlooked that those simplified boundary conditions have a strong impact on the near-wall turbulent quantities (Tiselj et al. [7], Flageul et al. [8]).

Accurately estimating those near-wall quantities is of prime importance for some configurations. As stated by Mosyak et al. [9]: *knowledge of the value of wall temperature fluctuations is very important for designing and operation of engineering devices*. As for industrial applications, the wall temperature fluctuations is one of the parameters that control the lifespan of nuclear plants and the schedule of the associated (costly) maintenance operations.

The structure of this paper is as follows. In the next section, the governing equations of the problem will be discussed. The third section is dedicated to the discontinuity of ε_θ, the dissipation rate associated with the temperature variance. In the fourth section, our DNS results are presented. The paper ends with a conclusion and some perspectives.

2 GOVERNING EQUATIONS

The fluid is assumed to be newtonian and the flow is considered incompressible. As the fluid density ρ is assumed constant, in the fluid domain (Ω_f), the continuity equation reads:

$$\partial_i u_i = 0 \tag{1}$$

Assuming the viscosity of the fluid is constant, the momentum equation in the fluid domain (Ω_f) reads:

$$\partial_t u_i = -\frac{\partial_j (u_i u_j) + u_j \partial_j u_i}{2} - \partial_i p + \nu \partial_{jj} u_i + f_i \tag{2}$$

where ν is the kinematic viscosity, the convective term is expressed using the skew-symmetric formulation and f_i is a source term.

In case of conjugate heat transfer, we note T_f (T_s) the fluid (solid) temperature, α_f (α_s) the thermal diffusivity and λ_f (λ_s) the thermal conductivity. Within this framework, the energy equation reads:

$$\begin{aligned}
\partial_t T_f &= -\partial_j (T_f u_j) + \alpha_f \partial_{jj} T_f + f_T \text{ in } \Omega_f \\
\partial_t T_s &= \alpha_s \partial_{jj} T_s \text{ in } \Omega_s \\
T_f &= T_s \text{ on } \partial\Omega_f \cap \partial\Omega_s \\
\lambda_f \partial_n T_f &= \lambda_s \partial_n T_s \text{ on } \partial\Omega_f \cap \partial\Omega_s
\end{aligned} \tag{3}$$

where f_T is a source term, Ω_f (Ω_s) is the fluid (solid) domain, $\partial\Omega_f \cap \partial\Omega_s$ the fluid-solid interface and ∂_n the derivative along the vector normal to the interface. The last 2 lines in (3) express the continuity of the temperature and heat flux across the interface.

According to Tiselj et al. [10], general conjugate heat transfer problems involving one fluid and one solid are described with 2 dimensionless numbers. In this study, we define G as the fluid-to-solid thermal diffusivity ratio and G_2 as the solid-to-fluid thermal conductivity ratio:

$$G = \frac{\alpha_f}{\alpha_s}, G_2 = \frac{\lambda_s}{\lambda_f} \tag{4}$$

Both can be combined to recover the thermal activity ratio K ($\frac{1}{K} = G_2\sqrt{G}$) defined by Geshev [11] and used by Tiselj et al. ([7], [10]).

The spectral compatibility condition from Flageul et al. [8] allows to demonstrate that for simple configurations, from the fluid perspective, the limit $G_2 \ll 1$ (insulating solid) is equivalent to an imposed heat flux at the fluid-solid interface. Similarly, the limit $G_2 \gg 1$ (conducting solid) is equivalent to an imposed temperature. This compatibility condition also shows that the thermal diffusivity ratio G has a complex impact on the temperature field, as it is related to the underlying unsteadiness. It is expected that for the parameter G_2, these limiting behaviours are quite general and not limited to simple configurations.

3 DISCONTINUITY OF ε_θ AT THE FLUID-SOLID INTERFACE.

Unsteady RANS models are the most frequently used in industrial applications. In the following, Reynolds averaged quantities are noted with an overbar ($\overline{T}$) while the associated fluctuating part is noted with a prime (T'). According to equations (3), the dissipation rate ε_θ associated with the temperature variance is:

$$\varepsilon_{\theta,f} = 2\alpha_f \overline{\nabla T'_f . \nabla T'_f} \text{ in } \Omega_f \text{ and } \varepsilon_{\theta,s} = 2\alpha_s \overline{\nabla T'_s . \nabla T'_s} \text{ in } \Omega_s \tag{5}$$

As the temperature is continuous on the fluid-solid interface, so is its gradient parallel to it. This leads to:

$$\frac{\varepsilon_{\theta,f}}{2\alpha_f} - \frac{\varepsilon_{\theta,s}}{2\alpha_s} = \overline{\partial_n T'_f \partial_n T'_f} - \overline{\partial_n T'_s \partial_n T'_s} \text{ on } \partial\Omega_f \cap \partial\Omega_s \tag{6}$$

Using the continuity of heat flux and $G_2 = \frac{\lambda_s}{\lambda_f}$, one gets:

$$\frac{\varepsilon_{\theta,f}}{2\alpha_f} - \frac{\varepsilon_{\theta,s}}{2\alpha_s} = \overline{\partial_n T'_f \partial_n T'_f} \left(1 - \frac{1}{G_2^2}\right) \text{ on } \partial\Omega_f \cap \partial\Omega_s \tag{7}$$

Using the definition of $\varepsilon_{\theta,f}$ from equation (5), one gets:

$$1 - \frac{\varepsilon_{\theta,s}}{2\alpha_s} \frac{2\alpha_f}{\varepsilon_{\theta,f}} = \frac{\overline{\partial_n T'_f \partial_n T'_f}}{\overline{\nabla T'_f . \nabla T'_f}} \left(1 - \frac{1}{G_2^2}\right) \text{ on } \partial\Omega_f \cap \partial\Omega_s \tag{8}$$

Using $G = \frac{\alpha_f}{\alpha_s}$ and $\frac{1}{K} = G_2\sqrt{G}$, one can conclude with:

$$\frac{\varepsilon_{\theta,s}}{\varepsilon_{\theta,f}} = \frac{\overline{\partial_n T'_f \partial_n T'_f}}{\overline{\nabla T'_f . \nabla T'_f}} K^2 + \left(1 - \frac{\overline{\partial_n T'_f \partial_n T'_f}}{\overline{\nabla T'_f . \nabla T'_f}}\right) \frac{1}{G} \text{ on } \partial\Omega_f \cap \partial\Omega_s \tag{9}$$

As the ratio $\frac{\overline{\partial_n T'_f \partial_n T'_f}}{\overline{\nabla T'_f . \nabla T'_f}}$ is bounded in $[0, 1]$, equation (9) can be interpreted as a convex combination between K^2 and $\frac{1}{G}$. Therefore, the discontinuity of ε_θ at the fluid-solid interface is bounded by K^2 and $\frac{1}{G}$. If the case is close to an imposed temperature one (conducting solid), the discontinuity scales with K^2. Oppositely, when the case is close to an imposed heat flux (insulating solid), the discontinuity scales with $\frac{1}{G}$. For all the intermediary cases, equation (9) shows that forecasting the discontinuity implies to forecast the relative contribution of the wall-normal part in the temperature gradient amplitude. This relative contribution is directly connected with the anisotropy of the fluctuating temperature gradient, a quantity that is not accessible for most of the (U)RANS turbulence models. Therefore, equation (9) is a challenge for existing (U)RANS models (Craft et al. [1]), and a call for new ones able to correctly handle cases with conjugate heat transfer.

It is important to stress that very few hypothesis are used when going from equations (3) and (5) to equation (9). The only mandatory hypothesis is that the vector normal to the fluid-solid interface and the gradient along it are well defined. This means that equation (9) holds on any smooth surface (flat or curved).

4 DNS RESULTS : TURBULENT CHANNEL FLOW.

Our DNS of the turbulent channel flow are performed with the open-source code Incompact3d. The code has High Performance Computing capabilities, is available at www.incompact3d.com and is developed at Imperial College London and University of Poitiers. High order finite difference compact schemes are used, combined with a direct spectral pressure solver, see Laizet et Lamballais [12] and Laizet et Li [13].

The main simulation parameters are recalled in Table 1 and compared with Tiselj et al. [7]. As described in Flageul et al. [8], the scalar diffusion scheme used is 4^{th} order accurate in the streamwise direction and 6^{th} order accurate in the others. The case and simulation setup are not fully described here as they are similar to the ones detailed in [8], except for the thermal properties ratio G and G_2. In the present study, the DNS performed with conjugate heat transfer are labelled CHT_{ij}. As indicated in Table 2, the index i and j stand for the ratio of thermal diffusivity and conductivity, respectively. The indexes can be equal to 0, 1 or 2, the corresponding thermal properties ratios being 0.5, 1 and 2, respectively. The results obtained with conjugate heat transfer are compared with the non-conjugate cases of locally imposed temperature ($isoT$) and locally imposed heat flux ($isoQ$) at the fluid boundary.

Table 1: Simulation parameters.

	Present	Tiselj et al. [7]
Prandtl	0.71	
Domain	[25.6, 2, 8.52]	[5π, 2, π]
Grid	[256, 193, 256]	[128, 97, 65]
Re_τ	149	150
Δy^+	[0.49, 4.8]	[0.08, 4.9]
$[\Delta_x^+, \Delta_z^+]$	[14.8, 5.1]	[18.4, 7.4]
Δt^+	0.02	0.12
Duration	29000	6000

Table 2: Case labels depending on the thermal properties ratios.

		G_2		
		0.5	1	2
G	0.5	CHT_{00}	CHT_{01}	CHT_{02}
	1	CHT_{10}	CHT_{11}	CHT_{12}
	2	CHT_{20}	CHT_{21}	CHT_{22}

In Figure 1, the near-wall values of the temperature variance $\overline{T'^2}$ and the associated dissipation rate ε_θ are visibly impacted by the thermal properties ratios G and G_2. As expected from the spectral compatibility condition in [8], in the fluid domain, the lower G_2, the closer to the imposed heat flux case ($isoQ$). For the cases with $G_2 = 0.5$ (labels CHT_{*0}, dotted lines), the near-wall dissipation rate in the fluid domain ($\varepsilon_{\theta,f}$) is very close to the imposed heat flux one ($isoQ$). Following that, one may say those conjugate cases are close to the imposed heat flux one. However, looking at the relative contribution of the wall-normal part in the temperature gradient, it is well above 0.5 for those cases, making the relative wall-normal contribution dominant and those cases closer to the imposed temperature one ($isoT$). This contradiction highlights the complexity of conjugate heat transfer and the difficulty one faces when trying to model it with a non-coupled approach and a steady boundary condition.

In Figure 2, the near-wall turbulent heat fluxes are also impacted by the thermal properties ratio G and G_2. As the turbulent heat fluxes vanish at the wall, this impact is not very visible, even using logarithmic axis. This impact is much more visible on the one-point correlation coefficient associated with those turbulent heat fluxes. The main trend for G_2 is confirmed by figure 2: the lower G_2, the closer to the imposed heat flux case. However, it is remarkable that for all the conjugate cases studied here, at the wall, the one-point correlation coefficients associated with the turbulent heat fluxes collapse towards the imposed heat flux case. According to Orlandi et al. [14], this seems to hold for ratios of thermal diffusivity G further away from unity.

5 CONCLUSION & PERSPECTIVES

In the third section, we have demonstrated that the dissipation rate associated with the temperature variance is discontinuous at the fluid-solid interface in case of conjugate heat-transfer for any smooth interface. We have also demonstrated that this discontinuity is directly related to the anisotropy of the fluctuating temperature gradient, according to equation (9). In the fourth section, we have presented some DNS results for the turbulent channel flow configuration with conjugate heat transfer that confirm the existence of such a discontinuity. In addition, our simulations have produced statistics in agreement with equation (9), the relative error being around 1 per 1000. We believe that the present analysis represents a step forward towards a better understanding and modelling of fluid-solid heat transfer.

Indeed, the present numerical results are limited to the turbulent channel flow configuration at $Re_\tau = 150$ and $Pr = 0.71$, with ratio of thermal properties G and G_2 close to unity. It would be interesting to obtain scalings for the discontinuity: how does it depends on the Reynolds number, on the Prandtl number and on the thermal properties ratios when they are further away from unity. We are currently studying the ability for wall-resolved LES to estimate that discontinuity. If such an estimation were to succeed, that would be a precious tool for anyone building a (U)RANS model or a LES wall-model adapted to conjugate heat transfer.

To the authors knowledge, there is currently no (U)RANS model that takes into account this discontinuity of the dissipation rate ε_θ at the fluid-solid interface. Given how important it is for some industrial applications to have an accurate prediction of the temperature fluctuations at the wall, we believe that this work will provide a solid ground for new models adapted to conjugate heat transfer.

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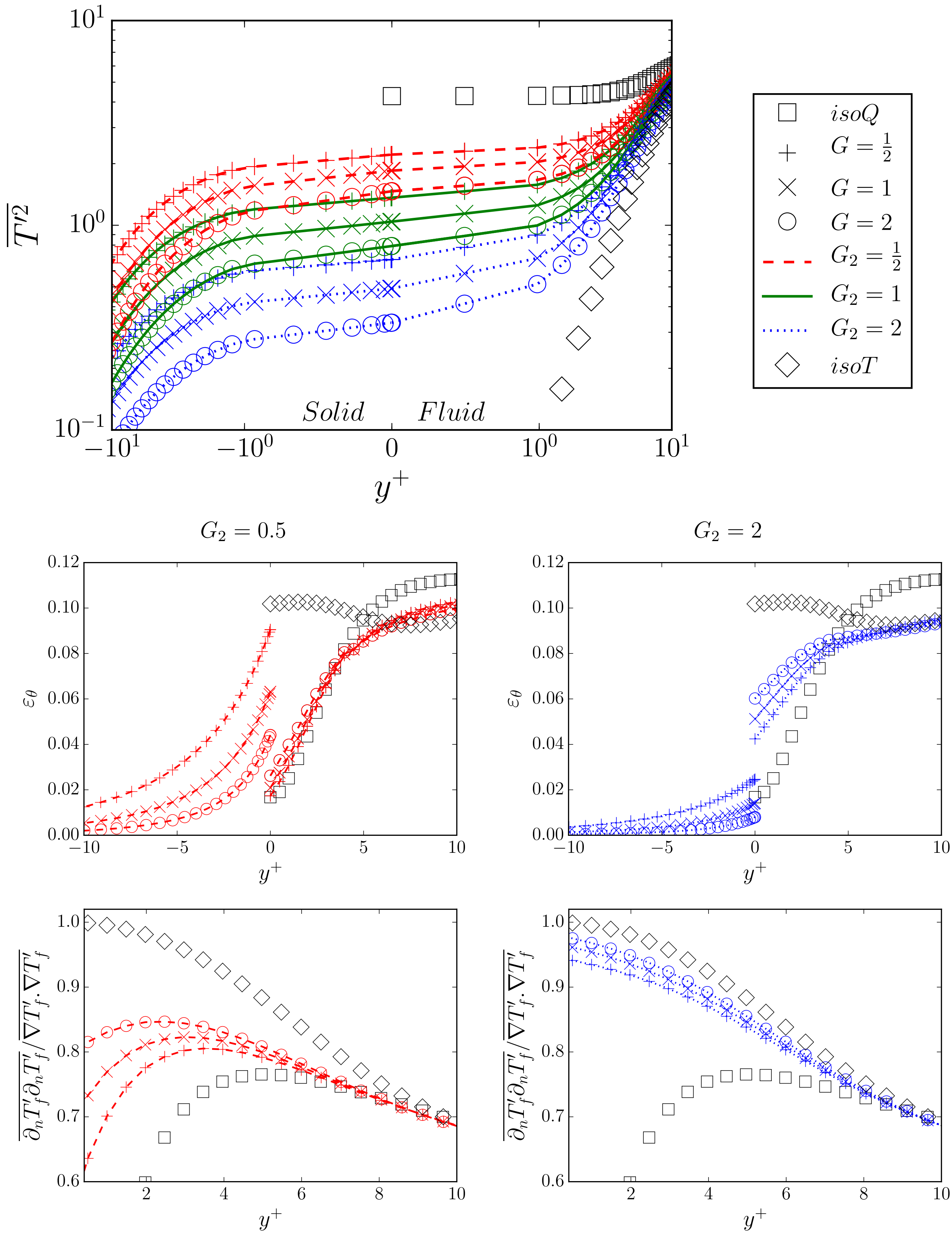

Figure 1: Top: temperature variance $\overline{T'^2}$. Middle: dissipation rate ε_θ. Bottom: relative contribution of the wall-normal part in the temperature gradient $\frac{\overline{\partial_n T'_f \partial_n T'_f}}{\overline{\nabla T'_f \cdot \nabla T'_f}}$. Left: CHT_{*0}. Right: CHT_{*2}.

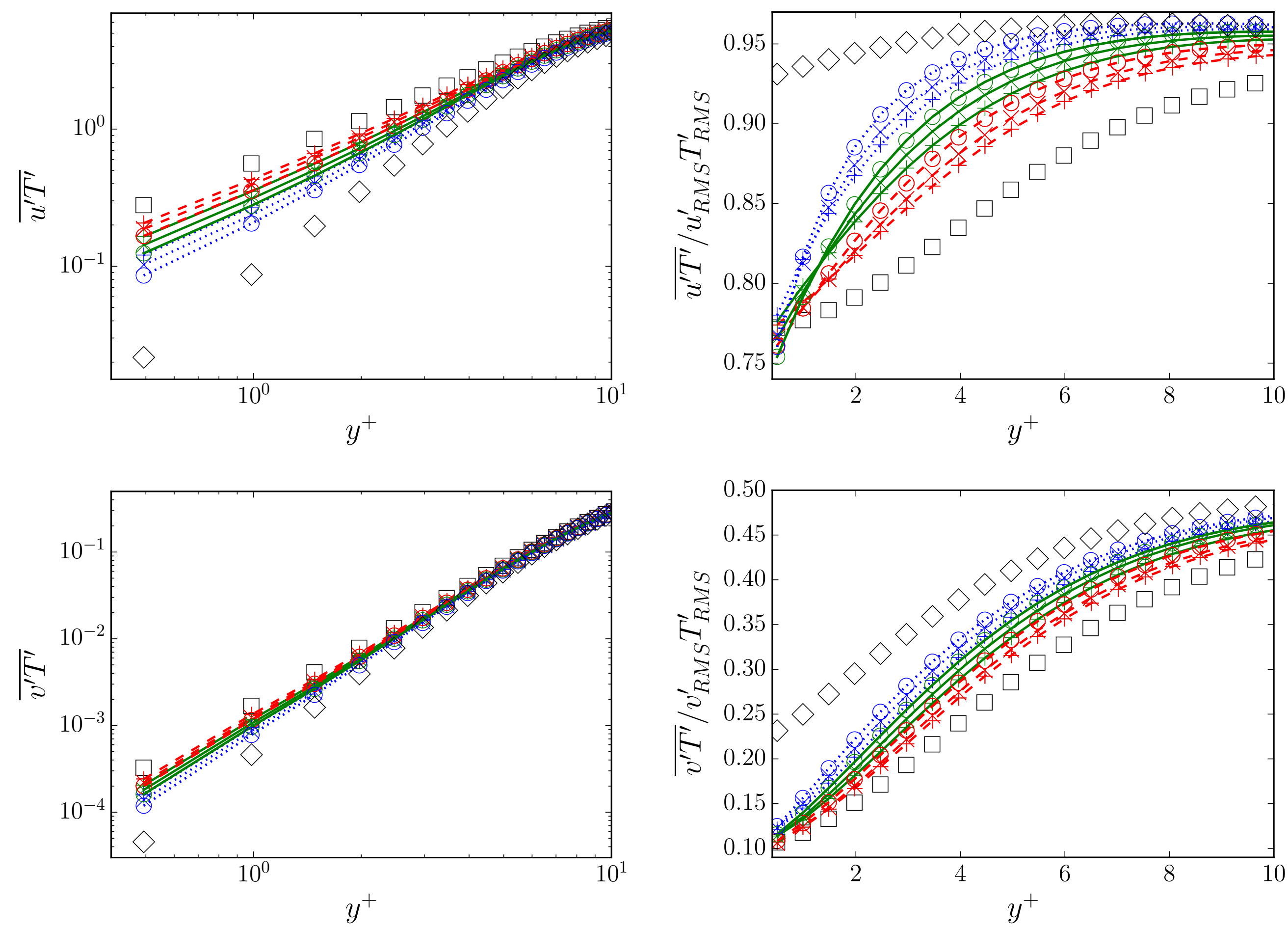

Figure 2: Top: streamwise heat flux. Bottom: wall-normal heat flux. Left: turbulent heat flux. Right: one-point correlation coefficient associated with the turbulent heat flux. Lines and symbols as in figure 1.

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