Evaluation of an ensemble based 4D Var assimilation
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Aims and model

- Objective:
  - Compare En4DVar with a classic 4DVar method

Incremental 4DVar vs En4DVar assimilation techniques

- Incremental 4DVar assimilation
  - Cost function using static covariance matrices $B$ and $R$
    \[ J(\delta X_0) = \frac{1}{2} \sum_{i=1}^{N} ||\delta X_0||_2^2 + \frac{1}{2} \sum_{i=1}^{N} ||E(\delta^3 X^0) + \partial_Z E(\delta^2 X_0) - Y||_R^2, \]
  - Adjoint equation determined by TAPENADE
    \[ -\partial_t \lambda + (\partial_t X_0^2) \lambda = \sum_{i=1}^{N} (\partial_t X_0^2)^R (Y - E(X^0))/||Y - E(X^0)||_R, \]
  - We deduce the gradient
    \[ \nabla J(\delta X_0) = \lambda(\delta b) \]

- En4DVar assimilation
  - Cost function using flow dependent background error covariance matrix within the context of the preconditioning techniques $\delta X_0 = B^{1/2} Z_0$.
    \[ J(\delta Z_0) = \frac{1}{2} \sum_{i=1}^{N} ||\delta Z_0||_2^2 + \frac{1}{2} \sum_{i=1}^{N} ||E(\delta^3 X^0) + \partial_Z E(\delta^2 X_0) - Y||_R^2, \]
  - where the $B^{1/2}$ matrix estimated from the difference between each ensemble member and ensemble mean
    \[ B^{1/2} = \frac{1}{\sqrt{N-1}}(X_0^1 - \bar{X}_0, ..., X_0^N - \bar{X}_0) \]
  - We estimate the evolution of the $B^{1/2}$ matrix from the evolution of the ensemble fields in observation space
    \[ \partial_t E(\delta^2 X_0) B^{1/2} = \frac{1}{\sqrt{N-1}}(H X_0^1 - H \bar{X}_0, ..., H X_0^N - H \bar{X}_0) \]
  - Localization technique used to eliminate sampling error in state space
    \[ P_0^1 = (C_1^1, B_1^1, ..., C_N^1, B_N^1) \]

Minimization performed with LBFGS algorithm: limited memory quasi Newton method

Results

- Synthetic data
  - Background initial condition
  - Exact initial condition
    \[ L = 25cm \]
    \[ W = 10cm \]
    \[ H_0 = 2cm \]
    \[ H_f = 2cm \]
    \[ H_s = 2cm \]
    \[ H_L = 7cm \]
    \[ U(x, y, t_0) = \delta(x, y) \]
    \[ U(L, t_0) = 0 \]
    \[ U(x, y, t_0) = 0 \]
    \[ V(x, y, t_0) = 0 \]

- We only possess the height observations given by the depth sensor (Kinect sensor)

Conclusions

- Sensibly the same computational time cost, En4DVar yields better results than the classic 4DVar assimilation when we have only height observations
- En4DVar is easy to implement for any given model. We gain a lot of time with the parallelization computing technique. En4DVar implemented with only one outer loop iteration and needs about 100 iterations for the optimization. Requires a lot of memory.
- 4DVar requires the tangent and adjoint operators. The assimilation converges with 3 outer loop iterations and requires less inner loop iterations for the optimization.

Figure: From left to right: free surface height obtained by the kinect, 4DVar and En4DVar at t=0.06s

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