Cognitive profile in elementary algebra: the PÉPITE test interface

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Abstract: The work presented here is part of a multidisciplinary project, called the PÉPITE project. Its aim is to set up a system to diagnose student profiles starting from a multidimensional analysis grid to evaluate their competences in elementary algebra. This paper deals with the design, the making and the evaluation of an interface in Computer Based Learning Environments. In it, we present the method adopted to set up software which proposes tasks to the students and collects data for the diagnosis module.

We consider here the problems linked to the re-use of didactic know-how based on pencil and paper tasks and in particular how to transfer these tasks to a data processing environment. These problems are considered from the interface designer’s point of view. In an attempt to make this clear, we make a distinction between ergonomic problems connected with the functioning of the interface and problems related to the field of application.

Keywords: secondary education, mathematics, interdisciplinary, Human Computer Interface, formative evaluation.

I.
II. INTRODUCTION

Studies of Human-Computer Interaction draw attention to the fact that the user of a technical device faces a double problem: first, transferring his knowledge of the task and second, learning how to use the system (SENACH, 1993). In Computer Based Learning Environments, these meaning transfer problems for the students are called by educational theorists «computational transposition» (BALACHEFF, 1994) (ARTIGUE, 1995): technical and physical limitations interfere with the student's knowledge at the levels both of representation and action. These limitations modify the perception of the effects of the actions. In our work, we use pedagogical material previously intended for a pencil and paper environment. This paper deals with the problems encountered by the designers of the Intelligent Tutoring System interface during the transfer from pencil and paper tasks to computerised tasks.

In this article, we consider these problems with reference to software created as part of the PÉPITE project, which aims to describe the students’ functioning in algebra, in order to establish their cognitive profile. The emphasis is laid on the method of design we adopted and the defining of evaluation criteria for the software. First we present an analysis of the pencil and paper tasks to be transferred and of the aims of the system (needs analysis and task analysis). We then present the design of the computerised tasks focusing on what we want to observe in the results produced by the students. Finally, we present experimentatal use of the software by the users.

I. THE BASIS OF THE PÉPITE PROJECT

The aim of the PÉPITE project is to build a computerised environment able to «model» the reasoning process of 15 year old students of French secondary schools (the year before the end of the secondary school studies certificate) in elementary algebra, at the beginning of

1 Involving, in computer science, Martial Vivet, Élisabeth DELOZANNE, Pierre Jacoboni and Stéphanie Jean from the LIUM (Laboratoire d’Informatique de l’Université du Maine) and, in didactics of mathematics, Michèle ARTIGUE and Brigitte GRUGEON from the DIDIREM laboratory of Paris VII.
the year. The LINGOT\textsuperscript{3} project which will follow the PÉPITE project will use this «modeling» to give the students appropriate learning situations likely to help the evolution of their knowledge. The idea is to seek out, in the student's way of functioning, the «grains» of knowledge (in French, the «pépites») to use as a basis for building on new knowledge.

According to (BARON and VIVET, 1995), the general problem of automatic diagnosis in an ITS is to infer information about the learner model from what is noticed about his behaviour. This is what we try to do through analysis and interpretation of the data collected during the interaction. In the PÉPITE project, we base our work on a rigorous didactic and cognitive study which has been validated (GRUGEON, 1995). This study enables us to build cognitive profiles of students, applying a multidimensional analysis grid to the answers given to a series of pencil and paper exercises (the pencil and paper tasks). In this part, we describe the basis of the project: the didactical analysis and the pencil and paper diagnosis tools created by Brigitte GRUGEON. We then present the general architecture of the PÉPITE project.

A. THE DIDACTICAL ANALYSIS

This research in the theory of mathematics learning sets out from the hypothesis of knowledge building: the students have built up pieces of knowledge which may be different from generally accepted knowledge. Consequently, the work of the students presents coherences and regularities which correspond to their personal knowledge. One of the results of this study\textsuperscript{3} is a tool enabling us to analyse the work produced in order to find starting points to modify their knowledge. This tool (cf. figure 1) combines a series of pencil and paper tasks with a multidimensional analysis grid making it possible to interpret the students’ work in order to establish their profile in elementary algebra.

1. The pencil and paper tasks

Three types of pencil and paper tasks are put to the students during a test:

\begin{itemize}
\item \begin{itemize}
\item PÉPITE stands for gold nugget and LINGOT for ingot.
\item This thesis also contains a study of institutional relations of the students to algebra which is outside matters here.
\end{itemize}
\end{itemize}
– technical exercises whose aim is to determine numeric calculating and formal manipulating procedures,

– recognition exercises whose aim is to determine how students identify and interpret algebraic expressions in an algebraic context or in connection with other contexts,

– modelling exercises whose aims are to ascertain whether the students use the expected algebraic type of treatment, how they translate problems in the algebraic context and how they use the tools adapted for solving the problems.

At this stage, the students’ answers to the exercises are analysed « by hand » by the teacher with the multidimensional analysis grid.

![Diagram](image)

**Figure 1:** The pencil and paper diagnosis tool. **Figure 2:** Architecture of the PÉPITE project.

1. The multidimensional analysis grid

This grid is made up of six components (GRUGEON, 1995):
the algebraic treatment component gives us the opportunity to determine the algebraic competence of the student compared with types of expected treatment, in terms of success / failure,

the arithmetic / algebra relationship component enables us to infer the meaning given by the students to the algebraic process and to compare it with the arithmetic process,

the operationality of formal manipulation of algebraic writings component aims to study the way students deal with algebraic expressions,

the articulation between the different contexts component allows us to identify preferred ways of dealing with and moving between the different semiotic contexts,

the role of algebra component is intended to describe the way the students deal with algebra,

the rationality in algebra component makes it possible to identify the use of algebra as a tool for generalisation and for proof.

A set of criteria is associated with each of these analysis components. During the correction of the tests, for each answer given by the student the teachers give global values defined by the different criteria of the analysis grid. Some of the global values are specified by local values linked to the exercise.

<table>
<thead>
<tr>
<th>An example of a modelling exercise: The conjurer exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>A conjurer is sure of himself while carrying out the following trick. He says to a player: « Think of a number, add 8, multiply by 3, subtract 4, add the number you thought of, divide by 4, add 2 and subtract the number you first thought of: you have the number 7. » Is this assertion true ? Justify your answer.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MÉRIÈME’S ANSWER</th>
<th>SÉBASTIEN’S ANSWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>I take number 2</td>
<td>Let x be a number</td>
</tr>
<tr>
<td>2+8=10</td>
<td>x+8</td>
</tr>
<tr>
<td>10x3=30</td>
<td>(x+8)3=3x+24</td>
</tr>
<tr>
<td>30-4=26</td>
<td>3x+24-4=3x+20</td>
</tr>
</tbody>
</table>
26+2=28  
3x+20+x=4x+20  
28+4=7  
(4x+20)/4=x+5  
7+2=9  
x+5+2=x+7  
9-2=7  
x+7-x=7  
The assertion is true  
The conjurer can be sure of himself because he will always find 7

**Figure 3: Two examples of students' production on a pencil and paper task.**

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>CRITERION</th>
<th>GLOBAL VALUE</th>
<th>LOCAL VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>interaction between different contexts</td>
<td>type of conversion</td>
<td>identifiable</td>
<td>step by step separated writing</td>
</tr>
<tr>
<td>interaction between different contexts</td>
<td>method of solving</td>
<td>arithmetic</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4: Representation of the analysis structure : Mérième’s case.**

In the example (cf. figure 3), the conjurer exercise is particularly concerned with the « type of conversion » criterion for the « interaction between different contexts » component. This criterion has three possible global values: « correct », « identifiable » and « unidentifiable ». The « identifiable » global value is further defined in this exercise by the local values « global linear writing with brackets » and « step by step separated writing », which make it possible to define the type of conversion used in the particular context of this exercise more precisely. For this criterion, the diagnosis has given the following two students the same « step by step separated writing » local value, but the « method of solving » criterion has an « arithmetic » value for Mérième (cf. figure 4) and an « algebraic » value for Sébastien.

1. **The students’ profiles**

Applying the analysis grid to the work produced by a student on the pencil and paper tasks gives a set of values for each student for each task. This result then needs important
theoretical analysis. The very precise description of the behaviour is too detailed to be used as it is by the teachers (or by a computer). A transversal analysis of the grid's results enables us to establish a higher level description: the «cognitive profiles» of the students. These profiles can be used to understand and modify the student's way of functioning and thus act against student under-achievement.

These profiles have three levels of description:

– a *quantitative description* of algebraic competence in terms of success rates for the technical exercises and the modelling exercises (cf. first part of figure 5),

– a *qualitative description* of functioning coherences, component by component, in terms of functioning modalities obtained by cross checking of values of some criteria on the whole set of exercises (cf. second part of figure 5),

– a description of flexibility between contexts, represented by a *diagram* (cf. figure 6).

<table>
<thead>
<tr>
<th></th>
<th>technical exercises</th>
<th>modelling exercises</th>
<th>recognition exercises</th>
</tr>
</thead>
<tbody>
<tr>
<td>algebraic treatment</td>
<td>40%</td>
<td>42%</td>
<td></td>
</tr>
<tr>
<td>arithmetic/algebraic relation</td>
<td></td>
<td>=: announcement of</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>the solution</td>
<td></td>
</tr>
<tr>
<td>operationality of formal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>manipulation of algebraic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>writings</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>role of algebra</td>
<td></td>
<td></td>
<td>no function</td>
</tr>
<tr>
<td>rationality of algebra</td>
<td></td>
<td></td>
<td>pragmatical proof</td>
</tr>
</tbody>
</table>

*Figure 5: An extract from Mérième's profile, success rate and functioning modalities*
This diagnostic tool, based on pencil and paper tasks, has been tested several times. In particular it was tested in June 1996 on 600 students of a third form class (14 year old students).

A. THE DIAGNOSIS IN PÉPITE

The PÉPITE project aims to automate the pencil and paper diagnosis tool invented by Brigitte GRUGEON. The architecture of PÉPITE contains three modules (cf. figure 2, § 2.1):

- PÉPITEST which offers the students an adaptation of the pencil and paper tasks to the computer and which collects their answers. The difficulty of creating PÉPITEST resides in the transfer of the pencil and paper tasks to the computer platform. This software has now been completed and its creation is the work which will be presented and discussed in this paper.

- PÉPIDIAG which interprets and codes the results produced by the students from the data furnished by PÉPITEST. It uses the multidimensional analysis grid and defines values given to criteria tested by the exercise.
– PÉPIPROFIL which, from the preceding codes, establishes the students’ profiles and presents them to the users (teachers or researchers). This last module has been developed but is not detailed here.

I. DESIGN OF PÉPI TEST

In this section, we present the design procedure used to perfect PÉPI TEST. We specify the dimensions, objectives and evaluation methods of the software. We then separate the ergonomic problems of using the system from those linked to the transfer of the tasks from the pencil and paper environment to the data processing environment.

A. DESIGN PROCESS

Many books about the ergonomics of the Human-Computer Interface recommend that evaluation should be considered as a « state of mind » (KOLSKI, 1993) which must express itself throughout the design of a system. This preoccupation with validating design choices and detecting problems of use as early as possible often leads to the adoption of an iterative design process based on the making of prototypes that are tested and then modified if necessary. Using prototypes enables us to meet the aim of a user-centered design: creating a system which is easy to learn and to use (PREECE et al. 1994). Another advantage of using prototypes is to facilitate communication with customers and within the multidisciplinary design team (SENACH, 1993), (KRIEF, 1992), (KOLSKI, 1993), (VAN-HEYLEN and HIRACLIDES, 1996) for example.

This process necessitates an early examination of the evaluation criteria and methods. SENACH distinguishes two principal aspects of the evaluation, the utility of the product and its usability (SENACH, 1993). The utility deals with the adequacy of the software to the high level objectives of the customer. The usability concerns the capacity of the software to allow the user to reach his objectives easily.

4 This distinction is introduced for the convenience of the presentation, the ergonomy is also concerned with the problems of tasks and not only with the superficial aspect of the interface.
As far as PÉPI TEST is concerned, the «user» is the student whose objective is to solve the exercises. The «customer» is the person or the system in charge of carrying out the diagnosis on the student’s productions. The usability of the software concerns the quality of the interface with regard to the ergonomic recommendations (BASTIEN and SCAPIN, 1993). Utility concerns the capacity of the software to take the student’s behaviour into account in order to establish the diagnosis. From the designer’s point of view, the problem lies with defining machine tasks that give equivalent data to the pencil and paper ones. The evaluation consists of specifying this equivalence.

The evaluation methods that we have selected for the usability dimension are the classical methods used for Human-Computer Interfaces design. For the utility dimension, we rely on theories of teaching mathematics (ROBERT, 1992) (ARTIGUE, 1990). During the design process, the validation consists of checking by case-studies that the students’ results on the PÉPI TEST tasks allow the theoreticians to apply the analysis grid to draw up the students’ cognitive profiles by hand.

A. Usability of PÉPI TEST

The quality of a Human-Computer Interface is not guaranteed by the simple fact that it meets certain style guidelines or respects a certain set of recommendations in the best possible way (SENACH, 1993). Nevertheless ergonomic criteria gather experience in these domains and constitute a guide for designers. We discuss their being taken into account in PÉPI TEST, while referring to (BASTIEN and SCAPIN, 1993), but we must not forget the specificity of PÉPI TEST as an assessment environment.

Guidance is particularly important in the case of PÉPI TEST. Each student will do the test only once, so there cannot be an initial contact session with the software. The student is guided by the instructions, by the structure of the screen, by modifications to the appearance of the cursor and by help-balloons.

Respecting the limitation of the workload criterion (conciseness, minimal actions and information density) is a problem in our context. Some functionalities of the software set up
to facilitate use (for example the drag and drop) can draw the attention of some students away from mathematical activities. This diversion to aspects of interface manipulation can disturb the diagnosis.

The objective being diagnosis and not learning, the student has explicit control over the system. As in the pencil and paper environment the student can change exercises and modify his answers when he wishes. He can also skip questions and go back to them later.

The adaptability criterion is taken into account on the one hand by flexibility and on the other hand by the experience of the user. In PÉPITEST, the student can answer the same question in different ways (for example by using, or not using, a graphic tool). For the student who has previous experience with computers, PÉPITEST proposes classical tools such as cut - and- paste.

Given that the objective is diagnosis, no protection is possible concerning error management. However the student can always correct his entries.

Special attention was paid to the maintenance of coherence and homogeneity in the interface with regard to the setting out of information, the stability of the screen and the use of physical control devices.

The names and codes used in PÉPITEST are those imposed by the mathematics education theorists. The individual coding of the interface is reduced to the icons used in the buttons of the toolbar.

The criterion of compatibility concerns the degree of similarity between the various work environments of the user. It refers to the transfer problems that we treat in the next paragraph.

The validation consists of submitting our prototypes to the judgement of experts (experts in ergonomics and experts in the domain: educational theorists and mathematics teachers), to informal tests by users, and finally to controlled experimentation in a high school class.

A. Utility of PÉPITEST

5 POLLIER, quoted by BALBO (BALBO, 1994 p.101), considers that three experts are enough to detect 85% of the errors of this level.
The activity of the student on the PÉPITEST tasks must furnish data allowing a theorist (and later the software PÉPIDIAG) to apply the multidimensional grid of analysis.

In the design phase of PÉPITEST, we retained the possibility for the theorists to carry out the coding of the data obtained as a validation criterion. The profiles thus obtained could be confirmed by the teacher of the class. According to our validation criterion, we have on the one hand to create PÉPITEST tasks as close as possible to the pencil and paper tasks in order to obtain the «same» results. On the other hand, we must take into account the computational transposition: the transfer of pencil and paper exercises and tools to a computational environment changes the test and has consequences for the students’ results.

PÉPITEST is a set of 23 very different exercises: from closed questions (such as Multiple Choice Questions) to totally open questions (such as the conjurer exercise presented on figure 1). At first sight, pencil and paper tasks such as Multiple Choice Questions do not give any transfer problems. However, tasks that involve the production of sentences or of mathematical expressions (in particular the modelling exercises) give problems. We might fear that the users will simplify their syntax while typing sentences with the keyboard. With regard to mathematical expressions, different authors (including ARTIGUE, 1995) have noticed that the transition for fractions or square roots from spatial writing (pencil and paper) to linear writing (the keyboard has only a fraction bar) distracts the students from mathematical tasks to low level tasks such as the use of brackets.

We expect the research to provide us with a body of information to study to what extent these expression constraints modify the data and change the diagnosis.

Indeed, each task gives special difficulties linked to the nature of the cognitive activities in play and specified in the analysis grid. The detailed study of each of these tasks is not our purpose here (JEAN, 1996). Here we will simply present an example.

A. AN EXAMPLE OF TRANSFER TASK

*Complete the table by writing a sentence translating each step of the calculation program opposite the corresponding algebraic expression.*
| step 1 | Let the initial number be x | x |
| step 2 | ............................................................. | - x + 3 |
| step 3 | ............................................................. | (-x + 3)^2 |
| step 4 | ............................................................. | 1/((-x + 3)^2 + 4) |

**Figure 7:** A pencil and paper task example.

<table>
<thead>
<tr>
<th>Figure 8: The same task in PÉPI TEST.</th>
</tr>
</thead>
</table>

This exercise (cf. figure 7) tries to identify in the student's work the conversion rules used to pass from the algebraic context to the natural language context. In the pencil and paper version, we raised only two cases: either the students did not treat the question at all, or they used a very limited number of terms, of which we established the exhaustive list. We have chosen to transfer this exercise proposing a set of terms to construct the sentences (cf. figure
8). The list of these terms is large enough to construct correct sentences, but also allows a number of errors, expected or not. This tool modifies the proposed activity: this is undoubtedly an aid provided to the students which does not however give any indication as to the answer. The exercise set up like this can help some students to begin their work without preventing them from giving erroneous answers.

At first sight, we expect more answers to this exercise in its PÉPITEST version than in its pencil and paper version.

I. RESEARCH PROJECT

PÉPITEST was the subject of a research project whose objectives were:
– to validate the interface from the point of view of its usability,
– to collect data on the test software and to compare it with the pencil and paper data, in particular for the modelling exercises and some exercises for which the PÉPITEST version is very different from the pencil and paper version,
– to check through case-studies that the work produced by the students on the PÉPITEST tasks allow the theorists to construct the students’ profiles « by hand ».

A. ORGANISATION OF THE EXPERIMENT

The experiment took place in October 1996 in a fifth form class of thirty-two students in a high school in the suburbs of Paris. The test lasted 1H 45 which is according to the usual timetable for the mathematics class. The students were put in two groups and each student had access to a computer.

These students were supposed to be accustomed to using a computer through technology classes at high school. It was impossible to organise a preliminary session with the software and its tools.

We collected the traces of the session, a file that records the students’ answers to the exercises (equivalent to the information collected in the pencil and paper test) as well as another file logging information on the use of tools and the timing, a questionnaire filled in
by each student, the sheets of notes taken by the observers during the experiment (three learning theorists, two computer scientists and the teacher of the class) and the rough copies produced by the students (if any).

A. RESULTS

We now present the results obtained concerning the three objectives that we fixed for this experiment: usability of the interface, differences between the two types of data from the exercises and possibility of constructing the cognitive profiles from the students’ productions.

1. Usability of the software

After the first half-hour, the questions concerning the use of software disappeared. The questions asked by the students during this period concerned the use of a computer (keyboard, new paragraph, drag and drop, selection of fields), the use of PÉPITEST tools (the eraser), mathematics (use of brackets, calculation, terminology) and the mathematics with PÉPITEST (typing mathematical expressions). For the last two points, difficulties naturally lasted longer for some students.

Handling the software through solving the exercises therefore took less than half an hour. Globally, the guidance setup (structure of the screen, cursors, help balloons) worked well. We plan nevertheless to reduce the learning time by proposing two or three screens presenting the few basic manipulations necessary to the use of PÉPITEST.

Finally, we noticed, as other research has already shown (DELOZANNE, 1994) (SCHNEIDERMAN, 1992) that the sophisticated tools invented by the designers are underused.

1. Differences between the different sets of results and the « manual » building up of profiles

We noted, while observing the students during the test and while studying the traces of the sessions, that the students behaved globally in a similar manner with PÉPITEST and
during the pencil and paper test: they did the test in order and did approximately the same number of exercises.

Some exercises are more attractive in PÉPITEST than in the pencil and paper test: actually the students did them easily. However others seem more boring in PÉPITEST because the students have to manipulate algebraic expressions in them, which is considerably harder with the computer. As we had foreseen, the typing of algebraic expressions gave problems to the students without however preventing them from writing them. Some exercises which were not much attempted in the pencil and paper test were more often attempted in PÉPITEST, in particular the exercise presented in figure 4. We did not note the opposite phenomenon.

Case-studies indicate that the results on the modelling exercises are similar and that in the unusual exercises PÉPITEST accentuates the difficulties of certain students. This sheds more light on the type of behaviour that we are trying to identify.

Finally and especially, on the first results analysed, the educational theorist was able to apply the analysis grid and to obtain cognitive profiles confirmed by the class teacher. This is undoubtedly the most interesting result from our point of view as designers.

I. CONCLUSION

In this paper, we have presented PÉPITEST software which collects data to diagnose the capacities of students in algebra. We have concentrated on the design problems and on evaluating the interface, relying on work in Computer Based Learning Environments and in the ergonomics of interfaces. PÉPITEST has been completed and successfully underwent the test of being used experimentally in class. This concludes the first prototyping cycle for the iterative design process (DELOZANNE, 1994) (VAN-HEYLEN and HIRACLIDES, 1996). In our design method, we laid great importance on defining the evaluation criteria of PÉPITEST which means specifying the equivalence between the data from the pencil and paper tasks and those of PÉPITEST. Our validation criterion of PÉPITEST consists of verifying that the data obtained with the software permits us to build up profiles equivalent to the pencil and
The validation of the whole PÉpite project consists of demonstrating that the profile built up by the machine with the data obtained by the software is equivalent to the profile built up by an educational theorist with the same data.

The first results presented here, and in particular the creation of a set of results from PÉPiTEST tasks, make it possible to consider the research necessary for the design of PÉpite’s module of diagnosis. Only this second phase will allow us, through case-studies and a statistical study, to validate the automatic diagnosis by comparing it with the manual diagnosis established from the pencil and paper results.

I. ACKNOWLEDGMENTS

We wish to thank the students of Georges Brassens high school of Villeneuve-Le-Roi who took part in the experimentation of PÉPiTEST, as well as Nicole Pernias, their mathematics teacher.

We also thank Gwenda and Philippe Daubias for their help in translating this paper.

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