When Hotelling meets Vickrey

Service timing and spatial asymmetry in the airline industry*

André de Palma,[†] Carlos Ordás Criado[‡] L. M. Randrianarisoa[§]

Abstract

This paper analyzes rivalry between transport facilities in a model that includes two sources of horizontal differentiation: geographical space and departure time. We explore how both sources influence facility fees and the price of the service offered by downstream carriers. Travellers' costs include a fare, a transportation cost to the facility and a schedule delay cost, which captures the monetary cost of departing earlier or later than desired. One carrier operates at each facility and schedules a single departure time. The interactions in the facility-carrier model are represented as a sequential three-stage game in fees, times and fares with simultaneous choices at each stage. We find that duopolistic competition leads to an identical departure time across carriers when their operational cost does not vary with the time of day, but generally leads to distinct service times when this cost is time dependent. When a facility possesses a location advantage, it can set a higher fee and its downstream carrier can charge a higher fare. Departure time differentiation allows the facilities and their carrier to compete along an additional differentiation dimension that can reduce or strengthen the advantage in location. By incorporating the downstream carriers into the analysis, we also find that a higher per passenger commercial revenue at *one* facility induces a lower fee charged by *both* facilities to their carrier and a lower fare charged by *both* carriers at their departure facility, while a lower marginal operational cost for *one* carrier implies a higher fee at its departure facility, a lower fee at the other facility served by the rival carrier and a lower fare at *both* facilities.

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[†]École Normale Supérieure de Cachan, Paris, France.

[‡]Corresponding author: carlos.ordas@ecn.ulaval.ca, Department of Economics, Université Laval, Québec, QC, Canada.

[§]Sauder School of Business, University of British Columbia, Vancouver, BC, Canada.

1 Introduction

Since airline deregulation in the US in the mid '70s and in Europe in the '90s, rivalry between airports and their carriers has intensified. Travellers flying from large metropolitan areas often have the opportunity to choose between multiple airports and airlines (in particular, between low-cost and legacy carriers). Key determinants of travellers' choice are the location of the departure airport, airline schedule and fares. In spatial oligopolies, such as multi-airport regions, flight providers can exploit the greater convenience of their location to increase prices and differentiate their service to gain travellers and raise profits. Flight schedules is one service-quality/**differentiation** dimension that is of paramount concern to passengers. Despite the existence of a large economic literature examining price and frequency rivalry in aviation, departure time competition has received much less attention. The purpose of this paper is to fill this gap by analyzing a model of schedule competition in which the price charged by each operator depends on their location and scheduling decisions.

While airline deregulation has lowered fares, opened new markets and stimulated air traffic, its impact on airline scheduling remains uncertain. According to empirical studies focusing on airline competition, deregulation greatly reduces schedule variety, as the titles of the papers “Why do all the flights leave at 8:00 am?” (Borenstein and Netz, 1999) or “Hotelling in the air ?” (Salvanes et al., 2005) suggest. Their findings tend to confirm that Hotelling’s principle of minimum differentiation applies in commercial aviation, i.e., that airlines tend to strategically schedule flights head to head (which is unlikely to be socially efficient). What these papers do not do, however, is specifically examine scheduling in multi-airport regions, even though multi-airport systems are destined to play a major role in the future development of air traffic worldwide.¹ Nor do they explore/**investigate** departure time differentiation between legacy and low-cost carriers.

When examining flight departure times in competitive multi-airport markets, we find both schedule clustering and schedule differentiation for different types of the

¹Bonnefoy et al. (2010) argue that the development of multi-airport systems is key to meet future air travel demand. Based on 2005 data, they identify 59 multi-airport systems worldwide where at least one primary airport (receiving more than 20% of the total passenger traffic) and one secondary airport (receiving between 1% and 20% of that total) serve commercial air traffic within a metropolitan region. They find that the most frequent system involves one primary and one secondary airport.

airports' ownership form. One example is the popular Boston - New York route out of Boston Logan International (BOS), a major US public airport. American Airlines and Delta schedule most of their daily flights at identical times between 6.00 a.m. and 8.00 p.m to JFK (John F. Kennedy Intl.) or LGA (La Guardia Intl.) while the low-cost carrier serving the same route (Jet Blue) departs at different times to JFK, LGA and EWR (Newark Intl.). In the German two-airport system composed of Frankfurt Airport (FRA) and the secondary airport Frankfurt-Hahn (HHN),² as of August 08, 2017, 14 out of the 25 routes offered at HHN by low-cost carriers were also offered at the main airport (FRA) by legacy carriers. Indeed, 93% of these (nonstop, one-way) flights depart at different times to the same destination. This underscores the need to provide a theoretical framework to explore the sources of these different outcomes.

This paper explores the mechanisms by which schedule differentiation occurs in a spatial setting where carriers compete on fares and schedules across airports, by highlighting the specific effects of travellers' schedule delays,³ *and* firms' time-related costs (i.e., the operational scheduling costs incurred by carriers along the day) on the market equilibrium. We draw on Hotelling (1929), the workhorse of location theory and industrial organisation, and Vickrey (1969), who first modelled consumer scheduling decisions, to develop a model that incorporates a fee-then-time-then-fare game in a two-airport system characterized by an asymmetric location of the airports.

De Palma and Leruth (1989) and De Borger and Van Dender (2006) consider sequential capacity-price games in spatial settings to investigate the rivalry between congestible facilities that can adjust their capacities. While these authors consider the facilities as final service providers, Basso and Zhang (2007) analyze rivalry in capacity and price between congestible facilities in a "vertical structure". They consider the facilities as input providers (upstream firms) that reach final consumers only through carriers (downstream firms). Brueckner (2002, 2009) and Pels and Verhoef (2004) extend the theory of congestion pricing developed for road traffic to congested airports when the final service providers – the carriers who want to depart/land at peak hours – have market power. Verhoef (2010) looks into alternative instruments (such as slots

²Frankfurt Airport (FRA)'s shareholder structure is mixed with public majority. The airport is listed on the German Stock Exchange and distributes dividends. A Chinese private conglomerate, HNA airport group co. ltd., is the majority shareholder of Frankfurt-Hahn (HHN) since August 2017.

³Schedule delay refers to the difference between a desired time of arrival or departure and the actual time.

sales and slots trading). These studies highlight that understanding airlines' scheduling decisions requires taking into account their time-related operational costs.

In a seminal paper, Panzar (1979) proposes a spatial model in which two profit-maximizing airlines each operate a single flight and consumers' generalized costs depend on fares, flight frequency and convex schedule delay costs (see also Douglas and Miller, 1974). Their analysis focuses on the fare-frequency equilibrium without explicitly addressing departure time competition. Encaoua et al. (1996) and Lindsey and Tomaszewska (1999) are the first to consider a time-then-fare game with quadratic schedule delay costs and uniformly-distributed, preferred departure times for the travellers in an airline competition model. However, these models do not consider the geographical space and the effects of airlines' scheduling on airports. With the same limits but closer to our setup, Van der Weijde et al. (2014) investigate several time-fare games for duopolistic travel operators by using a horizontal differentiation model with price-sensitive demands, uniform preferred times and asymmetric (piecewise linear) schedule delay costs. Departure times are treated as locations on a time interval and each operator schedules a single departure time. They show that the time-then-fare game leads to flights scheduled closer than socially optimal but not necessarily in minimum differentiation.⁴ Note that a large number of empirical work confirms that passengers' value of time significantly affects air travel demand, in addition to the cost/time to access the departure facility and to other characteristics of the trip.⁵

Our work is in the vein of the spatial approach. It borrows the “vertical structure” proposed by Basso and Zhang (2007) in a spatial setting but drops the congestion components to include schedule decisions in the spirit of Van der Weijde et al. (2014). In contrast to the existing literature, we *(i)* account for the existence of a spatial asymmetry in the location of one of the facilities, *(ii)* allow the time costs to differ across downstream firms and to impact both the fare and schedule decisions, *(iii)* remain agnostic about the shape of the distribution of travellers' desired departure times, *(iv)* consider the effect of commercial revenues on facilities' pricing decisions.

⁴Avoiding the spatial approach, Brueckner and Flores-Fillol (2007) and Brueckner (2010) study scheduling competition through frequency competition between suppliers in fare-frequency games. They consider that individuals care more about overall flight frequency rather than individual departure times. In particular, the 2007 model establishes that duopolistic competition does not lead to a sufficient choice set of flights from a travellers' perspective.

⁵See, among others, Pels et al. (2000, 2003); Adler et al. (2005); Brey and Walker (2011).

To keep the analysis tractable, we assume that only one carrier serves each facility at a single time. The interactions in this vertical facility-carrier framework are represented by a three-stage, subgame-perfect Nash game which is solved backwards.

The paper is organized as follows. The market equilibrium is determined in Section 2. Section 2.1 characterizes consumers' demand for the final service. Section 2.2 examines the carrier-rivalry subgame assuming either exogenous or endogenous service times and establishes carriers' equilibrium fare, departure time, demand and profit. Section 2.3 focuses on the facility-rivalry subgame and characterizes facilities' equilibrium fee, demand and profit. Section 3 proposes a welfare analysis of the market outcome by evaluating separately the access costs faced by the travellers for given facility locations and the time costs incurred by the travellers and carriers along the time space. The last two sections provide numerical results and a summary of our main findings along with possible extensions.

2 The Model

In a linear city of unit length, potential consumers are uniformly distributed with mass one. Two facilities ($i = 0, 1$) serve the city and a single carrier at each facility schedules a homogeneous service at time T_i during the operating hours of its departure facility. The opening and closing times of the facilities, denoted $\underline{T}, \bar{T} \in]0, 24[$, are exogenously given and such that $\underline{T} < \bar{T}$. The locations of two facilities are given, with facility 0 located at point h , with $h \in [0, \bar{h}]$, $\bar{h} < 1$, and facility 1 located at the end of the city at 1. In this setting, facility 0 and its carrier have a location advantage in the sense that, for equal prices, facility 0 faces a higher demand than its competitor. In what follows, we will mainly think of the service as being a trip or flight, carriers as airlines, facilities as airports and consumers as travellers.

2.1 Consumer choice

We assume that consumers select one airport and a flight on the basis of fare $\hat{p}_i$, transportation costs and schedule delay costs that capture the monetary value of the inconvenience caused by departing earlier or later than desired. Consumers' desired departure times, denoted by t , are heterogeneous and distributed according to a strictly

positive density $\rho(t)$ on the $[0, 24]$ time interval (referred to as the time line below). We denote by $F(t)$ the related cumulative distribution function (or CDF). Departure times are given to consumers and can differ across carriers. The total cost or "full fare" of the service for a potential consumer located at $x \in [0, 1]$, selecting facility i and with desired departure time t , is given by $\hat{p}_i + \hat{C}(T_i, t) + \frac{\theta}{2}d_i^2(x)$, where $\hat{p}_i \geq 0$ is the fare at facility i and $\theta \in [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}_+$ is the transportation cost per unit of *squared* distance, denoted $d_i^2(x)$, between consumer's location at x and facility i 's location where $d_0^2 = (x - h)^2$ and $d_1^2 = (1 - x)^2$. From the consumers' perspective, a quadratic transportation cost is justified when the marginal disutility to access the transport facility increases with distance.⁶ Term $\hat{C}(T_i, t) \equiv \hat{C}_i$ captures the schedule delay cost (in monetary units) incurred by a traveller for consuming the service offered at time T_i by the carrier operating at facility i . Following Van der Weijde et al. (2014) and for tractability reasons, we assume that consumers' schedule delay cost function is piecewise linear in t :

$$\hat{C}(T_i, t) = \hat{\beta}(t - T_i)\mathbf{1}_{t \geq T_i} + \hat{\gamma}(T_i - t)\mathbf{1}_{t < T_i}, \quad (1)$$

where $\hat{\beta} \in [\underline{\beta}, \bar{\beta}] \subset \mathbb{R}_+$ denotes the unit cost of departing earlier than desired, $\hat{\gamma} \in [\underline{\gamma}, \bar{\gamma}] \subset \mathbb{R}_+$ is its late counterpart and function $\mathbf{1}_A$ is an indicator function that equals 1 if condition A holds and 0 otherwise. A travel service (toward the same destination) is not necessarily scheduled at the same time of day across facilities. We treat hereafter the case $T_0 \leq T_1$.⁷ This schedule configuration allows to classify the travellers according to their departure time preferences into three categories: those with $t \leq T_0$ who prefer to depart earlier than the earliest service offered in the city, those with $t \in]T_0, T_1[$ who may incur early or late schedule delay depending on the chosen facility, and those with $t \geq T_1$ who prefer to depart later than the latest service offered.

If the service is consumed, the net benefit of travelling for a consumer with desired

⁶ In the Hotelling framework, quadratic transportation costs ensure continuous demands and concave profits in both prices at any pair of firm locations along the unit line, see D'Aspremont et al. (1979). This is not the case when consumers' transportation cost is linear. Hence, our setup allows consumers located at the left-hand side of facility 0 on the geographical line to consume the homogeneous product at facility 1 if their service cost is low enough.

⁷ Hereafter, we refer the reader to Appendix A for the more involved calculations of the equilibrium when $T_0 \leq T_1$ while the case $T_0 \geq T_1$, equally valid, is treated in Appendix B. The schedule differences arising from the model are discussed in detail in the time game.

time t , located at $x \in [0, 1]$, and departing facility i is given by:

$$\widehat{U}_i = \widehat{U} - \widehat{p}_i - \widehat{C}(T_i, t) - \frac{\theta}{2}d_i^2, \quad i = 0, 1,$$

where $\widehat{U}$ represents the gross benefit of the trip in monetary units and $\widehat{p}_i + \widehat{C}(T_i, t)$ is the ‘service cost’ incurred by the consumer for buying from carrier i , net of her transportation cost to reach the departure facility. We assume that consumer preferences for departure times are independent of the location in the city.⁸ The indifferent consumer $\tilde{x}(t)$ is determined by equalizing $\widehat{U}_0$ with $\widehat{U}_1$, that is:

$$\tilde{x}(t) = \frac{\widehat{p}_1 + \widehat{C}(T_1, t)}{\theta(1-h)} - \frac{\widehat{p}_0 + \widehat{C}(T_0, t)}{\theta(1-h)} + \frac{1+h}{2}. \quad (2)$$

The number of consumers with desired time t going to facility 0 (rather than 1) decreases in the service cost of its carrier ($\widehat{p}_0 + \widehat{C}_0$), increases in the service cost at the rival facility ($\widehat{p}_1 + \widehat{C}_1$) and increases with h if the inter-facility transportation cost is larger than the difference in service costs between facility 0 and 1. Furthermore, a higher transportation cost parameter (θ) induces more consumers at facility 0 if the service cost is lower at facility 1, i.e., $\widehat{p}_1 + \widehat{C}_1 < \widehat{p}_0 + \widehat{C}_0$.

Figure 1: Indifferent consumer along the geographical line

Figure 1 shows the indifferent consumer along with its service cost on the geographical line. We say that the market is covered for any given location of the facilities and

⁸This is consistent with Brey and Walker (2011) who find that party size and time zone change are the most influential variables of departure time preferences in air travel.

for any given departure time of the carriers if everyone consumes and if a strictly positive fraction of consumers depart each facility whatever their desired departure time. For this to occur, it suffices that the travellers located at $x = 0$ (resp. $x = 1$) with desired departure time $t = T_1$ (resp. $t = T_0$) chooses facility 0 (resp. facility 1), or

$$-\frac{\theta}{2}(1-h^2) < \hat{p}_1 - \hat{p}_0 - \hat{\beta}(T_1 - T_0) \leq \hat{p}_1 - \hat{p}_0 + \hat{\gamma}(T_1 - T_0) < \frac{\theta}{2}(1-h)^2. \quad (3)$$

The first inequality states that the service cost at facility 1 must be large enough to prevent all travellers located at $x = 0$ with desired time $t \geq T_1$ choosing facility 1. The same reasoning applies to the last inequality for those with $t \leq T_0$.⁹ Both conditions are maintained hereafter.

Given density $\rho(t)$ of desired departure times, aggregating the individual demands over the geographical and time lines, the market demand at each facility is:

$$\begin{aligned} D_0(\mathbf{p}, \mathbf{T}) &= \int_0^{24} \tilde{x}(t)\rho(t) dt = p_1 - p_0 + \frac{1+h}{2} + \Phi(\mathbf{T}), \\ D_1(\mathbf{p}, \mathbf{T}) &= 1 - D_0(\mathbf{p}, \mathbf{T}), \end{aligned} \quad (4)$$

where $\mathbf{p} \equiv (p_0, p_1)$ with $p_i = \hat{p}_i/\theta(1-h)$, $i = 0, 1$ and $\mathbf{T} \equiv (T_0, T_1)$. To simplify expressions and without loss of generality, we generally divide the fares and the schedule delay cost parameters by $\theta(1-h)$ and ‘drop the hats’. Term $\Phi(\mathbf{T})$, defined as

$$\Phi(\mathbf{T}) = \gamma(T_1 - T_0)m_\ell + (\beta T_0 + \gamma T_1)m_c - \beta(T_1 - T_0)m_r - (\beta + \gamma)\bar{t}_c \quad (5)$$

with $\beta = \hat{\beta}/\theta(1-h)$ and $\gamma = \hat{\gamma}/\theta(1-h)$, captures the (normalized) difference in schedule delay costs (SDC hereafter) at the market level and aggregates the individual SDC differences between facility 1 and 0 through the shares $m_\ell = \int_0^{T_0} \rho(t)dt$, $m_c = \int_{T_0}^{T_1} \rho(t)dt$ and $m_r = \int_{T_1}^{24} \rho(t)dt$, and through an average desired time $\bar{t}_c = \int_{T_0}^{T_1} t\rho(t)dt$.¹⁰ Clearly, a positive (negative) term $\Phi(\mathbf{T})$ gives facility 0 (facility 1) and its carrier a SDC advantage that renders facility 0 (facility 1) more attractive to travellers. When $T_0 = T_1 = T$, the SDC advantage is null, i.e., $\Phi(T, T) = 0$. As the SDC term will play a central role, we establish for later use the following properties.

⁹See Appendix A.2 for the detailed derivation of the covered market condition.

¹⁰As the uniform distribution is often used in Hotelling models as a tractable benchmark, an analytical expression for $\Phi(\mathbf{T})$ is given in (A.7) for $t \sim \mathcal{U}[0, 24]$.

Lemma 1. Consider a travel service scheduled at time T_0 (resp. T_1) at facility 0 (resp. facility 1) with $T_0 \leq T_1$. Let $F(t)$ be the CDF of consumers' desired departure time with $t \in [0, 24]$. Then:

$$\begin{aligned}\Phi_{T_0} &= \beta - (\beta + \gamma)m_\ell, & \Phi_{T_1} &= \gamma - (\beta + \gamma)m_r, \\ \Phi_{T_0, T_0} &= -(\beta + \gamma)\rho(T_0) < 0, & \Phi_{T_1, T_1} &= (\beta + \gamma)\rho(T_1) > 0,\end{aligned}$$

where $\Phi_{T_i} \equiv \partial\Phi(\mathbf{T})/\partial T_i$ and $\Phi_{T_i, T_i} \equiv \partial^2\Phi(\mathbf{T})/\partial T_i^2$. Setting Φ_{T_i} larger than zero yields

$$\Phi_{T_0} > 0 \quad \text{iff} \quad T_0 < F^{-1}\left(\frac{\beta}{\beta + \gamma}\right), \quad \Phi_{T_1} > 0 \quad \text{iff} \quad T_1 > F^{-1}\left(\frac{\beta}{\beta + \gamma}\right).$$

Proof. See Appendix A.3.

Hence, $\Phi(\mathbf{T})$ is concave in T_0 and convex in T_1 and remains so when $T_0 \geq T_1$.¹¹ Thus, the SDC advantage of facility 0 increases (decreases) in T_0 when the service scheduled by its carrier is below (above) the $\beta/(\beta + \gamma)$ -th quantile of consumers desired departure time distribution.¹² When facility 0's SDC advantage expands, more travellers choose facility 0 and less of them depart facility 1. Similarly, the SDC disadvantage of facility 1 increases (decreases) in T_1 when T_1 is above (below) time $F^{-1}[\beta/(\beta + \gamma)]$ and less (more) travellers select facility 1. If $\beta < \gamma$, time $F^{-1}[\beta/(\beta + \gamma)]$ is below consumers' median desired departure time and its exact location on the 24h-clock will further depend on the shape of $F(t)$.

We now characterize the change in market demand with respect to the fares, departure times and the location advantage of facility 0. From (4), we deduce that demand at a facility decreases in the fare of its carrier and increases in the fare of the carrier serving the rival. Differentiating consumer demands with respect to T_i and h , we get:

$$\begin{aligned}\frac{\partial D_0(\mathbf{p}, \mathbf{T})}{\partial T_i} &= \Phi_{T_i}(\mathbf{T}), & \frac{\partial D_1(\mathbf{p}, \mathbf{T})}{\partial T_i} &= -\frac{\partial D_0(\mathbf{p}, \mathbf{T})}{\partial T_i} \quad \text{for } i = 0, 1, \\ \frac{\partial D_0(\mathbf{p}, \mathbf{T})}{\partial h} &= \frac{1}{2} + \frac{p_1 - p_0 + \Phi(\mathbf{T})}{(1 - h)}, & \frac{\partial D_1(\mathbf{p}, \mathbf{T})}{\partial h} &= -\frac{\partial D_0(\mathbf{p}, \mathbf{T})}{\partial h}.\end{aligned} \tag{6}$$

Demands' sensitivity to changes in departure times follows from the discussion under Lemma 1. From (6), we can deduce that consumers' demand at facility 0 (facility 1) increases with its location advantage if the inter-facility transportation cost is larger

¹¹See Appendix B.3.

¹²When t is uniform on $[0, 24]$, then $F^{-1}[\beta/(\beta + \gamma)]$ is given by $24\beta/(\beta + \gamma)$.

(lower) than the difference in average service cost between facility 0 and 1.¹³ Having characterized consumers' problem, we turn to model the competition between carriers.

2.2 Carrier-rivalry game

We now focus on the time-then-fare game. Setting departure times before fares is the most common behaviour for carriers offering (air, rail, road, water) transportation services to individuals.¹⁴ Moreover, fares are generally easier to adjust than departure times. The two-stage decision process is solved backwards: we first maximize carrier profits with respect to fares and then analyze carrier timing decisions.

2.2.1 The fare game

This section analyzes carriers' market when departure times are given to carriers. Furthermore, we consider that carriers' operational time costs along the time of day are separable from other costs and we set carriers' fixed cost to zero. Then, the profit function of each carrier is:

$$\hat{\pi}_i(\mathbf{p}, \mathbf{T}, \boldsymbol{\tau}) = (\hat{p}_i - \hat{c}_i - \hat{\tau}_i)D_i(\mathbf{p}, \mathbf{T}) - \hat{K}(T_i), \quad i = 0, 1, \quad (7)$$

where $\hat{c}_i \geq 0$ is the marginal operational cost of the carrier serving facility i , $\hat{\tau}_i \geq 0$ is the per passenger fee charged by facility i to its carrier and $\hat{K}(T_i) \geq 0$ is the operational total time cost incurred by carrier i for departing its aircraft at time T_i . Given the departure times $\mathbf{T}$ and the fare of its rival, carriers simultaneously choose their fare to maximize profit. Solving the system of first-order conditions (FOCs) $\partial \hat{\pi}_i / \partial \hat{p}_i = 0$ for $i = 0, 1$ with respect to the fares leads to:

$$\begin{aligned} p_0^* &= \frac{2}{3}(c_0 + \tau_0) + \frac{1}{3}(c_1 + \tau_1) + \frac{3+h}{6} + \frac{1}{3}\Phi(\mathbf{T}), \\ p_1^* &= \frac{2}{3}(c_1 + \tau_1) + \frac{1}{3}(c_0 + \tau_0) + \frac{3-h}{6} - \frac{1}{3}\Phi(\mathbf{T}), \end{aligned} \quad (8)$$

¹³ Moreover, a higher transportation cost parameter induces a larger demand at facility 0 and a lower one at the rival if the service cost of facility 1 is larger than that of facility 0. See (A.8) in the Appendix for further details.

¹⁴ Typically, airlines will sign a (long-term) contract with an airport for the use of a gate or gates that may be available at the time the airline needs gate space.

where p_i^* , c_i and τ_i for $i = 0, 1$ stand for $\widehat{p}_i^*$, $\widehat{c}_i$ and $\widehat{\tau}_i$ divided by $\theta(1-h)$. The resulting vector of (normalized) equilibrium fares, denoted $\mathbf{p}^* \equiv (p_0^*, p_1^*)$, represents a Nash equilibrium.¹⁵ The first two terms on the RHS of Eqs. (8) are the (normalized) marginal costs of each carrier plus the usual duopolistic markup/markdown which is proportional to the marginal costs of the rival carrier serving the other facility. The third terms are a monopoly premium/penalty stemming from the location advantage/disadvantage of the facility at which a carrier operates. The last terms represent a markdown/markup related to the (normalized) SDC (dis)advantage of the carrier.

Fare expressions (8) can be further rearranged to analyze the difference in pricing and markup across carriers. Focusing first on the difference in (normalized) fares, setting p_0^* strictly larger than p_1^* and rearranging yields: $p_0^* > p_1^*$ iff

$$\Delta\tilde{c} < h + 2\Phi(\mathbf{T}), \quad (9)$$

where $\Delta\tilde{c} = (c_1 + \tau_1) - (c_0 + \tau_0)$ represents the (normalized) marginal costs (dis)advantage of carrier 0 with respect to its rival when $\Delta\tilde{c} > 0$ ($\Delta\tilde{c} < 0$). Equalizing the departure times across facilities and assuming no location advantage¹⁶ for facility 0 in (9), carrier 0 charges a higher fare than its rival if its marginal costs are higher ($\Delta\tilde{c} < 0$). In commercial aviation, this is typically the case of a legacy carrier that competes over a shared market with a lower marginal costs carrier departing the other facility. Setting $h > 0$, the location advantage of facility 0 allows its carrier to charge a higher fare than its rival, even if it has *lower* marginal costs ($\Delta\tilde{c} > 0$). Hence, in equilibrium, a carrier benefiting from the better location of its departure facility and with lower marginal costs than a rival carrier serving the other facility will charge a higher fare, if its advantage in location is large enough. The SDC term has a similar impact on carriers' fare as the location advantage and can strengthen or reduce a geographical (dis)advantage. The following proposition summarizes the above results.

Proposition 1. *Consider carrier 0 (resp. 1) that competes in fares with fixed scheduled times T_0 (resp. T_1), with $T_0 \leq T_1$. There exists a unique Nash equilibrium in fares given*

¹⁵The existence of the Nash equilibrium follows from the concavity in fares of the profit functions. Its uniqueness and stability are easy to verify (by using Vives, 1999, pp. 47-52). Fares can also be shown to be strategic complements in carrier decisions.

¹⁶Hereafter, the expressions 'no location advantage' or 'dropping the location advantage' mean setting $h = 0$. Note that $h = 0$ implies *maximum* geographical differentiation between the facilities and their carrier while $T_0 = T_1$ implies *minimum* schedule differentiation.

by (8). In equilibrium, carrier 0 charges a higher fare than its rival if (9) holds, i.e., if its location and schedule delay cost advantages offset its marginal costs advantage.

To show that Proposition 1 holds when $T_0 \geq T_1$, use demands (B.2) in the profit functions (7) and apply the above derivation steps. The SDC difference $\Phi(\mathbf{T})$ in the related expressions is replaced by its counterpart $\Phi^{sym}(\mathbf{T})$ given in Eq. (B.3).

Defining the (normalized) markup of carrier i in equilibrium as $m_i^* = p_i^* - (c_i + \tau_i)$ for $i = 0, 1$, setting m_0^* strictly larger than m_1^* and rearranging yields: $m_0^* > m_1^*$ iff

$$\Delta\tilde{c} > -\frac{h}{2} - \Phi(\mathbf{T}). \quad (10)$$

Assuming no location and SDC advantages, $\Delta\tilde{c}$ in (10) becomes positive which means that the markup of carrier 0 exceeds its rival's markup if its marginal costs are lower. Further setting $h > 0$, the advantage in location of facility 0 allows its carrier to receive a higher markup than its rival, even if its carrier has *higher* marginal costs ($\Delta\tilde{c} < 0$) than its competitor at the rival facility. Therefore, the markup of a carrier serving a facility endowed with a better location can be higher than the markup of the rival, lower marginal costs carrier if its marginal costs disadvantage is not excessive. Considering the full expression (10), the SDC advantage has a similar impact on carrier markups as the location advantage and can either expand or reduce facility 0's location advantage.

We now explore the impact of an increase in T_i and in h on the equilibrium fares with the following expressions:

$$\frac{\partial p_0^*}{\partial T_i} = \frac{1}{3}\Phi_{T_i}(\mathbf{T}), \quad \frac{\partial p_1^*}{\partial T_i} = -\frac{\partial p_0^*}{\partial T_i}, \quad i = 0, 1, \quad (11)$$

$$\frac{\partial \hat{p}_0^*}{\partial h} = -\frac{\theta(1+h)}{3} < 0, \quad \frac{\partial \hat{p}_1^*}{\partial h} = -\frac{\theta(2-h)}{3} < 0. \quad (12)$$

The sensitivity of the (normalized) fares to changes in departure times can be analyzed in light of Lemma 1: p_0^* increases in T_0 (resp. T_1) if the service of the carrier serving facility 0 (resp. facility 1) is scheduled earlier (resp. later) than the $\beta/(\beta+\gamma)$ th quantile of consumers' desired departure time distribution. The reasoning for p_1^* is analogous. Next, notice that (12) are calculated on $\hat{p}_i^*$ and not on p_i^* . As $h \in [0, 1]$, the equilibrium fares are decreasing in h at both facilities. A shorter inter-facility distance enhances the rivalry between the carriers and reduces the equilibrium fares at both facilities.

Substituting the equilibrium fares $\mathbf{p}^*$ in consumer demands (4), carriers' marginal costs become explicit in the *equilibrium* demands, that is:

$$D_0^*(\mathbf{T}, \boldsymbol{\tau}) = \frac{1}{6}[3 + h + 2\Delta\tilde{c} + 2\Phi(\mathbf{T})], \quad D_1^*(\mathbf{T}, \boldsymbol{\tau}) = 1 - D_0^*(\mathbf{T}, \boldsymbol{\tau}), \quad (13)$$

where $D_0^* \in]0, 1[$ requires $|\frac{1}{3}[h + 2\Delta\tilde{c} + 2\Phi(\mathbf{T})]| < 1$. As outlined in Basso and Zhang (2007), in equilibrium, the marginal costs of a carrier have the same effects on the equilibrium demand as those of fares: higher marginal costs for a carrier induce a lower equilibrium demand at its facility and a larger one at the rival facility. Dropping the location and SDC advantage terms, and equalizing marginal costs across carriers, the market demand is evenly shared across facilities. A larger demand at a facility goes hand-in-hand with a larger markup for its carrier than its rival's markup.¹⁷

Regarding the effects of a change in T_0 and T_1 on the equilibrium demands, one can readily see in (13) that $\partial D_0^*(\mathbf{T}, \boldsymbol{\tau})/\partial T_i$ and $\partial D_1^*(\mathbf{T}, \boldsymbol{\tau})/\partial T_i$ for $i = 0, 1$ leads to, respectively, $\partial p_0^*/\partial T_i$ and $\partial p_1^*/\partial T_i$ in (11). Thus, departure times have exactly the same impact on the equilibrium demands as on the equilibrium fares. Next, differentiating the equilibrium demands with respect to h , setting the resulting expression larger than zero and rearranging, we obtain:¹⁸

$$\frac{\partial D_0^*(\mathbf{T}, \boldsymbol{\tau})}{\partial h} > 0 \quad \text{iff} \quad \frac{\theta}{2}(1 - h)^2 > -\Delta\hat{\tilde{c}} - \hat{\Phi}(\mathbf{T}), \quad (14)$$

where $\Delta\hat{\tilde{c}} = \theta(1 - h)\Delta\tilde{c}$ and $\hat{\Phi}(\mathbf{T}) = \theta(1 - h)\Phi(\mathbf{T})$, and the reverse holds for $\partial D_1^*/\partial h$. Thus, if the carrier that benefits from a location advantage (carrier 0) is more competitive in marginal and schedule delay costs than its rival (both $\Delta\hat{\tilde{c}}$ and $\hat{\Phi}(\mathbf{T}) > 0$), increasing h increases the demand at its facility at the expense of its rival. If carrier 0 is less competitive in marginal and schedule delay costs (both $\Delta\hat{\tilde{c}}$ and $\hat{\Phi}(\mathbf{T}) < 0$), increasing h increases demand at its facility if the inter-facility transportation cost is larger than carrier 0's marginal and schedule delay costs disadvantages. Taking the perspective of the carrier serving the most remote facility, carrier 1 captures part of its 'rival's backyard' when h increases if its marginal costs and SDC advantages fully compensate the transportation cost from facility 0 to facility 1.

¹⁷ To see this, use (13), set $D_0^* > D_1^*$ and rearrange to get (10).

¹⁸ Replacing the difference in carriers' marginal costs by the difference in fares, (14) is identical to $\partial D_0(\mathbf{p}, \mathbf{T})/\partial h$ in (A.8).

In equilibrium, the profits (7) can be written as:

$$\pi_i^*(\mathbf{T}, \boldsymbol{\tau}) = D_i^{*2}(\mathbf{T}, \boldsymbol{\tau}) - K(T_i), \quad i = 0, 1, \quad (15)$$

where $\pi_i^* = \widehat{\pi}_i/\theta(1-h)$ and $K(T_i) = \widehat{K}(T_i)/\theta(1-h)$. As expected, the *equilibrium* profit of a carrier depends upon all determinants of the equilibrium demand at its departure facility (in particular the equilibrium fare, the fee and the departure time set at the other facility) minus carrier's own total operational time costs. Ignoring the latter term for now and setting $D_0^{*2} > D_1^{*2}$, we can readily exploit a result obtained for the equilibrium demands under (13): $\pi_0^* > \pi_1^*$ if the markup of carrier 0 exceeds carrier 1's markup. Thus, a larger markup for a carrier implies a larger demand and profit than its rival when carriers' time costs are null. The following proposition summarizes these results.

Proposition 2. *Consider carrier 0 (resp. 1) departing its facility at times T_0 (resp. T_1), with $T_0 \leq T_1$ and let $K(T_0) = K(T_1) = 0$. In equilibrium, the markup fare of carrier 0, its demand and profit are higher than its rival's if (10) holds, i.e., if its marginal costs disadvantage does not offset its location and schedule delay costs advantages.*

Again, to prove that Proposition 2 holds when $T_0 \geq T_1$, use the symmetric demands (B.2) in the profit functions (7) and follow the same derivation steps. The SDC difference $\Phi(\mathbf{T})$ is replaced by its counterpart $\Phi^{sym}(\mathbf{T})$ given in Eq. (B.3).

Turning to the comparative statics of the profits with respect to T_i and h , we get:

$$\frac{\partial \pi_i^*(\mathbf{T}, \boldsymbol{\tau})}{\partial T_j} = \frac{2}{3} \Phi_{T_j} D_i^*(\mathbf{T}, \boldsymbol{\tau}) - \frac{\partial K(T_i)}{\partial T_i}, \quad \text{for } i, j = 0, 1 \text{ and for } i \neq j, \quad (16)$$

$$\begin{aligned} \frac{\partial \widehat{\pi}_0^*(\mathbf{T}, \boldsymbol{\tau})}{\partial h} &= \frac{\theta}{3} \left[-\frac{(1+3h)}{2} + \Delta\tilde{c} + \Phi(\mathbf{T}) \right] D_0^*(\mathbf{T}, \boldsymbol{\tau}), \\ \frac{\partial \widehat{\pi}_1^*(\mathbf{T}, \boldsymbol{\tau})}{\partial h} &= \frac{\theta}{3} \left[\frac{3h-5}{2} - \Delta\tilde{c} - \Phi(\mathbf{T}) \right] D_1^*(\mathbf{T}, \boldsymbol{\tau}). \end{aligned} \quad (17)$$

Regarding the impact of a change in the departure times on carrier profits in (16), ignoring $\partial K(T_i)/\partial T_i$, we notice that $\text{sgn}(\partial \pi_i^*/\partial T_j) = \text{sgn}(\Phi_{T_j})$. Hence, Lemma 1 applies again. The effect of a change in the location of facility 0 on carrier profits is explored through the “competition” and “demand” effects.¹⁹ Whether an increase

¹⁹The competition and demand effects related to h are given by $\frac{\partial \widehat{\pi}_i^*}{\partial h} = D_i^* \frac{\partial \widehat{p}_i^*}{\partial h} + (\widehat{p}_i^* - \widehat{c}_i) \frac{\partial D_i^*}{\partial h}$.

in demand at *one* facility compensates the decrease in the fare of its carrier when h increases depends on the relative (dis)advantages identified above. Fig. 2 summarizes the net effect of a RHS move of h on carrier profits along the $\Delta\tilde{c}$ line.

Figure 2: Effect of a positive change in h on π_i^*

Having characterized the fare game for given departure times, we proceed to consider the time game for a simultaneous choice of the departure times.

2.2.2 The time game

Scheduling a transport service at the most appropriate time of the day is a central element of carriers' planning. A large number of operational research professionals have been developing methods to optimize airline schedules since the 1950s. Etschmaier and Mathaisel (1985) describe the general procedure solved in mathematical programming as follows: given (i) a set of demand functions and associated revenues for every passenger origin-destination pair over the time of day (and the day of week of the planning cycle), (ii) route characteristics (distance, times and operating restrictions), (iii) aircraft characteristics and operating costs, and (iv) operating and managerial constraints; find a set of flights with associated assignments of aircraft *and times of departure and arrival* which maximize profits. In daily operations, optimized schedules are rarely executed as planned due to unexpected disruptions of the transport service (weather conditions, unscheduled maintenance) or to congestion caused by operations of other flights at the departure or arrival airport, that propagate through the network during the day. Scheduling a transport service at a specific time of the day may be more or less costly in terms of logistics, and may depend on carriers' business model.²⁰

We assume that carriers can decompose their operational time costs $K(T_i)$ additively into a fixed component and a cost that varies along the time of day over all

²⁰ Compared to traditional legacy carriers, low-cost carrier's operations are often characterized by the use of smaller aircrafts, a more intense daily use of their fleet with less idle times, aircrafts and crew returned to a base airport, which reduces maintenance and accommodation costs (see IATA (2005), Gross and Schröder (2007) or Bley and Buermann (2007, pp.59-62) for a discussion on the strategic use of schedules by low-cost airlines to reduce their operational costs).

feasible service times $T_i \in [\underline{T}, \bar{T}]$ at their departure facility. We posit that the time-varying cost is either null or linearly increasing (or decreasing) in T_i . Thus, carriers' total operational time costs are given by:

$$\begin{aligned} K(T_0) &= K_0 + k_0 T_0, \quad \text{with } K_0 \geq 0, \quad k_0 \neq 0, \quad T_0 \in [\underline{T}, \bar{T}], \\ K(T_1) &= K_1 + k_1 T_1, \quad \text{with } K_1 \geq 0, \quad k_1 \neq 0, \quad T_1 \in [\underline{T}, \bar{T}], \end{aligned} \quad (18)$$

where $K(T_i) = \widehat{K}(T_i)/\theta(1-h)$. Notice that a positive marginal time cost for a carrier favours a departure time closer to the opening hour of its facility while a negative one favours a service scheduled closer to the closing hour. In what follows, we assume that facilities operate 24 hours a day (i.e., $\underline{T}$ and $\bar{T}$ are not binding). Given the endogenous fares (8), we can focus on maximizing profits (15) with respect to T_i . Combining (16) with (18) and assuming for now the existence of interior and unique solutions on $[0, 24]$, setting $\partial\pi_i^*/\partial T_i = 0$ for $i = 0, 1$ yields the following first-order conditions (FOCs):

$$\begin{aligned} \frac{\partial\pi_0^*(\mathbf{T}, \boldsymbol{\tau})}{\partial T_0} &= \frac{2}{3} [\beta - (\beta + \gamma)m_\ell] D_0^*(\mathbf{T}, \boldsymbol{\tau}) - k_0 = 0, \\ \frac{\partial\pi_1^*(\mathbf{T}, \boldsymbol{\tau})}{\partial T_1} &= -\frac{2}{3} [\gamma - (\beta + \gamma)m_r] D_1^*(\mathbf{T}, \boldsymbol{\tau}) - k_1 = 0, \end{aligned} \quad (19)$$

and, by rearranging these expressions, we can characterize the candidate departure times by the following implicit functions:

$$F(T_i) = \frac{\beta}{\beta + \gamma} - \frac{3k_i}{2(\beta + \gamma)D_i^*(\mathbf{T}, \boldsymbol{\tau})}, \quad i = 0, 1, \quad (20)$$

where $F(T_0) \equiv m_\ell$, $F(T_1) \equiv 1 - m_r$ and $k_i = \partial K(T_i)/\partial T_i$ denotes carrier i 's (normalized) marginal time cost along the time of day. The second-order conditions (SOCs) that need to be satisfied are:

$$\begin{aligned} \frac{\partial^2\pi_0^*(\mathbf{T}, \boldsymbol{\tau})}{\partial T_0^2} &= -\frac{2}{3}(\beta + \gamma)\rho(T_0)D_0^*(\mathbf{T}, \boldsymbol{\tau}) + \frac{2}{9}[\beta - (\beta + \gamma)m_\ell]^2 < 0, \\ \frac{\partial^2\pi_1^*(\mathbf{T}, \boldsymbol{\tau})}{\partial T_1^2} &= -\frac{2}{3}(\beta + \gamma)\rho(T_1)D_1^*(\mathbf{T}, \boldsymbol{\tau}) + \frac{2}{9}[\gamma - (\beta + \gamma)m_r]^2 < 0. \end{aligned} \quad (21)$$

Consider first the case where carriers' time cost is constant along the time of day ($k_i = 0$ for $i = 0, 1$). FOCs (20) lead to the best timing response functions of each

carrier to their rival's schedule, and solving these yields:

$$F(T_0) = F(T_1) = \frac{\beta}{\beta + \gamma} \Rightarrow T^*|_{k_0=k_1=0} = F^{-1}\left(\frac{\beta}{\beta + \gamma}\right). \quad (22)$$

Clearly, the response functions in (22) are identical across carriers and depend only on consumers' unit schedule delay costs and on the CDF $F(t)$. As the equilibrium departure time of a carrier does not depend on the schedule of its rival, each carrier has a dominant strategy which represents a unique and interior Nash equilibrium. Carrier 0 schedules its service to maximize its SDC advantage while carrier 1 set its departure time to minimize its SDC disadvantage. Moreover, the last RHS terms in the SOC's (21) are null when the FOCs are satisfied and the parameter restrictions below (13) become $D_0^*(\mathbf{T}^*, \boldsymbol{\tau}) \in]0, 1[$ if $|\frac{1}{3}[h + 2\Delta\tilde{c}]| < 1$. When these restrictions hold and given that $\rho(T_i)$, β and γ are positive, this solution maximizes carrier profits.

Turning to the case where carriers' time costs vary with the time of day ($k_i \neq 0$ for $i = 0, 1$), assuming that the solution of system (20) – a system of two nonlinear functions in T_i – yields a unique intersection at coordinate $\mathbf{T}^* = (T_0^*, T_1^*) \in]0, 24]^2$ with $T_i^* \equiv T_i^*(\boldsymbol{\tau}, k_i)$, we need to ensure that this equilibrium maximizes carrier profits. Using the SOC's (21), the last terms on the RHS no longer vanish at the stationary points and profit maximization requires bounded marginal time costs such that:

$$k_i^2 < \frac{4}{3}\rho(T_i^*)(\beta + \gamma)D_i^{*3}(\mathbf{T}^*, \boldsymbol{\tau}), \quad i = 0, 1, \quad (23)$$

where k_i enters both sides of the inequality. In what follows we posit that k_i satisfies (23) and yields a unique and interior Nash equilibrium.²¹

Given that $F(t)$ is strictly increasing in its argument, we can use (20) to characterize all possible schedule configurations across carriers in equilibrium with respect to $T^*|_{k_0=k_1=0}$. By applying the implicit function theorem to (20), we can show that the equilibrium departure times T_i^* for $i = 0, 1$ are decreasing in k_i when (23) holds.²²

²¹When $k_i \neq 0$ for $i = 0, 1$, the existence, uniqueness and stability of the Nash equilibrium in departure times on $]0, 24]^2$ must be investigated numerically. Assuming that this equilibrium exists and is unique, setting $0 < F(T_i^*) < 1$ for $i = 0, 1$ we can derive bounds around k_i *conditional upon* $D_0^*(\mathbf{T}^*, \boldsymbol{\tau}) \in]0, 1[$ and which will depend on $D_0^*(\mathbf{T}^*, \boldsymbol{\tau})$. These bounds are available upon request. In Appendix A.4, we derive explicit bounds for k_i as well as *existence conditions* for interior solutions in departure times when $\rho(t) = \mathcal{U}[0, 24]$.

²²That is, $dT_i^*/dk_i < 0$ for $i = 0, 1$, see Appendix A.5.

Next, we deduce from (20) that a carrier schedules its service earlier (resp. later) than $T^*|_{k_0=k_1=0}$ when its marginal time cost is positive (resp. negative), and even earlier (resp. later) when its marginal time cost is large (resp. large in absolute value) or its equilibrium demand at its departure facility is low. Setting $F(T_0^*) = F(T_1^*)$, we can further characterize the principle of minimum differentiation in departure times. When the marginal time cost-demand ratio is equal for both carriers, i.e., when $k_0/D_0^*(\mathbf{T}^*, \boldsymbol{\tau}) = k_1/D_1^*(\mathbf{T}^*, \boldsymbol{\tau})$, the carriers schedule their service at the same time, earlier than $T^*|_{k_0=k_1=0}$ when $k_0, k_1 > 0$ and later than $T^*|_{k_0, k_1=0}$ when $k_0, k_1 < 0$. This situation could arise across rival carriers sharing evenly the market and facing the same marginal time cost ($k_0 = k_1 \neq 0$). Aside from the case where carriers' marginal time cost is exactly proportional to their equilibrium demand, we can infer from (20) that the existence of operational costs that vary with the time of day implies distinct service times in equilibrium.²³ The shape of $F(t)$ exerts an additional agglomeration/deglomeration force driving service time locations closer/farther away from $T^*|_{k_0=k_1=0}$ on the time line. When the configuration of marginal time costs induces $T_0^* \geq T_1^*$, carrier 0 schedules its service later than carrier 1 and the time game is solved by using the expressions provided in Appendix B.4. Appendix C numerically explores the stability of the departure time equilibria for all combinations of positive, negative and null marginal time costs across carriers when $F(t)$ is uniform. The following proposition summarizes our findings in the time game.

Proposition 3. *Consider two competing carriers ($i = 0, 1$) which set simultaneously a single departure time $T_i \in]0, 24[$ prior to setting fares. Let carriers' operational time costs be given by $K(T_i) = K_i + k_i T_i$ for $i = 0, 1$. Then:*

1. *if $k_0 = k_1 = 0$, both carriers schedule their service at the $\beta/(\beta + \gamma)$ th quantile of consumers' desired time distribution. As these optimal departure times are strictly dominant strategies for both carriers, the Nash equilibrium is unique and interior.*
2. *when carriers' time cost varies along the time of day, if a unique Nash equilibrium in departure times (denoted $\mathbf{T}^*$) exists, carriers schedule their service at the same time if $k_0/k_1 = D_0^*(\mathbf{T}^*, \boldsymbol{\tau})/D_1^*(\mathbf{T}^*, \boldsymbol{\tau})$ and at different times otherwise.*

To prove that Proposition 3 holds when $T_0 \geq T_1$, use the equilibrium demands (B.5) in the profit functions (15) and follow the same derivation steps (see Appendix B.4).

²³See Appendix A.6 for the detailed derivation.

The main takeaway of the time game for a two-airport region is that schedule differentiation is observed in equilibrium when competition occurs across airports which differ both in terms of demand and operational costs conditions for the airlines. Schedule clustering occurs when they are symmetric. Several factors favour departure times set early in the day or late at night in equilibrium. On the demand side, we can identify a strongly asymmetric schedule delay cost function and a large mass of consumers preferring early or late departure. When $k_i \neq 0$, any structural parameter penalizing demand at a facility favours early/late scheduling of flights. On the supply side, we can mention a large marginal time cost (in absolute value) along the day for a carrier.

Inserting the equilibrium service times $\mathbf{T}^*$ in demands (13) and in profits (15) yields:

$$\begin{aligned} D_0^*(\mathbf{T}^*, \boldsymbol{\tau}) &= \frac{1}{6} \left[3 + h + 2\Delta\tilde{c} + 2\Phi(\mathbf{T}^*) \right], \quad D_1^*(\mathbf{T}^*, \boldsymbol{\tau}) = 1 - D_0^*(\mathbf{T}^*, \boldsymbol{\tau}), \\ \pi_i^*(\mathbf{T}^*, \boldsymbol{\tau}) &= D_i(\mathbf{T}^*, \boldsymbol{\tau})^2 - K(T_i^*), \quad i = 0, 1. \end{aligned}$$

Having characterized the carriers' market, we turn to the facility-rivalry game.

2.3 Facility-rivalry game

Consider two facilities, with fixed capacity, that compete with each other in the fees charged per passenger to their downstream firm. Assume that the facilities derive a per passenger commercial revenue from services provided by concessionaires to travellers. Without loss of generality, we set their marginal operational cost to zero and denote their fixed cost by $\hat{F}_i \geq 0$ for $i = 0, 1$. Thus, the facility maximization problem reads:

$$\max_{\hat{\tau}_i} \hat{\Pi}_i(\mathbf{T}, \hat{\boldsymbol{\tau}}) = (\hat{\tau}_i + \hat{\omega}_i) D_i^*(\mathbf{T}, \boldsymbol{\tau}) - \hat{F}_i, \quad i = 0, 1, \quad (24)$$

where $\hat{\tau}_i \geq 0$ denotes the per passenger fee at facility i and $\hat{\omega}_i \geq 0$ is the per passenger commercial revenue. In the last resolution stage of the game, facilities take carrier fares and departure times $\mathbf{T}$ as given. Departure times can be either given to the carriers (as in Section 2.2.1) or optimally chosen (as in Section 2.2.2). In the latter case, when the time costs of a carrier vary with the time of day ($k_i \neq 0$) its equilibrium departure time in (20) is an implicit expression which depends on the (normalized) fees $\boldsymbol{\tau}$. Solving (24) analytically is not tractable. However, in all other cases, maximizing facility profits

is straightforward and follows the same resolution steps as the carriers' fare game. If the facilities simultaneously choose their fees to maximize profit, solving the system of FOCs with respect to the fees yields:

$$\begin{aligned}\tau_0^* &= \frac{1}{3} [(c_1 - c_0) - (2\omega_0 + \omega_1)] + \frac{9 + h}{6} + \frac{1}{3}\Phi(\mathbf{T}), \\ \tau_1^* &= \frac{1}{3} [(c_0 - c_1) - (\omega_0 + 2\omega_1)] + \frac{9 - h}{6} - \frac{1}{3}\Phi(\mathbf{T}),\end{aligned}\tag{25}$$

where τ_i^* and ω_i are $\widehat{\tau}_i^*$ and $\widehat{\omega}_i$ divided by $\theta(1 - h)$, and $\boldsymbol{\tau}^* \equiv (\tau_0^*, \tau_1^*)$ represents a Nash equilibrium in fees. The optimal fee of a facility is decreasing in the marginal operational cost of its carrier and increasing in that of the carrier serving its rival. This result, identical to Basso and Zhang (2007), also stresses that a facility captures a fraction (1/3) of its carrier's operational cost advantage. Next, a higher per passenger revenue at *one* facility induces a lower fee at *both* facilities.²⁴ Notice that, in equilibrium, a facility shares part (2/3) of its own per passenger commercial revenue with its carrier and pushes the rival to reduce its fee by an amount that is proportional to its own per passenger revenue (1/3). The h term is the monopoly premium/penalty related to the location advantage/disadvantage of the facility and $\Phi(\mathbf{T})$ captures the SDC (dis)advantage due to potential differences in service times across carriers. Moreover, the facility fees capture the same share of the marginal operational and SDC advantages from the carriers.

Substituting the equilibrium fees (25) into demands (13) yields:

$$\begin{aligned}D_0^{f,*}(\mathbf{T}) &= \frac{1}{18} [9 + h + 2(c_1 - c_0) + 2(\omega_0 - \omega_1) + 2\Phi(\mathbf{T})], \\ D_1^{f,*}(\mathbf{T}) &= 1 - D_0^{f,*}(\mathbf{T}).\end{aligned}\tag{26}$$

The equilibrium demand for a facility decreases in the marginal operational cost of its carrier and in the per passenger commercial revenue of its rival; and increases in the marginal operational cost of the carrier serving its rival, in its own per passenger commercial revenue and in the SDC advantage of its carrier. Note that, while per passenger commercial revenues help to keep the equilibrium fees low at both facilities,

²⁴ Substituting the optimal fees (25) in the equilibrium fares (8), we can establish how per passenger commercial revenues affect the equilibrium fares and carrier profits. Further replacing these fares in the covered market condition (3) allows to express the latter condition in terms of the exogenous parameters of the model. See Appendix A.7.

a per passenger commercial revenue advantage allows a facility to increase its demand at the expense of its rival.

With the above equilibrium fees and demands at hand, we obtain the facility profits in equilibrium given by:

$$\Pi_i^*(\mathbf{T}) = 3D_i^{*2}(\mathbf{T}) - F_i, \quad i = 0, 1. \quad (27)$$

Similarly to the carrier market analysis, the differences in fees, demands and profits between facility 0 and 1 can be analyzed as follows:

$$\begin{aligned} \tau_0^* > \tau_1^* & \text{ iff } \Delta c > -\frac{h}{2} - \Phi(\mathbf{T}) + \frac{\Delta\omega}{2}, \\ D_0^{f,*}(\mathbf{T}) > D_1^{f,*}(\mathbf{T}) & \text{ iff } \Delta c > -\frac{h}{2} - \Phi(\mathbf{T}) - \Delta\omega, \\ \Pi_0^*(\mathbf{T}) > \Pi_1^*(\mathbf{T}) & \text{ iff } \Delta c > -\frac{h}{2} - \Phi(\mathbf{T}) - \frac{3}{2}\Delta F - \Delta\omega, \end{aligned} \quad (28)$$

where $\Delta c = c_1 - c_0$, $\Delta\omega = \omega_0 - \omega_1$, $\Delta F = F_1 - F_0$ with $F_i = \hat{F}_i/\theta(1-h)$. These delta terms represent (normalized) costs or commercial revenue advantages for facility 0/1 when they are positive/negative while $\Phi(\mathbf{T})$ captures the usual (normalized) SDC advantage. Setting $h = \Phi(\mathbf{T}) = \Delta\omega = \Delta F = 0$ in (28), in equilibrium, facility 0 charges a higher fee and receives a larger demand and profit than its rival when the marginal operational cost of its carrier is lower than that of the carrier serving the rival facility. Introducing a location (or a SDC or a fixed-cost) advantage for facility 0, a facility can charge a higher fee than its rival and get higher demand and profit even if the marginal operational cost of its carrier is larger than that serving the rival. This happens when the marginal operational cost disadvantage of its carrier is not below $-\frac{h}{2}$ (or below $-\Phi(\mathbf{T})$ or below $-\frac{3}{2}\Delta F$, respectively). Setting $h = \Phi(\mathbf{T}) = \Delta F = 0$ in (28), we can focus on the effect of the per passenger commercial revenue advantage on the differences in fees, demands and profits across facilities. We deduce that a facility charges a higher fee than its rival if the marginal operational cost advantage of its carrier with respect to the carrier serving the rival outweighs half of its per passenger commercial revenue advantage; a facility faces a larger demand and receives a larger profit than its rival if the marginal operational cost *disadvantage* of its carrier with respect to the carrier serving the rival offsets its per passenger commercial revenue

advantage. The following proposition summarizes the results of the facility-rivalry subgame.

Proposition 4. *Consider two facilities that compete in fees charged to their carrier characterized by constant operational time costs along the time of day. Further assume that the carriers schedule their service such that $T_0 \leq T_1$. Then, (1) there exists a unique Nash Equilibrium in fees given by (25), (2) in equilibrium, demands and profits are given by (26) and (27), respectively. Facility 0 charges a higher fee, faces a larger demand and receives a larger profit than its rival if the three inequalities in (28) hold.*

To prove that Proposition 4 also holds when $T_0 \geq T_1$, use the equilibrium demands (B.5) in the profit functions (24) and follow the same steps. The SDC difference $\Phi(\mathbf{T})$ in the related expressions will be replaced by its counterpart $\Phi^{sym}(\mathbf{T})$ given in Eq. (B.3).

Having characterized the duopolistic outcome of the three-stage game, we now explore regulator's problem.

3 Welfare analysis

This section sets up the welfare criteria by which regulating authorities may evaluate existing or new airport locations along the geographic space or the lack/excess of schedule variety in the timing of the transport service. In the transportation industry, the location of the facilities is often decided by regional authorities in order to minimize consumers' access cost. These authorities have little control over schedule decisions which require broad coordination between a variety of agents (carriers, facility managers, regional and national authorities, international transport regulator) and may need to obey international standards. Hence, we treat the location and schedule decisions of the social planner independently of one another.²⁵

Choosing the geographical location of firms/facilities that minimizes the average total transportation cost is a standard problem in Hotelling's model. Assuming equal marginal operational costs across carriers and equal marginal commercial revenues across facilities, the optimal location of facility 0 and 1 can be easily shown to be at coordinates $(1/4, 3/4)$ when they are simultaneously chosen, at $1/3$ for facility 0 when

²⁵The scaling factor $1/\theta(1-h)$ is not needed in this section; hence, all 'hat' magnitudes represent unscaled monetary values.

facility 1 is located at 1, and at $(2 + h)/3$ for facility 1 when facility 0 is located at h with $0 \leq h < 1$.

For the schedule regulator, choosing the departure times that minimize the average total time cost of society (denoted $\widehat{SC}^R$ below), ignoring geographical locations, requires finding the departure time of each carrier that minimizes the area under consumer schedule delay cost functions (1) plus carriers' total time cost in (18), that is:

$$\min_{T_0, T_1} \widehat{SC}^R = \widehat{SDC}(\mathbf{T})^R + \widehat{K}(\mathbf{T})^R, \quad (29)$$

where

$$\begin{aligned} \widehat{SDC}(\mathbf{T})^R = & \int_0^{T_0} \widehat{\gamma}(T_0 - t)\rho(t) dt + \int_{T_0}^{\tilde{t}} \widehat{\beta}(t - T_0)\rho(t) dt + \\ & \int_{\tilde{t}}^{T_1} \widehat{\gamma}(T_1 - t)\rho(t) dt + \int_{T_1}^{24} \widehat{\beta}(t - T_1)\rho(t) dt, \end{aligned} \quad (30)$$

and where $\widehat{K}^R(\mathbf{T}) = \widehat{K}_0 + \widehat{k}_0 T_0 + \widehat{K}_1 + \widehat{k}_1 T_1$, $T_0 \leq T_1$ and $\tilde{t} = (\widehat{\beta}T_0 + \widehat{\gamma}T_1)/(\widehat{\beta} + \widehat{\gamma})$ in (30) is the abscissa of the intersection between the schedule delay cost functions (1). The FOCs of (29) are given by:²⁶

$$\frac{\partial \widehat{SC}(\mathbf{T})^R}{\partial T_0} = \widehat{\gamma}m_\ell - \widehat{m}_\ell \widehat{\beta} + \widehat{k}_0 = 0, \quad \frac{\partial \widehat{SC}(\mathbf{T})^R}{\partial T_1} = \widehat{m}_r \widehat{\gamma} - \widehat{\beta}m_r + \widehat{k}_1 = 0, \quad (31)$$

where $\widehat{m}_\ell = \int_{T_0}^{\tilde{t}} \rho(t)dt$ and $\widehat{m}_r = \int_{\tilde{t}}^{T_1} \rho(t)dt$. Using $\rho(t) = \mathcal{U}[0, 24]$ in (30) for analytical tractability in (30) and solving the FOCs, we obtain the socially optimal departure times:

$$T_0^R = \frac{12\widehat{\beta}}{\widehat{\beta} + \widehat{\gamma}} - a_0\widehat{k}_0 - b_0\widehat{k}_1, \quad T_1^R = 12 \left[1 + \frac{\widehat{\beta}}{\widehat{\beta} + \widehat{\gamma}} \right] - a_1\widehat{k}_0 - b_1\widehat{k}_1, \quad (32)$$

where $a_i, b_i > 0$ for $i = 0, 1$. Assuming symmetric unit schedule delay costs and null marginal time costs ($\widehat{\beta} = \widehat{\gamma}$ and $\widehat{k}_0 = \widehat{k}_1 = 0$) in (32), the regulator sets $T_0^R = 6$ (i.e., the first quartile of the $\mathcal{U}[0, 24]$ distribution) and $T_1^R = 18$ (i.e., its third quartile), a classical result of Hotelling's simultaneous location-game with symmetric transportation costs. Focusing on the case where $\widehat{k}_0 = \widehat{k}_1 = 0$ with $\widehat{\beta} \neq \widehat{\gamma}$, denoting the related

²⁶These derivations are lengthy but straightforward. See Appendix A.8 for the sufficient conditions related to (29) as well as for further analytical details related to (32). The situation where $T_0 \geq T_1$ is analyzed in Appendix B.5.

socially optimal service times by $T_i^R|_{\hat{k}_0=\hat{k}_1=0}$ for $i = 0, 1$ and by using the duopolistic equilibrium departure times $T^*|_{\hat{k}_0=\hat{k}_1=0}$ in (22) with the uniform distribution, we deduce that:

$$\begin{aligned}\Delta T_0 &= T_0^R|_{\hat{k}_0=\hat{k}_1=0} - T^*|_{\hat{k}_0=\hat{k}_1=0} = -12 \frac{\hat{\beta}}{\hat{\beta} + \hat{\gamma}}, \\ \Delta T_1 &= T_1^R|_{\hat{k}_0=\hat{k}_1=0} - T^*|_{\hat{k}_0=\hat{k}_1=0} = 12 \frac{\hat{\gamma}}{\hat{\beta} + \hat{\gamma}}.\end{aligned}$$

Hence, duopolistic competition results in departure times which are later (earlier) than socially optimal at facility 0 (facility 1) when the marginal time costs are null across carriers. The schedule regulator could either impose the optimal departure times given by (32) or equalize the marginal social time costs with carriers' marginal time costs.

A detailed analysis of the policy instruments – prices or quantities ? – that would allow a regulator to achieve the optimal level of schedule variety in the two-airport region goes beyond the scope of the present paper.²⁷ From an applied perspective, the schedule regulator would need to assess the level of schedule variety in its multi-airport region. It may be that the differences in demand and operational time costs across airlines – two main drivers of schedule differentiation in our model – lead to a reasonable level of schedule variety, which is not too far from the social optimum.

The following proposition summarizes the above results.

Proposition 5. *Consider a regulator that chooses the departure time of the carrier that operates at each facility (T_0^R and T_1^R with $T_0^R \leq T_1^R$) over the $[0, 24]$ time interval to minimize the average total time cost (29). Assume a uniform distribution of consumers' desired departure time. Then, (1) if the time costs of the carriers are constant along the time of day, the socially optimal departure times are such that $T_0^R < T^*|_{\hat{k}_0=\hat{k}_1=0} < T_1^R$, (2) if the time costs of the carriers vary along the time of day, the socially optimal departure times of the carriers are given in (32).*

The distributional impacts of an optimal location of facility 0 and of a socially optimal schedule for both carriers are briefly analyzed below.

²⁷When discussing airport congestion management, Brueckner (2009) argues that congestion pricing (i.e., a time of day charge) is “politically infeasible” and that current slot systems “may be better than recognized”.

4 Numerical analysis

In this section, the three-stage game is solved for given parameter values by assuming null marginal time costs for the carriers (i.e., $\hat{k}_0 = \hat{k}_1 = 0$). Under this assumption, a unique Nash equilibrium is guaranteed in each subgame and the solutions follow from the closed-form expressions given above. We can thus focus on the effects of the location and schedule delay cost advantages on the market equilibrium without discussing the sources of the differences in departure times.²⁸ In this section, we use the “hat notation” and express cost parameters, fares, fees and profits in unscaled monetary units. We posit that $\theta/2 = \$130$, $h = \{0, 0.25, 0.33\}$, $\hat{\beta} = \$5$ and $\hat{\gamma} = \$7$,²⁹ $t \sim \mathcal{U}[0, 24]$, $\hat{c}_0 = \$10$, $\hat{c}_1 = \$8$, $\hat{\omega}_0 = \$20$ and $\hat{\omega}_1 = \$18$. Without loss of generality, we set all fixed costs to zero ($\hat{K}_i = \hat{F}_i = 0$ for $i = 0, 1$). These assumptions are representative of typical two-airport systems in which the airports differ in the location of the primary (more central) airport, the commercial revenues received from travellers and the competitiveness of their carrier. We posit that a legacy carrier (carrier 0) endowed with higher marginal operational costs operates from a primary facility (facility 0) endowed with a higher per passenger commercial revenue and competes with a lower marginal cost carrier (carrier 1, an LCC) that operates from a secondary facility (facility 1). Two independent regulators then evaluate the market outcome based on their own assessment of the average access and time costs incurred by the agents.

Table 1 shows the market equilibria for different levels of schedule variety. By Proposition 3.1, duopolistic competition results in identical departure time across carriers and in a travel service scheduled in the morning, at the quantile $\hat{\beta}/(\hat{\beta} + \hat{\gamma}) = 5/(5 + 7) = 41.\bar{6}\%$ of the $\mathcal{U}[0, 24]$ distribution: $T_0^* = T_1^* = 41.\bar{6}\% \times 24 = 10$, which is a dominant strategy for both carriers. When we combine maximum spatial differentiation between the airports with minimum differentiation in departure times in the first column of results in Table 1 (Column 1), the expected outcome from location theory is confirmed: equilibrium prices and profits as well as average transportation and time costs of society are the largest as compared to any other differentiation context (in Columns 2 to 5). Column 1 also illustrates how the markups identified in

²⁸ Appendix C provides a detailed numerical analysis of several time games where the operational costs of the carriers vary along the time of day.

²⁹ These unit schedule delay costs’ values are slightly lower than those used in Van der Weijde et al. (2014) but closer to the willingness to pay found by Brey and Walker (2011) for air travellers.

Table 1: Market and regulated equilibria with null marginal costs

	$T_0^* = T_1^* = 10$ $h = 0$ $\widehat{\Phi} = 0$	$T_0^* = T_1^* = 10$ $h = 0.25$ $\widehat{\Phi} = 0$	$T_0 = 7 ; T_1^* = 10$ $h = 0.25$ $\widehat{\Phi} = -2.25$	$T_0 = 0 ; T_1^* = 10$ $h = 0.25$ $\widehat{\Phi} = -25$	$T_0^S = 5 ; T_1^S = 17$ $h^S = 0.33$ $\widehat{\Phi} = 6$
Facilities					
$(\widehat{\tau}_0^*, \widehat{\tau}_1^*)$	(370, 372)	(280.6, 266.4)	(279.9, 267.1)	(272.3, 274.7)	(252.9, 231.7)
$(\widehat{\Pi}_0^*, \widehat{\Pi}_1^*)$	(195, 195)	(154.5, 138.2)	(153.7, 139.0)	(146.0, 146.5)	(142.5, 119.3)
Carriers					
$(\widehat{p}_0^*, \widehat{p}_1^*)$	(510, 510)	(390.8, 369.2)	(389.8, 370.2)	(379.7, 380.3)	(353.8, 323.0)
$(\widehat{\pi}_0^*, \widehat{\pi}_1^*)$	(65, 65)	(51.5, 46.1)	(51.2, 46.3)	(48.7, 48.8)	(47.5, 39.8)
Consumers					
(D_0^*, D_1^*)	(0.5, 0.5)	(0.514, 0.486)	(0.513, 0.487)	(0.499, 0.501)	(0.522, 0.478)
Schedule delay cost [†]	35	35	28.1	26.5	17.5
Transportation cost [†]	10.8	5.3	5.3	5.3	4.8
Total Cost [†]	45.8	40.3	33.4	31.8	22.3

Notes: All simulations assume that $t \sim \mathcal{U}[0, 24]$, $\widehat{\beta} = 5$, $\widehat{\gamma} = 7$, $\frac{\theta}{2} = 130$, $\widehat{c}_0 = 10$, $\widehat{c}_1 = 8$, $\widehat{k}_0 = \widehat{k}_1 = 0$, $\widehat{\omega}_0 = 20$, $\widehat{\omega}_1 = 18$, $\widehat{K}_i = \widehat{F}_i = 0$ for $i = 0, 1$. [†]The average total schedule delay and transportation costs of consumers are computed from a regulator's perspective.

Section 2 affect the market equilibrium: the interplay between airlines' marginal costs and airports' commercial revenues lead to equal equilibrium fares for travellers and equal demands and profits for firms but differing equilibrium fees faced by airlines.³⁰

Columns 2 to 5 emphasize the role played by the location and schedule delay cost advantages in softening the increase in competition that results from a closer proximity between the two airports. As compared to Column 1, closer proximity between the two airports induce a lower average access cost for the travellers and lower equilibrium prices and profits. The latter result stresses that the competition effect dominates the demand effect when airport 0 is more centrally located and Fig. 2 explains why this occurs. Note however that prices and profits drop more at the less central airport in Column 2 as compared to Column 1 due to the location advantage of the central airport (about -28% for $(\widehat{\tau}_1, \widehat{p}_1)$ versus -24% for $(\widehat{\tau}_0, \widehat{p}_0)$, and -30% for $(\widehat{\Pi}_1, \widehat{\pi}_1)$ versus -21% for $(\widehat{\Pi}_0, \widehat{\pi}_0)$) while the reverse holds in Column 4 due to the schedule delay cost advantage at airport 1 that offsets the location advantage of airport 0 (about -25% for $(\widehat{\tau}_1, \widehat{p}_1)$ versus -26% for $(\widehat{\tau}_0, \widehat{p}_0)$, and -25% for $(\widehat{\Pi}_1, \widehat{\pi}_1)$ versus -26% for $(\widehat{\Pi}_0, \widehat{\pi}_0)$). A similar outcome (lower prices and asymmetric impacts across firms depending on their competitive advantages) would result from decreasing the transportation cost

³⁰Propositions 1, 2 and 4.2 are particularly easy to verify for Column 1. By way of example, note that $\Delta\widehat{c} = -\Delta\widehat{\omega} < -\Delta\widehat{\omega}/2$. Proposition 4.2 implies $D_0^{f,*} = D_1^{f,*}$, $\widehat{\Pi}_0^* = \widehat{\Pi}_1^*$ and $\widehat{\tau}_0^* < \widehat{\tau}_1^*$, which hold in Column 1.

parameter θ .³¹

Column 5 illustrates a two-airport system where the location of the central facility minimizes travellers' costs while the schedules are set to minimize travellers' time costs. As shown, the average time cost reduction associated with schedule regulation can be substantial. However, in a real two-airport region, the potential benefits of schedule regulation should be contrasted with the cost of implementing such regulation and scaled by the number of flights that occur.

5 Conclusion

This paper proposes a framework to analyze the rivalry in prices and in service times between facilities, for the case in which the facilities provide an input to downstream firms that sell the final service to consumers at a specific time of day. The model allows (i) asymmetries in the location of the facilities along the geographical space and in consumers' valuation of schedule delays, (ii) a general distribution of consumers' desired departure time and (iii) heterogeneous operational costs related to the scheduling of the service along the time of day by carriers. This setup is used to explain the competition in departure times and fares between two types of airlines (legacy *versus* low-cost carriers) facing different operational time costs along the day at their departure airport.

We find that accounting for the operational costs incurred by the airlines in the timing of the service is essential for identifying the level of differentiation in departure times in duopolistic airport systems: (i) when the operational cost of the airlines does not vary with the time of day, the flight is scheduled at the same time across airports, (ii) when this cost varies with the time of day and is *identical across airlines*, differences in equilibrium demands across airports suffice to generate (moderately) distinct departure times, (iii) when this cost varies with the time of day *and differs across airlines*, the flight schedules generally differ and the level of schedule differentiation is proportional to airlines' marginal time cost and to airports' demand. By letting the distribution of consumers' preferred times be general, we show explicitly how this distribution interacts with their valuation of schedule delays and carriers' marginal time cost to determine the equilibrium departure times, fares, fees, demands and profits at

³¹To see this, it suffices to compute $\partial \hat{p}_i^* / \partial \theta$, $\partial \hat{D}_i^* / \partial \theta$ and $\partial \hat{\pi}_i^* / \partial \theta$ for $i = 0, 1$ and to construct the counterpart to Fig. 2.

each airport.

The paper also identifies the markups of the firms operating in the vertical structure. In equilibrium, we establish that a higher per passenger commercial revenue at *one* airport induces a lower per passenger fee charged by *both* airports to their airline and a lower fare charged by *both* airlines at their departure airport. A lower marginal operational cost for *one* airline implies a higher fee charged at its departure airport, a lower fee at the other airport served by the rival airline and a lower fare at *both* airports. When an airport is more conveniently located for travellers, it can set a higher fee and its airline can charge a higher fare. Differentiation in departure times allows the airport and its airline to compete in an additional differentiation dimension that can reduce or strengthen the advantage in location.

This model could be extended in a number of directions. Stackelberg games would clearly refine our results regarding strategic behaviours. Considering heterogeneous transportation costs toward the facilities would allow to better characterize the role played by the location advantage. Allowing multiple departure times in the spirit of Lindsey and Tomaszewska (1999) would help to design realistic schedule policies to improve social welfare. Future research may want to consider price-elastic demands for the individual consumers as in Van der Weijde et al. (2014) and to conduct simulations based on alternative distributions of travellers' desired departure time as our simulations focus on the uniform shape.

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A Derivations assuming $T_0 \leq T_1$

A.1 Demands (4)

The net benefit of travelling from each facility for a consumer located at $x \in [0, 1]$ with desired time $\underline{t} \in [0, T_0]$ is given by:

$$\begin{aligned}\widehat{U}_0^\ell &= \widehat{U} - \widehat{p}_0 - \frac{\theta}{2}(x - h)^2 - \widehat{\gamma}(T_0 - t), \\ \widehat{U}_1^\ell &= \widehat{U} - \widehat{p}_1 - \frac{\theta}{2}(1 - x)^2 - \widehat{\gamma}(T_1 - t),\end{aligned}$$

where superscript ℓ designates magnitudes related to consumers with t at the LHS of T_0 . Solving $\widehat{U}_0^\ell - \widehat{U}_1^\ell$ for x leads to the indifferent consumer:

$$\tilde{x}^\ell = \frac{1}{\theta(1 - h)}[\widehat{p}_1 - \widehat{p}_0 + \widehat{\gamma}(T_1 - T_0)] + \frac{1 + h}{2}. \quad (\text{A.1})$$

As consumers are uniformly distributed with density one ($f(x) = 1$) on $x \in [0, 1]$, consume a single unit of the good, and given a distribution $\rho(t)$ of desired departure times, market demands for those with $t \in [0, T_0]$ are given by:

$$\begin{aligned}D_0^\ell &= \int_0^{T_0} \int_0^{\tilde{x}^\ell} f(x)\rho(t) \, dx \, dt = \tilde{x}^\ell \int_0^{T_0} \rho(t) \, dt \\ &= \left[p_1 - p_0 + \gamma(T_1 - T_0) + \frac{1 + h}{2} \right] m_\ell, \\ D_1^\ell &= m_\ell - D_0^\ell = \left[p_0 - p_1 - \gamma(T_1 - T_0) + \frac{1 - h}{2} \right] m_\ell,\end{aligned} \quad (\text{A.2})$$

where m_ℓ denotes the share of consumers with $t \in [0, T_0]$ as defined under (5). Following the same reasoning for the consumers with $\underline{t} \in]T_0, T_1[$, we get:

$$\begin{aligned}\widehat{U}_0^c &= \widehat{U} - \widehat{p}_0 - \frac{\theta}{2}(x - h)^2 - \widehat{\beta}(t - T_0), \\ \widehat{U}_1^c &= \widehat{U} - \widehat{p}_1 - \frac{\theta}{2}(1 - x)^2 - \widehat{\gamma}(T_1 - t), \\ \tilde{x}^c(t) &= \frac{1}{\theta(1 - h)}(\widehat{p}_1 - \widehat{p}_0) + \frac{\widehat{\beta}T_0 + \widehat{\gamma}T_1 - (\widehat{\gamma} + \widehat{\beta})t}{\theta(1 - h)} + \frac{(1 + h)}{2};\end{aligned} \quad (\text{A.3})$$

$$\begin{aligned}
D_0^c &= \int_{T_0}^{T_1} \tilde{x}^c(t) \rho(t) dt = \left[p_1 - p_0 + (\gamma T_1 + \beta T_0) + \frac{1+h}{2} \right] m_c - (\beta + \gamma) \bar{t}_c, \\
D_1^c &= m_c - D_0^c = \left[p_0 - p_1 - (\gamma T_1 + \beta T_0) + \frac{1-h}{2} \right] m_c + (\beta + \gamma) \bar{t}_c,
\end{aligned} \tag{A.4}$$

where superscript c designates magnitudes related to consumers with t between T_0 and T_1 , m_c is the share of consumers with $t \in]T_0, T_1[$ and $\bar{t}_c$ is the expected desired departure time defined under (5). Turning to those with $t \in [T_1, 24]$, we get:

$$\begin{aligned}
\hat{U}_0^r &= \hat{U} - \hat{p}_0 - \frac{\theta}{2}(x-h)^2 - \hat{\beta}(t-T_0), \\
\hat{U}_1^r &= \hat{U} - \hat{p}_1 - \frac{\theta}{2}(1-x)^2 - \hat{\beta}(t-T_1), \\
\tilde{x}^r &= \frac{1}{\theta(1-h)} [\hat{p}_1 - \hat{p}_0 - \hat{\beta}(T_1 - T_0)] + \frac{(1+h)}{2};
\end{aligned} \tag{A.5}$$

$$\begin{aligned}
D_0^r &= \tilde{x}^r \int_{T_1}^{24} \rho(t) dt = \left[p_1 - p_0 - \beta(T_1 - T_0) + \frac{1+h}{2} \right] m_r, \\
D_1^r &= \left[p_0 - p_1 + \beta(T_1 - T_0) + \frac{1-h}{2} \right] m_r,
\end{aligned} \tag{A.6}$$

where superscript r designates magnitudes related to consumers with t at the RHS of T_1 and m_r is the share of consumers with $t \in [T_1, 24]$ as defined under (5). Aggregating the demands over the entire $[0, 24]$ segment – setting $D_0 = D_0^\ell + D_0^c + D_0^r$, using (A.2), (A.4) and (A.6) and recalling that $m_\ell + m_c + m_r = 1$ – we get D_0 in (4). Further using the covered condition, we get $D_1 = 1 - D_0$.

Focusing on the SDC difference term (5), by collecting the T_0 and T_1 terms and dispatching the average central terms $\bar{t}_c$, we obtain:

$$\Phi(\mathbf{T}) = \underbrace{(\gamma m_\ell + \gamma m_c - \beta m_r) T_1 - \gamma \bar{t}_c}_{C_1^M} - \underbrace{[(\gamma m_\ell - \beta m_c - \beta m_r) T_0 + \beta \bar{t}_c]}_{C_0^M},$$

where C_i^M is the average *normalized* schedule delay cost difference related to each departure time T_i for $i = 0, 1$. Note that when $t \sim \mathcal{U}[0, 24]$ then:

$$\Phi^U(\mathbf{T}) = \frac{\beta + \gamma}{48} T_1^2 + \beta(T_0 - T_1) - \frac{\beta + \gamma}{48} T_0^2. \tag{A.7}$$

Moreover, $\Phi^U(\mathbf{T})$ is identical whether $T_0 \leq T_1$ or $T_0 \geq T_1$. Next, given that $\Phi(\mathbf{T}) \equiv [\hat{C}_1^M - \hat{C}_0^M] / \theta(1 - h)$, by using (4), calculating $\partial D_0 / \partial \eta > 0$ for $\eta = \{h, \theta\}$ and rearranging the expressions, we get:

$$\begin{aligned} \frac{\partial D_0(\mathbf{p}, \mathbf{T})}{\partial h} > 0 \quad \text{iff} \quad \frac{\theta}{2}(1 - h)^2 > (\hat{p}_0 + \hat{C}_0^M) - (\hat{p}_1 + \hat{C}_1^M), \\ \frac{\partial D_0(\mathbf{p}, \mathbf{T})}{\partial \theta} > 0 \quad \text{iff} \quad \hat{p}_1 + \hat{C}_1^M < \hat{p}_0 + \hat{C}_0^M, \end{aligned} \quad (\text{A.8})$$

where $(\hat{p}_0 + \hat{C}_0^M) - (\hat{p}_1 + \hat{C}_1^M) \equiv -(\hat{p}_1 - \hat{p}_0) - \hat{\Phi}(\mathbf{T})$ and $\partial D_1(\mathbf{p}, \mathbf{T}) / \partial \eta = -\partial D_0(\mathbf{p}, \mathbf{T}) / \partial \eta$. The link between (A.8) and the results under (2) should be clear.

A.2 Covered market condition (3)

To establish market conditions which guarantee that $0 < \tilde{x}(t) < 1$ for $t \in [0, 24]$, use (A.1), (A.3) and (A.5) and note that $\tilde{x}^r(t) \leq \tilde{x}^c(t) \leq \tilde{x}^\ell(t)$. Then, setting $0 < \tilde{x}^r(t) \leq \tilde{x}^\ell(t) < 1$ yields (3). Note that this condition could be further relaxed to allow all travellers with $t \geq T_1$ (resp. $t \leq T_0$) to depart facility 1 (resp. facility 0). These situations are not analyzed here but Appendix A.7 discusses the parameter conditions under which these cases are more likely to happen.

A.3 Proof of Lemma 1

To compute $\partial \Phi(\mathbf{T}) / \partial T_i$ for $i = 0, 1$, note first that:

$$\begin{aligned} \frac{\partial m_l}{\partial T_0} &= \rho(T_0), & \frac{\partial m_c}{\partial T_0} &= -\rho(T_0), & \frac{\partial m_r}{\partial T_0} &= 0, & \frac{\partial \bar{t}_c}{\partial T_0} &= -T_0 \rho(T_0), \\ \frac{\partial m_l}{\partial T_1} &= 0, & \frac{\partial m_c}{\partial T_1} &= \rho(T_1), & \frac{\partial m_r}{\partial T_1} &= -\rho(T_1), & \frac{\partial \bar{t}_c}{\partial T_1} &= T_1 \rho(T_1). \end{aligned}$$

Using the above, we obtain:

$$\frac{\partial \Phi(\mathbf{T})}{\partial T_0} = \beta - (\beta + \gamma)m_\ell, \quad \frac{\partial \Phi(\mathbf{T})}{\partial T_1} = \gamma - (\beta + \gamma)m_r, \quad (\text{A.9})$$

$$\frac{\partial^2 \Phi(\mathbf{T})}{\partial T_0^2} = -(\beta + \gamma)\rho(T_0), \quad \frac{\partial^2 \Phi(\mathbf{T})}{\partial T_1^2} = (\beta + \gamma)\rho(T_1), \quad (\text{A.10})$$

where we use $m_c + m_r = 1 - m_\ell$ to get $\partial\Phi(\mathbf{T})/\partial T_0$ and $m_\ell + m_c = 1 - m_r$ to get $\partial\Phi(\mathbf{T})/\partial T_1$. Setting each first derivative in (A.9) equal to zero leads to the departure times $T_0^* = T_1^* = F^{-1}[\beta/(\beta + \gamma)]$, where we use $\gamma/(\beta + \gamma) = 1 - \beta/(\beta + \gamma)$ to obtain T_1^* .

A.4 Existence of the departure time equilibrium when $\rho(t) = \mathcal{U}[0, 24]$

When $\rho(t) = \mathcal{U}[0, 24]$, by using $\Phi^U(T_0, T_1)$ in (A.7) and demands (13) in carrier profits (15), these profits can be shown to be two quartic functions in T_i that reads:

$$\pi_i(T_0, T_1) = f_i + e_i T_i + d_i T_i^2 + b T_i^3 + a T_i^4, \quad i = 0, 1, \quad (\text{A.11})$$

where coefficients $a = (\beta + \gamma)^2/20736$ and $b = -\beta(\beta + \gamma)/216$ are identical across carriers and depend only on the (normalized) unit schedule delay costs, coefficients d_i, e_i and f_i are carrier-specific and depend on T_{-i} and other parameters of the model.³² Let $T_i, T_{-i} \in \mathbb{R}$. The coefficient of leading term T_i^4 being positive ($a > 0$), we deduce that $\lim_{T_i \rightarrow \pm\infty} \pi_i(T_i, T_{-i}) = +\infty$. For a unique and interior profit-maximizing departure time $T_i^* \in]0, 24[$ to exist for $i = 0, 1$, the profit functions need to be locally concave on $[0, 24]$. They must be ‘W-shaped’ and thus possess three critical points, two of which are minima and one is a local maximum. Below we provide a set of parameter restrictions which ensure that this local maximum exists, is unique and interior on the $[0, 24]$ time interval and yields positive profits for $i = 0, 1$. To establish this, we impose

$$\begin{aligned} (i) \quad & \frac{\partial^2 \pi_i(T_0, T_1)}{\partial T_i^2} < 0, \quad \forall T_i \in]T_{i,a}^{**}, T_{i,b}^{**}[\subset \mathbb{R} \text{ with } T_i = 0 \in]T_{i,a}^{**}, T_{i,b}^{**}[, \\ (ii) \quad & \left. \frac{\partial \pi_i(T_0, T_1)}{\partial T_i} \right|_{T_i=0} > 0, \quad \left. \frac{\partial \pi_i(T_0, T_1)}{\partial T_i} \right|_{T_i=24} < 0 \quad \text{and} \\ (iii) \quad & \pi_i(T_0, T_1)|_{T_i=0} > 0 \end{aligned}$$

for $i = 0, 1$. Condition (i) ensures the existence of a locally quasi-concave profit function for each carrier over the real interval $[T_{i,a}^{**}, T_{i,b}^{**}] \subset \mathbb{R}$ which includes $T_i = 0$. On this interval, the *first derivative* of the profit function is decreasing. Condition (ii) guarantees that this first derivative is: positive at $T_i = 0$, null at the local maximum

³²The detailed expressions of this Appendix are available upon request.

T_i^* and negative on $]T_i^*, 24]$. This necessarily implies the existence of a single stationary point $T_i^* \in]0, 24[$ which is the profit-maximizing departure time for each carrier. Condition (iii) makes sure that the maximized profit for $i = 0, 1$ is positive because $\pi_i(T_0, T_1)|_{T_i=T_i^*}$ is above $\pi_i(T_0, T_1)|_{T_i=0} > 0$.

Starting with (i), by (A.11) we have:

$$\partial^2 \pi_i / \partial T_i^2 = \bar{d}_i + \bar{b}T_i + \bar{a}T_i^2, \quad i = 0, 1,$$

where the coefficient of the leading terms T_i^2 is positive ($\bar{a} = (\beta + \gamma)^2/1728 > 0$). We need parameter restrictions which ensure the existence of two real solutions (or roots) to $\partial^2 \pi_i(T_0, T_1)/\partial T_i^2 = 0$, denoted $T_{i,a}^{**}$ and $T_{i,b}^{**}$ and such that $T_{i,a}^{**} < T_{i,b}^{**}$ and $\partial^2 \pi_i / \partial T_i^2 < 0$, $\forall T_i \in]T_{i,a}^{**}, T_{i,b}^{**}[$. These roots are the inflection points of the quartic profit functions. A quadratic function has two real solutions if its discriminant satisfies $\Delta_i = \bar{b}^2 - 4\bar{a}\bar{d}_i > 0$. As $\bar{a}, \bar{b}^2 > 0$, we can impose $\bar{d}_i < 0$. Term $\bar{d}_i$ can be shown to be itself a quadratic function in T_{-i} such that $\bar{d}_i = \bar{\bar{d}}_i + \bar{\bar{b}}T_{-i} + \bar{\bar{a}}T_{-i}^2$ with $\bar{\bar{a}} = -(\beta + \gamma)/5184 < 0$. Let the discriminant of $\bar{d}_i$ be given by $\Delta_{\bar{d}_i}$. Setting $\Delta_{\bar{d}_i} < 0$ for $i = 0, 1$ guarantees that $\text{sgn}(\bar{d}_i) = \text{sgn}(\bar{\bar{a}})$ and implies $\bar{d}_i < 0, \forall T_{-i} \in \mathbb{R}$. This restriction leads to the following first condition:

$$\begin{aligned} \beta, \gamma > 0, \quad 0 \leq h < 1, \quad T_0, T_1 \in \mathbb{R} \quad \text{and} \\ \frac{72\beta^2 - (\beta + \gamma)(3 + h)}{2(\beta + \gamma)} < \Delta\tilde{c} < \frac{-72\beta^2 + (\beta + \gamma)(3 - h)}{2(\beta + \gamma)}. \end{aligned} \quad (\text{A.12})$$

Next, solving $\partial^2 \pi_i(T_0, T_1)/\partial T_i^2 = 0$ for $i = 0, 1$ yields:

$$T_{0,a,b}^{**} = \frac{24\beta}{\beta + \gamma} \mp \frac{\sqrt{A}}{\sqrt{3}(\beta + \gamma)}, \quad T_{1,a,b}^{**} = \frac{24\beta}{\beta + \gamma} \mp \frac{\sqrt{A'}}{\sqrt{3}(\beta + \gamma)}$$

where $T_{i,a}^{**} < T_{i,b}^{**}$ because

$$\begin{aligned} A &= 24 [24\beta^2 + (3 + h + 2\Delta\tilde{c})(\beta + \gamma)] - 48\beta(\beta + \gamma)T_1 + (\beta + \gamma)^2T_1^2 > 0, \\ A' &= 24 [24\beta^2 + (3 - h - 2\Delta\tilde{c})(\beta + \gamma)] - 48\beta(\beta + \gamma)T_0 + (\beta + \gamma)^2T_0^2 > 0. \end{aligned}$$

We can further show that $T_{i,a}^{**} < 0$ and $T_{i,b}^{**} > 0$ when (A.12) holds so that $T_i = 0 \in]T_{i,a}^{**}, T_{i,b}^{**}[$ for $i = 0, 1$.

For satisfying (ii), note first that $\partial \pi_i(T_0, T_1)/\partial T_i|_{T_i=0} = e_i$ in (A.11) and we can show that $e_i = d'_i + b'T_{-i} + a'T_{-i}^2$ with $a' = \beta(\beta + \gamma)/216 > 0$. Then, the first

derivative of the profit function evaluated at $T_i = 0$ for $i = 0, 1$ appears to be, again, a quadratic function in T_{-i} and the coefficient of the leading term T_{-i}^2 – identical across carriers – is positive. To ensure that $e_i > 0$, $\forall T_{-i} \in \mathbb{R}$, we require its discriminant to be lower than zero so that $\text{sgn}(e_i) = \text{sgn}(a')$. The same reasoning applies to $\partial\pi_i(T_0, T_1)/\partial T_i|_{T_i=24} < 0$. In the latter case, the coefficient of the leading term T_{-i}^2 can be shown to be negative ($a' = -\gamma(\beta + \gamma)/216 < 0$) and so must be set the related discriminant. Combining the resulting restrictions for $i = 0, 1$ yields to the following set of conditions:

$$\begin{aligned} \frac{8\gamma^3}{3(\beta + \gamma)} - \frac{(3 + h + 2\Delta\tilde{c})\gamma}{9} < k_0 < -\frac{8\beta^3}{3(\beta + \gamma)} + \frac{(3 + h + 2\Delta\tilde{c})\beta}{9}, \\ \frac{8\gamma^3}{3(\beta + \gamma)} - \frac{(3 - h - 2\Delta\tilde{c})\gamma}{9} < k_1 < -\frac{8\beta^3}{3(\beta + \gamma)} + \frac{(3 - h - 2\Delta\tilde{c})\beta}{9}. \end{aligned} \quad (\text{A.13})$$

Turning to (iii), $\pi_i(T_0, T_1)|_{T_i=0}$ is clearly equivalent to f_i in (A.11), where f_i can be shown to be a fourth-degree polynomial in T_{-i} which does not depend on k_i . Further setting the fixed time costs $K_i = 0$ for simplicity, when (A.12) holds, we can show that $f_i > 0$ if:³³

$$0 < \beta < \frac{1}{24} \quad \text{and} \quad \gamma > 0 \quad \text{or when} \quad \beta > \frac{1}{24} \quad \text{and} \quad \gamma > \beta(24\beta - 1). \quad (\text{A.14})$$

Replacing $\beta, \gamma > 0$ in (A.12) by (A.14) guarantees that $\pi_i(T_0, T_1)|_{T_i=0} > 0$ for $i = 0, 1$.

To sum up, (A.12), (A.13) and (A.14) ensure the existence a unique profit-maximizing $T_i^* \in]0, 24[$ for each carrier with positive profits. Notice that these conditions are expressed in terms of scaled parameters. Multiplication by $\theta(1 - h)$ leads to their unscaled counterpart which can be used in Section 4 and Appendix C. These interior solutions do not guarantee the *uniqueness of the Nash equilibrium in departure times*. These additional conditions are excessively technical. Alternatively, one can simply use the parameter values which satisfy the existence conditions, solve system $\partial\pi_i(T_0, T_1)/\partial T_i = 0$ for $i = 0, 1$ and draw the time reaction functions that fall within the $[0, 24]^2$ space (one per carrier). This approach is more insightful and allows to graphically explore the uniqueness of the Nash equilibrium and its local stability. Such numerical analysis is applied in Appendix C.

³³This result is obtained with Mathematica after solving: `Reduce[ $f_i > 0 \ \&\& \ (\text{A.12})$ ]`. Plotting f_i for $i = 0, 1$ under the above parameter restrictions yields a quartic function which is positive $\forall T_{-i} \in \mathbb{R}$.

A.5 Differentiation of the FOCs (20) w.r.t. k_i

To show that $dT_i^*/dk_i < 0$, rearrange first (20) as

$$G(T_i, k_i) \equiv F(T_i) + \frac{3k_i}{2(\beta + \gamma)D_i^*(\mathbf{T}, \boldsymbol{\tau})} - \frac{\beta}{\beta + \gamma} = 0.$$

The implicit function theorem implies that $dT_i/dk_i = -(G_{k_i}/G_{T_i})$. After some algebra, we get:

$$\frac{dT_i}{dk_i} \equiv -\frac{G_{k_i}}{G_{T_i}} = \frac{3D_i^*(\mathbf{T}, \boldsymbol{\tau})}{3k_i \frac{\partial D_i^*(\mathbf{T}, \boldsymbol{\tau})}{\partial T_i} - \rho(T_i)2(\beta + \gamma)D_i^{*,2}(\mathbf{T}, \boldsymbol{\tau})}. \quad (\text{A.15})$$

Next, we deduced from (13) that $\partial D_i^*(\mathbf{T}, \boldsymbol{\tau})/\partial T_i = \frac{1}{3}\Phi_{T_i}(\mathbf{T})$ for $i = 0, 1$ and by Lemma 1 we have $\Phi_{T_0}(\mathbf{T}) = \beta - (\beta + \gamma)m_\ell$ and $\Phi_{T_1}(\mathbf{T}) = \gamma - (\beta + \gamma)m_r$. Assuming the existence of an interior and unique Nash equilibrium $\mathbf{T}^* = (T_0^*, T_1^*)$, FOCs (19) satisfy

$$\beta - (\beta + \gamma)m_\ell = \frac{3k_0}{2D_0^*(\mathbf{T}^*, \boldsymbol{\tau})} \quad \text{and} \quad \gamma - (\beta + \gamma)m_r = \frac{3k_1}{2D_1^*(\mathbf{T}^*, \boldsymbol{\tau})}, \quad (\text{A.16})$$

Thus, $\partial D_i^*(\mathbf{T}, \boldsymbol{\tau})/\partial T_i = \frac{1}{3}\Phi_{T_i}(\mathbf{T})$ becomes $\partial D_i^*(\mathbf{T}^*, \boldsymbol{\tau})/\partial T_i = k_i/2D_i^*(\mathbf{T}^*, \boldsymbol{\tau})$. Replacing the latter expressions in (A.15) for $i = 0, 1$, the denominator of (A.15) becomes:

$$\frac{3k_i^2}{2D_i^*(\mathbf{T}^*, \boldsymbol{\tau})} - \rho(T_i^*)2(\beta + \gamma)D_i^{*,2}(\mathbf{T}^*, \boldsymbol{\tau}), \quad i = 0, 1. \quad (\text{A.17})$$

Assuming that (23) is satisfied in equilibrium for $i = 0, 1$, the above expression is negative. That is, setting (A.17) < 0 yields (23). Thus, given that the numerator of (A.15) is positive and its denominator is negative in equilibrium, we deduce that $dT_i^*/dk_i < 0$.

A.6 Schedule differentiation with time-varying costs

If the solution to the system of FOCs (20) or (B.7) yields a unique and interior Nash equilibrium $\mathbf{T}^*$ and if the market is covered, i.e., if $D_0^*(\mathbf{T}^*, \boldsymbol{\tau}) = 1 - D_1^*(\mathbf{T}^*, \boldsymbol{\tau}) \in]0, 1[$, we can use these FOCs to establish the conditions under which the equilibrium demands and carriers' marginal time cost lead to $T_0^* \lesseqgtr T_1^*$. That is, setting $F(T_0^*) \lesseqgtr F(T_1^*)$

yields:

$$-\frac{k_0}{D_0^*(\mathbf{T}^*, \boldsymbol{\tau})} \leq -\frac{k_1}{D_1^*(\mathbf{T}^*, \boldsymbol{\tau})} \Leftrightarrow -\frac{1 - D_0^*(\mathbf{T}^*, \boldsymbol{\tau})}{D_0^*(\mathbf{T}^*, \boldsymbol{\tau})} \leq -\frac{k_1}{k_0}, \quad (\text{A.18})$$

where, slightly abusing of notation, $D_i^*(\mathbf{T}^*, \boldsymbol{\tau})$ for $i = 0, 1$ is (13) if we set $F(T_0^*) \leq F(T_1^*)$ and (B.5) if we impose $F(T_0^*) \geq F(T_1^*)$. We can now analyze the above inequalities when $k_0, k_1 > 0$, $k_0 > 0$ and $k_1 < 0$, $k_0 < 0$ and $k_1 > 0$, and $k_0, k_1 < 0$. Rearranging (A.18) yields:

$$\begin{aligned} (i) \text{ if } k_0, k_1 > 0, \quad D_0^*(\mathbf{T}^*, \boldsymbol{\tau}) &\leq \frac{k_0}{k_0 + k_1} \Rightarrow T_0^* \leq T_1^*, \\ (ii) \text{ if } k_0 < 0, k_1 < 0, \quad D_0^*(\mathbf{T}^*, \boldsymbol{\tau}) &\leq \frac{|k_0|}{|k_0| + |k_1|} \Rightarrow T_0^* \geq T_1^*, \\ (iii) \text{ if } k_0 > 0 \text{ and } k_1 < 0, \quad \frac{1 - D_0^*(\mathbf{T}^*, \boldsymbol{\tau})}{D_0^*(\mathbf{T}^*, \boldsymbol{\tau})} &> -\frac{|k_1|}{k_0} \Rightarrow T_0^* < T_1^*, \\ (iv) \text{ if } k_0 < 0 \text{ and } k_1 > 0, \quad \frac{1 - D_0^*(\mathbf{T}^*, \boldsymbol{\tau})}{D_0^*(\mathbf{T}^*, \boldsymbol{\tau})} &> -\frac{k_1}{|k_0|} \Rightarrow T_0^* > T_1^*, \end{aligned}$$

where, slightly abusing of notation, $D_0^*(\mathbf{T}^*, \boldsymbol{\tau})$ denotes (13) if $F(T_0^*) \leq F(T_1^*)$ and (B.5) if $F(T_0^*) \geq F(T_1^*)$. Hence, in the cases (i) or (ii), the schedule differentiation pattern depends on whether the equilibrium demand $D_0^*(\mathbf{T}^*, \boldsymbol{\tau})$ is lower than, equal to or larger than the marginal time cost shares $k_0/(k_0 + k_1)$ or $|k_0|/(|k_0| + |k_1|)$. Cases (iii) and (iv) highlight that $k_0 > 0$ and $k_1 < 0$ necessarily leads to $T_0^* < T_1^*$ while $k_0 < 0$ and $k_1 > 0$ implies $T_0^* > T_1^*$.

A.7 Substituting the equilibrium fees in the equilibrium fares and in the covered market condition

Substituting the fees (25) in the fares (8) and rearranging yields:

$$\begin{aligned} p_0^{**} &= \frac{1}{9}[(5c_0 + 4c_1) - (5\omega_0 + 4\omega_1)] + \frac{18 + 2h}{9} + \frac{4}{9}\Phi(\mathbf{T}), \\ p_1^{**} &= \frac{1}{9}[(4c_0 + 5c_1) - (4\omega_0 + 5\omega_1)] + \frac{18 - 2h}{9} - \frac{4}{9}\Phi(\mathbf{T}). \end{aligned} \quad (\text{A.19})$$

Hence, in equilibrium, a lower marginal operational cost for *one* carrier at a facility induces a lower fare at *both* facilities; a higher per passenger commercial revenue at *one* facility reduces fare charged by *both* carriers at their departure facility. Substituting the fees (25) in carrier markups, $m_i^{**} = p_i^* - (c_i + \tau_i^*)$ for $i = 0, 1$ leads to:

$$\begin{aligned} m_0^{**} &= \frac{1}{9}[(c_1 - c_0) + (\omega_0 - \omega_1)] + \frac{9+h}{18} + \frac{1}{9}\Phi(\mathbf{T}), \\ m_1^{**} &= \frac{1}{9}[(c_0 - c_1) + (\omega_1 - \omega_0)] + \frac{9-h}{18} - \frac{1}{9}\Phi(\mathbf{T}). \end{aligned} \quad (\text{A.20})$$

Thus, a lower marginal operational cost for *one* carrier induces a higher markup for that carrier and a lower markup for the rival carrier serving the other facility. Similarly, a higher per passenger commercial revenue at *one* facility induces a higher markup for its carrier and a lower markup for the rival carrier serving the other facility. Furthermore, by using (A.20), we can rewrite (10) in terms of commercial revenues as:

$$m_0^{**} - m_1^{**} > 0 \quad \text{iff} \quad \Delta c > -\frac{h}{2} - \Phi(\mathbf{T}) - \Delta\omega, \quad (\text{A.21})$$

where $\Delta c = c_1 - c_0$ and $\Delta\omega = \omega_0 - \omega_1$. Recall that c_i and ω_i for $i = 0, 1$ are $\widehat{c}_i$ and $\widehat{\omega}_i$ divided by $\theta(1-h)$.

Moreover, when the marginal time cost of the carriers is null ($\widehat{k}_0 = \widehat{k}_1 = 0$) or when the departure times are given to the carriers, we can substitute the fares (A.19) in (3) and express *explicitly* the covered market condition in terms of the exogenous parameters of the model. Recalling that $\widehat{p}_i^{**} = \theta(1-h)p_i^{**}$, note first that:

$$\widehat{p}_1^{**} - \widehat{p}_0^{**} = \frac{1}{9}[(\widehat{c}_1 - \widehat{c}_0) + (\widehat{\omega}_0 - \widehat{\omega}_1) - 8\widehat{\Phi}(\mathbf{T}) - 4h\theta(1-h)]. \quad (\text{A.22})$$

Substituting (A.22) in (3), the covered market condition (3) becomes:

$$\begin{aligned} & -\frac{\theta}{2}(1-h)(9+h) < \\ & \Delta\widehat{c} + \Delta\widehat{\omega} - 8\widehat{\Phi}(\mathbf{T}) - 9\widehat{\beta}(T_1 - T_0) \leq \Delta\widehat{c} + \Delta\widehat{\omega} - 8\widehat{\Phi}(\mathbf{T}) + 9\widehat{\gamma}(T_1 - T_0) \\ & < \frac{\theta}{2}(1-h)(9-h), \end{aligned} \quad (\text{A.23})$$

with $\Delta\widehat{c} = \widehat{c}_1 - \widehat{c}_0$, $\Delta\widehat{\omega} = \widehat{\omega}_0 - \widehat{\omega}_1$. Further replacing $\mathbf{T}$ by $\mathbf{T}^*$, one can get rid of the endogenous departure times. Sticking to the exogenous departure times' case, we

notice that the market is more likely to be covered when θ is large, $h, \hat{\beta}, \hat{\gamma}$ are low, $\Delta\hat{c} \approx -\Delta\hat{\omega}$ and when $\hat{\Phi}(\mathbf{T}) \approx 0$.

A.8 Social time cost minimization (29)

By using the first derivatives (31), we deduce the following second and cross-partial derivatives:

$$\begin{aligned}\frac{\partial^2 \widehat{SC}^R(\mathbf{T})}{\partial T_0^2} &= (\hat{\beta} + \hat{\gamma})\rho(T_0) - \frac{\hat{\beta}^2}{\hat{\beta} + \hat{\gamma}}\rho(\tilde{t}), & \frac{\partial^2 \widehat{SC}^R(\mathbf{T})}{\partial T_1^2} &= (\hat{\beta} + \hat{\gamma})\rho(T_1) - \frac{\hat{\gamma}^2}{\hat{\beta} + \hat{\gamma}}\rho(\tilde{t}), \\ \frac{\partial^2 \widehat{SC}^R(\mathbf{T})}{\partial T_i \partial T_{-i}} &= -\frac{\hat{\beta}\hat{\gamma}}{\hat{\beta} + \hat{\gamma}}\rho(\tilde{t}) \text{ for } i = 0, 1.\end{aligned}$$

Setting the second derivatives larger than 0 and rearranging yields:

$$\frac{\partial^2 \widehat{SC}^R(\mathbf{T})}{\partial T_0^2} = \frac{(\hat{\beta} + \hat{\gamma})^2}{\hat{\beta}^2} > \frac{\rho(\tilde{t})}{\rho(T_0)}, \quad \frac{\partial^2 \widehat{SC}^R(\mathbf{T})}{\partial T_1^2} = \frac{(\hat{\beta} + \hat{\gamma})^2}{\hat{\gamma}^2} > \frac{\rho(\tilde{t})}{\rho(T_1)}.$$

Minimization requires the determinant of the Hessian matrix to be positive, i.e.,

$$2\hat{\beta}\hat{\gamma}\rho(T_0)\rho(T_1) + \hat{\beta}^2\rho(T_1)[\rho(T_0) - \rho(\tilde{t})] + \hat{\gamma}^2\rho(T_0)[\rho(T_1) - \rho(\tilde{t})] > 0.$$

From the above, we deduce that if $\rho(t) \sim \mathcal{U}[0, 24]$, then $\rho(t) = 1/24, \forall t \in [0, 24]$ and (T_0^R, T_1^R) in (32) is a unique minimum over the feasible times. Note that, by using this uniform distribution in (30), we obtain the following expression for $\widehat{SC}^R(\mathbf{T})$:

$$\widehat{SC}^R = \frac{\hat{\gamma}(2\hat{\beta} + \hat{\gamma})T_0^2 + \hat{\beta}(\hat{\beta} + 2\hat{\gamma})T_1^2 - 48\hat{\beta}(\hat{\beta} + \hat{\gamma})T_1 - 2\hat{\beta}\hat{\gamma}T_0T_1 + 576\hat{\beta}(\hat{\beta} + \hat{\gamma})}{48(\hat{\beta} + \hat{\gamma})} + \hat{K}^R(\mathbf{T}).$$

Moreover, note that $a_0 = \frac{12(\hat{\beta}+2\hat{\gamma})}{\hat{\gamma}(\hat{\beta}+\hat{\gamma})} > 0$, $b_0 = a_1 = \frac{12}{\hat{\beta}+\hat{\gamma}} > 0$ and $b_1 = \frac{12(2\hat{\beta}+\hat{\gamma})}{\hat{\beta}(\hat{\beta}+\hat{\gamma})} > 0$ and that the following identity leads to T_1^R in (32): $12 \left[1 + \frac{\hat{\beta}}{\hat{\beta}+\hat{\gamma}} \right] \equiv 12 \left[\frac{2\hat{\beta}}{\hat{\beta}+\hat{\gamma}} + \frac{\hat{\gamma}}{\hat{\beta}+\hat{\gamma}} \right]$.

B Derivations assuming $T_0 \geq T_1$

In this Appendix, we denote the counterpart of the expressions derived assuming $T_0 \leq T_1$ with the superscript ‘sym’.

B.1 Counterpart of demands (4)

When $T_0 \geq T_1$, $D_0^{\ell, \text{sym}}$ and $D_0^{r, \text{sym}}$ are given by replacing the shares m_ℓ in (A.2) and m_r in (A.6) by $m_\ell^{\text{sym}} = \int_0^{T_1} \rho(t)dt$ and $m_r^{\text{sym}} = \int_{T_0}^{24} \rho(t)dt$, respectively. Regarding $D_0^{c, \text{sym}}$, applying the same reasoning as in Appendix A.1, we get:

$$\begin{aligned}\widehat{U}_0^c &= \widehat{U} - \widehat{p}_0 - \frac{\theta}{2}(x - h)^2 - \widehat{\gamma}(T_0 - t), \\ \widehat{U}_1^c &= \widehat{U} - \widehat{p}_1 - \frac{\theta}{2}(1 - x)^2 - \widehat{\beta}(t - T_1), \\ \tilde{x}^{c, \text{sym}}(t) &= \frac{1}{\theta(1 - h)}(\widehat{p}_1 - \widehat{p}_0) - \frac{(\widehat{\beta}T_1 + \widehat{\gamma}T_0) - (\widehat{\beta} + \widehat{\gamma})t}{\theta(1 - h)} + \frac{(1 + h)}{2}; \\ D_0^{c, \text{sym}} &= \left[p_1 - p_0 - (\beta T_1 + \gamma T_0) + \frac{1 + h}{2} \right] m_c^{\text{sym}} + (\beta + \gamma) \bar{t}_c^{\text{sym}}, \\ D_1^{c, \text{sym}} &= \left[p_0 - p_1 + (\beta T_1 + \gamma T_0) + \frac{1 - h}{2} \right] m_c^{\text{sym}} - (\beta + \gamma) \bar{t}_c^{\text{sym}},\end{aligned}\tag{B.1}$$

where $m_c^{\text{sym}} = \int_{T_1}^{T_0} \rho(t)dt$ and $\bar{t}_c^{\text{sym}} = \int_{T_1}^{T_0} t \rho(t)dt$. Given that $D_0^{\text{sym}} = D_0^{l, \text{sym}} + D_0^{c, \text{sym}} + D_0^{r, \text{sym}}$ and $D_1^{\text{sym}} = 1 - D_1^{\text{sym}}$, we have the following aggregate demands:

$$\begin{aligned}D_0^{\text{sym}} &= p_1 - p_0 + \frac{1 + h}{2} + \Phi^{\text{sym}}(\mathbf{T}), \\ D_1^{\text{sym}} &= p_0 - p_1 + \frac{1 - h}{2} - \Phi^{\text{sym}}(\mathbf{T}),\end{aligned}\tag{B.2}$$

where the counterpart of the SDC difference (5) is given by:

$$\Phi^{\text{sym}}(\mathbf{T}) = \gamma(T_1 - T_0)m_l^{\text{sym}} - (\beta T_1 + \gamma T_0)m_c^{\text{sym}} - \beta(T_1 - T_0)m_r^{\text{sym}} + (\beta + \gamma)\bar{t}_c^{\text{sym}},\tag{B.3}$$

where $m_l^{\text{sym}} + m_c^{\text{sym}} + m_r^{\text{sym}} = 1$.

B.2 Counterpart of the covered market condition (3)

Following the same steps as in Appendix A.2, the covered market condition (3) is:

$$-\frac{\theta}{2}(1-h^2) < \hat{p}_1 - \hat{p}_0 - \hat{\gamma}(T_0 - T_1) \leq \hat{p}_1 - \hat{p}_0 + \hat{\beta}(T_0 - T_1) < \frac{\theta}{2}(1-h)^2. \quad (\text{B.4})$$

B.3 Proof of Lemma 1 when $T_0 \geq T_1$

Consider $\Phi^{sym}(\mathbf{T})$ in (B.3). To compute $\partial\Phi^{sym}(\mathbf{T})/\partial T_i$ for $i = 0, 1$, use terms m_l^{sym} , m_c^{sym} , m_r^{sym} and $\bar{t}_c^{sym}$ defined in Appendix B.1. Note first that:

$$\begin{aligned} \frac{\partial m_l^{sym}}{\partial T_0} &= 0, & \frac{\partial m_c^{sym}}{\partial T_0} &= \rho(T_0), & \frac{\partial m_r^{sym}}{\partial T_0} &= -\rho(T_0), & \frac{\partial \bar{t}_c^{sym}}{\partial T_0} &= T_0 \rho(T_0), \\ \frac{\partial m_l^{sym}}{\partial T_1} &= \rho(T_1), & \frac{\partial m_c^{sym}}{\partial T_1} &= -\rho(T_1), & \frac{\partial m_r^{sym}}{\partial T_1} &= 0, & \frac{\partial \bar{t}_c^{sym}}{\partial T_1} &= -T_1 \rho(T_1). \end{aligned}$$

Using the above, we obtain:

$$\begin{aligned} \frac{\partial \Phi^{sym}(\mathbf{T})}{\partial T_0} &= (\beta + \gamma)m_r^{sym} - \gamma, & \frac{\partial \Phi^{sym}(\mathbf{T})}{\partial T_1} &= (\beta + \gamma)m_\ell^{sym} - \beta, \\ \frac{\partial^2 \Phi^{sym}(\mathbf{T})}{\partial T_0^2} &= -(\beta + \gamma)\rho(T_0), & \frac{\partial^2 \Phi^{sym}(\mathbf{T})}{\partial T_1^2} &= (\beta + \gamma)\rho(T_1). \end{aligned}$$

where we use $m_\ell^{sym} + m_c^{sym} = 1 - m_r^{sym}$ to get $\partial\Phi^{sym}(\mathbf{T})/\partial T_0$ and $m_c^{sym} + m_r^{sym} = 1 - m_\ell^{sym}$ to get $\partial\Phi^{sym}(\mathbf{T})/\partial T_1$. Again, setting each first derivative equal to zero leads to $T_0^* = T_1^* = F^{-1}[\beta/(\beta + \gamma)]$.

B.4 Counterpart of the time game of Section 2.2.2

When $T_0 \geq T_1$, the counterpart of the equilibrium demands (13) are given by:

$$\begin{aligned} D_0^{sym,*}(\mathbf{T}, \boldsymbol{\tau}) &= \frac{1}{6}[3 + h + 2\Delta\tilde{c} + 2\Phi^{sym}(\mathbf{T})], \\ D_1^{sym,*}(\mathbf{T}, \boldsymbol{\tau}) &= \frac{1}{6}[3 - h - 2\Delta\tilde{c} - 2\Phi^{sym}(\mathbf{T})], \end{aligned} \quad (\text{B.5})$$

and FOCs (19) become:

$$\begin{aligned}\frac{\partial \pi_0^{sym,*}(\mathbf{T}, \boldsymbol{\tau})}{\partial T_0} &= \frac{2}{3} [(\beta + \gamma) m_r^{sym} - \gamma] D_0^{sym,*}(\mathbf{T}, \boldsymbol{\tau}) - \frac{\partial K(T_0)}{\partial T_0} = 0, \\ \frac{\partial \pi_1^{sym,*}(\mathbf{T}, \boldsymbol{\tau})}{\partial T_1} &= -\frac{2}{3} [(\beta + \gamma) m_l^{sym} - \beta] D_1^{sym,*}(\mathbf{T}, \boldsymbol{\tau}) - \frac{\partial K(T_1)}{\partial T_1} = 0,\end{aligned}$$

where m_r^{sym} and m_l^{sym} , defined in Appendix B.1, denote the share of travellers with desired times earlier than T_1 and later than T_0 , respectively. Consider the case where $\partial K(T_i)/\partial T_i = k_i = 0$ for $i = 0, 1$. Solving the above system of FOCs with respect to T_i yields the counterpart of the equilibrium departure times (22):

$$F(T_0) = F(T_1) = \frac{\beta}{\beta + \gamma} \Rightarrow T^{sym,*}|_{k_0=k_1=0} = F^{-1}\left(\frac{\beta}{\beta + \gamma}\right). \quad (\text{B.6})$$

Turning to the case where $k_0, k_1 \neq 0$ in (18), the counterpart of (20) is:

$$F(T_i) = \frac{\beta}{\beta + \gamma} - \frac{3k_i}{2(\beta + \gamma)D_i^{sym,*}(\mathbf{T}, \boldsymbol{\tau})}, \quad i = 0, 1. \quad (\text{B.7})$$

Assuming that a unique and interior solution exists for each T_i in (B.7), profit maximization requires the SOC to hold, that is:

$$k_i^2 < \frac{4}{3} \rho(T_i^{sym,*})(\beta + \gamma) D_i^{sym,*3}(\mathbf{T}^{sym,*}, \boldsymbol{\tau}).$$

B.5 Counterpart of the social time cost minimization (29)

When $T_0 \geq T_1$, the $\widehat{SDC}^R(\mathbf{T})$ term (29) becomes:

$$\begin{aligned}\widehat{SDC}^{sym,R}(\mathbf{T}) &= \int_0^{T_1} \widehat{\gamma}(T_1 - t) \rho(t) dt + \int_{T_1}^{\tilde{t}^{sym}} \widehat{\beta}(t - T_1) \rho(t) dt + \\ &\quad \int_{\tilde{t}^{sym}}^{T_0} \widehat{\gamma}(T_0 - t) \rho(t) dt + \int_{T_0}^{24} \widehat{\beta}(t - T_0) \rho(t) dt,\end{aligned}$$

where $\tilde{t}^{sym} = (\hat{\gamma}T_0 + \hat{\beta}T_1)/(\hat{\beta} + \hat{\gamma})$. Then, the counterpart of FOCs (29) are:

$$\begin{aligned}\frac{\partial \widehat{SC}^{sym,R}(\mathbf{T})}{\partial T_0} &= \hat{\gamma}\tilde{m}_r^{sym} - \hat{\beta}m_r^{sym} + \hat{k}_0 = 0, \\ \frac{\partial \widehat{SC}^{sym,R}(\mathbf{T})}{\partial T_1} &= \hat{\gamma}m_\ell^{sym} - \hat{\beta}\tilde{m}_\ell^{sym} + \hat{k}_1 = 0,\end{aligned}\tag{B.8}$$

where $\tilde{m}_r^{sym} = \int_{\tilde{t}^{sym}}^{T_0} \rho(t)dt$, $\tilde{m}_\ell^{sym} = \int_{T_1}^{\tilde{t}^{sym}} \rho(t)dt$ and where m_r^{sym} and m_ℓ^{sym} are given in Appendix B.1. Using (B.8), we deduce the following second and cross-partial derivatives:

$$\begin{aligned}\frac{\partial^2 \widehat{SC}^R(\mathbf{T})}{\partial T_0^2} &= (\hat{\beta} + \hat{\gamma})\rho(T_0) - \frac{\hat{\gamma}^2}{\hat{\beta} + \hat{\gamma}}\rho(\tilde{t}^{sym}), & \frac{\partial^2 \widehat{SC}^R(\mathbf{T})}{\partial T_1^2} &= (\hat{\beta} + \hat{\gamma})\rho(T_1) - \frac{\hat{\beta}^2}{\hat{\beta} + \hat{\gamma}}\rho(\tilde{t}^{sym}), \\ \frac{\partial^2 \widehat{SC}^{sym,R}(\mathbf{T})}{\partial T_i \partial T_{-i}} &= -\frac{\hat{\beta}\hat{\gamma}}{\hat{\beta} + \hat{\gamma}}\rho(\tilde{t}^{sym}), & i &= 0, 1.\end{aligned}$$

Setting the second derivatives larger than 0 and rearranging yields:

$$\frac{\partial^2 \widehat{SC}^{sym,R}(\mathbf{T})}{\partial T_0^2} = \frac{(\hat{\beta} + \hat{\gamma})^2}{\hat{\gamma}^2} > \frac{\rho(\tilde{t}^{sym})}{\rho(T_0)}, \quad \frac{\partial^2 \widehat{SC}^{sym,R}(\mathbf{T})}{\partial T_1^2} = \frac{(\hat{\beta} + \hat{\gamma})^2}{\hat{\beta}^2} > \frac{\rho(\tilde{t}^{sym})}{\rho(T_1)}.$$

Minimization requires the determinant of the Hessian matrix to be positive, i.e.,

$$2\hat{\beta}\hat{\gamma}\rho(T_0)\rho(T_1) + \hat{\beta}^2\rho(T_0)[\rho(T_1) - \rho(\tilde{t}^{sym})] + \hat{\gamma}^2\rho(T_1)[\rho(T_0) - \rho(\tilde{t}^{sym})] > 0.$$

When $t \sim \mathcal{U}[0, 24]$, then $\rho(t) = 1/24, \forall t \in [0, 24]$ and the sufficient condition for (global) minimum holds. With the latter distribution, the counterpart of the SC^S function in (29) is:

$$\widehat{SC}^{sym,R} = \frac{\hat{\beta}(\hat{\beta} + 2\hat{\gamma})T_0^2 - 48\hat{\beta}(\hat{\beta} + \hat{\gamma})T_0 + \hat{\gamma}(2\hat{\beta} + \hat{\gamma})T_1^2 - 2\hat{\beta}\hat{\gamma}T_0T_1 + 576\hat{\beta}(\hat{\beta} + \hat{\gamma})}{48(\hat{\beta} + \hat{\gamma})} + \widehat{K}^R(\mathbf{T}).$$

Solving the FOCs yields the socially optimal departure times:

$$T_i^{sym,R} \Big|_{\hat{k}_0, \hat{k}_1 \neq 0} = T_i^{sym,R} \Big|_{\hat{k}_0 = \hat{k}_1 = 0} - a_i^{sym}\hat{k}_0 - b_i^{sym}\hat{k}_1, \quad i = 0, 1,$$

where $T_0^{sym,S}|_{\hat{k}_0=\hat{k}_1=0} = 12 \left[1 + \frac{\hat{\beta}}{\hat{\beta}+\hat{\gamma}} \right]$, $T_1^{sym,S}|_{\hat{k}_0=\hat{k}_1=0} = \frac{12\hat{\beta}}{\hat{\beta}+\hat{\gamma}}$, $a_0^{sym} = \frac{12(2\hat{\beta}+\hat{\gamma})}{\hat{\beta}(\hat{\beta}+\hat{\gamma})} > 0$, $b_0^{sym} = a_1^{sym} = \frac{12}{\hat{\beta}+\hat{\gamma}} > 0$ and $b_1^{sym} = \frac{12(\hat{\beta}+2\hat{\gamma})}{\hat{\gamma}(\hat{\beta}+\hat{\gamma})} > 0$. Again, setting $\hat{k}_0 = \hat{k}_1 = 0$ simplifies the analysis. Then, by using (B.6) and the uniform distribution, we get:

$$\begin{aligned}\Delta T_0^{sym} &= T_0^{sym,R}|_{\hat{k}_0=\hat{k}_1=0} - T^{sym,*}|_{\hat{k}_0=\hat{k}_1=0} = 12 \frac{\hat{\gamma}}{\hat{\beta} + \hat{\gamma}}, \\ \Delta T_1^{sym} &= T_1^{sym,R}|_{\hat{k}_0=\hat{k}_1=0} - T^{sym,*}|_{\hat{k}_0=\hat{k}_1=0} = -12 \frac{\hat{\beta}}{\hat{\beta} + \hat{\gamma}}.\end{aligned}$$

C The time game with time-varying operational costs

Below we numerically solve several time games under the assumptions given below Fig. C.1 for all marginal time cost pairs of the set $\hat{k}_0 \times \hat{k}_1$ with $\hat{k}_i \in \{-1, 0, 1\}$, $i = 0, 1$. As t is uniformly distributed, the reader can check that the existence conditions (A.12), (A.13) and (A.14) hold.

Fig. C.1 shows the best response functions of each carrier to its rival's schedule, denoted $BR_i(\hat{k}_i)$, $i = 0, 1$, for the above combinations of marginal time costs. The “NE” intersections represent the Nash equilibria in departure times. To further highlight demand's role on schedule differentiation, we also compute the equilibria obtained by assuming $\hat{k}_0 = \hat{k}_1 > 0$, $\hat{k}_0 = \hat{k}_1 < 0$ and $\hat{k}_i = -\hat{k}_{-i} > 0$ for $i = 0, 1$. The numerical values of the departure times, fares, demands and profits are available upon request from the authors.

When the marginal time cost is null for both carriers, their reaction functions $BR_0(\hat{k}_0 = 0)$ and $BR_1(\hat{k}_1 = 0)$ are constant and intersect at point $NE(\hat{k}_0 = \hat{k}_1 = 0) = (10, 10)$. When the time cost increases in the time of day for carrier 0, its reaction function, denoted $BR_0(\hat{k}_0 = +1)$, moves leftwards and becomes convex in the departure time of its rival. The intersection between $BR_0(\hat{k}_0 = +1)$ and $BR_1(\hat{k}_1 = 0)$ yields a unique Nash equilibrium $NE(\hat{k}_0 = +1, \hat{k}_1 = 0) = (4, 10)$. Hence, carrier 0 schedules its service earlier in the morning while carrier 1 departs at 10. The remaining Nash equilibria can be analyzed similarly.

Consider now the case where the marginal time cost is *the same* across carriers and different from zero ($\hat{k}_0 = \hat{k}_1 \neq 0$). These equilibria lie along the dotted lines $T_i^*(\hat{k}_0 = \hat{k}_1 < 0)$ and $T_i^*(\hat{k}_0 = \hat{k}_1 > 0)$, close to the $T_0 = T_1$ diagonal. Both services

Figure C.1: Nash equilibria in departure times and time reaction functions

Notes: all simulations assume: $h = 0.25$, $t \sim \mathcal{U}[0, 24]$, $\hat{\beta} = 5$, $\hat{\gamma} = 7$, $\frac{\theta}{2} = 130$, $\hat{c}_0 = 10$, $\hat{c}_1 = 8$, $\hat{\tau}_0 = 280$, $\hat{\tau}_1 = 266$, $\hat{K}_i = 0$ for $i = 0, 1$.

are scheduled *later/earlier* than 10 when the time costs *decrease/increase* in the time of day. As carrier 1 receives a *lower* demand in equilibrium in this game, its departure time moves farther away to the RHS of 10 than that of its rival when the marginal time costs are negative (then, $T_0^* < T_1^*$), and farther away to the LHS of 10 when the marginal time costs are positive (then, $T_0^* > T_1^*$). The proximity of the lines $T_i^*(\hat{k}_0 = \hat{k}_1 < 0)$ and $T_i^*(\hat{k}_0 = \hat{k}_1 > 0)$ to the diagonal $T_0 = T_1$ is due to the closeness of the equilibrium demand of the carriers. Notice in Fig. C.1 that any small deviation from the equilibrium for a carrier along its reaction function drives its departure time back to the initial equilibrium. This confirms the stability in schedule competition of the carrier-rivalry game when travellers' preferred departure times are uniformly dis-

tributed and it stresses that the departure times are strategic complements. A similar reaction space with right-shifted time reaction functions is obtained when $\hat{\beta} = \hat{\gamma}$ or when $\hat{\beta} > \hat{\gamma}$.

D Supplementary material not for publication

This Appendix provides derivations made to answer some specific comments of the referees, material available upon request, numerical checks performed to verify the analytical results and results/figures not included in the paper.

- Section D.1, pp. 2-3, includes derivations to the attention of the referees.
- Section D.2, pp. 5-6, gathers the main material removed from the first version and not included in the Appendix of the revised version.
- Section D.3, pp. 7-16, focuses on the theoretical results and offers additional analytical details for some derivations.
- Section D.4, pp. 17-29, presents computational aspects and numerical checks related to Section 4 and Appendix C.

As compared to the Supplementary material of the first version, note that Sections D.1 and D.2 are new. Sections D.3 and D.4 are very similar to the previous version; they have been adapted to better match the revised version of the paper.

D.1 Analytical derivations for the referees

D.1.1 For referee # 1, analytics of our answer to comment # 5

“To see why no assumption is required on whether $\beta \lesseqgtr \gamma$ without making lengthy derivations, we can focus on the differences in schedule delay cost for the indifferent consumer given in Eqs.(A.1), (A.3) and (A.5) of Appendix A.1. These equations, which are fundamental to derive the market demands, remain unchanged whether $\beta < \gamma$ or $\beta > \gamma$. They also hold when $\beta = \gamma$, one just needs to collect terms and new expressions, similar to former ones, follow immediately. Thus, the relative levels of β and γ have no impact on later derivations, *as long as the market cover condition holds*. And again, one can easily see that $\beta \lesseqgtr \gamma$ neither changes fundamentally the covered market condition. Please refer to Appendix D.1.1 in the Supplementary material for further details.”

To see the above, focus on the terms which capture the differences in schedule delay costs in the indifferent consumer expressions $\tilde{x}(t)$, given by (A.1), (A.3) and (A.5):

$$\Delta C(T_0, T_1, t) = C_1(T_1, t) - C_0(T_0, t) = \begin{cases} \gamma(T_1 - T_0) & \text{if } t \in [0, T_0], \\ \beta T_0 + \gamma T_1 - (\beta + \gamma)t & \text{if } t \in]T_0, T_1[, \\ -\beta(T_1 - T_0) & \text{if } t \in [T_1, 24]. \end{cases}$$

Function $\Delta C(T_0, T_1, t)$, a piecewise function in t with given T s, remains unchanged whether $\beta < \gamma$ or $\beta > \gamma$. When $\beta = \gamma$, ΔC becomes

$$\Delta C(T_0, T_1, t) = C_1(T_1, t) - C_0(T_0, t) = \begin{cases} \gamma(T_1 - T_0) & \text{if } t \in [0, T_0], \\ \gamma(T_0 + T_1) - 2\gamma t & \text{if } t \in]T_0, T_1[, \\ -\gamma(T_1 - T_0) & \text{if } t \in [T_1, 24]. \end{cases}$$

Plotting ΔC along t (see Fig. D.1 below) makes clear that these schedule delay cost differences are not fundamentally altered by the assumptions made on the relative levels of β and γ .

Similarly the covered market condition (3), replicated below, remains unchanged whether $\beta < \gamma$ or $\beta > \gamma$.

$$-\frac{\theta}{2}(1 - h^2) < \hat{p}_1 - \hat{p}_0 - \hat{\beta}(T_1 - T_0) \leq \hat{p}_1 - \hat{p}_0 + \hat{\gamma}(T_1 - T_0) < \frac{\theta}{2}(1 - h)^2.$$

When $\beta = \gamma$, (3) simply becomes

$$-\frac{\theta}{2}(1 - h^2) < \hat{p}_1 - \hat{p}_0 - \hat{\gamma}(T_1 - T_0) \leq \hat{p}_1 - \hat{p}_0 + \hat{\gamma}(T_1 - T_0) < \frac{\theta}{2}(1 - h)^2.$$

Figure D.1: Differences in schedule delay costs for the indifferent consumer

where $\tilde{t} = (\beta T_0 + \gamma T_1) / (\beta + \gamma)$.

We thus conclude that as long as the covered market condition holds, the relative levels of β and γ play no major role in the derivations. What is more crucial for the existence and stability of the equilibrium is the shape of the distribution of consumers' preferred time $F(t)$. This is stressed when we solve the time game in Section 2.2.2.

D.1.2 For referee # 2

Below, we compute the partial derivatives of the equilibrium fares, demands and profits with respect to the transportation cost parameter θ .

$$\begin{aligned}
\frac{\partial \hat{p}_0^*}{\partial \theta} &= \frac{1}{6}(3+h)(1-h), & \frac{\partial \hat{p}_1^*}{\partial h} &= \frac{1}{6}(3-h)(1-h) \\
\frac{\partial D_0^*(\mathbf{T}, \boldsymbol{\tau})}{\partial \theta} &= -\frac{1}{3\theta^2(1-h)}(\Delta \hat{c} - \hat{\Phi}(\mathbf{T})), & \frac{\partial D_1^*(\mathbf{T}, \boldsymbol{\tau})}{\partial \theta} &= -\frac{\partial D_0^*(\mathbf{T}, \boldsymbol{\tau})}{\partial \theta} \\
\frac{\partial \hat{\pi}_0^*(\mathbf{T}, \boldsymbol{\tau})}{\partial \theta} &= (1-h) \left[\frac{(3+h)}{6} - \frac{1}{3}(\Delta \hat{c} - \Phi(\mathbf{T})) \right] D_0^*(\mathbf{T}, \boldsymbol{\tau}), \\
\frac{\partial \hat{\pi}_1^*(\mathbf{T}, \boldsymbol{\tau})}{\partial \theta} &= (1-h) \left[\frac{(3-h)}{6} + \frac{1}{3}(\Delta \hat{c} + \Phi(\mathbf{T})) \right] D_1^*(\mathbf{T}, \boldsymbol{\tau}),
\end{aligned} \tag{D.1}$$

where $h, D_i^* \in [0, 1]$ for $i = 0, 1$. We deduce that:

$$\begin{aligned}
\frac{\partial \hat{p}_i^*}{\partial \theta} &> 0, \\
\frac{\partial D_0^*(\mathbf{T}, \boldsymbol{\tau})}{\partial \theta} &> 0 \quad \text{iff} \quad \Delta \hat{c} < -\hat{\Phi}(\mathbf{T}), \\
\frac{\partial \hat{\pi}_0^*(\mathbf{T}, \boldsymbol{\tau})}{\partial \theta} &> 0 \quad \text{iff} \quad \Delta \hat{c} < \frac{(3+h)}{2} - \Phi(\mathbf{T}), \\
\frac{\partial \hat{\pi}_1^*(\mathbf{T}, \boldsymbol{\tau})}{\partial \theta} &> 0 \quad \text{iff} \quad \Delta \hat{c} > \frac{(h-3)}{2} - \Phi(\mathbf{T}).
\end{aligned} \tag{D.2}$$

Thus, the equilibrium fares decrease when θ decreases. The impacts of an increase in θ on the equilibrium demand and profits depend on the marginal operational cost, location and schedule delay cost advantages. To illustrate the effects on carriers' equilibrium profits, we can use $\partial \hat{\pi}_i^* / \partial \theta$ and deduce the following plot:

Effect of a positive change in θ on π_i^*

$$\begin{array}{c}
\uparrow \hat{\pi}_0^*, \downarrow \hat{\pi}_1^* \qquad \qquad \uparrow \hat{\pi}_0^*, \uparrow \hat{\pi}_1^* \qquad \qquad \downarrow \hat{\pi}_0^*, \uparrow \hat{\pi}_1^* \\
\hline
\frac{h-3}{2} - \Phi(\mathbf{T}) \qquad \qquad \frac{3+h}{2} - \Phi(\mathbf{T}) \qquad \qquad \rightarrow \Delta \hat{c}
\end{array}$$

D.2 Main material removed from the first version

D.2.1 Plot of the schedule delay cost functions (1)

Figure D.2 shows the schedule delay costs faced by a consumer for carrier 0 departing at time T_0 and carrier 1 at time T_1 .

Figure D.2: Traveller's schedule delay costs for $\hat{\beta} < \hat{\gamma}$

D.2.2 Indifferent consumer and the time line

Fig. D.3 illustrates the covered market condition and establishes the link between the geographic and time dimensions.

Figure D.3: Indifferent consumer and the time line

For given fares, departure times, parameters and given a desired time t , the travellers located to the left-hand side (LHS) of the solid black *indifference line* on Fig. D.3 depart facility 0 and those to the right-hand side (RHS) depart facility 1. By way of example, when the indifferent consumer prefers departing at time T_0 or earlier, $\tilde{x}(t) = \tilde{x}(T_0)$ and a larger share of consumers choose facility 0 located at $x = h$ and a positive fraction of them, $1 - \tilde{x}(T_0)$, leave from facility 1 located at $x = 1$. Integrating over the geographic space for any given t , we get the demand functions $D_0(t) = \tilde{x}(t)$ and $D_1(t) = 1 - \tilde{x}(t)$.

The broken shape of the indifference line is related to the piecewise schedule delay cost function $\hat{C}_i(T_i, t)$. When $\hat{p}_0$ (resp. $\hat{p}_1$) increases, all else equal, the indifference line moves to the LHS (resp. RHS). From Fig. D.3, we deduce the following lemma.

Lemma 2. *If the consumers located at $x = 0$ (resp. $x = 1$) with desired departure time $t = T_0$ (resp. $t = T_1$) select facility 1 (resp. facility 0), then no consumer chooses facility 0 (resp. facility 1).*

D.2.3 The explanations below Fig. 2, replicated below, have been dropped:

Effect of a positive change in h on π_i^*

$$\begin{array}{c} \downarrow \hat{\pi}_0^*, \uparrow \hat{\pi}_1^* \quad \quad \downarrow \hat{\pi}_0^*, \downarrow \hat{\pi}_1^* \quad \quad \uparrow \hat{\pi}_0^*, \downarrow \hat{\pi}_1^* \\ \hline \frac{3h-5}{2} - \Phi(\mathbf{T}) \quad \quad \frac{1+3h}{2} - \Phi(\mathbf{T}) \quad \rightarrow \Delta \tilde{c} \end{array}$$

Assuming strictly positive transportation cost parameter θ and equilibrium demands, we can focus on the terms within brackets in (17) to determine under which conditions the demand effect dominates the competition effect when h expands. We deduce that $\hat{\pi}_0^*$ increases in h if $\Delta \tilde{c} > (1 + 3h)/2 - \Phi(\mathbf{T})$. Hence, carrier 0 needs a large marginal costs advantage ($\Delta \tilde{c} > (1 + 3h)/2$) to increase its profit when h increases, unless its SDC advantage is large enough ($\Phi(\mathbf{T}) > (1 + 3h)/2$). The same interpretation holds regarding the condition needed for the profit of carrier 1 to increase in h , i.e., $\Delta \tilde{c} < (3h - 5)/2 - \Phi(\mathbf{T})$.

D.2.4 All plots in former Section 4.1 are now included in this Supplementary material in Section D.4.4

D.2.5 Table A.1 in former Appendix A is now included in this Supplementary material in Section D.4.8

D.3 Additional analytical results

D.3.1 Further analytical details for the fare and fee subgames

Consider the profit functions (7). Solving $\partial \widehat{\pi}_i / \partial \widehat{p}_i = 0$ for $i = 0, 1$ with respect to $\widehat{p}_i$ gives the reaction function of each carrier to its rival's fare:

$$\begin{aligned} p_0 &= \frac{1}{2} \left[p_1 + c_0 + \tau_0 + \frac{1+h}{2} + \Phi(\mathbf{T}) \right], \\ p_1 &= \frac{1}{2} \left[p_0 + c_1 + \tau_1 + \frac{1-h}{2} - \Phi(\mathbf{T}) \right], \end{aligned}$$

where p_i, c_i and τ_i stand for $\widehat{p}_i, \widehat{c}_i$ and $\widehat{\tau}_i$ divided by $\theta(1-h)$. If carrier 1 increases its fare $\widehat{p}_1$, the best response of carrier 0 is to increase its own fare $\widehat{p}_0$ and the same holds for carrier 0, which highlights that fares are strategic complements. Solving this system of FOCs yields the Nash equilibrium (8). The unicity and local stability of this equilibrium are quite obvious. Formally, by using Vives (1999, pp.47-52), we have

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial p_i^2} + \left| \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \right| &= -\frac{1}{\theta(1-h)} < 0, \quad \text{for } i, j = 0, 1 \text{ and for } i \neq j, \\ \left| \frac{\partial^2 \pi_0}{\partial p_0^2} \frac{\partial^2 \pi_1}{\partial p_1^2} \right| - \left| \frac{\partial^2 \pi_0}{\partial p_0 \partial p_1} \frac{\partial^2 \pi_1}{\partial p_0 \partial p_1} \right| &= \frac{3}{\theta^2(1-h)^2} > 0. \end{aligned}$$

The first inequality is a sufficient condition for uniqueness (based on a contraction approach, see Vives (1999), page 47) while the second inequality (evaluated at the equilibrium fares) ensures local stability (see Vives (1999), page 51).

As outlines in Section 2.3 of the paper, the fee game exactly follows the same resolutions steps as the fare game and yields similar results. In particular, the reader can verify that the fee reaction functions are:

$$\begin{aligned} \tau_0 &= \frac{1}{2} \left[\tau_1 + c_1 - c_0 - \omega_0 + \frac{3+h}{2} + \Phi(\mathbf{T}) \right], \\ \tau_1 &= \frac{1}{2} \left[\tau_0 + c_0 - c_1 - \omega_1 + \frac{3-h}{2} - \Phi(\mathbf{T}) \right]. \end{aligned}$$

D.3.2 Bounds for k_i when $F(t)$ is unspecified.

As outlined in footnote 21 of the paper, when $F(t)$ is unspecified, we can derive bounds around k_i conditional upon $D_i^*(\mathbf{T}^*, \boldsymbol{\tau}) \in]0, 1[$. Let's rewrite below the FOCs (20) of the paper:

$$\begin{aligned} F(T_0^*) &= \frac{\beta}{\beta + \gamma} - \frac{3k_0}{2(\beta + \gamma)D_0^*(\mathbf{T}^*, \boldsymbol{\tau})}, \\ F(T_1^*) &= \frac{\beta}{\beta + \gamma} - \frac{3k_1}{2(\beta + \gamma)D_1^*(\mathbf{T}^*, \boldsymbol{\tau})}. \end{aligned}$$

Recalling that $D_1^*(\mathbf{T}^*, \boldsymbol{\tau}) = 1 - D_0^*(\mathbf{T}^*, \boldsymbol{\tau})$ and denoting $D_i^*(\mathbf{T}^*, \boldsymbol{\tau})$ by D_i^* , $i = 0, 1$ for simplicity, we can set $0 < F(T_0^*) < 1$ and rearrange to get:

$$\begin{aligned} -\frac{3k_0}{2\gamma} &< \frac{3k_0}{2\beta} < D_0^* \quad \text{when } k_0 > 0, \\ -\frac{3|k_0|}{2\beta} &< \frac{3|k_0|}{2\gamma} < D_0^* \quad \text{when } k_0 < 0. \end{aligned}$$

Next, setting $0 < F(T_1^*) < 1$ and rearranging leads to:

$$\begin{aligned} D_0^* &< 1 - \frac{3k_1}{2\beta} < 1 + \frac{3k_1}{2\gamma} \quad \text{when } k_1 > 0, \\ D_0^* &< 1 - \frac{3|k_1|}{2\gamma} < 1 + \frac{3|k_1|}{2\beta} \quad \text{when } k_1 < 0, \end{aligned}$$

where we used $D_1^* = 1 - D_0^*$. Then, we deduce that:

- when $k_0, k_1 > 0$, $\frac{3k_0}{2\beta} < D_0^* < 1 - \frac{3k_1}{2\beta}$. Rearranging the lower and upper bounds yields $\boxed{\frac{1}{\beta}(k_0 + k_1) < \frac{2}{3}}$;
- when $k_0 > 0, k_1 < 0$, $\frac{3k_0}{2\beta} < D_0^* < 1 - \frac{3|k_1|}{2\gamma}$. Rearranging the lower and upper bounds yields $\boxed{\frac{k_0}{\beta} + \frac{|k_1|}{\gamma} < \frac{2}{3}}$;
- when $k_0 < 0, k_1 > 0$, $\frac{3|k_0|}{2\gamma} < D_0^* < 1 - \frac{3k_1}{2\beta}$. Rearranging the lower and upper bounds yields $\boxed{\frac{|k_0|}{\gamma} + \frac{k_1}{\beta} < \frac{2}{3}}$;
- when $k_0 < 0, k_1 < 0$, $\frac{3|k_0|}{2\gamma} < D_0^* < 1 - \frac{3|k_1|}{2\gamma}$. Rearranging the lower and upper bounds yields $\boxed{\frac{1}{\gamma}(|k_0| + |k_1|) < \frac{2}{3}}$.

Note that the symmetric FOCs (B.7) lead to the same bounds.

D.3.3 Detailed derivation of Appendix A.4

When $\rho(t) = \mathcal{U}[0, 24]$, by using $\Phi^U(T_0, T_1)$ in (A.7) and demands (13) in profits (15), the profits $\pi_i(T_0, T_1)$ are two quartic functions in T_i that reads:³⁴

$$\pi_i(T_0, T_1) = f_i + e_i T_i + d_i T_i^2 + b T_i^3 + a T_i^4, \quad i = 0, 1, \quad (\text{D.3})$$

where coefficients $a = (\beta + \gamma)^2/20736$ and $b = -\beta(\beta + \gamma)/216$ are identical across carriers and depend only on the (normalized) unit schedule delay costs, coefficients d_i, e_i and f_i are carrier-specific and depend on T_{-i} and other parameters of the model. Let $T_i, T_{-i} \in \mathbb{R}$. The coefficient of leading term T_i^4 being positive ($a > 0$), we deduce that $\lim_{T_i \rightarrow \pm\infty} \pi_i(T_i, T_{-i}) = +\infty$. For a unique and interior profit-maximizing departure time $T_i^* \in]0, 24[$ to exist for $i = 0, 1$, the profit functions need to be locally concave on $[0, 24]$. They must be ‘W-shaped’ and thus possess three critical points, two of which are minima and one is a local maximum. To have a sense of these shapes, we show below the profit of carrier 0 along with its first and second derivatives for the parameters used in the time game of Appendix C.³⁵

The expressions of the first, second and cross-partial derivatives are explicitly given below:

$$\begin{aligned} \frac{\partial \pi_0(T_0, T_1, k_0)}{\partial T_0} &= \frac{\beta(3 + h + 2\Delta\tilde{c})}{9} - k_0 - \frac{2\beta^2}{9} T_1 + \frac{\beta(\beta + \gamma)}{216} T_1^2 \\ &\quad + \left(\frac{1152\beta^2 - 24(\beta + \gamma)(3 + h + 2\Delta\tilde{c}) + 48\beta(\beta + \gamma)T_1 - (\beta + \gamma)^2 T_1^2}{5184} \right) T_0 \\ &\quad - \frac{\beta(\beta + \gamma)}{72} T_0^2 + \frac{(\beta + \gamma)^2}{5184} T_0^3, \end{aligned} \quad (\text{D.4})$$

$$\begin{aligned} \frac{\partial \pi_1(T_0, T_1, k_1)}{\partial T_1} &= \frac{\beta(3 - h - 2\Delta\tilde{c})}{9} - k_1 - \frac{2\beta^2}{9} T_0 + \frac{\beta(\beta + \gamma)}{216} T_0^2 \\ &\quad + \left(\frac{1152\beta^2 - 24(\beta + \gamma)(3 - h - 2\Delta\tilde{c}) + 48\beta(\beta + \gamma)T_0 - (\beta + \gamma)^2 T_0^2}{5184} \right) T_1 \\ &\quad - \frac{\beta(\beta + \gamma)}{72} T_1^2 + \frac{(\beta + \gamma)^2}{5184} T_1^3, \end{aligned} \quad (\text{D.5})$$

³⁴The use of a computer algebra system, such as Mathematica, is helpful here to avoid calculation mistakes. The full quartic expressions are not shown as they are long and not of direct interest.

³⁵See also Section D.4.6 of this Supplementary material for further computational and numerical details.

$$\frac{\partial^2 \pi_0(T_0, T_1, k_0)}{\partial T_0^2} = \frac{1152\beta^2 - 24(\beta + \gamma)(3 + h + 2\Delta\tilde{c}) + 48\beta(\beta + \gamma)T_1 - (\beta + \gamma)^2 T_1^2}{5184} \quad (D.6)$$

$$- \frac{\beta(\beta + \gamma)}{36} T_0 + \frac{(\beta + \gamma)^2}{1728} T_0^2,$$

$$\frac{\partial^2 \pi_1(T_0, T_1, k_1)}{\partial T_1^2} = \frac{1152\beta^2 - 24(\beta + \gamma)(3 - h - 2\Delta\tilde{c}) + 48\beta(\beta + \gamma)T_0 - (\beta + \gamma)^2 T_0^2}{5184} \quad (D.7)$$

$$- \frac{\beta(\beta + \gamma)}{36} T_1 + \frac{(\beta + \gamma)^2}{1728} T_1^2.$$

$$\frac{\partial^2 \pi_0(T_0, T_1, k_0)}{\partial T_0 \partial T_1} = -\frac{2\beta^2}{9} + \frac{\beta(\beta + \gamma)}{108} T_0 + \left(\frac{\beta(\beta + \gamma)}{108} - \frac{(\beta + \gamma)^2 T_0}{2592} \right) T_1, \quad (D.8)$$

$$\frac{\partial^2 \pi_1(T_0, T_1, k_1)}{\partial T_1 \partial T_0} = -\frac{2\beta^2}{9} + \frac{\beta(\beta + \gamma)}{108} T_1 + \left(\frac{\beta(\beta + \gamma)}{108} - \frac{(\beta + \gamma)^2 T_1}{2592} \right) T_0. \quad (D.9)$$

We proceed by providing a set of parameter restrictions which ensure that the local maximum exists, is unique and interior on the $[0, 24]$ time interval and yields positive profits for $i = 0, 1$. To establish this, we impose

- (i) $\frac{\partial^2 \pi_i(T_0, T_1)}{\partial T_i^2} < 0, \forall T_i \in]T_{i,a}^{**}, T_{i,b}^{**}[\subset \mathbb{R}$ with $T_i = 0 \in]T_{i,a}^{**}, T_{i,b}^{**}[$,
- (ii) $\left. \frac{\partial \pi_i(T_0, T_1)}{\partial T_i} \right|_{T_i=0} > 0, \left. \frac{\partial \pi_i(T_0, T_1)}{\partial T_i} \right|_{T_i=24} < 0$ and
- (iii) $\pi_i(T_0, T_1)|_{T_i=0} > 0$

for $i = 0, 1$. Condition (i) ensures the existence of a locally quasi-concave profit function for each carrier over the real interval $[T_{i,a}^{**}, T_{i,b}^{**}] \subset \mathbb{R}$ which includes $T_i = 0$. On this interval, the first derivative of the profit functions is decreasing. Condition (ii) guarantees that the first derivative of the profit function is: positive – and decreasing by (i) – at $T_i = 0$, null at the local maximum T_i^* and negative on $]T_i^*, 24]$. This necessarily implies the existence of a single stationary point $T_i^* \in]0, 24[$ which is the profit-maximizing departure time for each carrier given its rival's time departure time. Condition (iii) makes sure that the maximized profits are positive because $\pi_i(T_0, T_1)|_{T_i=T_i^*}$ is above $\pi_i(T_0, T_1)|_{T_i=0} > 0$.

Starting with (i), by (D.6)-(D.7) we have:

$$\partial^2 \pi_i / \partial T_i^2 = \bar{d}_i + \bar{b}T_i + \bar{a}T_i^2, \quad i = 0, 1,$$

where the coefficient of the leading terms T_i^2 is positive ($\bar{a} = (\beta + \gamma)^2 / 1728 > 0$). We need parameter restrictions which ensure the existence of two real solutions (or roots) to $\partial^2 \pi_i(T_0, T_1) / \partial T_i^2 = 0$, denoted $T_{i,a}^{**}$ and $T_{i,b}^{**}$ below and such that $T_{i,a}^{**} < T_{i,b}^{**}$

and $\partial^2 \pi_i / \partial T_i^2 < 0$, $\forall T_i \in]T_{i,a}^{**} < T_i, T_{i,b}^{**}[$. These roots are the inflection points of the quartic profit functions. A quadratic function has two real solutions if its discriminant satisfies $\Delta_i = \bar{b}^2 - 4\bar{a}\bar{d}_i > 0$. As $\bar{a}, \bar{b}^2 > 0$, we can impose $\bar{d}_i < 0$. As we can see in (D.6)-(D.7), term $\bar{d}_i$ is itself a quadratic function in T_{-i} such that $\bar{d}_i = \bar{\bar{d}}_i + \bar{b}T_{-i} + \bar{a}T_{-i}^2$ with $\bar{a} = -(\beta + \gamma)/5184 < 0$. Let the discriminant of $\bar{d}_i$ be given by $\Delta_{\bar{d}_i}$. Setting $\Delta_{\bar{d}_i} < 0$ for $i = 0, 1$ guarantees that $\text{sgn}(\bar{d}_i) = \text{sgn}(\bar{a})$ and implies $\bar{d}_i < 0, \forall T_{-i} \in \mathbb{R}$. Note, however, that setting $\bar{d}_i < 0$ is more restrictive than imposing $\Delta_i > 0$ directly as $\bar{b}^2 > 4\bar{a}\bar{d}_i$ allows $4\bar{a}\bar{d}_i < 0$ but also $b^2 > 4\bar{a}\bar{d}_i > 0$. Below, we show the first condition with the two different approaches:

$$\beta, \gamma > 0, \quad 0 \leq h < 1, \quad T_0, T_1 \in \mathbb{R} \quad \text{and} \quad (\text{D.10})$$

$$\frac{72\beta^2 - (\beta + \gamma)(3 + h)}{2(\beta + \gamma)} < \Delta\tilde{c} < \frac{-72\beta^2 + (\beta + \gamma)(3 - h)}{2(\beta + \gamma)}, \quad \text{or} \quad (\text{D.11})$$

$$\frac{12\beta^2}{(\beta + \gamma)} + \frac{3 - h}{2} - \beta T_0 + \frac{\beta + \gamma}{48} T_0^2 < \Delta\tilde{c} < -\frac{12\beta^2}{(\beta + \gamma)} - \frac{3 + h}{2} + \beta T_1 - \frac{\beta + \gamma}{48} T_1^2. \quad (\text{D.12})$$

Restriction (D.11) follows from setting $\bar{d}_i < 0$ while (D.12) stems from $\bar{b}^2 > 4\bar{a}\bar{d}_i$. An important advantage of (D.11) over (D.12) is that it does not depend on the departure time T_{-i} set by the rival carrier.

Next, solving $\partial^2 \pi_i(T_0, T_1) / \partial T_i^2 = 0$ for $i = 0, 1$ yields

$$T_{0,a,b}^{**} = \frac{24\beta}{\beta + \gamma} \mp \frac{\sqrt{A}}{\sqrt{3}(\beta + \gamma)}, \quad T_{1,a,b}^{**} = \frac{24\beta}{\beta + \gamma} \mp \frac{\sqrt{A'}}{\sqrt{3}(\beta + \gamma)}$$

with $T_{i,a}^{**} < T_{i,b}^{**}$ because

$$A = 24 [24\beta^2 + (3 + h + 2\Delta\tilde{c})(\beta + \gamma)] - 48\beta(\beta + \gamma)T_1 + (\beta + \gamma)^2 T_1^2 > 0, \\ A' = 24 [24\beta^2 + (3 - h - 2\Delta\tilde{c})(\beta + \gamma)] - 48\beta(\beta + \gamma)T_0 + (\beta + \gamma)^2 T_0^2 > 0.$$

Note that (D.11) guarantees that $T_{i,a}^{**} < 0 < T_{i,b}^{**}$ while (D.12) ensures $T_{i,b}^{**} > 0$ only but allows $T_{i,a}^{**}$ to be positive or negative. To ensure that $T_{i,a}^{**} < 0$ in the latter context, we need to replace (D.12) by:

$$\frac{24\beta^2}{(\beta + \gamma)} - \frac{3 + h}{2} + \beta T_1 - \frac{\beta + \gamma}{48} T_1^2 < \Delta\tilde{c} < -\frac{24\beta^2}{(\beta + \gamma)} + \frac{3 - h}{2} - \beta T_0 + \frac{\beta + \gamma}{48} T_0^2. \quad (\text{D.13})$$

Regarding (ii), note first that $\partial \pi_i(T_0, T_1) / \partial T_i|_{T_i=0}$ for $i = 0, 1$ corresponds to term e_i in (D.3), which is explicitly given by the *constants* in (D.4)-(D.5). We notice that $e_i = d'_i + b'T_{-i} + a'T_{-i}^2$, where $a' = \beta(\beta + \gamma)/216 > 0$. Then, e_i appears to be, again, a quadratic function in T_{-i} and the coefficient of the leading term T_{-i}^2 – identical across carriers – is positive. To ensure that $e_i > 0$, $\forall T_{-i} \in \mathbb{R}$, we require its discriminant to be lower than zero so that $\text{sgn}(e_i) = \text{sgn}(a')$. The same reasoning applies for

$\partial\pi_i(T_0, T_1)/\partial T_i|_{T_i=24} < 0$. In the latter case, the coefficient of the leading term T_{-i}^2 can be shown to be negative ($a' = -\gamma(\beta + \gamma)/216 < 0$) and so must be set the related discriminant. Combining the resulting restrictions for $i = 0, 1$ yields to the following set of conditions:

$$\begin{aligned} \frac{8\gamma^3}{3(\beta + \gamma)} - \frac{(3 + h + 2\Delta\tilde{c})\gamma}{9} < k_0 < -\frac{8\beta^3}{3(\beta + \gamma)} + \frac{(3 + h + 2\Delta\tilde{c})\beta}{9}, \\ \frac{8\gamma^3}{3(\beta + \gamma)} - \frac{(3 - h - 2\Delta\tilde{c})\gamma}{9} < k_1 < -\frac{8\beta^3}{3(\beta + \gamma)} + \frac{(3 - h - 2\Delta\tilde{c})\beta}{9}. \end{aligned} \quad (\text{D.14})$$

Turning to (iii), $\pi_i(T_0, T_1)|_{T_i=0}$ is clearly equivalent to f_i in (D.3) and can be shown to be:

$$\begin{aligned} f_0 &= -K_0 + \frac{(3 + h + 2\Delta\tilde{c})^2}{36} - \frac{\beta(3 + h + 2\Delta\tilde{c})}{9}T_1 \\ &\quad + \frac{48\beta^2 + (\beta + \gamma)(3 + h + 2\Delta\tilde{c})}{432}T_1^2 - \frac{\beta(\beta + \gamma)}{216}T_1^3 + \frac{(\beta + \gamma)^2}{20736}T_1^4, \\ f_1 &= -K_1 + \frac{(-3 + h + 2\Delta\tilde{c})^2}{36} - \frac{\beta(3 - h - 2\Delta\tilde{c})}{9}T_0 \\ &\quad + \frac{48\beta^2 + (\beta + \gamma)(3 - h - 2\Delta\tilde{c})}{432}T_0^2 - \frac{\beta(\beta + \gamma)}{216}T_0^3 + \frac{(\beta + \gamma)^2}{20736}T_0^4. \end{aligned}$$

Thus, f_i is a fourth-degree polynomial in T_{-i} which does not depend on k_i . Setting the fixed time costs $K_i = 0$ for simplicity, if we impose (D.13), the positivity conditions are intractably large. By contrast, when (D.11) holds, $f_i > 0$ only requires replacing $\beta, \gamma > 0$ in (D.10) by:

$$0 < \beta < \frac{1}{24} \quad \text{and} \quad \gamma > 0 \quad \text{or} \quad \beta > \frac{1}{24} \quad \text{and} \quad \gamma > \beta(24\beta - 1). \quad (\text{D.15})$$

The above result is derived by using Mathematica and by solving: `Reduce`[$f_i > 0$ && (D.11) && $\beta > 0$ && $\gamma > 0$ && $0 \leq h < 1$ && $T_{-i} \in \mathbb{R}$]. Plotting f_i for $i = 0, 1$ under the above parameter restrictions yields a quartic function which is positive $\forall T_{-i} \in \mathbb{R}$.

To sum up, (D.11), (D.14) and (D.15) ensure the existence a unique profit-maximizing $T_i^* \in]0, 24[$ for each carrier with positive profits. Notice that these conditions are expressed in terms of scaled parameters. In the simulations of Section C (see footnote ?? in particular) we multiply all existence conditions by $\theta(1 - h)$ to get their unscaled counterpart. Details on these adjustments are given in Section D.4.6 of this Supplementary material.

Of course, these interior solutions do not guarantee the *uniqueness of the Nash equilibrium in departure times*. A contraction approach would require to find parameter

restrictions such that the following sufficient conditions hold for $\forall T_i \in]0, 24[$:

$$\frac{\partial^2 \pi_0}{\partial T_0^2} + \left| \frac{\partial^2 \pi_0}{\partial T_0 \partial T_1} \right| < 0, \quad \frac{\partial^2 \pi_1}{\partial T_1^2} + \left| \frac{\partial^2 \pi_1}{\partial T_1 \partial T_0} \right| < 0.$$

Replacing the second and cross-derivatives in the uniqueness conditions by their expression leads to a large set of parameter restrictions which are excessively technical. Alternatively, one can simply use the parameter values which satisfy the existence conditions, solve $\partial \pi_i(T_0, T_1)/\partial T_i = 0$ for $i = 0, 1$ and draw the time reaction functions that fall within the $[0, 24]^2$ space (one per carrier). This approach is more insightful and allows to graphically explore the uniqueness of the Nash equilibrium and its local stability. This is the path we take in the simulations of Section C and for which we provide further details in Section D.4.6 of this Supplementary material.

D.3.4 Detailed derivation of Appendix A.5

In this Appendix, we establish in greater detail $dT_i^*/dk_i < 0$. Rearrange first the FOCs (20) as:

$$G(T_i, k_i) \equiv F(T_i) + \frac{3k_i}{2(\beta + \gamma)D_i^*(\mathbf{T}, \boldsymbol{\tau})} - \frac{\beta}{\beta + \gamma} = 0.$$

The implicit function theorem implies that $dT_i/dk_i = -(G_{k_i}/G_{T_i})$. Note first that:

$$G_{k_i} = \frac{3}{2(\beta + \gamma)D_i^*(\mathbf{T}, \boldsymbol{\tau})}, \quad \frac{1}{G_{T_i}} = \frac{2(\beta + \gamma)D_i^{*,2}(\mathbf{T}, \boldsymbol{\tau})}{\rho(T_i)2(\beta + \gamma) - 3k_i \frac{\partial D_i^*(\mathbf{T}, \boldsymbol{\tau})}{\partial T_i}},$$

Thus, $dT_i/dk_i = -(G_{k_i}/G_{T_i})$ is:

$$\frac{dT_i}{dk_i} \equiv -\frac{G_{k_i}}{G_{T_i}} = \frac{3D_i^*(\mathbf{T}, \boldsymbol{\tau})}{3k_i \frac{\partial D_i^*(\mathbf{T}, \boldsymbol{\tau})}{\partial T_i} - \rho(T_i)2(\beta + \gamma)D_i^{*,2}(\mathbf{T}, \boldsymbol{\tau})}. \quad (\text{D.16})$$

Next, we deduced from (13) that $\partial D_i^*(\mathbf{T}, \boldsymbol{\tau})/\partial T_i = \frac{1}{3}\Phi_{T_i}(\mathbf{T})$ for $i = 0, 1$ and by Lemma 1 we have $\Phi_{T_0}(\mathbf{T}) = \beta - (\beta + \gamma)m_\ell$ and $\Phi_{T_1}(\mathbf{T}) = \gamma - (\beta + \gamma)m_r$. Assuming the existence of an interior and unique Nash equilibrium in departure time, FOCs (19) satisfy

$$\beta - (\beta + \gamma)m_\ell = \frac{3k_0}{2D_0^*(\mathbf{T}^*, \boldsymbol{\tau})} \quad \text{and} \quad \gamma - (\beta + \gamma)m_r = \frac{3k_1}{2D_1^*(\mathbf{T}^*, \boldsymbol{\tau})}, \quad (\text{D.17})$$

and $\partial D_i^*(\mathbf{T}, \boldsymbol{\tau})/\partial T_i = \frac{1}{3}\Phi_{T_i}(\mathbf{T})$ becomes $\partial D_i^*(\mathbf{T}^*, \boldsymbol{\tau})/\partial T_i = k_i/2D_i^*(\mathbf{T}^*, \boldsymbol{\tau})$. Replacing the latter expressions in (D.16) for $i = 0, 1$, the denominators become:

$$\begin{aligned} & \frac{3k_0^2}{2D_0^*(\mathbf{T}^*, \boldsymbol{\tau})} - \rho(T_0^*)2(\beta + \gamma)D_0^{*,2}(\mathbf{T}^*, \boldsymbol{\tau}), \\ & \frac{3k_1^2}{2D_1^*(\mathbf{T}^*, \boldsymbol{\tau})} - \rho(T_1)2(\beta + \gamma)D_1^{*,2}(\mathbf{T}^*, \boldsymbol{\tau}). \end{aligned} \quad (\text{D.18})$$

Assuming that (23) is satisfied in equilibrium for $i = 0, 1$, both expressions in (D.18) are negative. That is, setting the expressions in (D.18) < 0 , we get (23). Thus, given that the numerator in (D.16) is positive and the denominator is negative in equilibrium, we deduce that $dT_i^*/dk_i < 0$.

D.3.5 Socially optimal locations (3 Welfare analysis)

This analysis posits that the regulator choosing the optimal location of the facilities ignores all time and fixed costs (the schedule delay costs of the consumers, the cost of the carriers and the fixed cost of the facilities). In this context, the ‘socially indifferent’ consumer, denoted $\tilde{x}^S$, is obtained by adding $\hat{\pi}_0 = \hat{p}_0 - \hat{c}_0 - \hat{\tau}_0$ and $\hat{\Pi}_0 = \hat{\tau}_0 + \hat{\omega}_0$ to $\hat{U}_0$, and $\hat{\pi}_1 = \hat{p}_1 - \hat{c}_1 - \hat{\tau}_1$ and $\hat{\Pi}_1 = \hat{\tau}_1 + \hat{\omega}_1$ to $\hat{U}_1$, where $\hat{U}_i$ for $i = 0, 1$ are given above (2). Denoting the social utilities U_0^S and U_1^S , solving $U_0^S - U_1^S$ for x and assuming that $\hat{c}_i$ and $\hat{\omega}_i$ for $i = 0, 1$ are equal across carriers and facilities, we get $\tilde{x}^S = \frac{1-b+a}{2}$. Let $a \in [0, 1]$ be the location of facility 0 and $1 - b$ represent the location of facility 1 for $b \in [0, 1]$ with $a + b \leq 1$. In the Hotelling model with quadratic transportation costs, the locations that minimize the average total transportation cost are given by solving:

$$\min_{a,b} TC^S = \frac{\theta}{2} \left[\int_0^a (a-x)^2 dx + \int_a^{\tilde{x}^S} (x-a)^2 dx + \int_{\tilde{x}^S}^{1-b} ((1-b)-x)^2 dx + \int_{1-b}^1 (x-(1-b))^2 dx \right].$$

Integrating the above expressions, we get:

$$\frac{a^3}{3} + \frac{(1-b-a)^3}{12} + \frac{b^3}{3}.$$

The related FOCs are:

$$\frac{\partial TC^S}{\partial a} = a^2 - \frac{(1-b-a)^2}{4} = 0, \quad (D.19)$$

$$\frac{\partial TC^S}{\partial b} = b^2 - \frac{(1-b-a)^2}{4} = 0. \quad (D.20)$$

Solving the system of FOCs gives $a^* = b^*$. Replacing b^* in (D.19) and a^* in (D.20) drives to $(a^S, 1-b^S) = (\frac{1}{4}, \frac{3}{4})$. Given that $\frac{\partial^2 TC^S}{\partial a^2} = 2a + \frac{(1-b-a)}{2}$, $\frac{\partial^2 TC^S}{\partial b^2} = 2b + \frac{(1-b-a)}{2}$ and $\frac{\partial^2 TC^S}{\partial a \partial b} = \frac{\partial^2 TC^S}{\partial b \partial a} = \frac{1-b-a}{2}$, the Hessian determinant is $|H| = a + b - (a-b)^2$. Given that $\frac{\partial^2 TC^S}{\partial a^2} > 0$, $\frac{\partial^2 TC^S}{\partial b^2} > 0$, and $|H| > 0, \forall a, b \in [0, 1]$, the stationary point is a minimum. If the social planner relocates only facility 0, setting $1-b = 1$, solving the quadratic equation (D.19) and discarding the negative solution yields $a^S|_{b=1} = \frac{1}{3}$. Conducting the same exercise with (D.20) when only facility 1 is relocated, given $a = h \geq 0$, we get the local minimum $1-b^S|_{a=h} = \frac{2+h}{3}$.

D.3.6 Socially inefficient schedules when $\widehat{k}_0, \widehat{k}_1 \neq 0$ (not included in 3 Welfare analysis)

Here, we generalize the derivation of $\Delta T_i = T_i^S|_{\widehat{k}_0=\widehat{k}_1=0} - T_i^*|_{\widehat{k}_i=0}$, $i = 0, 1$ below (32) for the case where $\widehat{k}_0, \widehat{k}_1 \neq 0$. By using $\rho(t) = \mathcal{U}[0, 24]$ and rearranging T_i^* in (20), we get:

$$T_i^* = 24 \left[\frac{\beta}{\beta + \gamma} - \frac{3k_i}{2(\beta + \gamma)D_i^*(\mathbf{T}^*, \boldsymbol{\tau})} \right].$$

Then, subtracting T_i^* from T_i^S in (32), ΔT_i becomes:

$$\begin{aligned} \Delta T_0 &= T_0^S - T_0^* = -12 \left[\frac{\widehat{\beta}}{\widehat{\beta} + \widehat{\gamma}} + \frac{\widehat{\beta}D_0^* + \widehat{\gamma}(2D_0^* - 3)\widehat{k}_0}{\widehat{\gamma}(\widehat{\beta} + \widehat{\gamma})D_0^*} \right] - b_0\widehat{k}_1, \\ \Delta T_1 &= T_1^S - T_1^* = 12 \left[\frac{\widehat{\gamma}}{\widehat{\beta} + \widehat{\gamma}} + \frac{\widehat{\beta}(3 - 2D_1^*) - \widehat{\gamma}D_1^*\widehat{k}_1}{\widehat{\beta}(\widehat{\beta} + \widehat{\gamma})D_1^*} \right] - a_1\widehat{k}_0, \end{aligned}$$

where $D_i^* \equiv D_i^*(\mathbf{T}^*, \boldsymbol{\tau})$. Setting carriers' marginal time cost to zero ($\widehat{k}_0 = \widehat{k}_1 = 0$), we get the expressions below (32).

Consider now the case where $T_0 \geq T_1$. Clearly, $T_i^{sym,*}$ in (B.7) can be rearranged as above for the uniform distribution and $D_i^*(\mathbf{T}^*, \boldsymbol{\tau})$ becomes $D_i^{sym,*}(\mathbf{T}^*, \boldsymbol{\tau})$. Then, subtracting $T_i^{sym,*}$ from $T_i^{sym,S}$ in page 44, we obtain:

$$\begin{aligned} \Delta T_0^{sym} &= T_0^{sym,S} - T_0^{sym,*} = 12 \left[\frac{\widehat{\gamma}}{\widehat{\beta} + \widehat{\gamma}} + \frac{\widehat{\beta}(3 - 2D_0^{sym,*}) - \widehat{\gamma}D_0^{sym,*}\widehat{k}_0}{\widehat{\beta}(\widehat{\beta} + \widehat{\gamma})D_0^{sym,*}} \right] - b_0\widehat{k}_1, \\ \Delta T_1^{sym} &= T_1^{sym,S} - T_1^{sym,*} = -12 \left[\frac{\widehat{\beta}}{\widehat{\beta} + \widehat{\gamma}} + \frac{\widehat{\beta}D_1^{sym,*} + \widehat{\gamma}(2D_1^{sym,*} - 3)\widehat{k}_1}{\widehat{\gamma}(\widehat{\beta} + \widehat{\gamma})D_1^{sym,*}} \right] - a_1\widehat{k}_0, \end{aligned}$$

where $D_i^{sym,*} \equiv D_i^{sym,*}(\mathbf{p}^*, \mathbf{T}^{sym,*}, \boldsymbol{\tau})$. Setting carriers' marginal time cost to zero ($\widehat{k}_0 = \widehat{k}_1 = 0$), we get the expressions ΔT_i^{sym} , $i = 0, 1$, in page 44.

D.4 Additional numerical checks

D.4.1 Shape of the schedule delay cost function $\Phi(\mathbf{T})$ for $t \sim \mathcal{U}[0, 24]$

In (A.7) we have:

$$\Phi^U(\mathbf{T}) = \frac{\beta + \gamma}{48} T_1^2 - \beta T_1 + \beta T_0 - \frac{\beta + \gamma}{48} T_0^2. \quad (\text{D.21})$$

We can deduce the following first and second derivatives :

$$\begin{aligned} \frac{\partial \Phi^U(\mathbf{T})}{\partial T_0} &= \beta - \frac{\beta + \gamma}{24} T_0, & \frac{\partial \Phi^U(\mathbf{T})}{\partial T_1} &= \gamma - \frac{\beta + \gamma}{24} T_1, \\ \frac{\partial^2 \Phi^U(\mathbf{T})}{\partial T_0^2} &= -\frac{\beta + \gamma}{24} < 0, & \frac{\partial^2 \Phi^U(\mathbf{T})}{\partial T_1^2} &= \frac{\beta + \gamma}{24} > 0. \end{aligned} \quad (\text{D.22})$$

Fig. D.4 shows the $\Phi^U(\mathbf{T})$ and $\Phi^{sym,U}(\mathbf{T})$ obtained with the parameters used in the simulations ($\theta = 260$, $h = 0.25$, $\hat{\beta} = 5$, $\hat{\gamma} = 7$) of Section 4. The light gray area corresponds to $\Phi^U(\mathbf{T})$ and the dark gray area is $\Phi^{sym,U}(\mathbf{T})$. As expected, both functions are concave in T_0 and convex in T_1 . The first derivatives are null when $T_0^U = T_1^U = \frac{24\beta}{\beta + \gamma} = 10$.

Figure D.4: $\Phi^U(\mathbf{T})$ when $t \sim \mathcal{U}[0, 24]$

D.4.2 Numerical checks of the covered market condition (3) for Section 4

When the marginal time cost of the carriers is null ($\widehat{k}_0 = \widehat{k}_1 = 0$) or when the departure times are given, the covered market condition is shown in Appendix A.7 to be:

$$\begin{aligned} & -\frac{\theta}{2}(1-h)(9+h) < \\ & \Delta\widehat{c} + \Delta\widehat{\omega} - 8\widehat{\Phi}(\mathbf{T}) - 9\widehat{\beta}(T_1 - T_0) \leq \Delta\widehat{c} + \Delta\widehat{\omega} - 8\widehat{\Phi}(\mathbf{T}) + 9\widehat{\gamma}(T_1 - T_0) \quad (\text{D.23}) \\ & < \frac{\theta}{2}(1-h)(9-h), \end{aligned}$$

where all parameters (or departure times) are exogenous. By using the parameters and departure times of Table 1, one can verify that (D.23) is satisfied in each column of the table. One may rather want to check whether the maximal admissible difference in schedules ($T_0 = 0$ and $T_1 = 24$) would comply with (D.23). The latter schedules would *violate* the covered market condition. By adjusting the parameters values (e.g., increasing the unit transportation cost parameter (θ), one can allow (D.23) to be satisfied $\forall T_i \in [0, 24]$.

Instead of using (D.23), the reader can use the equilibrium fares to verify that the covered market condition holds in equilibrium *ex post*. Doing this for the equilibria shown in Table 1, recalling that the covered market condition is given by

$$-\frac{\theta}{2}(1-h^2) < \underbrace{\widehat{p}_1 - \widehat{p}_0 - \widehat{\beta}(T_1 - T_0)}_{(B)} \leq \underbrace{\widehat{p}_1 - \widehat{p}_0 + \widehat{\gamma}(T_1 - T_0)}_{(A)} < \frac{\theta}{2}(1-h)^2.$$

and by using $\widehat{\beta} = 5, \widehat{\gamma} = 7, \frac{\theta}{2} = 130$ and the equilibrium fares and departure times in Table 1, we get:

- Column 1: $h = 0 \Rightarrow -\frac{\theta}{2}(1-h^2) = -130$ and $\frac{\theta}{2}(1-h)^2 = 130$.
 $\widehat{p}_1 - \widehat{p}_0 = 510 - 510 = 0, T_1 - T_0 = 10 - 10 = 0$
 $\Rightarrow -130 < (B) = 0 \leq (A) = 0 < 130$.
- Columns 2-4: $h = 0.25 \Rightarrow -\frac{\theta}{2}(1-h^2) = -121.9$ and $\frac{\theta}{2}(1-h)^2 = 73.1$. Then,

Column 2:

$$\begin{aligned} \widehat{p}_1 - \widehat{p}_0 &= 369.2 - 390.8 = -21.6, T_1 - T_0 = 10 - 10 = 0 \\ \Rightarrow -121.9 < (B) &= -21.6 \leq (A) = -21.6 < 73.1. \end{aligned}$$

Column 3:

$$\begin{aligned} \widehat{p}_1 - \widehat{p}_0 &= 370.2 - 389.8 = -19.6, T_1 - T_0 = 10 - 7 = 3 \\ \Rightarrow -121.9 < (B) &= -19.6 - 5 \times 3 = -34.6 \leq (A) = -19.6 + 7 \times 3 = 1.4 < 73.1. \end{aligned}$$

Column 4:

$$\begin{aligned}\hat{p}_1 - \hat{p}_0 &= 380.3 - 379.7 = 0.6, \quad T_1 - T_0 = (10 - 0) = 10 \\ \Rightarrow -121.9 < (B) &= 0.6 - 5 \times 10 = -49.4 \leq (A) = 0.6 + 7 \times 10 = 70.6 < 73.1.\end{aligned}$$

- Column 5: $h = 0.33 \Rightarrow -\frac{\theta}{2}(1 - h^2) = -115.8$ and $\frac{\theta}{2}(1 - h)^2 = 58.4$
 $\hat{p}_1 - \hat{p}_0 = 323 - 353.8 = -30.8, \quad T_1 - T_0 = 12$
 $\Rightarrow -115.8 < (B) = -30.8 - 5 \times 12 = -90.8 \leq (A) = -30.8 + 7 \times 12 = 53.2 < 58.4.$

D.4.3 Numerical checks of selected propositions and theoretical results for Section 4

Let's illustrate that the equilibrium fares, fees and profits obtained in Column 1 of Table 1 are consistent with our propositions. By Proposition 4.2, facility 0 charges a lower fee than the rival facility if the marginal operational cost advantage of its carrier exceeds half of its per passenger commercial revenue advantage ($\Delta\hat{c} < \frac{\Delta\hat{\omega}}{2}$). Carrier 0 has an operational cost disadvantage ($\Delta\hat{c} = -2$) which is lower than (half of) the commercial revenue advantage of its facility ($\frac{\Delta\hat{\omega}}{2} = \frac{2}{2}$). Thus, $\hat{\tau}_0^* < \hat{\tau}_1^*$ as we can see in Column 1. Proposition 4.2 also states that profits are equal across facilities when $\Delta\hat{c} = -\Delta\hat{\omega}$. This condition holds here as well and $\hat{\Pi}_0^* = \hat{\Pi}_1^*$. Carriers' outcome also follows from Propositions 1 and 2: given that $h = \hat{\Phi} = 0$ in Column 1, fares and profits are identical across carriers when the marginal costs advantage of carrier 0 is null. As the reader can verify, $(\hat{c}_1 + \hat{\tau}_1) - (\hat{c}_0 + \hat{\tau}_0) = 380 - 380 = 0$, which implies $\hat{p}_0^* = \hat{p}_1^*$ and $\hat{\pi}_0^* = \hat{\pi}_1^*$. Moreover, given that D_0^* increases in h (and D_1^* drops) between Columns 1 and 2 of Table 1, let's show that the condition in (A.8) for $\partial D_0 / \partial h > 0$ holds at $h = 0$ and at $h = 0.25$.

Recall that, by (A.8), $\frac{\partial D_0}{\partial h} > 0$ iff $\frac{\theta}{2}(1 - h)^2 > \hat{p}_0 + \hat{C}_0^M - (\hat{p}_1 + \hat{C}_1^M)$.

- Column 1: $h = 0 \Rightarrow \frac{\theta}{2}(1 - h)^2 = 130$.
 $\hat{p}_0 - \hat{p}_1 = 510 - 510 = 0, \quad \hat{\Phi} = \hat{C}_0^M - \hat{C}_1^M = 0,$
 $\Rightarrow 130(1 - 0)^2 = 130 > 510 - 510 = 0$
 $\Rightarrow \partial D_0 / \partial h > 0$ holds.
- Column 2: $h = 0.25 \Rightarrow \frac{\theta}{2}(1 - h)^2 = 73.1$.
 $\hat{p}_0 - \hat{p}_1 = 390.8 - 369.2 = 21.6, \quad \hat{\Phi} = \hat{C}_0^M - \hat{C}_1^M = 0,$
 $\Rightarrow 130(1 - 0.25)^2 = 73.1 > 390.8 - 369.2 = 21.6$
 $\Rightarrow \partial D_0 / \partial h > 0$ holds.

Fig. D.5 below, not included in the final version of paper, summarizes the main results of Propositions 1 and 2 graphically. It represents conditions (9) and (10) along

the marginal costs' advantage. The figure posits $h + 2\Phi(\mathbf{T}) > -(h/2) - \Phi(\mathbf{T})$, which implies $\Phi(\mathbf{T}) > -(h/2)$. Of course, this inequality could be reversed as nothing ensures that the schedule delay cost disadvantage of carrier 0 is lower than $-(h/2)$. In the figure, we notice that the former assumption rules out the situation where $\hat{p}_0 < \hat{p}_1$, $D_0^* < D_1^*$ and $\hat{\pi}_0 < \hat{\pi}_1$. The latter case does happen if we assume, instead, that $\Phi(\mathbf{T}) < -(h/2)$ and adapt Fig. D.5 accordingly. In that case, $\hat{p}_0 > \hat{p}_1$, $D_0^* > D_1^*$ and $\hat{\pi}_0 > \hat{\pi}_1$ will not occur. With this in mind, we proceed to checking whether these conditions do hold in the simulations.

Figure D.5: Carrier optimal fares, markups, demands and profits

In Column 1 of Table 1, we have: $\hat{p}_0^* = \hat{p}_1^*$, $D_0^* = D_1^*$ and $\hat{\pi}_0^* = \hat{\pi}_1^*$. Thus we expect $-\frac{h}{2} - \Phi(\mathbf{T}) = \Delta\tilde{c} = h + 2\Phi(\mathbf{T})$ to hold. That is indeed the case as $\Delta\tilde{c} = \frac{1}{\theta(1-h)}((\hat{c}_1 + \hat{\tau}_1) - (\hat{c}_0 + \hat{\tau}_0)) = 0$ and $h = \Phi(\mathbf{T}) = 0$. In Column 2 of Table 1, we have: $\hat{p}_0^* > \hat{p}_1^*$, $D_0^* > D_1^*$ and $\hat{\pi}_0^* > \hat{\pi}_1^*$. Thus we expect $-\frac{h}{2} - \Phi(\mathbf{T}) < \Delta\tilde{c} < h + 2\Phi(\mathbf{T})$ to hold. Here, $h = 0.25$ and $\Phi(\mathbf{T}) = 0$, thus:

$$\begin{aligned} \Rightarrow \Delta\tilde{c} &= \frac{1}{\theta(1-h)}((\hat{c}_1 + \hat{\tau}_1) - (\hat{c}_0 + \hat{\tau}_0)) = 0.005 \cdot (-16.2) = -0.08 \\ \Rightarrow h + 2\Phi(\mathbf{T}) &= 0.25 \\ \Rightarrow -\frac{h}{2} - \Phi(\mathbf{T}) &= -0.125 \\ \Rightarrow -\frac{h}{2} - \Phi(\mathbf{T}) < \Delta\tilde{c} < h + 2\Phi(\mathbf{T}) &\text{ holds.} \end{aligned}$$

Turning to Column 4 in Table 1, we have $\hat{p}_0^* < \hat{p}_1^*$, $D_0^* < D_1^*$ and $\hat{\pi}_0^* < \hat{\pi}_1^*$. This is precisely the case that Fig. D.5 rules out by assumption and that we discussed above the figure. Again, given that $\Phi(\mathbf{T}) = \frac{\hat{\Phi}}{\theta(1-h)}$ and that $\Delta\tilde{c} = \frac{1}{\theta(1-h)}((\hat{c}_1 + \hat{\tau}_1) - (\hat{c}_0 + \hat{\tau}_0))$, we can use the relevant magnitudes in Column 4 to verify that $h + 2\Phi(\mathbf{T}) < \Delta\tilde{c} < -\frac{h}{2} - \Phi(\mathbf{T})$:

- Column 4: $h = 0.25$, $\Phi(\mathbf{T}) = \frac{\hat{\Phi}}{\theta(1-h)} = \frac{-25}{260(1-0.25)} = -0.13$ and $\Delta\tilde{c} = \frac{(\hat{c}_1 + \hat{\tau}_1) - (\hat{c}_0 + \hat{\tau}_0)}{\theta(1-h)} = \frac{0.4}{260(1-0.25)} = 0.002$

$$\Rightarrow h + 2\Phi(\mathbf{T}) = -0.006 \text{ and } -\frac{h}{2} - \Phi(\mathbf{T}) = 0.005. \text{ Thus, } -0.006 < 0.002 < 0.005.$$

Obviously, $\Phi(\mathbf{T}) = -0.13 < -(h/2) = -0.125$ holds as well here.

Thus, $\hat{p}_0^* < \hat{p}_1^*$, $D_0^* < D_1^*$ and $\hat{\pi}_0^* < \hat{\pi}_1^*$ in Column 4 is consistent with our theoretical results.

Next, recall that Fig. 2, replicated below, establishes conditions related to the competition and demand effects when h varies.

$$\begin{array}{c} \downarrow \hat{\pi}_0^*, \uparrow \hat{\pi}_1^* \qquad \qquad \downarrow \hat{\pi}_0^*, \downarrow \hat{\pi}_1^* \qquad \qquad \uparrow \hat{\pi}_0^*, \downarrow \hat{\pi}_1^* \\ \hline \frac{3h-5}{2} - \Phi(\mathbf{T}) \qquad \qquad \frac{1+3h}{2} - \Phi(\mathbf{T}) \qquad \qquad \rightarrow \Delta\tilde{c} \end{array}$$

In Section 4 of the paper, we observe that the competition effect dominates the demand effect along h in all simulations (see, e.g., Fig. D.6 or Fig. D.8). Thus we can verify easily if our theoretical results are corroborated by simulations. Again, note that $\Phi = \hat{\Phi}(\mathbf{T}) = 0$ in Columns 1 and 2. Thus, $\frac{3h-5}{2} < \Delta\tilde{c} < \frac{1+3h}{2}$ should hold at $h = 0$ and at $h = 0.25$:

- Column 1: $h = 0 \Rightarrow \frac{3h-5}{2} = -2.5$ and $\frac{1+3h}{2} = 0.5$. $(\hat{c}_1 + \hat{\tau}_1) - (\hat{c}_0 + \hat{\tau}_0) = 0$, then $\Delta\tilde{c} = 0$. Thus, $-2.5 < 0 < 0.5$.
- Column 2: $h = 0.25 \Rightarrow \frac{3h-5}{2} = -2.1$ and $\frac{1+3h}{2} = 0.9$, $(\hat{c}_1 + \hat{\tau}_1) - (\hat{c}_0 + \hat{\tau}_0) = -16.2$, $\Delta\tilde{c} = -0.083$. Thus, $-2.1 < -0.08 < 0.9$,

$\pi_0^* \downarrow$ and $\pi_1^* \downarrow$ in Columns 1 and 2 when h increases, which confirms our theoretical results numerically.

D.4.4 Figures related to the equilibria shown in Section 4

The first two top plots in the figures below show how fares and profits vary along h under the assumptions made in Column 2 of Tab. 1. The two middle plots display carriers' profits along $T_i, i = 0, 1$ keeping h at 0.25. The two bottom plots correspond to the fares and profits along h in the situation of Column 4. The plots in the next page are the facilities' counterpart.

Figure D.6: Carrier fares and profits as a function of h when service times are equal

Figure D.7: Carrier profits as a function of departure times

Figure D.8: Carrier fares and profits as a function of h when service times are strongly differentiated

Figure D.9: Facility fees and profits as a function of h when service times are equal

Figure D.10: Facility profits as a function of departure times

Figure D.11: Facility fees and profits as a function of h when service times are strongly differentiated

D.4.5 Existence conditions of Appendix A.4 for the carrier-rivalry game of Appendix C.

Let's show that the existence conditions (A.12), (A.13) and (A.14) hold in the carrier-rivalry game analyzed in Section C. As the simulations rely on unscaled parameter values, the existence conditions are multiplied by $\theta(1-h) = 260(1-0.25) = 195$. Starting with (A.14) and (A.12), we get:

$$\begin{aligned} 0 < \beta\theta(1-h) &< \frac{\theta(1-h)}{24} \quad \text{and} \quad \gamma\theta(1-h) > 0, \\ 0 < 5 &< \frac{195}{24} = 8.125 \quad \text{and} \quad 7 > 0, \\ \frac{72\beta^2 - (\beta + \gamma)(3+h)}{2(\beta + \gamma)}\theta(1-h) &< \Delta\tilde{c}\theta(1-h) < \frac{-72\beta^2 + (\beta + \gamma)(3-h)}{2(\beta + \gamma)}\theta(1-h), \\ -241.875 &< \Delta\tilde{c} < 193.125. \end{aligned}$$

The first two lines ensure that (A.14) holds, so the profits are positive in equilibrium. As $\Delta\tilde{c} = -16$, the two last lines ensure that (A.12) holds as well, which guarantees the local concavity of the profit functions. Regarding (A.13), we have:

$$\begin{aligned} \left(\frac{8\gamma^3}{3(\beta + \gamma)} - \frac{(3+h+2\Delta\tilde{c})\gamma}{9} \right) \theta(1-h) &< k_0 \theta(1-h) < \left(-\frac{8\beta^3}{3(\beta + \gamma)} + \frac{(3+h+2\Delta\tilde{c})\beta}{9} \right) \theta(1-h) \\ -2.01 &< \hat{k}_0 < 1.88 \\ \left(\frac{8\gamma^3}{3(\beta + \gamma)} - \frac{(3-h-2\Delta\tilde{c})\gamma}{9} \right) \theta(1-h) &< k_1 < \left(-\frac{8\beta^3}{3(\beta + \gamma)} + \frac{(3-h-2\Delta\tilde{c})\beta}{9} \right) \theta(1-h) \\ -1.88 &< \hat{k}_1 < 1.48 \end{aligned}$$

Thus, setting $k_i \in [-1, 1]$ for $i = 0, 1$ guarantees the existence of a unique and profit-maximizing departure time $T_i^* \in]0, 24[$ for each carrier.

D.4.6 Time reaction functions for the time game of Appendix C.

We establish the uniqueness and local stability of the *Nash equilibria* in departure times by plotting the locally concave profit functions and the related reaction functions. In Fig. D.12 below, we first show the profit functions $\hat{\pi}_0(T_0 \in [0, 24], T_1 = 24, \hat{k}_0 \in [-1, 0, 1])$ and $\hat{\pi}_1(T_0 = 0, T_1 \in [0, 24], \hat{k}_1 \in [-1, 0, 1])$ obtained with the simulation parameters under Fig. C.1 from the main text. Given that $\hat{\Phi}(T_0, T_1) \equiv \hat{\Phi}^{sym}(T_0, T_1)$ when $\rho(t) = \mathcal{U}[0, 24]$, then $\hat{\pi}_i \equiv \hat{\pi}_i^{sym}$ for $i = 0, 1$ (see below Eq. (A.7) or footnote 29). Thus, distinguishing the case $T_0 \leq T_1$ from $T_0 \geq T_1$ is not relevant.

Figure D.12: Profit functions related to Fig. C.1 and Table D.1.

$$\hat{\pi}_0(T_0, T_1 = 24, \hat{k}_0 \in [-1, 1], \hat{\beta} = 5, \hat{\gamma} = 7) \quad \hat{\pi}_1(T_0 = 0, T_1, \hat{k}_1 \in [-1, 1], \hat{\beta} = 5, \hat{\gamma} = 7)$$

$$\hat{\pi}_0(T_0, T_1 = 24, \hat{k}_0 \in [-1, 1], \hat{\beta} = 7, \hat{\gamma} = 7) \quad \hat{\pi}_1(T_0 = 0, T_1, \hat{k}_1 \in [-1, 1], \hat{\beta} = 7, \hat{\gamma} = 7)$$

$$\hat{\pi}_0(T_0, T_1 = 24, \hat{k}_0 \in [-1, 1], \hat{\beta} = 7, \hat{\gamma} = 5) \quad \hat{\pi}_1(T_0 = 0, T_1, \hat{k}_1 \in [-1, 1], \hat{\beta} = 7, \hat{\gamma} = 5)$$

Notes: We assume: $h = 0.25$, $\frac{\theta}{2} = 130$, $t \sim \mathcal{U}[0, 24]$, $\hat{c}_0 = 10$, $\hat{c}_1 = 8$, $\hat{\tau}_0 = 280$, $\hat{\tau}_1 = 266$, $\hat{\omega}_0 = 20$ and $\hat{\omega}_1 = 18$.

As the reader can see in Fig. D.12, the profits are concave on the $[0, 24]$ time interval. Therefore, a unique maximum exists over that range for each carrier and we can exclude corner profits dominating the local maximum. We also numerically verified that the first derivative of $\hat{\pi}_i(T_i, T_{-i}, \hat{k}_i)$ with respect to T_i is positive (negative) when evaluated at the lower (upper) bound of the $[0, 24]$ interval for all relevant T_{-i} and $\hat{k}_i$ magnitudes.

We now explore the first-order conditions by focusing on the main case treated in Section C, that is $\hat{\beta} = 5, \hat{\gamma} = 7$ and $\hat{k}_i \in \{-1, 0, 1\}$ for $i = 0, 1$. The first derivatives of the profit functions are of the form:

$$\begin{aligned} \frac{\partial \hat{\pi}_0(T_0, T_1, \hat{k}_0)}{\partial T_0} &= m_0 - 195\hat{k}_0 - lT_1 + gT_1^2 + (-\tilde{f}_0 + \tilde{e}T_1 - \tilde{d}T_1^2)T_0 - bT_0^2 + aT_0^3, \\ \frac{\partial \hat{\pi}_1(T_0, T_1, \hat{k}_1)}{\partial T_1} &= m_1 - 195\hat{k}_1 - lT_0 + gT_0^2 + (-\tilde{f}_1 + \tilde{e}T_0 - \tilde{d}T_0^2)T_1 - bT_1^2 + aT_1^3, \end{aligned}$$

where all equation parameters are strictly larger than 0. Notice that the only coefficients which differ across carriers are m_i and f_i . One can check that this is consistent with the algebraic expressions (D.4) and (D.5) provided in Section D.3.3 of this Supplementary material. Fig. D.13 below shows the shapes of the above first derivatives. Solving the FOCs with respect to the departure time for each carrier, we get three distinct solutions or ‘reaction functions’ for each carrier (not shown here but apparent because of the three intersections of the first derivatives with the T_i axis). Only one of these falls within the $]0, 24[$ space and maximizes the profit of each carrier.

Figure D.13: First derivatives of the profit functions.

Notes: We assume: $h = 0.25, \frac{\theta}{2} = 130, t \sim \mathcal{U}[0, 24], \hat{\beta} = 5, \hat{\gamma} = 7, \hat{c}_0 = 10, \hat{c}_1 = 8, \hat{\tau}_0 = 280, \hat{\tau} = 266, \hat{\omega}_0 = 20$ and $\hat{\omega}_1 = 18$.

In Fig. D.14 below, we further show the three interior solutions (or reaction functions) $T_i^*(T_{-i}, \hat{k}_i)$, $i = 0, 1$, obtained for the cases $k_i \in \{-1, 0, 1\}$. Merging the LHS

and RHS plots into a single plot (appropriately inverting the axes) leads to Fig. C.1 in Section C that we use in the paper to establish the uniqueness of the Nash equilibria and their local stability (see the related comments in the second paragraph of page ?? of the paper).

Figure D.14: Time reaction functions used to build Fig. C.1.

Notes: We assume: $h = 0.25$, $\frac{\theta}{2} = 130$, $t \sim \mathcal{U}[0, 24]$, $\hat{\beta} = 5$, $\hat{\gamma} = 7$, $\hat{c}_0 = 10$, $\hat{c}_1 = 8$, $\hat{\tau}_0 = 280$, $\hat{\tau} = 266$, $\hat{\omega}_0 = 20$ and $\hat{\omega}_1 = 18$.

D.4.7 Fare reaction functions for the time game of Appendix C

The figures below are the fare reaction functions related to the Nash equilibria shown in Fig. C.1 of Section C. To keep the figure readable, we plot only selected equilibria.

Figure D.15: Nash Equilibria in fares: reaction functions for the carrier-rivalry game

Notes: all simulations assume: $h = 0.25$, $t \sim \mathcal{U}[0, 24]$, $\hat{\beta} = 5$, $\hat{\gamma} = 7$, $\frac{\theta}{2} = 130$, $\hat{c}_0 = 10$, $\hat{c}_1 = 8$, $\hat{\tau}_0 = 280$, $\hat{\tau}_1 = 266$, $\hat{\omega}_0 = 20$ and $\hat{\omega}_1 = 18$.

D.4.8 Numerical Nash equilibria for the time game of Appendix C

Table D.1 below reports the fare and departure time equilibria found in the carrier-rivalry game of Appendix C. It also shows equilibria obtained assuming $\hat{\beta} = \hat{\gamma}$ or $\hat{\beta} > \hat{\gamma}$. The reader can verify that the covered market condition (3) holds in all cases.³⁶

Table D.1: Carrier-rivalry equilibria with time-varying marginal costs for the carriers

	$T_0 \leq T_1$				
	Carrier marginal time costs				
	$\hat{k}_0 = 0$ $\hat{k}_1 = 0$	$\hat{k}_0 = 0$ $\hat{k}_1 = -1$	$\hat{k}_0 = +1$ $\hat{k}_1 = 0$	$\hat{k}_0 = +1$ $\hat{k}_1 = -1$	$\hat{k}_0 = -1$ $\hat{k}_1 = -1$
	$\hat{\beta} = 5, \hat{\gamma} = 7$				
(T_0^*, T_1^*)	(10, 10)	(10, 16.4)	(4, 10)	(4.2, 16.2)	(15.8, 16.2)
$(\hat{p}_0^*, \hat{p}_1^*)$	(390.3, 368.7)	(393.7, 365.3)	(387.3, 371.7)	(390.7, 368.3)	(390.7, 368.3)
(D_0^*, D_1^*)	(0.514, 0.486)	(0.532, 0.468)	(0.499, 0.501)	(0.516, 0.484)	(0.516, 0.484)
$(\hat{\pi}_0^*, \hat{\pi}_1^*)$	(51.6, 46.0)	(55.2, 59.1)	(44.5, 49.0)	(47.8, 61.8)	(67.8, 61.8)
	$\hat{\beta} = 7, \hat{\gamma} = 7$				
	(T_0^*, T_1^*)	(12, 12)	(12, 17.5)	(6.9, 12)	(7, 17.3)
	$(\hat{p}_0^*, \hat{p}_1^*)$	(390.3, 368.7)	(393.2, 365.8)	(387.7, 371.3)	(390.6, 368.4)
	(D_0^*, D_1^*)	(0.514, 0.486)	(0.529, 0.471)	(0.501, 0.499)	(0.516, 0.484)
	$\hat{\beta} = 7, \hat{\gamma} = 5$				
	(T_0^*, T_1^*)	(14, 14)	(14, 20.4)	(8, 14)	(8.2, 20.2)
	$(\hat{p}_0^*, \hat{p}_1^*)$	(390.3, 368.7)	(393.7, 365.3)	(387.3, 371.7)	(390.7, 368.3)
	(D_0^*, D_1^*)	(0.514, 0.486)	(0.532, 0.468)	(0.499, 0.501)	(0.516, 0.484)
	$T_0 \geq T_1^\dagger$				
	Carrier marginal time costs				
	$\hat{k}_0 = 0$ $\hat{k}_1 = 0$	$\hat{k}_0 = 0$ $\hat{k}_1 = +1$	$\hat{k}_0 = -1$ $\hat{k}_1 = 0$	$\hat{k}_0 = -1$ $\hat{k}_1 = +1$	$\hat{k}_0 = +1$ $\hat{k}_1 = +1$
	$\hat{\beta} = 5, \hat{\gamma} = 7$				
$(T_0^{sym,*}, T_1^{sym,*})$	(10, 10)	(10, 3.6)	(16, 10)	(15.8, 3.8)	(4.2, 3.8)
$(\hat{\pi}_0^{sym,*}, \hat{\pi}_1^{sym,*})$	(51.6, 46.0)	(55.2, 39.1)	(64.5, 49.0)	(67.8, 41.8)	(47.8, 41.8)
	$\hat{\beta} = 7, \hat{\gamma} = 7$				
	(T_0^*, T_1^*)	(12, 12)	(12, 6.5)	(17.1, 12)	(17, 6.7)
	$(\hat{\pi}_0^{sym,*}, \hat{\pi}_1^{sym,*})$	(51.6, 46.0)	(54.6, 36.7)	(66.1, 48.5)	(68.9, 39.0)
	$\hat{\beta} = 7, \hat{\gamma} = 5$				
$(T_0^{sym,*}, T_1^{sym,*})$	(14, 14)	(14, 7.6)	(20, 14)	(19.8, 7.8)	(8.2, 7.8)
$(\hat{\pi}_0^{sym,*}, \hat{\pi}_1^{sym,*})$	(51.6, 46.0)	(55.2, 35.1)	(49.0, 68.5)	(71.8, 37.8)	(43.8, 37.8)

Notes: all simulations assume: $h = 0.25$, $t \sim \mathcal{U}[0, 24]$, $\frac{\theta}{2} = 130$, $\hat{c}_0 = 10$, $\hat{c}_1 = 8$, $\hat{\tau}_0 = 280$, $\hat{\tau}_1 = 266$, $\hat{K}_i = 0$ for $i = 0, 1$. [†] The equilibrium fares $(\hat{p}_0^*, \hat{p}_1^*)$ and demands (D_0^*, D_1^*) when $T_0 \geq T_1$ are identical to those reported in the same column for $T_0 \leq T_1$ due to $\Phi^U(\mathbf{T}) \equiv \Phi^{sym,U}(\mathbf{T})$.

³⁶ When $\hat{k}_0 = -\hat{k}_1 = 1.13$ in Fig. C.1, note that $T_0^* = 3.4$, $T_1^* = 17$, $\hat{p}_0^* = 390.8$ and $\hat{p}_1^* = 368.2$. Then, the larger term within inequality (3) is close to the upper bound. Similarly, when $-\hat{k}_0 = \hat{k}_1 = 1.13$, then $T_0^* = 16.6$, $T_1^* = 3$, $\hat{p}_0^* = 390.8$ and $\hat{p}_1^* = 368.2$, and the lower term within inequality (B.4) is close to its lower bound. Thus, $T_i^*(\hat{k}_0 = -\hat{k}_1)$ and $T_i^*(-\hat{k}_0 = \hat{k}_1)$ are bounded within the $[0, 24]^2$ space to ensure a covered market.

D.4.9 Checking footnote 36

Footnote 36 states: "When $\widehat{k}_0 = -\widehat{k}_1 = 1.13$, then $T_0^* = 3.4$, $T_1^* = 17$, $\widehat{p}_0^* = 390.8$ and $\widehat{p}_1^* = 368.2$, and the larger term within inequality (3) is close to the upper bound. Similarly, when $-\widehat{k}_0 = \widehat{k}_1 = 1.13$, then $T_0^* = 16.6$, $T_1^* = 3$, $\widehat{p}_0^* = 390.8$ and $\widehat{p}_1^* = 368.2$, and the lower term within inequality (B.4) is close to its lower bound. Thus, $T_i^*(\widehat{k}_0 = -\widehat{k}_1)$ and $T_i^*(-\widehat{k}_0 = \widehat{k}_1)$ are bounded within the $[0, 24]^2$ space to ensure a covered market."

Lets show that this statement is true. Given $h = 0.25$, $\widehat{\beta} = 5$, $\widehat{\gamma} = 7$, $\frac{\theta}{2} = 130$, we get:

- Bounds of (3) and (B.4): $-\frac{\theta}{2}(1 - h^2) = -121.9$ and $\frac{\theta}{2}(1 - h)^2 = 73.1$. Then,
when $\widehat{k}_0 = -\widehat{k}_1 = 1.13$: $\widehat{p}_1 - \widehat{p}_0 = 368.2 - 390.8 = -22.6$, $T_1 - T_0 = 17 - 3.4 = 13.6$
 $\Rightarrow -121.9 < -22.6 - 5 \times 13.6 = -90.6 \leq \underline{-22.6 + 7 \times 13.6 = 72.6} < 73.1$.
when $-\widehat{k}_0 = \widehat{k}_1 = 1.13$: $\widehat{p}_1 - \widehat{p}_0 = 368.2 - 390.8 = -22.6$, $T_0 - T_1 = 16.6 - 3 = 13.6$
 $\Rightarrow -121.9 < \underline{-22.6 - 7 \times 13.6 = -117.8} \leq -22.6 + 5 \times 13.6 = 45.4 < 73.1$.

This result stresses that widely separated departure times violate the covered market conditions (3) and (B.4).

D.5 Main notations

$\widehat{U}$: consumer's gross benefit,

$\frac{\theta}{2}$: unit transportation cost,

x : location of the consumer over the $[0, 1]$ geographical space,

h : location of facility 0 over the $[0, 1]$ geographical space,

1 : location of facility 1 over the $[0, 1]$ geographical space,

t : consumer's desired service time,

$\widehat{\beta}$: consumer's unit cost for early schedule delay (in monetary units),

$\widehat{\gamma}$: consumer's unit cost for late schedule delay (in monetary units),

T_i : departure time scheduled at facility i , $i = 0, 1$,

$\widehat{C}(T_i, t)$: schedule delay cost incurred by a consumer with desired time t departing facility i at time T_i ,

m_l, m_c, m_r : fraction of consumers having departure time preference earlier than T_0 , between T_0 and T_1 and later than T_1 , respectively,

$\widehat{p}_i$: fare of the carrier serving facility i at time T_i ,

$\widehat{c}_i$: marginal operational cost of carrier i , $i = 0, 1$,

$\widehat{\tau}_i$: fee charged by facility i to its carrier, $i = 0, 1$,

$\widetilde{c}_i = (c_i + \tau_i)$: normalized marginal costs of carrier i , $i = 0, 1$,

$\widehat{K}_i$: fixed time cost incurred by carrier i , $i = 0, 1$,

$\widehat{k}_i$: marginal time cost incurred by carrier i along the time of day,

$\widehat{K}(T_i)$: total time cost incurred by carrier i at time T_i , $i = 0, 1$,

$\widehat{\omega}_i$: per passenger commercial revenue of facility i , $i = 0, 1$,

D_i : market demand at facility i , $i = 0, 1$,

$\widehat{\pi}_i$: profit of carrier i , $i = 0, 1$,

$\widehat{\Pi}_i$: profit of facility i , $i = 0, 1$,

$\widehat{F}_i$: fixed cost of facility i , $i = 0, 1$.