Evaluation of an ensemble based 4D Var assimilation

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Aims and model

- Objective:
  - Compare En4DVar with a classic 4DVar method

- Incremental 4DVar vs En4DVar assimilation techniques

  - Incremental 4DVar assimilation
    - Cost function using static covariance matrices $\mathbb{B}$ and $\mathbb{R}$:
      \[
      J(\delta x_0) = \frac{1}{2} \sum_{i=0}^{N} \sum_{m=1}^{M} (\delta x_{0m})^2 + \frac{1}{2} \sum_{i=0}^{N} \sum_{m=1}^{M} (\mathbb{H}(\mathbb{X}^m) + \partial_x \mathbb{H}\partial_y(\delta x_0, \mathbb{X}^m)) - \mathbb{Y}_{|\mathbb{R}}.
      \]
    - Adjoint equation determined by TAPENADE:
      \[
      -\partial_x \lambda + \partial_x \mathbb{H}^\dagger = \sum_{i=1}^{N} (\partial_x \mathbb{H})^\dagger (\mathbb{Y} - \mathbb{H}(\mathbb{X}^m))
      \]
      $\lambda(t) = 0$
    - We deduce the gradient
      \[
      \nabla J(\delta x_0) = \lambda(b)
      \]

  - En4DVar assimilation
    - Cost function using flow dependent background error covariance matrix within the context of the preconditioning techniques $\delta X_0 = \mathbb{B}^{1/2}Z_0$:
      \[
      J(\delta Z_0) = \frac{1}{2} \sum_{i=0}^{N} \sum_{m=1}^{M} (\delta Z_{0m})^2 + \frac{1}{2} \sum_{i=0}^{N} \sum_{m=1}^{M} (\mathbb{H}(\mathbb{X}^m) + \partial_x \mathbb{H}\partial_y(\delta z_{0}, \mathbb{X}^m)) - \mathbb{Y}_{|\mathbb{R}},
      \]
    - Where the $\mathbb{B}^{1/2}$ matrix is estimated from the difference between each ensemble member and ensemble mean
      \[
      \mathbb{B}^{1/2} = \frac{1}{\sqrt{N-1}} (\mathbb{X}^0 - \bar{X}^0, ..., \mathbb{X}^N - \bar{X}^0)
      \]
    - We estimate the evolution of the $\mathbb{B}^{1/2}$ matrix from the evolution of the ensemble fields in observation space
      \[
      \partial_x \partial_y \mathbb{B}^{1/2} = \frac{1}{\sqrt{N-1}} \mathbb{H}(\mathbb{X}^0 - \bar{X}^0, ..., \mathbb{X}^N - \bar{X}^0)
      \]
    - Localization technique used to eliminate sampling error in state space
      \[
      (\mathbb{P}^0_{\mathbb{B}} = ([\mathbb{C}^{1/2} : \mathbb{B}^{1/2}, ..., \mathbb{C}^{1/2} : \mathbb{B}^{1/2}])
      \]

- Minimization performed with LBFGS algorithm: limited memory quasi Newton method

Results

- Synthetic data

  - Background initial state (left) and True initial state (right)

    - Background initial condition
      - $L = 25$ cm
      - $W = 10$ cm
      - $H_0 = 2$ cm
      - $\Delta H = 0.25$ cm
      - $\bar{U}(x, y, z_0) = \bar{c}_2(x, y)$
      - $\bar{U}(x, L, z_0) = 0$
      - $\bar{V}(x, y, z_0) = 0$
      - $\bar{V}(W, y, z_0) = \bar{c}_2(x, y)$
    - Vertical velocity RMS

- Experimental data

  - We only possess the height observations given by the depth sensor (Kinect sensor)

Conclusions

- Sensibly the same computational time cost, En4DVar yields better results than the classic 4DVar assimilation when we have only height observations
- En4DVar is easy to implement for any given model. We gain a lot of time with the parallelization computing technique. En4DVar implemented with only one outer loop iteration and needs about 100 iterations for the optimization. Requires a lot of memory.
- 4DVar requires the tangent and adjoint operators. The assimilation converges with 3 outer loop iterations and requires less inner loop iterations for the optimization.

Assimilation with height observations only
- En4DVar is slightly better than 4DVar
- Same computation time
- En4DVar requires more memory space
- Assimilation with height and velocity observations
- En4DVar requires much more computation time
- En4DVar leads to better results with a higher truncation mode
- Higher truncation mode demands higher computation time and memory needs