Plane wave spectra estimation in 2D reverberation chamber

K. Nafkha, E. Richalot, S. Mengué and O. Picon

Université Paris-Est Marne-la-Vallée, Laboratoire ESYCOM

Bât. Copernic, 5 bd Descartes, 77454 Marne-la-Vallée cedex 2, France

Abstract:

In this article, we study the angular plane waves expansion of the standing electric field within a reverberation chamber. For this purpose, we adapted a high resolution Root-MUSIC based method to the context of the reverberation chamber by using a decorrelation procedure to estimate the angles of arrival and complex amplitudes of the plane waves. Using FDTD simulations to get the standing electric field cartographies, we demonstrate, at low frequencies, that directions of arrival are slightly affected by the stirrer rotation whereas the wave amplitudes are strongly perturbed. This result indicates that stirrer works as a plane wave powers tuner.

Key-words: reverberation chamber; directions-of-arrival; MUSIC; mode stirring; FDTD

I. Introduction

Reverberation chambers (RC) constitute a performing tool for radiated emissions and immunity measurements. They are electrically large multimoded cavities that use either electronic (Hill, 1994) or mechanical (Corona et al., 1996) stirring to reproduce, on the equipment under test (EUT), an average homogeneous and isotropic field. While fields within an RC are often described through their statistical distributions, it has been shown that they can be expanded on their angular plane wave spectra (Hill, 1998) (Moglie & Pastore, 2006).

As plane wave distributions are often used to model indoor and urban propagation channels, it is of great interest to use a similar representation in an RC in order to relate its performance to practical multipath environments where devices under test are used.

The estimation of the directions of arrival (DOA) of wave fronts on an array of sensors is a common problem for many fields including radar, sonar (Iwata et al., 2001), radio astronomy, and mobile communications (Fuhl et al., 1997). Fourier analysis is the simplest technique to determine the plane wave angular distribution in space; however, it suffers from a poor angular resolution for a limited observation surface (as in an RC), generates secondary lobes that disturb the result interpretation and necessitates a high number of sensors.

Other algorithms have been proposed to increase the angular resolution over conventional Fourier analysis. Super resolution techniques are advanced approaches possessing theoretically an infinite asymptotic resolving power: two sources can be theoretically resolved as closed and as weak as they can be. Limitations of resolving power come only from limited observation time. These techniques include multiple signal classification (MUSIC) (Schmidt, 1986) (Laghmardi et al. 2009), Root-MUSIC (Barabell, 1983) (Hwang et al., 2008) and estimation of signal parameters via rotational invariance techniques (ESPRIT) (Roy et al., 1989). We choose Root-MUSIC, one of the most powerful approaches, that returns the estimated discrete angular spectrum. This algorithm provides asymptotically unbiased estimates of the directions of arrival.

Super resolution techniques are based on eigen-decomposition of the autocorrelation matrix generated by a received signal vector. As this decomposition is possible only with a full rank matrix, the plane waves impinging on the array must be not fully correlated. We will see that, in an RC, received signals are highly correlated. Therefore, we perform a matrix preprocessing that restores the matrix rank.

The algorithm used to estimate the plane wave spectrum within the working volume of a reverberation chamber is presented in Section II. This algorithm is independent of the way signals on sensors are obtained, using simulations or measurements. Only 2D-FDTD simulation results are used in this paper. After its validation for the canonical case of an empty rectangular cavity, our method is applied to a reverberation chamber equipped with a fixed stirrer and leads to very accurate results (Section III). By rotating the stirrer, we then analyze the plane wave spectrum variation in the RC at frequencies below the lowest useable frequency (LUF). The results obtained show that, whereas directions of arrival stay approximately constant, the plane wave powers are highly affected by the stirrer rotation (Section IV). We conclude that, at low frequencies, the stirrer principally works as tuner of plane wave powers.

II. Plane wave estimation algorithm

MUSIC goniometer processes the signals received on a sensor array to compute the parameters of multiple incident wavefronts, which are the directions of arrival, the amplitudes and the phases of the waves. Root-MUSIC consists of a modification of this algorithm that improves the resolution performance, and is only applicable with an equally spaced sensor array.

II.1. The received signal

Let us consider a linear equally spaced array of N identical omnidirectional sensors constituting the receiving antenna (Figure 1). P plane wave fronts impinge on this array from directions θ_i , $i \in \{1, \dots, P\}$, defined from the array normal. These plane waves, generally considered as coming from P far-field point sources, are supposed to be narrowband and uncorrelated. The signal received by a sensor is a linear combination of the P plane wave

contributions with added noise. Thus, the N -dimensional vector $X(t_m)$ of the signals received by each sensor at time t_m , also called time observation, can be written as:

$$X(t_m) = A S(t_m) + B(t_m) \quad (1)$$

where S represents the complex amplitudes of the incident wave fronts (we take the phase reference point at sensor 1), and B describes the noise at the sensors. The $N \times P$ rectangular matrix A is the directional matrix whose columns are the P directional vectors.

Each directional vector $a(\theta_i)$ is obtained by illuminating the N -sensors linear array by a unit wave plane of direction of arrival θ_i , $i \in \{1, \dots, P\}$. For sensors along the y -axis with inter-element spacing d (Figure 1) and λ the plane wave wavelength, the directional vector is given by :

$$a(\theta_i) = (1 \ e^{j2\pi d \sin(\theta_i)/\lambda} \ e^{j4\pi d \sin(\theta_i)/\lambda} \ \dots \ e^{j(N-1)2\pi d \sin(\theta_i)/\lambda})^T.$$

II.2. The Root-MUSIC algorithm

The $N \times N$ sample covariance matrix of the X vector is:

$$R_{xx} \stackrel{\Delta}{=} E(X(t).X(t)^H) = \frac{1}{M} \sum_{m=1}^M X(t_m).X(t_m)^H \quad (2)$$

where $(.)^H$ denotes the Hermitian transpose, $E(.)$ the expectation operator and M the number of observed samples.

Under the basic assumption that the incident signals and the noise are uncorrelated, R_{xx} is a full rank matrix which can be written:

$$R_{xx} = A.S^2.A^H + \Sigma \quad (3)$$

where $S^2 = E(S(t).S(t)^H)$ is a diagonal matrix containing the plane wave powers and $\Sigma = E(B(t).B(t)^H)$. In the following, the noise is assumed to be gaussian and spatially white so that $\Sigma = \sigma^2 I$, where I is the unit matrix.

If the number P of incident waves is less than the number of sensors N , then the minimum eigenvalue of $A.S^2.A^H$ is zero, which occurs $(N-P)$ times. According to Eq. (3) and to the

eigenvector orthogonality property, the eigenvectors of R_{xx} corresponding to the highest eigenvalues form a basis of the signal subspace, whereas the $(N-P)$ others form a basis of the noise subspace. As a consequence, signal and noise subspaces are orthogonal. Therefore, the product between an eigenvector of the noise-subspace and a vector $a(\theta)$ is null for all direction θ corresponding to an incident wave. This orthogonality is used in MUSIC by defining a pseudo-spectrum $P(\theta)$ as:

$$P(\theta) = \frac{1}{a(\theta)^H V_N V_N^H a(\theta)} \quad (4)$$

where $V_N = [e_{P+1}, e_{P+2}, \dots, e_N]$ is the matrix of noise-subspace eigenvectors. This function is maximal when θ corresponds to a signal incidence direction.

An alternative to the tracing of the MUSIC pseudo-spectrum consists of determining the poles of this function. This is the strategy adopted in Root-MUSIC (Barabell, 1983).

The denominator of $P(\theta)$ can be written:

$$D(\theta) = \sum_{k=1}^N \sum_{n=1}^N e^{-j \frac{2\pi d \sin(\theta)}{\lambda} (k-1)} \cdot G_{kn} \cdot e^{j \frac{2\pi d \sin(\theta)}{\lambda} (n-1)} = \sum_{l=-N+1}^{N-1} g_l e^{-j \frac{2\pi d \sin(\theta)}{\lambda} l} \quad (5)$$

where $G = V_N V_N^H$, and g_l is the sum of G -coefficients along the l^{th} diagonal.

If we define the polynomial:

$$D(z) = \sum_{l=-N+1}^{N-1} g_l z^{-l} \quad (6),$$

then the research of the directions of arrival is tantamount to one of the roots of $D(z)$ on the unit circle (Castanié, 2006). The Root-MUSIC algorithm is known to offer better resolution performance and to require a lower signal-to-noise ratio than MUSIC itself.

II.3. Plane wave decorrelation procedure

The presented algorithm in II.2 has been developed in the case of several independent time observations $X(t_m)$. We will show that an appropriate pre-processing makes it possible to use Root-MUSIC in the case of a single time observation.

In our study of the reverberation chamber, the signal is analysed at a resonant frequency, so that we only have a single snapshot of the data. It implies that the phase differences between the incident plane waves are fixed, so that the signals are seen as highly correlated. Moreover, an RC is classically excited by a single antenna, so that the plane waves are all generated by the same source and not P uncorrelated sources as required for the application of MUSIC. When the sources (or plane waves) are highly correlated, the rank of R_{xx} matrix is no longer the number P of directions of arrival, but the number of groups of coherent sources that are uncorrelated with each other (Grenier, 1994). As a result, the rank of R_{xx} matrix is in our case equal to one (Gabriel, 1980) as all plane waves are fully correlated, and the estimation of the P directions of arrival is impossible using the Root-MUSIC algorithm.

To overcome this problem, a spatial smoothing is applied to the data vector (Shan et al., 1985). This operation guaranties the nonsingularity of the covariance matrix. For an array of identical sensors, the duality between time and space can be used. To obtain several samples, we take a subarray of Q adjacent sensors out of the whole array, from one edge of the array, and write the measured field values into a Q -dimensional vector X_1 . The second subarray is obtained by shifting the first one by one sensor. The corresponding values are stored into X_2 . This subarray translation introduces a phase difference between the observation vectors X_1 and X_2 , and is equivalent to a temporal shift. The input vectors X_i obtained from the $(N-Q+1)$ subarrays constitute the observed samples used to calculate the covariance matrix R_{xx} as the average of the elementary covariance matrices of each X_i vector, using Eq. (2). It has been shown by Shan et al. (Shan et al., 1985) that if the number of subarrays is greater than or equal to the number of signals, then the modified covariance matrix is non-singular; thus

Root-MUSIC can be successfully apply to the smoothed covariance. In the studied examples, we will take subarrays having the recommended length $N/2$.

Thus the use of translated subarrays to average the covariance matrix compensates the absence of time variation with the frequential signal. To improve the correlation matrix properties and to enhance the method accuracy, a second treatment is performed to the covariance matrix after spatial smoothing. As we will see, this method artificially increases the number of sensors, so that the fall of the sensor number due to the use of subarrays is partially compensated.

When signals are completely uncorrelated, the correlation matrix is Toeplitz. We perform a second processing to reduce the coefficients variation on R_{xx} -matrix diagonals (Jansson & Stoica, 1999) (Phaisal-atsawasenee & Suleesathira, 2006). It consists of averaging the previous covariance matrix and the one obtained with the same sensors but ranged in decreasing order, from N to 1. We notice here R_F the first covariance matrix, and R_B the second one. The average covariance matrix R_{FB} is given by:

$$R_{FB} = (R_F + J R_F^H J)/2 \quad (7),$$

where J is an $N \times N$ exchange matrix given by

$$J = \begin{pmatrix} 0 & 0 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & \dots & 0 \\ 1 & 0 & \dots & 0 \end{pmatrix} \quad (8).$$

This averaging process leads to a hermitian and centro-symmetric matrix.

II.4. Plane waves number estimation

Root-MUSIC algorithm is a parametric method requiring prior knowledge of the number of plane waves to separate. In the canonical case of an empty parallelepipedic reverberation chamber (without stirrer), the number of plane waves is known analytically, but after the

stirrer introduction, this number is no longer known theoretically. To overcome this problem, we refer to the information theory criterion of Rissanen and Schwartz (Barron et al., 1998) (Wax & Kailath, 1985) (Athanasios et al., 2001). According to this criterion, the number of plane waves is the k value which minimizes the MDL (Minimum Description Length) quantity defined as:

$$MDL(k) = -M \cdot \log \left(\frac{\prod_{i=k+1}^N \hat{\lambda}_i}{\left(\frac{1}{N-k} \cdot \sum_{i=k+1}^N \hat{\lambda}_i \right)^{N-k}} \right) + \frac{1}{2} \cdot k \cdot (2N - k) \cdot \log M \quad (9)$$

for k varying from 1 to $N-1$. N is the number of sensors, M the number of time observations and $\hat{\lambda}_{i, 1 \leq i \leq N}$ are R_{xx} -matrix eigenvalues.

Because of the decorrelation procedure, we have $N/2$ sensors by subarray and the parameter k varies from 1 to $N/2-1$. The number of time observations is $M = N/2-1$.

II.5. Plane wave complex amplitudes

After the determination of the number of waves using the MDL criterion, the use of the previously presented Root-MUSIC algorithm associated to the decorrelation procedure leads to directions of arrival. However, for a line of sensors parallel to y -axis, only k_y component of wave vectors can be determined, and the angles of incidence are extracted using the equation $k_y = k_0 \cdot \sin \theta$, where k_0 is the free space propagation constant. One k_y value corresponds to two directions of arrival, θ and $(\pi - \theta)$, and both possible directions are considered while evaluating wave amplitudes. To resolve this ambiguity on symmetric directions of arrival, a second line of sensors parallel to the first one is used.

By neglecting the noise, Eq. (1) becomes $X = A S$, where $(N \times P)$ rectangular matrix A is calculated analytically using the found directions of arrival. X is a $2N$ -dimensional vector containing the observations on the two sensor arrays. $A^H A$ matrix is non singular because of

the orthogonality of $a(\theta_i)$ vectors, so by a simple matrix inversion we can find the plane waves complex amplitudes vector S :

$$S = (A^H A)^{-1} A^H X \quad (10)$$

Thus, amplitudes and phases of the received plane waves are known without any ambiguity.

It has to be noticed that, as standing waves are generated within the cavity, if θ_i is a DOA and A_i the associated complex amplitude, then $(\theta_i - \pi)$ is also a DOA and the associated amplitude is the conjugate of A_i : $A.e^{-j(k_x x + k_y y)} + A^*.e^{j(k_x x + k_y y)}$ is real at every point (x, y) . These two waves correspond to forward and backward travelling waves on the same propagation axis.

This study could be extended to the 3 D case of electromagnetic field distribution, where two orthogonal lines of sensors should be used to estimate correctly the elevation and azimuthal angles (Harabi et al., 2007), however a particular attention should be taken to the orientation of sensors (antennas oriented along each axis) for electric field polarization consideration.

In this paper, we validate plane wave extraction technique in a 2D case in order to avoid considering the electric field polarization and for computation time reduction.

III. Simulations results

III.1. 2D FDTD simulations

In order to reduce the computation time, simulations are performed in this paper with a 2D RC. The simulated structure presents a cylindrical symmetry along z-axis. The fields are independent of z axis, so that only TM modes are excited with an electric field along z-axis. A 2D-FDTD tool (Taflove, 1995) is used to calculate the field distribution within the 2D reverberation chamber.

The simulated chambers, with and without stirrer, are of dimensions $a=3.105\text{m}$ along x-axis and $b=2.475\text{m}$ along y-axis as shown in Figure 1. The classical formula $6 \times f_0$ (f_0 = empty

cavity cut-off frequency) (Mittra & Trost, 1997) for the LUF of chamber leads to 465MHz. A fine mesh of 414×330 rectangular Yee's cells is used. Chamber walls as well as the stirrer are taken to be perfectly conducting, so that the boundary condition is that electric field tangential components are equal to zero on their surfaces. The cavity excitation is performed by 10 punctual sources placed along a wall. This choice of numerous sources aims for a correct excitation of all the modes, whereas with a single punctual source a very weak amplitude is observed for a mode presenting a node near the excitation point. To obtain the frequential spectrum, a Dirac excitation imposes firstly on each of the sources a unit amplitude on E_z component at $t=0$. Once the studied resonant frequency is chosen, the field cartography is obtained with a sinusoid excitation at the chosen frequency to avoid the excitation of adjacent modes. We can notice that, even if 10 punctual sources are used, the excitations are not independent, so that the correlation problem mentioned in Section II.3 remains unchanged and the decorrelation procedure is used.

Once electric field cartography is determined, the complex electric field is known on each cell of the mesh; so that each cell can be considered as an electric field sensor. To extract the directions of arrival, we use E_z -field real values from a line of 330 cells parallel to y-axis as an observation vector. Spatial smoothing applied to decorrelate the signals on the 330 sensors of the array is performed by using subarrays of 165 sensors.

As already seen in Section II.5, the extraction of DOA from a single line of sensors does not allow distinguishing θ and $(\pi-\theta)$. To resolve this ambiguity, another line of sensors, parallel to and close to the first one, is used. The distance between the two sensor lines is $5 \times d$, where d refers to intersensor distance which is equal to FDTD-cell dimension of 7.5 cm. The two lines are placed far away from the stirrer at $x_1 = 3m$ and $x_2 = 3.0375m$ from the origin as presented in Figure 1.

The working volume of a reverberation chamber is the particular volume where the field is statistically isotropic and uniform when the RC is well stirred (beyond the LUF). It implies that plane waves have no preferred propagation direction, field polarisation nor phase. This means that, in this volume where the device under test is placed, propagation direction angles, polarisation angles and phases are uniformly distributed. In our 2D case, we define the working surface as being distant of $\lambda_{\text{LUF}}/3$ from the cavity walls as well as from the stirrer (Figure 1), where λ_{LUF} is the wavelength in free space at the LUF. We focus on the field properties on this particular surface of dimensions 1.778m along x-axis and 2.055m along y-axis.

Once the spectrum is estimated from 2D-FDTD simulation results, we measure its accuracy by reconstructing the electric field cartography on the working surface from the estimated directions of arrival, amplitudes and phases. This reconstructed cartography is compared to the one originally obtained from FDTD simulation, and the absolute difference between both electric fields is calculated on the working surface. We will use as an indicator the relative error obtained by normalizing this absolute difference by the average amplitude of the initial electric field on the whole working surface.

III.2. Plane wave spectrum in the empty RC

To validate our approach, an empty rectangular cavity is examined, as the fields are known analytically in this case. We consider a cavity of dimensions a along x-axis and b along y-axis, with a coordinate system origin placed at its corner. The TM_{mn0} modes, which satisfy the boundary conditions, have an E_z -field of the following form:

$$E_z^{mn} = E_z^0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \quad (11)$$

with a corresponding resonance frequency:

$$f^{mn} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \quad (12)$$

where c is the light speed in vacuum.

By expanding the sine terms on exponentials, the resonant mode can be written as a superposition of four plane waves.

$$E_z^{mn} = -\frac{E_z^0}{4} \left(e^{j\left(\frac{m\pi}{a}x + \frac{n\pi}{b}y\right)} - e^{j\left(\frac{m\pi}{a}x - \frac{n\pi}{b}y\right)} - e^{j\left(-\frac{m\pi}{a}x + \frac{n\pi}{b}y\right)} + e^{j\left(-\frac{m\pi}{a}x - \frac{n\pi}{b}y\right)} \right) \quad (13)$$

The plane wave vectors are of the form $\vec{k} = (k_x, k_y)^T = (\pm m\pi/a, \pm n\pi/b)^T$, and the angles of incidence defined from x-axis are given by $\theta = \arctan(k_y/k_x)$ and $\theta = \arctan(k_y/k_x) + \pi$.

To validate our method, we examine the TM_{680} mode of theoretical resonance frequency 564.88MHz. Following the previous analytic equations, one finds four plane waves of directions of arrival $\theta_1 = 59.13^\circ$, $\theta_2 = -59.13^\circ$, $\theta_3 = 120.87^\circ$, $\theta_4 = -120.87^\circ$, of the same amplitude equal to $0.25 \times E_z^0$.

The simulation of the empty rectangular cavity using the FDTD method leads to a resonance frequency of 564.97MHz, and the electric field cartography is normalized to have a maximal E_z amplitude equal to one ($E_z^0 = 1$ Volt/meter). The number of plane waves, theoretically equal to 4 for an empty cavity, is overestimated by MDL criterion as 20 planes waves are estimated; this overestimation is due to numerical noise that disturbs the waves number estimation process.

The estimated DOA corresponding to the two highest eigenvalues are $\theta_1 = 59.06^\circ$, $\theta_2 = -59.06^\circ$, $\theta_3 = 120.94^\circ$, $\theta_4 = -120.94^\circ$ with the same amplitudes of 0.2493. The relative errors on directions of arrival and amplitudes are respectively about 0.1% and 0.3%. The amplitudes of the 16 other plane waves are very low, with a ratio between their amplitude and the one of the first modes below -41dB. This confirms that these modes can be considered as noise. The precision of our approach is also evaluated by comparing the initial and reconstructed

cartographies on the working surface. The mean relative error, calculated as the average relative error on the whole working surface, is less than 1.5 %.

FDTD simulations as well as the DOA extraction method have been validated in this simple case where the angular spectrum is known analytically. With the study of an RC equipped with a fixed stirrer, we will now validate our plane wave spectrum estimation technique in a more realistic cavity.

III.3. Plane wave spectrum in the RC equipped with a fixed stirrer

The reverberation chamber is simulated with an oblique stirrer, rotated to 45° from the axes (Figure 1). We examine the field at the resonance frequency of 327.5MHz. The same lines of sensors as in the previous case are used to estimate the angular spectrum. After computing the smoothed bidirective covariance matrix, we use the MDL criterion which estimates 20 plane waves. The amplitudes of the 20 plane waves found by our algorithm are less disparate than for the empty cavity: the ratio between the maximal and the minimal amplitudes is of 2.7. It seems to indicate the MDL criterion doesn't overestimate the number of waves in this case.

As the solution is not analytically known with a stirrer, the accuracy of the spectrum evaluation can only be evaluated by the comparison between the initial and the reconstructed cartographies of E_z -field. As the mean relative error found is less than 3%, the method is considered as validated and can be used to study the variation of the plane wave spectrum due to the stirrer rotation.

IV. Study of the stirrer rotating effect

Two fundamental parameters of the RC operation are deduced from FDTD simulations: the frequency spectrum which is the Fourier transform of the impulse response and the

electromagnetic field cartography at each resonant frequency. By rotating the stirrer around its axis by steps of 2.5° from 0° to 180° (and not 360° for symmetry considerations), it is possible to follow the variation of resonant frequencies on a stirrer rotation and to deduce from it E_z -field cartographies for each stirrer position.

We choose to follow the variation of a mode of resonant frequencies inferior than the LUF ([323MHz-328 MHz], see Figure 2). Its frequency excursion is about 5MHz. In order to explain the influence this frequency variation can have on the cavity response, we also present the resonant frequency variation of the two adjacent modes. Their resonant frequency excursions are about 7 MHz. Although the relative resonant frequency variations are small (about 2 %), the related influence on the field properties is important. Thus a given frequency, for example 327.3MHz, can correspond to the resonant frequency of two different modes for different stirrer positions (see Figure 2), and these two modes will present very different field distribution. Moreover, this effect increases with the frequency due to the increase of the resonant mode density. The choice of a low resonant frequency in the paper is justified by the low density of modes and by the low density of plane waves on angular spectrum that permits an easier analysis than at high frequencies.

For each stirrer position, once the resonant frequency and the associated E_z -cartography are determined, we use the presented plane wave estimation algorithm to estimate all the plane wave parameters (DOA, complex amplitude).

For a better legibility, we focus on the DOA comprised between 0° and 90° . Five plane waves are found on this interval upon 20 waves in total. We first of all examine the plane wave DOA as a function of the stirrer position (Figure 3). We observe that directions of arrival are slightly affected by the stirrer rotation. The whole couples of DOA and associated amplitude, found on a complete stirrer rotation, are represented in a polar diagram (Figure 4), where the angle indicates the DOA and the radius the plane wave power. In agreement with the remark

drawn with Figure 3, we can notice that DOA are grouped together around 5 principal directions. On the contrary, a large variation of wave powers is observed, with a variation of the wave amplitudes that reaches more than 45dB. We conclude that, below the LUF, the stirrer can be used as plane wave power tuner.

V. Conclusion

This paper presents a method to estimate the plane wave spectrum within a reverberation chamber. In our frequential study of the cavity, Root-MUSIC method cannot be directly applied because of the high correlation between wave fronts. Particular attention is paid to this problem, and a pre-processing is applied to decorrelate the received signals. The study of the canonical case of the empty cavity shows that directions of arrival, amplitudes and phases of the plane waves are obtained very precisely with relative errors below 0.3%. The accuracy of this method is also demonstrated for an RC equipped with a stirrer, with a mean difference between the cartography issued from FDTD and the one reconstructed from the extracted wave parameters below 3% at about 565MHz.

With this method we attempt to contribute to a better understanding of the stirring process within an RC. Thus a study of the stirrer rotation effect on plane wave DOA and powers is presented. For the studied frequency band, situated below the LUF, the stirrer rotation generates a large variation of wave powers whereas its effect on DOA distribution is very low.

Figure 1. Geometry of the 2D RC studied

Figure 2. Resonant frequency variation versus stirrer rotation angle

Figure 3. Directions of arrival of 5 plane waves for all stirrer positions

Figure 4. Polar diagram of plane wave directions of arrival and powers on a stirrer rotation

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