The eyes of French mathematicians on Tullio Levi-Civita - the case of hydrodynamics (1900-1930)

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1. Introduction

Tullio Levi-Civita (1871-1943), professor at the University of Rome from 1918 onwards, was a prominent Italian mathematician of the first part of the 20th century. He gave remarkable contributions to various mathematical fields, such as general relativity, the three-body problem, differential geometry, and hydrodynamics. French scholars generally appreciated Levi-Civita's work – for instance, his work on differential geometry influenced Élie Cartan and his school, and his papers on the regularization of the three-body were essential for Jean Chazy's research concerning the long-term behaviour of the solutions of the three-body problem. Henri Villat and other French mathematicians assimilated Levi-Civita's works on hydrodynamics and took inspiration from them, especially from those concerning the wake hypothesis and wave theory. Villat's thesis focused on the so-called Levi-Civita method for deducing the general integral of any plane motion with wake under some conditions on the shape of the body (Levi-Civita, 1907a). In 1929 Villat, who was professor at the Sorbonne, was appointed director of the Institute of fluid mechanics (*Institut de mécanique des fluides*) of Paris, created by the Ministry of aeronautics. In the twenties and thirties, Levi-Civita in Italy and Villat in France were the two reference points for hydrodynamics in their respective countries, although Villat had an institutional role much more important than that played by Levi-Civita in Italy.

In this article we aim at giving an image of Levi-Civita's work on hydrodynamics and of its reception in France, starting from his personal and scientific relationship with Villat. In particular, we focus on the following questions: How did French scholars assess Levi-Civita's work? How did he influence French studies on hydrodynamics, and particularly those of Villat and his students? We point out that Villat supervised a large number of theses during the years between the two world wars and some of his students spent a period of study with Levi-Civita at the University of Rome.[1]

In this paper we argue that Levi-Civita's aim is to found hydrodynamics on rigorous mathematical bases by avoiding the use of physical evidence often invoked by his time scholars. We shall show that his approach to hydrodynamics has been transmitted to the French school headed by Villat.

Our analysis makes use of letters and unpublished documents mainly contained in Levi-Civita's Archive (Library of the Accademia dei Lincei, Rome), Villat's Archive (Archives of the French Academy of Sciences, Paris), and Boussinesq's Archive (Institute de France, Paris). Such documents allow us to provide a fresh picture of Levi-Civita's relations with France. In particular, we observe that Villat was undoubtedly his French privileged correspondent; nevertheless, concerning hydrodynamics, Levi-Civita also exchanged letters with Joseph Hadamard, Marcel Brillouin, Marie-Louise Jacotin-Dubreil, Robert Mazet, and others.

We mention that Levi-Civita's scientific and personal relationships with his French colleagues were not affected by political opinions. In the twenties, the French generally shared feelings and policy of ostracism against scholars from Central Powers, to which Levi-Civita opposed his internationalist ideals. Levi-Civita was against imperialism and wars, as he often claimed in his correspondence. Just after the First World War, in a letter dated December 9, 1920, he wrote to his German colleague Arnold Sommerfeld: "I have always been, and not only in science, a convinced internationalist and, in consequence of this ideal, I consider au dessus de la mêlée all nationalisms indistinctly. I have the same opinion about the nationalisms preceding and following the horrible war, which upset Europe in a so ruinous way."[2] In 1922, in opposition to the decision to exclude Axis scientists from international meetings strongly supported by France, Levi-Civita organized, together with the director of the Laboratory of Aeronautics in Aachen, Theodor von Kármán, the first International Congress of Applied Mechanics opened to scholars from all countries.[3]

[1] Villat and Henri Cartan supervised the largest number of thesis in the interwar period, see (Leloup, 2009).

[2] The letter is contained in the Archives of the Deutsches Museum, and published in (Nastasi, Tazzioli, 2005; 224).

[3] On Levi-Civita's attitude towards German and Austrian scholars see (Nastasi, Tazzioli, 2013), (Nastasi, Tazzioli, 2014). For the organisation of the first International Congress of Applied Mechanics see (Battimelli, 1988).

Section 2 gives a brief – and not exhaustive – overview on Levi-Civita's life and research. The following sections will focus on his contribution to hydrodynamics and how French scholars assimilated his ideas. Section 3 illustrates Levi-Civita's contributions to plane motions with wake, especially related to the works by Marcel Brillouin and Villat. Section 4 focuses on Levi-Civita's role as a master at the University of Rome; it describes the work of Levi-Civita on wave theory and jets, which attracted many students to Rome – Rockefeller fellows, post-doc students, and students at a distance who just exchanged letters with Levi-Civita without meeting him personally. Section 5 contains concluding remarks.

2. A biographical sketch of Tullio Levi-Civita

Levi-Civita graduated from the University of Padua in 1892 with a thesis on the theory of invariants supervised by Gregorio Ricci Curbastro (1853-1925). Levi-Civita's professional biography is divided into two great periods – the period at the University of Padua, where he was appointed professor of rational mechanics in 1897, and the period at the University of Rome, from 1918 onwards, where he contributed to create a mathematical school at an international level. Nevertheless, even during the time spent in Padua, Levi-Civita supervised some theses, especially in the field of hydrodynamics (see sections 3, 4).

Levi-Civita's main research fields are: the theory of relativity, the three-body problem, adiabatic invariants, analytic mechanics, hydrodynamics, differential geometry, and tensor calculus.[4] His contributions are essential for the development of some branches of these theories. Each of these subjects attracted the most interest of French mathematicians, who sometimes strictly followed Levi-Civita's research.

On the eve of World War I, Levi-Civita had already published relevant papers, especially in the fields of differential geometry, hydrodynamics and the three-body problem, and was considered as one of the most prominent Italian mathematicians. For these reasons, he was asked to move to the Italian capital to join the other Roman mathematicians and to improve the scientific level of the University of Rome. In 1909 Vito Volterra (1860-1940), who had a great esteem of Levi-Civita from the early years of his scientific career, together with his colleague Guido Castelnuovo (1865-1952), offered him the opportunity to get a chair at the University of Rome.[5] At the time Levi-Civita declined – maybe his family ties prevented him

[4] For a more detailed analysis of Levi-Civita's work see (Nastasi, Tazzioli, 2005).
[5] The letter by Castelnuovo to Levi-Civita is dated August 27, 1909 and is published in (Nastasi, Tazzioli, 2000; 260-261).

from accepting the offer. Immediately after the war, however, in 1918, various personal reasons induced Levi-Civita to leave Padua and join the Roman group. In 1919, he was in fact appointed professor of higher analysis *per chiara fama*, and two years later he moved to the chair of mathematical physics.

In the twenties, also the two algebraic geometers, Federigo Enriques (1871-1946) and Francesco Severi (1879-1961), joined Levi-Civita, Guido Castelnuovo, and Vito Volterra at the University of Rome – Castelnuovo and Volterra being in Rome since 1891 and 1900 respectively. The *International Education Board* (IEB), created by the Rockefeller Foundation in 1923 to encourage scientific studies, judged Rome University as an international centre of excellence in mathematics.[6] George D. Birkhoff (1884-1944), a distinguished American mathematicians of the first half of the 20th century and a close friend of Levi-Civita, addressed to the director of the Paris office of the IEB, Augustus Trowbridge (1870-1934), his Report on European mathematical centres based on his trip through Europe of 1926: "The numerical strength of the mathematical group and the power of tradition at Paris, Göttingen and Rome far transcend those at the other centers named." (Siegmund-Schultze, 2001; 268) Already in 1923, Birkhoff suggested Levi-Civita as one of the advisors for mathematics in Europe – he also mentioned Borel (France), Brouwer (Netherlands), Hardy (Great Britain), Mittag-Leffler (Sweden), Nörlund (Denmark), de la Vallée Poussin (Belgium) (see Sigmund-Schultze, 2001; 37). There were several reasons to mention Levi-Civita as one of the main references, together with Borel, Mittag-Leffler and de la Vallée Poussin who were pillars of European mathematics. First, since the early years of the twentieth century, Birkhoff and Levi-Civita shared common scientific interests, in particular both contributed to the three-body problem; secondly, the scientific interests of Levi-Civita were in the forefront of mathematics of his time, in particular his studies on the theory of general relativity and his related work on differential geometry and tensor calculus; finally, his well-known scientific internationalism was another aspect surely taken into account.

In the twenties and thirties, Levi-Civita and his Roman colleagues welcomed many foreign students, often Rockefeller fellows, from several countries including several French (see Section 4), and made mathematical studies in Rome really international. In addition to his position of advisor for IEB, Levi-Civita was the reference point in Italy of Ernest Vessiot (1865-1952), the director of the *Ecole Normale Supérieure* (ENS) of Paris. Vessiot often recommended to Levi-Civita his students of the ENS. In a letter to Levi-Civita, dated August

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[6] On the role of the IEB in the internationalization of mathematics between the two world wars see (Sigmund-Schultze, 2001).

8th, 1925, Vessiot wrote that "he was pleased" to have obtained grants for Michel Guérard des Lauriers (1898-1988), Robert Mazet (see section 4.2) and André Weil (1906-1998), three ENS students, who would have spent a period of study at the University of Rome. Vessiot asked Levi-Civita to supervise his students' research and to introduce them to the other colleagues at the University of Rome.[7] He remarked that André Weil was an "exceptional" student.[8]

Levi-Civita's scientific reputation in Italy and abroad as a mathematician explains, at least partially, his success as a *Master* at the University of Rome and his wide network of mathematicians, physicists and more generally intellectuals, who were in touch or corresponded with him for personal and scientific reasons. In 1922 he was awarded the Sylvester Medal by the Royal Society of London, and in 1930 elected a foreign member of the Royal Society. He was often invited to give lectures in all Europe – including in the Soviet scientific institutes (for instance, the Institute of Tensorial Calculus, the Institute of Aerodynamics, and the Institute of Theoretical Astronomy) – US and South America. In France, he became a member of the Academy of Sciences in 1911, and was awarded the title of Doctor honoris causa from the University of Toulouse in 1929[9] and from the University of Paris (Sorbonne) in 1933.[10] From 1929 onwards, Henri Villat invited him several times to the Institute of fluid mechanics. In 1932 and 1934 the celebrated *Séminaire Hadamard* devoted a series of conferences to Levi-Civita's (and his students') works on adiabatic invariants.

In 1931, despite his hostility to the fascist regime, Levi-Civita decided to swear a loyalty oath to Fascism. He made that difficult decision to save the Roman school of mathematics. Various sources support this argument. In some letters, Guido Fubini (1879-1943) and Volterra showed him their utmost gratitude for his sacrifice – the oath to Fascism – in the name of the

[7] For instance, on September 24th, 1937, Vessiot recommended to Levi-Civita his student Christian Pauc: "M. Pauc travaille dans le champ de la théorie des ensembles, et les éléments de sa thèse sont soumis actuellement à votre collègue M. Fréchet ; de sorte qu'il est, dans une certaine nature, libre d'étudier d'autres sujets dans lesquels la théorie des ensembles pourrait servir d'instrument utile. Il pense qu'il serait intéressant pour lui d'être mis, à cet effet, en relation avec votre collègue M. Severi. Voudriez-vous être assez aimable pour mettre ces jeunes gens en rapport avec lui, et pour présenter, à cette occasion, à Monsieur Severi mes meilleurs souvenirs ?"
[8] On August 8th, 1925, Vessiot wrote to Levi-Civita: "Notre troisième boursier est M. *Weil* ; il sort cette année de l'Ecole Normale ; c'est un esprit d'une précocité et d'une faculté d'assimilation exceptionnelles, car il n'a pas encore 20 ans. Il travaille sans doute sur la direction de M. Volterra, à qui il a été présenté dernièrement. Mais je serai très heureux que vous vous vouliez bien vous intéresser aussi à lui." The letters by Vessiot to Levi-Civita are contained in Fondo Levi-Civita, Biblioteca dell'Accademia dei Lincei, Rome.
[9] See the report by Adolph Buhl in *Enseignement Mathématique*, vol. 28, 1929, p. 129-132.
[10] Villat was charged to write the "Report", which the Dean of the Faculty of Sciences of Paris read during the celebration. See the letter by Villat to Levi-Civita on March 3rd, 1933 (in Nastasi, Tazzioli, 2003; 397-398).

school.[11] Moreover, according the witness of Gino Arrighi (1906-2001), Levi-Civita swore also by following Volterra's explicit advice – Volterra and Levi-Civita shared the responsibility of the Roman school and did not want to betray their students, but to give them the necessary support in such a difficult period.[12]

In September, 1938 Italian government promulgated the Laws for the defence of the race, according to which Jews were dismissed from public positions. In particular, Jewish university professors were excluded from universities, academies and cultural institutions. Levi-Civita was forced to retire and prevented from participating in scientific meetings. Moreover, he was expelled from the editorial board of the *Zentralblatt füt Mathematik* – he was the only Italian! – and Francesco Severi and Enrico Bompiani (1889-1975) took his place. That replacement of Levi-Civita was a scandal and produced a strong international reaction. Some members of the editorial staff of the journal resigned, and among them the director Otto Neugebauer (1899-1990). The project of abstracting and reviewing mathematical journals moved to US where *Mathematical Reviews* was founded.[13]

In the thirties, in spite of his difficult personal and scientific situation, Levi-Civita remained faithful to the ideal of scientific internationalism and helped colleagues and students victims of anti-Semitism. Thanks to him, several of them found positions in the US or South America.[14] His death occurred in December, 1941 and was ignored by the Italian mathematical community that did not officially commemorate Levi-Civita.

3. The D'Alembert paradox and the wake hypothesis. Villat and Levi-Civita.

In this section we analyse some aspects of the scientific relationship between Levi-Civita and his French colleagues with regard to hydrodynamics and, in particular, to wake theory.[15] We focus on Villat's PhD thesis (Villat, 1911) directly inspired by Levi-Civita's ideas, in order to assess French reception of Levi-Civita's work on hydrodynamics. We shall notice that

[11] See for instance the letter by Fubini to Levi-Civita dated December 1, 1931 (in Nastasi, Tazzioli, 2003; 124) and the letter by Volterra to Levi-Civita dated February 23, 1932 (in Nastasi, Tazzioli, 2000; 151).

[12] The testimony by Gino Arrighi is reported in Lettera Matematica Pristem, vol. 7, February 1993. Arrighi wrote: "L'argomento sostenuto da Volterra – che ebbe valore essenziale e determinante – era che quando Levi-Civita avesse lasciato l'Università, come comportava il rifiuto a giurare, si sarebbe creato un vuoto incolmabile nella scuola matematica di Roma, mentre gli scolari che lo avevano seguito sino ad allora non dovevano assolutamente essere abbandonati. Queste considerazioni sulla scuola - mi assicurava Levi-Civita - furono quelle che decisero il successivo suo comportamento dinanzi all'odiosa prescrizione."

[13] This episode is reported in detail in (Sigmund-Schultze, 1994).

[14] For instance Guido Fubini, Alessandro Terracini, Berud Steinlerger, Enrico Volterra. On that subject see the epistolary exchanges with Levi-Civita published in (Nastasi, Tazzioli, 2000).

[15] For a general overview on the history of hydrodynamics see (Darrigol, 2005).

Boussinesq had certainly encouraged Villat to do his thesis on hydrodynamic issues, but it was M. Brillouin who suggested to his young student the subject and closely followed his work. We argue that Levi-Civita and his French colleagues shared a similar approach to hydrodynamics based on mathematical and rigorous methods.

3.1 Levi-Civita's works on wake theory

"The group of experimental facts, which the longest disconcerted mathematicians, concerns the drag opposed by a fluid to the advancement of a solid body fully immersed in it" ["Le group des faits expérimentaux, qui a le plus longuement déconcerté des mathématiciens, est celui qui se rattache à la résistance qui oppose un fluide à l'avancement d'un solide entièrement baigné par celui-ci"] – wrote Villat in his survey on recent advances of hydrodynamics published in 1918 (Villat, 1918; 44). In particular, D'Alembert proved that in a perfect fluid the drag force is zero on a body moving with constant velocity relative to the fluid. That statement, which is in contradiction to the observation of non-zero drag on bodies moving relative to fluids, is called the D'Alembert paradox. Besides his own papers and those published by his *patron* (supervisor) Marcel Brillouin (1854-1948), Pierre Duhem (1861-1916) and Jacques Hadamard (1865-1963), in his work Villat mainly cited Italian mathematicians, and Levi-Civita in particular.

In 1901 Levi-Civita published his first note on hydrodynamics aiming at reassessing the D'Alembert paradox and succeeded in explaining it by introducing the "wake hypothesis", according to which there are two different regions in the fluid – inside and outside the wake – separated by a surface of discontinuity.[16] He deduced "theoretically" – as he was proud to claim – "Newton's law on incompressible fluids (drag is proportional to the square of velocity)" (Levi-Civita, 1901; 130).

The relevance of that result is corroborated by an exchange of letters between Levi-Civita and Hadamard, who since 1901 was editing his lectures on hydrodynamics given in the academic years 1898-99 and 1899-1900 and then published in 1903 as *Leçons sur la propagation des ondes et les équations de l'hydrodynamique* (Hadamard, 1903). Hadamard moved a critical objection to Levi-Civita's wake hypothesis (19 April, 1902): "there is one case in which I cannot agree with you: that of liquids. It seems to me that the true theory of the phenomenon cannot be found (for liquids) in discontinuities of the kind you introduce. Indeed, these discontinuities should propagate being affected at different moments by

[16] For a detailed analysis of Levi-Civita's papers on the wake hypothesis and motions with wake see (Nastasi, Tazzioli, 2006).

different molecules (unless your surface […] has the shape of a cylinder with generators parallel to the motion), which is impossible in the case of liquids. On the contrary, I believe like you that in the case of gases one must introduce the discontinuities." (Nastasi, Tazzioli, 2006; 87-88) We do not have Levi-Civita's answer, but in a following undated letter Hadamard recognised that Levi-Civita was right, and that his "objection" to the hypothesis on the existence of a discontinuity in the case of the liquids was "a simple inadvertence".

In another letter on February 19, 1903, Hadamard asked Levi-Civita some questions about "phenomena presented by fluids and which rational mechanics fails to explain" concerning friction and (above all) "the mixing of layers, local turbulence or *swirls*".[17] In 1907 Levi-Civita published a paper in the *Rendiconti del Circolo matematico di Palermo* that partially answers to these questions (Levi-Civita, 1907a). By starting from the wake hypothesis introduced in his 1901 paper, Levi-Civita deduced the general integral of any plane motion involving a wake if the shape of the body moving in the fluid (supposed as incompressible) is polygonal or curvilinear with a unique angular point *O* called bow. In his proof, he used an adequate conformal transformation of complex analysis by starting from some works by von Helmholtz, Kirchhoff, Rayleigh and Joukowski.[18] Levi-Civita dealt with the wake problem in a "purely mathematical way", but he remarked that "actual applications to ship movements" are very important. The latter question – he pointed out – has not been based on "scientific foundations" yet. Levi-Civita used the wake hypothesis already described in his 1901 paper, though actually "the nature of discontinuity is more complicated than the one supposed theoretically". In this regard, he quoted the photographs of the wake Étienne Marey had taken and published some years before (Marey, 1900), and added in a footnote some remarks on the actual surfaces of discontinuity that recall Prandtl's boundary-layer concept (Prandtl, 1905).[19] In particular, Levi-Civita pointed out, the wake is not infinitely long and not even rigidly attached to the body.

Levi-Civita's ideas in hydrodynamics – especially his wake hypothesis – influenced the work of many students, and scholars in general, who read and followed his research. At the University of Padua Levi-Civita had given rise to an Italian school of hydrodynamics, mainly

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[17] For the complete text of the letters see (Nastasi, Tazzioli, 2006; 88-90).

[18] On the polemics about drag measurements that poorly agreed with late nineteenth-century hydrodynamic theories by Helmholtz, Kirchhoff and Rayleigh, see (Darrigol, 2008). For a detailed analysis of Levi-Civita's 1907 paper (Levi-Civita, 1907a) see (Nastasi, Tazzioli, 2006).

[19] On Prandtl's theory see for instance the Part III of (Anderson, 1997). In his footnote, Levi-Civita (1907, 522) wrote: "New vortex rings must continually come out from the first part of the surface of discontinuity, in contact with the body, descending along the surface, in substitution for those which separate at the end."

formed by mathematicians. Some of them generalized his methods and contributed to obtain new results. Umberto Cisotti (1882-1946) extended the validity of D'Alembert's paradox to bodies with various shapes and motions (Cisotti 1905-1906); Tommaso Boggio (1877-1963) and Emilio Almansi (1869-1948) published some papers in which they improved and simplified Levi-Civita's 1907 method (Boggio, 1910) (Almansi 1909-1910).

In his lectures held at the University of Pavia in the academic year 1918-19 and published in 1922, Cisotti gives an interesting overview of the knowledge on plane hydrodynamics in that period. He significantly writes in the Preface to his book: "The development of plane hydrodynamics in recent years – due for the most part by Italian contribution, concerning both the development of methods and the large number of results – made me feel that time is appropriate not only to bring the echo of the school, but also to extend the knowledge to a wider circle of scientists who are outside the field of mathematics, more particularly among the specialists of hydraulic disciplines." (Cisotti 1921-1922, I; V) Besides Boggio, Almansi and Cisotti himself, we also mention Gustavo Colonnetti (1886-1968), Bruto Caldonazzo (1886-1960), Attilio Palatini (1889-1949), and Giuseppe Picciati (1868-1908), who generalized Levi-Civita's results on plane motions with wake, and whose contributions are collected and organized in Cisotti's book *Idromeccanica piana* (Cisotti, 1921-1922, I). They all belonged to the Levi-Civita's school of hydrodynamics.

3.2 Levi-Civita's influence in France. Villat's thesis

In France, Levi-Civita's 1907 paper inspired in particular Brillouin and Villat, who tried to improve his method and to adapt it to actual cases. Henri Villat (1879-1972), who entered the ENS in 1899, defended his PhD thesis in 1911. Brillouin and Boussinesq were among his advisors. From 1902 to 1911, Villat taught in Caen, his native town – before at the Lycée and then at the University as *chargé de conférences*. During the years he spent in Caen, Villat prepared his thesis and exchanged letters with Brillouin and Boussinesq. His correspondence allows us to reconstruct the genesis of Villat's thesis, the choice of the subject, the difficulties of the task, and the achievement of the final result. Moreover, it enlightens the point of view of French mathematicians on Levi-Civita's approach to hydrodynamics.

Marcel Brillouin (1854-1948), who was professor of theoretical physics at the *Collège de France* from 1900 to 1931, held a series of lectures on general properties of fluids and the kinetic theory of gas when Villat was a student of ENS.[20] Brillouin lectured on the theory of "Jets" in the academic year 1908-1909, and on "Viscous fluids" the year later. Villat explicitly mentioned Brillouin's lectures of 1909 in his thesis (Villat, 1911; 205). As it appears by reading the notes of his course, Brillouin deeply analysed Levi-Civita's 1907 paper in March 1909, and other papers by Italian mathematicians in the following months, such as (Boggio, 1910) and (Picciati, 1907) concerning the motion of a sphere on a viscous fluid.[21]

Another French *master* of hydrodynamics, Joseph Boussinesq (1842-1929), highly appreciated the Italian works on that subject. Boussinesq got the chair of calculus of probability and mathematical physics at the Sorbonne in 1896 by succeeding Poincaré, and was one of the reference points for hydrodynamics in France. Villat and Boussinesq exchanged letters in spring 1910 in which Boussinesq "provided Villat with a precious reference to Italian literature which he had not read due to the language in which they were written" (Aubin, 2010; 6). Boussinesq mentioned " the Italians"[22] and explicitly referred to the works by Levi-Civita and Almansi (very probably (Levi-Civita 1907a) and (Almansi 1909-1910)). In his five notes, Almansi deals with the action of a fluid on a body plunged in it; he considers the very general problem, but he is forced to introduce special assumptions in order to obtain some interesting results. Actually, Villat was already aware of the Italian papers cited in Boussinesq's letters: "Besides the papers in Rendiconti you have suggested, I have already study during the last months the beautiful papers you mentioned", he wrote to Boussinesq on May 1910.[23] In the same letter, Villat illustrates his work project with the utmost detail. Though Villat had already read the Italian works cited by Bousinnesq, he was certainly comforted by the fact that a recognized expert in hydrodynamics showed him the same way he had begun to follow some time before.

Both Boussinesq[24] and Brillouin put in evidence that the Italian authors had restricted their attention to perfect fluids when it was obvious that other physical aspects – such as small speed, vortices, and so on – played an essential role. Nevertheless, they were aware that "the

[20] The manuscript of Brillouin's course (1898-1900) by Henri Bénard is contained at the Library of the Institut de France (Code: MS 5592).

[21] The manuscript of Brillouin's course at Collège de France of the academic year 1908-1909 entitled "Jets" is kept at the Library of the Institut de France (Code: MS 5601). In March 1909 Brillouin developed a detailed analysis of Levi-Civita's 1907 paper on hydrodynamics (Levi-Civita, 1907a). The year later (1909-1910), Brillouin lectured on the subject "Viscosité" (Code: MS5602).

[22] Boussinesq to Villat, letter dated May 11th, 1910 (Fonds Villat, Archives de l'Académie des Sciences, Paris).

[23] "A part les articles des Rendiconti que vous m'indiquez, j'ai déjà étudié pendant ces derniers mois les beaux mémoires dont vous me parlez." The letters by Villat to Boussinesq are contained in Papiers Boussinesq, MS4229, Bibliothèque de l'Institut de France, Paris.

[24] On Boussinesq and his relationship with Villat see (Aubin, 2010).

Italians" dealt with perfect fluids in the hope of finding a generalization to concrete fluids later on – their choice of working with perfect fluids being imposed by the mathematical difficulty of the general problem.

In his lectures of 1909, Brillouin criticized some points of Levi-Civita's article and his letters to Villat[25] pointed out his "physical" approach in contrast with Levi-Civita's point of view. In a letter dated 27 September, 1910, Brillouin remarked: "Your plan is very interesting and will take you very far, even without adding anything to Levi-Civita."[26] And he pointed out that he was trying to improve Levi-Civita's work. In fact, Levi-Civita's method was too general and applicable to cases that were not *physically* possible. In his 1911 note (Brillouin, 1911b), Brillouin analysed Levi-Civita's wake hypothesis by trying to define its limits of validity. He proved that, if the usual assumptions on wake are valid, the discontinuity surfaces must be infinite, otherwise some points of fluid have a negative pressure. Duhem proposed to denote that statement as "the Brillouin paradox" and extended it from two to three dimensions (Duhem, 1914).

In another letter to Villat dated 10 June, 1910, Brillouin suggested analysing Levi-Civita's memoir in details but avoiding "the Italian fashion of publication"; he mainly referred to the lack of examples.[27] Some remarks are required to put in perspective the contribution of Levi-Civita to hydrodynamics and his concerns on the status of the discipline. Levi-Civita's approach to hydrodynamics is indeed the same he shows towards other fields of mathematical physics – he aims to found physical principles on rigorous mathematical theory. Levi-Civita elaborates the rigorous mathematical theory by starting from ideal fluids. He is aware that it is only the first stage – even if an important stage – towards attaining the final goal. In his paper on the wake hypothesis, Levi-Civita remarks that, "because of the difficulty of the question, it is convenient to study the problem in two dimensions" (Levi-Civita, 1907a; 521); in that special case, he can apply complex analysis, which is the key method for solving the general integral of plane motions with wake. However, he adds that "in spite of such a simplification, we do not miss the practical interest", and gives the example of a ship whose motion can be seen on horizontal sections. (Levi-Civita, 1907a; 521) He is then actually concerned with the applications of his results. Furthermore, we shall see that in some cases he takes inspiration

[25] The letters by Marcel Brillouin and Levi-Civita to Villat are kept in Dossier Villat, Archives de l'Académie des Sciences, Paris.

[26] "Votre plan est très intéressant, et vous amènera fort loin, même sans rien ajouter à Levi-Civita".

[27] Brillouin wrote to Villat (June 10th, 1910): "toutefois je vous conseille de ne pas trop imiter le mode de publication des Italiens, qui modifient infiniment peu l'exposition déjà connue, la généralisent un peu, et finalement ne traitent aucun exemple ou groupe d'exemples nouveaux."

from experimental facts and tries to deduce them from his new theoretical principles. (See section 4.2)

His French colleagues appreciate Levi-Civita's work on hydrodynamics, and try to improve it by making it applicable to concrete cases. After all, this is the usual way to proceed in mathematical physics today; at the beginning of the 20th century, however, scientists – and especially specialists of hydraulics and engineers – often preferred to deal with hydrodynamic theories empirically and often rejected mathematical rigor.

In a letter on June 20th, Brillouin persuaded his student Villat to focus on some special topics of Levi-Civita's 1907 paper, which "deserve" the most attention. In Brillouin's opinion, in his thesis Villat had to re-deal with, develop and generalize such crucial points of (Levi-Civita, 1907a). Brillouin added that a mathematician specialist in the theory of complex functions, as Villat was, could be able to make important advances in "that sense" – in improving Levi-Civita's work.

Villat followed Brillouin's advice and actually generalized some Levi-Civita's results in his dissertation. In the first part of his work, he established the general integral of a permanent plane motion in a fluid bounded by an infinite wall, and where a given body is immersed. Villat's solution is expressed in terms of elliptic functions, and becomes Levi-Civita's formula when the distance from obstacle and wall tends to infinity. The second part of Villat's thesis is devoted to solving the following question: "To find the motion and all its elements, if the obstacle shape is given." (Villat, 1911a; 206). It is the inverse of Levi-Civita's problem – for every regular analytical function in a circle that assumes real values on the diameter, an irrotational motion with wake exists (the shape of the obstacle is deduced a posteriori). Villat considered the case of an indefinite fluid, and found a new arbitrary function that allows us to find the general solution of the problem in a simple way, and whose expression is linked to the obstacle shape. In Roy's opinion, that is "one of the fundamental merits" of Villat's memoir (Roy, 1972). In the conclusive section, Villat showed some remarkable examples. Even though he tried to extend Levi-Civita's results by making applications and showing actual examples, Villat's mathematical methods are directly inspired by Levi-Civita's ones.

Villat sent his memoir to Levi-Civita, who answered with his usual modesty (letter dated June 19th, 1911): "I am very pleased that the perfection of the Helmholtz-Kirchhoff method that I have indicated has given rise to many scientific studies". (In Nastasi, Tazzioli, 2003; 373) The rich epistolary exchanged between Villat and Levi-Civita began in 1911 and went on until 1939.[28]

3.3 Henri Villat

Villat is the French colleague with whom Levi-Civita had a privileged relationship, both personal and scientific. In the years 1911-1912 their letters mainly concern some points of Levi-Civita's 1907 method and Villat's thesis, and especially focus on a special functional relation, due to Ulisse Dini (1845-1918), and its application to hydrodynamics. This formula connects the values of a function *f* on the circumference of a circle with the values of its normal derivative on the same circumference, if *f* is assumed to be harmonic in the circle.[29] We note *en passant* that their discussion is also connected to both the theory of integral equations and the use of the Green function for solving the Dirichlet problem. Villat published several notes (Villat, 1911b, 1911c, 1912) concerning the Dirichlet principle in circle, annulus and other special figures, and exchanged some letters with Émile Picard (1856-1941) on that subject. On October 25th, 1911 Picard wrote to Villat: "Dear Sir, yesterday I communicated your note to our Academy [Académie des Sciences]. Your elegant results strongly interested me [he refers to (Villat, 1911d)]. Taking the question in its most general form, for my part I get a lot less simple analysis […]"[30] In the years 1906-1920, Boggio, Marcolongo, Orlando, Lauricella and other Italian mathematicians often applied the theory of integral equations in order to solve questions of mathematical physics, including heat theory, theory of elasticity, and hydrodynamics.[31] In particular, Boggio clearly highlighted the fundamental links between hydrodynamic problems and the so-called Fredholm method for solving some special classes of integral equations.[32]

After defending his PhD thesis, Villat got a position as assistant professor (*maître de conférence*) at the University of Montpellier. Villat played a vey important role in applied hydrodynamics and aeronautics in France after World War I – in 1920, just after World War I,

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[28] The letters by Villat to Levi-Civita are kept in Fondo Levi-Civita; the letters by Villat to Levi-Civita are contained in Fonds Villat, Archives de l'Académie des Sciences, Paris. Their correspondence (43 letters) is published in (Nastasi, Tazzioli, 2003 ; 371-410).

[29] For a detailed analysis of the correspondence between Villat and Levi-Civita in the couple of years 1911-1912 and its connection to Dini's formula, see (Nastasi, Tazzioli, 2006 ; 97-108). See (Nastasi, Tazzioli, 2004) for a historical analysis of the Dirichlet problem in connection to problems of mathematical physics.

[30] "Cher Monsieur, j'ai communiqué hier votre note à l'Académie. Vos résultats élégants m'ont vivement intéressé. En prenant la question sous sa forme la plus générale, j'ai de mon côté une analyse beaucoup moins simple…". The letters by Emile Picard to Villat are contained in Fonds Villat, Archives de l'Académie des Sciences, Paris.

[31] See (Archibald, Tazzioli, 2013) for a detailed analysis.

[32] See (Nastasi, Tazzioli, 2006; 106-108).

he moved to Strasbourg, when the University of Strasbourg became the symbol of the French victory against the Germans,[33] obtained the chair of mechanics at the Sorbonne by succeeding to Paul Painlevé (1863-1933) in 1927, was appointed director of the Institute of fluid mechanics in 1929, and became professor of aerodynamics at the School of Aeronautics in 1930.[34]

In the twenties and thirties Villat became more and more interested in questions of applied hydrodynamics both for intellectual reasons and for his institutional position as the director of the Institute of fluid mechanics. In spite of that, Villat often mobilized mathematical tools in his works on fluid dynamics, such as Dini's formula or the theory of integral equations. In particular, he investigated some questions connected to the first section of his dissertation and deduced the explicit solution for an obstacle of any shape, discussed the uniqueness of the solution, and established some necessary conditions for avoiding solutions that are physically impossible (Villat, 1921). In another paper he succeeded in applying integral equations to fluids with vortices (Villat, 1923). His taste for abstract mathematical methods, especially related to functional analysis and the theory of complex functions, explains his admiration for the work of Levi-Civita and his school. However, while Levi-Civita showed some attention to applications especially concerning hydraulic experiments, Villat's works did not explicitly refer to experimental facts.

Moreover, some of Villat's students analysed topics connected to Levi-Civita's hydrodynamic research. For instance the thesis by René Thiry (1886-1968) was directly inspired by "an important Memoir of Levi-Civita [Levi-Civita 1907a]" and its "important developments" made by Villat and Brillouin (Thiry, 1921; 1). He started from Villat's statement that hydrodynamic equations can lead to more than one solution – in particular, if the obstacle is made by two concurrent segments with concavity towards the current, then two solutions are possible and neither is better a priori that the other (Villat, 1914). Thiry proved that, under certain conditions, the solutions constitute a continuous succession between the two solutions deduced by Villat. Thiry's research is clearly inspired by Levi-Civita's search of rigour apparently shared by his *patron* Villat.

We mention that, again in the forties when he was already an authority in the field of aerodynamics, Villat seems more interested in mathematical rigour rather than in experimental data. His lectures on the theory of viscous fluids published in 1943 (Villat,

[33] On the University of Strasbourg and its strategic importance in France see (Crawford, Olff-Nathan, 2005).

[34] A wide image of French mathematics and Villat's role in the interwar period is in (Gispert, Leloup, 2009). An analysis of Villat's role as a specialist of fluid-dynamics and the director of the Institute of fluid mechanics is in (Aubin, 2010). For a survey on the history of aeronautics in France see (Weber, 2008).

1943) put in evidence the "extreme" difficulties of research involving viscosity that make the approach to these issues "very dangerous". Villat pointed out that there were several "rudimentary" books about viscous fluids that are not rigorous at all from the mathematical point of view; here "the experimenter" generally misses the crucial points. On the contrary, the aim of Villat's book is to specify some "essential points" of the theory, in order "to facilitate theoretical research without which any practical application will always devoid of real meaning" (Villat, 1943; V). Anyway, he added he did not intend to replace Brillouin's 1907 treatise on viscosity (Brillouin, 1907), whose approach was "rather experimental", with his 1943 lectures. Once more, Villat's point of view on *mathematical* hydrodynamics seems to be close to that of Levi-Civita.

4. The wave theory. Levi-Civita as a Master at the University of Rome

In Section 4.1 we illustrate Levi-Civita's original works on wave theory, which mainly concern progressive permanent waves and the existence proof of the irrotational wave in a canal with infinite depth. We focus on some works of his French students, which he often supervised in collaboration with other French colleagues – such as Villat, M. Brillouin, Hadamard, and Painlevé. Section 4.2 is devoted to Levi-Civita's work on the theory of jets, often neglected by historians of science, and its influence in France.

4.1. Levi-Civita's contribution to wave theory

Levi-Civita's interest in waves starts very early, when he was professor at the University of Padua, and concerns the very definition of "wave". In a letter to Volterra (dated April 26th, 1903), Levi-Civita wrote: "Before leaving [for the Easter holidays] I started and now go on studying wave theory […] – it is the subject I shall develop in my course of higher mechanics. I met, however, a preliminary difficulty […] concerning the lack of a definition of wave, both general and rigorous." (Nastasi, Tazzioli, 2000; 75-76) Levi-Civita's interest in wave theory remains central in his research throughout his scientific career. Again in 1935, in a fundamental article written in collaboration with Ugo Amaldi (1875-1957) for the *Enciclopedia Italiana*, the authors observe how it is difficult to give a definition of wave motion, which on the one hand embraces all cases where waves are physically detectable, and on the other hand is mathematically rigorous (Amaldi, Levi-Civita, 1935). We point out that, in his research on hydrodynamics – both on the wake hypothesis and wave theory – Levi-

Civita often contributed by founding some intuitive concepts on rigorous mathematical formalism.

In 1907, Levi-Civita published a paper on progressive permanent waves in a canal with horizontal bed – "progressive" means that the motion appears as stationary to an observer moving with the apparent translation of the fluid (Levi-Civita, 1907b). In this research Levi-Civita considered vertical motions and included gravity as an actual force. So, he considered the velocity potential φ with $d\varphi = udx + vdy$ (u, v the velocity components of the point (x, y)) in a suitable region, and the stream function ψ such that $d\psi = -vdx + udy$. He introduced the complex variable z, and the functions f and w:

$$
\begin{aligned}
z &= x + iy; \\
w &= u - iv; \\
f &= \varphi + i\psi
\end{aligned}
$$

such that $w = \frac{df}{dz}$. By following step by step his method illustrated in (Levi-Civita, 1907a), he still applied complex analysis and proved that f is a conformal map, in order to deduce the rigorous solution of wave motion in a finite depth canal. Finally, he reduced his original problem to the solution of a mixed equation (differential and finite difference equation) related to the holomorphic function $w(f)$.

Levi-Civita's results on various mathematical fields, including hydrodynamics, allowed him to be awarded the prestigious *Premio Reale* from the Academy of Lincei in 1907 (*ex aequo* with Federigo Enriques). His contributions to wave theory inspired several scholars. In particular his paper (Levi-Civita, 1907b), together with some contributions published by his students and colleagues on the same topics, influenced the French René Risser (1870-1958). A *polytechnicien*, Risser soon turned his interests into actuarial science. In 1898 he founded and directed, together with Lucien March, the *Statistique Générale de la France*. In 1907 he was employed in the Ministry of Labour, where he became the director of the actuarial service in 1912. In parallel to his career as an actuary, Risser obtained his doctorate with a thesis on hydrodynamics (Risser, 1925) inspired by Levi-Civita's ideas on progressive permanent waves. Risser's work provides a historical overview on wave theory, and especially refers to the works of Boussinesq, and Levi-Civita and his students. Moreover, his dissertation deals with actual applications by considering some special examples, such as waves produced by an emerging cylinder whose generatrixes are perpendicular to the axis of the canal and as large as the canal width. Risser is proud to send his work to Levi-Civita, whom he had met in Italy during World War I.[35] In fact during the war Risser had served as an artillery officer and received the War Cross for his military actions. He also contributed to war work by improving ballistic tables for several artillery pieces.

After defending his PhD thesis, Risser took part in the International Congress of Mathematicians in Bologna (1928), where he met again Levi-Civita and his wife; just after the Congress he wrote him (letter dated December 1st): “I regret not having been your student, because you are not only incomparable as a master, but also a friend to all those who work with you.”[36] Risser went on working on hydrodynamics and in 1935 published a paper, still inspired “by Hadamard, Levi-Civita and their students”, in the book for celebrating the Jubilee of Marcel Brillouin (Risser, 1935). In the same years, Risser contributed to Borel’s *Traité du calcul des probabiblités et de ses applications* by publishing in 1936 (together with Claude-Emile Traynard) the 4th issue on the principles of mathematical statistics.[37]

Levi-Civita’s approach focuses on the use of functional and complex analysis as prominent methods for dealing with hydrodynamic problems in general, and permanent waves in particular. As already mentioned, Dini’s formula in functional analysis has a central tool in some contributions by Levi-Civita and Villat (see section 3.3). Levi-Civita devoted a note, communicated to the Academy of Lincei, to Dini’s formula and its deep connections to an analytic method due to Lord Rayleigh and related to the solitary wave (Levi-Civita, 1911). A solitary wave is a wave that propagates without any temporal evolution in shape or size. It consists of a single intumescence and was observed and experimentally studied by Scott Russell and theoretically (in an approximating way) by Boussinesq and Lord Rayleigh. Alexander Weinstein (1897-1979), a Rockefeller student of Levi-Civita at the University of Rome, obtained further approximations for solitary waves by improving Boussinesq-Lord Rayleigh ones (Weinstein, 1926).

In (Levi-Civita, 1912) Levi-Civita analysed permanent waves in a canal submitted to certain physical conditions. He extended Lord Rayleigh’s well-known theorem – according to which the global transport of fluid indefinitely increases with time – to permanent waves in a canal with finite or infinite depth. Nevertheless, his main contribution to wave theory concerns

[35] Risser wrote to Levi-Civita on May 20th, 1925: “Cher Maître, Dans les premiers jours de Janvier 1918, j’ai eu l’honneur de vous être présenté à Padoue par un ami de ma famille, alors que je venais d’être détaché en mission à l’armée d’Italie par Monsieur le Général Bourgeois aujourd’hui membre de l’Institut, et à cette époque grand Directeur de l’Artillerie.” The letters by Risser to Levi-Civita are contained in Fondo Levi-Civita.

[36] “Je regrette de n’avoir point été votre élève, car vous êtes non seulement le maitre incomparable, mais aussi l’ami de tous ceux qui travaillent avec vous.”

[37] On Risser’s contribution to statistics see (Bustamante, Cléry, Mazliak, 2015).

irrotational waves with finite amplitude (Levi-Civita, 1925) – in particular, it deals with periodic waves that propagate without changing their shape. A periodic wave that attracted Levi-Civita's interest is given by trochoidal waves found by Gerstner (1802) and then by Rankine (1863) independently.[38] The wave profile is that of a trochoid and the fluid particles describe little circles decreasing with the distance from water surface.[39] As in Figure 1 the *limit* wave is a cycloid. Gerstner's waves have interesting applications to hydraulics and nautical science.

The fluid particles have, however, a turbulent (or rotational) character, while in a perfect fluid – where only conservative forces act – only irrotational motions are admitted; a mathematical inconvenient that "detracts somewhat from the physical interest of the result" (Lamb, 1932; 421), as also pointed out by Amaldi and Levi-Civita in their 1935 paper published in the Italian Encyclopaedia (Amaldi, Levi-Civita, 1935).

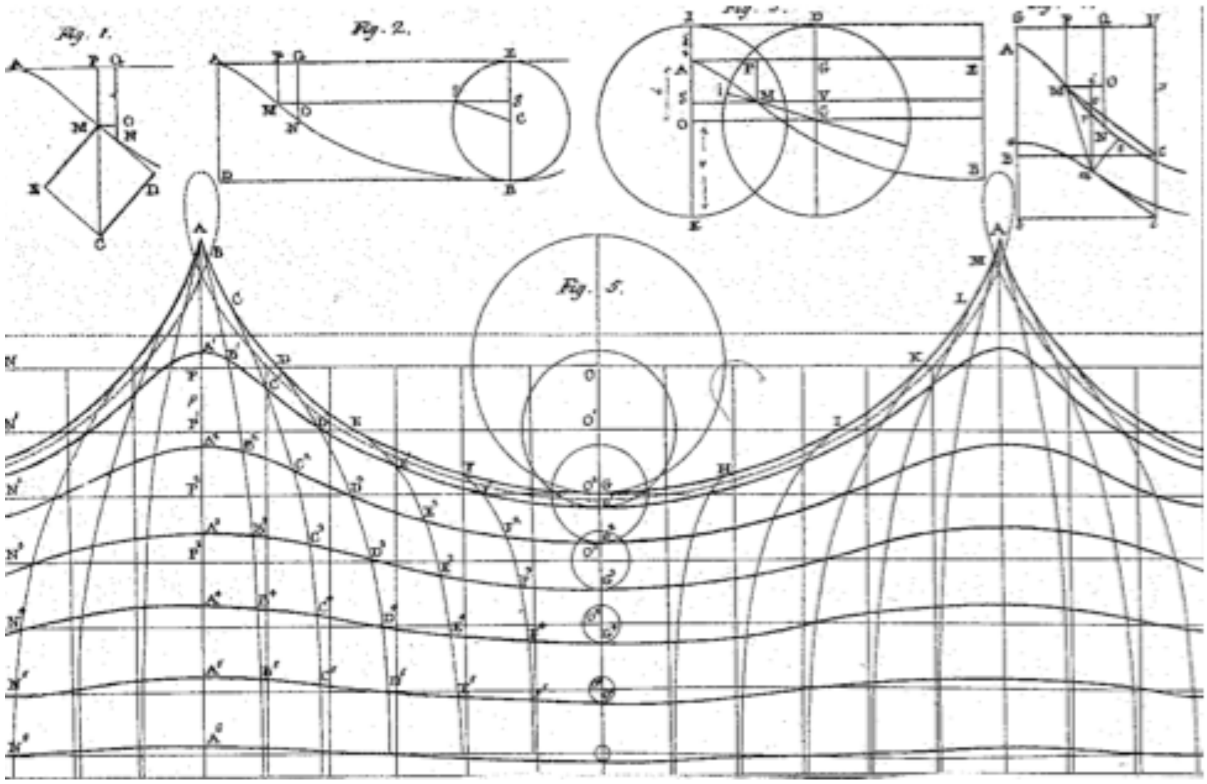

Figure 1
Gerstner's waves. The lines A^i B^i C^i… represent possibly wave profiles; the circles represent the orbits of fluid particles; the remaining lines represent the successive forms of a line of particles that is vertical when passed by a crest or a through. (Darrigol, 2005; 74)

[38] For a survey of Gerstner's waves see (Darrigol, 2005 ; 72-75).
[39] A trochoid is the curve described by a fixed point on the radius of a circle when the circle rolls without sliding along a straight line.

In 1847 Stokes had elaborated an approximating method for calculating periodic permanent irrotational waves by using a series development. Lord Rayleigh obtained more precise approximated solutions. However, their methods did not prove the convergence of the approximating series. In his 1925 paper, Levi-Civita solved the problem with the utmost rigour, by means of a new approximating expression that he called the "stokian" (in honour of Stokes). In Lamb's words, Levi-Civita closed "an historic controversy" (Lamb, 1932; 420) by proving the existence of the irrotational wave in a canal with infinite depth and deducing simple mathematical formulae linking together height, length, transport, and velocity of propagation of waves. An important stage towards that result is represented by an article, published in *Mathematische Annalen* in 1922, that concerns the functional equation representing periodic waves in a canal of infinite depth (Levi-Civita, 1922a).

Of course, the extension of his 1925 existence proof to more general cases was central for Levi-Civita's research, and some of his students dealt with it. His Rockefeller student Dirk J. Struik (1894-2000) was able to generalize his result to canals with finite depth (Struik, 1926), and Marie-Louise Dubreil-Jacotin (1905-1972) proved the existence of infinite rotational waves (Dubreil-Jacotin, 1934). Dubreil-Jacotin, graduated from the ENS and followed Villat's course on fluid mechanics in Paris, got a grant that allowed her to collaborate with the meteorologist Vilhelm Bjerknes (1862-1951) in Oslo. In 1930 she married the mathematician Paul Dubreil (1904-1994) and the same year the couple travelled in Germany – where they met, among others, Emmy Noether (1882-1935) and David Hilbert (1862-1943) – and spent a period in Rome, where Paul Dubreil was a Rockefeller fellow and collaborated with the group of algebraic geometry. In his souvenirs, Paul Dubreil wrote about his wife's research: "Having studied the work [(Levi-Civita, 1925)] my wife arrived in Rome with a curious remark: one of the conditions introduced by Levi-Civita, *the absence of mass transport in the deeper layers*, was not satisfied by the wave of Gerstner, also called cycloid wave, which is an exact solution as well, but not irrotational.[40] [...] That remark on *the deeper layers* was indeed the starting point of my wife's thesis – by assuming a hypothesis valid for both the wave of Gerstner and that of Levi-Civita, she rigorously establishes the existence of an infinite number of waves including the two mentioned above." (Dubreil, 1983 ; 69)

Marie-Louise Dubreil proved that statement for the two cases of finite and infinite depth of the canal; for both cases she reduced the problem to a system of integral equations. She

[40] We point out that Paul Dubreil denotes as "Gerstner wave" the limit of the trochoidal waves.

defended her PhD thesis at the University of Paris in 1934 – Vessiot, Julia, and Villat were the members of the PhD committee – and published her memoir in the *Journal des mathématiques pures et appliquées* directed by Villat. Immediately after obtaining her doctorate, she was put in charge of the *Cours Peccot* at the prestigious *Collège de France* by giving lectures on wave theory. In this regard, she wrote on October 31st, 1935 to Levi-Civita: "Having been asked to give the course *Peccot* at the *Collège de France* this winter, I have to hold a series of lectures on waves. In this regard, I plan to talk a little also of the solitary wave." At this point, she asked Levi-Civita for some references on that subject. In other letters, she sent him some notes he communicated to the Academy of Lincei. Throughout her letters to Levi-Civita (Nastasi, Tazzioli, 2003; 159-172) it is possible to follow some aspects of her scientific career and also the difficulties linked to the fact that she got positions at provincial universities and came back to Paris only in 1956.[41]

In conclusion of the section, we mention Levi-Civita's treatises on wave theory. Since Levi-Civita's activity at the University of Rome focused on the creation and development of an international mathematical school, it is not surprising that he published most of his university treatises during this period.

In 1921 Levi-Civita held a series of lectures in Barcelona on various questions of classic and relativistic mechanics, including a lecture on waves in canals.[42] Such lectures have been collected in a book and immediately published by the Institute of Catalan Studies (Levi-Civita, 1922b); two years later, they appeared in Italian and in German translation. Though some French scholars wished a French translation as well, it has never appeared. In this regard, Risser wrote to Levi-Civita (30 May 1925): "I am convinced that your book entitled « Questions concerning mechanics and relativity » [(Levi-Civita, 1922b)] would have the most success here; for my part, I deeply regret that my very imperfect knowledge of Italian language does not allow me at all an immediate reading, and I admit that I would like to have a translation. Such a translation would have the advantage that all French teachers could profit of your remarkable work on hydrodynamics and mechanics of relativity…"[43]

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[41] We mention that some years after her doctorate, she turned her interests to algebra by following Emmy Noether's ideas.

[42] In 1921 Esteve Terradas invited Levi-Civita for giving a series of lectures in the "Cursos Monogràfics d'Alts Estudis i d'Intercanvi" in Barcelona. The Catalan Institute organized the "Cursos Monogràfics" from 1915 to 1925 and invited, among the others, Jacques Hadamard, Hermann Weyl, Arnold Sommerfeld, and Albert Einstein. On the subject see (Sallent Del Colombo, Roca i Rosell, 2006).

[43] "Je suis convaincu que votre traité intitulé « Questions touchant à la mécanique et à la relativité » aurait ici le plus grand succès ; pour ma part je regrette profondément que ma connaissance très imparfaite de la langue italienne ne m'en permette point une lecture immédiate et j'avoue que je serais ravi d'en avoir une traduction.

Another book on theoretical hydrodynamics was, however, translated in French. This treatise is a course held by Levi-Civita at the Mathematical Seminar of the University of Rome on the theory of characteristics during the academic year 1930-31 (Levi-Civita, 1931). These lectures, drafted by one of Levi-Civita's students Giovanni Lampariello (1903-1964), provide an account of the theory of differential equations, and focus on the problem of Cauchy and its exceptional cases naturally leading to the idea of characteristics. Levi-Civita analyses characteristics in relation to waves of discontinuity, and elucidates some compatibility conditions at the wave front. In special cases, Levi-Civita proves the existence theorems and then introduces the concept of "characteristic manifold" by following Hadamard's theory and language.[44] The theory is well developed and illustrated by examples from hydrodynamics, optics and from the theory of de Broglie waves.

It was Marcel Brelot (1903-1987), *normalien* and then Rockefeller student of Vito Volterra in Rome (1929-30), who proposed the French translation of his treatise on the theory of characteristics to Levi-Civita. In a letter, dated Berlin, December 26th 1931, Brelot informed Levi-Civita that a scientific commission recommended his book (Levi-Civita, 1931) for a French translation.[45] In his letter he also mentioned that Mussolini was aware of the decision and that a similar Italian scientific commission had been created. At the end of his letter, Brelot wrote about "a strange decision of the Italian government, which of course leaves all astonished abroad […]" Apparently he referred to the loyalty oath to the Fascist regime, instituted in that period and that all university professors were required to take. We remark that Volterra had to retire as a consequence of his refusal.

4.2 Jets

Levi-Civita published just one contribution not related to wave theory during the long period he spent at the University of Rome – it is a lecture held at the mathematical and physical

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Une semblable traduction aurait l'avantage de faire bénéficier tous les professeurs français de votre connaissance si remarquable de l'Hydrodynamique et de la mécanique de la Relativité... "

[44] For details on Hadamard's theory see the chapter "Lines on the horizon. Hadamard and Fréchet, readers of Volterra" by Guerraggio, Jaëck, Mazliak in this book.

[45] More in detail, in his letter Brelot wrote: "Hanno creato, poco fa, una commissione di scienziati in Francia per scegliere le opere francesi e straniere degne di essere tradotte in diverse lingue; il signor Paul Gaultier direttore della "Revue Scientifique" si occupa dell'impressione. E' andato poco fa a Roma, ha parlato col Mussolini e una commissione italiana è stata anche scelta nello stesso scopo; forse che Lei lo sa? Il signor Gaultier sta questi giorni a Berlino per occuparsi della stessa cosa in Germania, e riunire scienziati di tutti i campi. E' venuto a vedermi questa mattina perché le commissioni italiana e francese desiderano far tradurre la sua opera "Caratteristiche dei sistemi differenziali e propagazione ondosa", e per questa traduzione in francese, il Prof. Picard mi ha proposto." The letter is contained in Fondo Levi-Civita.

Seminar of Milan in 1931, entitled "On liquid jets". He gave the same lecture at the Institute of fluid mechanics of Paris and published it in 1932 in French translation in the *Journal de mathématiques pures et appliquées*, directed by Villat, and in Polish translation in 1933. We shall analyse the content of that memoir (Levi-Civita, 1932) in order to discuss, once more, Levi-Civita's approach to hydrodynamics. We shall see that Levi-Civita was very interested in applying his results to hydraulics, and in deducing concrete results from a theory founded on solid mathematical bases. However, actual issues in hydrodynamics generally lead to very complicated mathematical problems and, therefore, simplified assumptions are required in order to find mathematical solutions. Of course, such assumptions do not generally describe concrete problems – that is quite natural in mathematical physics!

In his 1932 paper Levi-Civita writes: "Eminent specialists in hydraulics and physicists have been doing research on experimental study of liquid jets for a long time", and quotes the works by Borda, Bidone, Bazin, Plateau, Tyndall, Rayleigh, and others. However, a mathematical theory of this phenomenon has not been elaborated yet and that is the aim of the paper. (Levi-Civita, 1932; 37) In the conclusive section, Levi-Civita discusses his theoretical results and relates them to experiments done in the hydraulic laboratory of the engineering school of Rome (Levi-Civita, 1932; 55-56).

In his paper, Levi-Civita deals with the theory of jets in 3 dimensions, and deduces results in agreement with experimental data. Once more, Levi-Civita tries to found hydrodynamic principles on rigorous mathematical bases. Before him, fluid particles were considered as independent one of the other, and fluid pressure was supposed constant during the motion (it is called *free regime*). Nevertheless, such assumptions are true only for stationary motion. In the case of thin jets, the internal pressure is much higher than atmospheric pressure (rather than equal as in free regime). That is the *linear regime* analysed in Levi-Civita's paper by using an analogy between hydrodynamics and mechanics – the liquid jet is indeed interpreted as a flexible and inextensible wire; the pressure of the jet as the pressure of the wire; the continuity equation as an equation between the derivatives of the wire tensions, and so on. Therefore, he deduces the hydrodynamic equations of the jet from the corresponding differential equations of classical mechanics. These equations, in the case of a stationary regime, lead to the following statement: "For usual heavy jets [...], jets take the form of a catenary with the concavity downwards (while a heavy wire in equilibrium turns its concavity upward)." (Levi-Civita, 1932; 193)

Already in a paper published in 1905, Levi-Civita (Levi-Civita, 1905) dealt with two-dimensional jets, called "vena contracta". As Weinstein wrote in his commemorative memoir on Levi-Civita's contribution to hydrodynamics: "More than a century ago and only a few years before T. Levi-Civita was born, H. von Helmholtz gave the first mathematical example of a two-dimensional *jet*, the so-called vena contracta." And he added: "However, it was only in 1905-1908 that fundamental progress was made by Levi-Civita". (Weinstein, 1973; 269) Progress concerns mathematical rigour especially.

In modern terms, "vena contracta" denotes the reduction in the area/diameter of a fluid jet after it emerges from an orifice in a tank (see Figure 2). The maximum contraction takes place at a section slightly downstream of the orifice, where the jet is more or less horizontal.

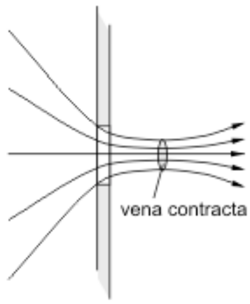

Figure 2

In his note, Levi-Civita remarked: "The phenomenon of vena contracta has been the subject of experimental studies that, at least for the needs of hydraulics, can be said to be exhaustive. Not so advanced is the theoretical study". He just quoted the classic papers by Helmholtz and Stokes, which gave the complete theory of two-dimensional jets issuing from "long enough" nozzles. But what happens if the nozzle has a closed border with shape, for example, circular or square?

Levi-Civita considered the coefficient of contraction, defined as the ratio between the area of the jet at the vena contracta to the area of the orifice. In equation :

C_c = Area at vena contracta/ Area of orifice

Until then, in fact, physicists had only recognised that the coefficient of contraction for ordinary nozzles (independent of what happens downstream) is always > ½, and becomes very close to ½ for orifices for a cylindrical mouth (Borda's tube). It is possible, and it is the purpose of Levi-Civita's work, to deduce a general and rigorous formula leading to these statements and to prove that the coefficient of contraction is less than ½ if the mouth of the internal orifice is divergent (Figure 3).

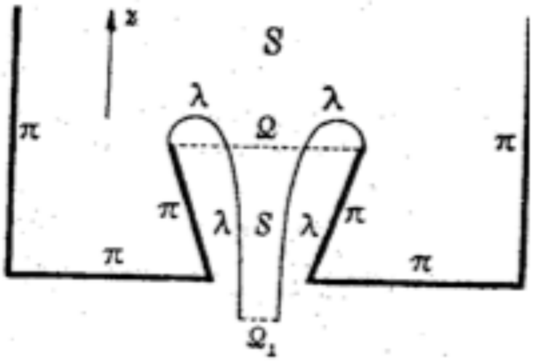

Figure 3
In the figure π is the plane boundary, Ω is the orifice, Ω_1 is the *contracta* section, λ is the vein surface between the orifice Ω and Ω_1, *S* the whole space filled by the fluid that has as boundary $\Omega_1 + \lambda + \pi$ (in Levi-Civita, 1905; 461)

Nevertheless, Levi-Civita was forced to introduce some hypotheses: in the tank S the fluid motion is irrotational and permanent (then the velocity potential is submitted to some conditions) and no external forces act. Moreover, for experimental confirmations he refers to some specialists of hydraulics at the University of Rome and Padua, in particular to his colleague Giacinto Turazza (1853-1925),[46] the director of the *Gabinetto di Idraulica* in Padua. Some years later, one of Levi-Civita's students, Cisotti, improves the theory of two-dimensional jets by using the method of conformal representation developed by Levi-Civita in his article on the wake (Levi-Civita, 1907a), briefly described in Section 3.1. In his book on two-dimensional hydrodynamics (Cisotti, 1921-1922, II), Cisotti applies Levi-Civita's method in order to deduce the general solution of irrotational two-dimensional motions of a fluid passing through an orifice. Cisotti highlight that any analytic function regular in a circle, which is real on the real axis and remains finite and continuous on the circumference, corresponds to a motion analytically possible (the shape of the walls is deduced a posteriori); however, further conditions are necessary if the motion is required to be *physically* possible.

[46] His father was Domenico Turazza (1813-1892), professor of geodesy and hydrometry at the University of Padua from 1842. Domenico Turazza founded the *Scuola di Applicazione per gli Ingegneri* in Padua that later became the Faculty of Engineering. Author of a famous treatise on hydraulics, D. Turazza wrote in the Preface to his book that hydraulics is "a pure experimental science", and pointed out that his book is not "a mathematical work" but it concerns "applicable experimental hydraulics" addressed "to engineers" (Turazza, 1867; VI-VII). On the attitude of Italian engineers and mathematicians towards rational hydraulics in the 19th century see (Garibaldi 1994).

The duality between mathematical and physical solution often arises in works of this period. Even in his 1907 paper (Levi-Civita, 1907a) Levi-Civita found solutions mathematically possible but that, by a closer examination, are not admissible from the physical point of view. M. Brillouin highlighted this aspect (Brillouin 1911b), as we pointed out in section 3.2.

Still devoted to the principles of hydrodynamics is the article on Torricelli's theorem that Levi-Civita published on the *Comptes Rendus* of the Academy of Sciences of Paris in 1913. The paper aims at extending Torricelli's theorem to more general case: "The speed of the jet outflowing through a small nozzle is expressed, by using Torricelli's theorem, in the form

(*) $v^2 = gh$

h being the level of the orifice below the free surface of the liquid." (Levi-Civita, 1913 ; 481)

As Levi-Civita claimed, the classic proof concerns the stationary regime only. "I am not aware that the validity of (*) has been pointed out when the motion starts, that is at the moment in which the fluid begins to outflow through a nozzle suddenly opened in the wall of a tank containing a liquid at rest." (Levi-Civita, 1913; 481) By applying the energy theorem only, Levi-Civita is able to prove that formula (*) defines "in a rigorous way" the initial velocity for each element dW, where W is the width of the orifice.

Levi-Civita's ideas on jet theory influenced several scholars in Italy and abroad. Here, we shall consider the case of the French Phd-student, Robert Mazet.

Robert Mazet (1903-1991) graduated from the ENS of Paris and was a student of Villat and Painlevé. He spent two years at the University of Rome (from 1925 to 1927) with Levi-Civita and Volterra.

Both Painlevé and Vessiot recommended Mazet to Levi-Civita. Painlevé wrote to Levi-Civita that "his student" Mazet knew very well "such questions" of fluid mechanics, since he had attended his lectures on fluid resistance at the Sorbonne and drafted them for publication (Painlevé, 1930).[47] Vessiot, the director of the ENS, recommended Mazet as one of his *normaliens*. Vessiot remarked in his letter to Levi-Civita (August 8, 1925) that though Mazet's subject for his thesis was not yet "delimitated", he had attended Painlevé's lectures on fluid dynamics and read the memoirs of Villat and Thiry.[48] Finally, in 1925 Mazet left for

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[47] On October 17th, 1925, Painlevé wrote to Levi-Civita: "Je recommande tout particulièrement à votre bon accueil mon élève, Monsieur MAZET, qui se rend en Italie pour préparer une thèse sur la mécanique des fluides. J'ai chargé Monsieur Mazet de rédiger et publier le cours que j'ai fait l'année dernière sur la mécanique des fluides. Il connaît très bien ces questions, et sera à même de comprendre et d'appliquer les conseils que vous pourriez lui donner." The letter is in Fondo Levi-Civita.

[48] In particular, Vessiot wrote to Levi-Civita: "M. *Mazet* n'a pas encore un sujet aussi délimité ; mais il a déjà bien étudié la dynamique des fluides et surtout le problème du sillage ; il a lu, en particulier les mémoires de M.

Rome with a David-Weill fellowship, and the year later he was happy to come back to Rome as a Rockefeller fellow.[49] Levi-Civita supervised Mazet's research. From 1926 to 1928, Mazet published many notes on issues related to the theory of two-dimensional jets in the *Comptes Rendus* of the French Academy of Sciences and in the *Rendiconti* of the Academy of Lincei, communicated by Levi-Civita.

Mazet defended his PhD thesis at the Sorbonne in 1929. In his thesis (Mazet, 1929) he collected and developed the content of his published notes. Mazet mainly investigated two-dimensional flows of a liquid through an orifice, oscillations of a fluid in communicating vessels, and motions of a permanent liquid under the action of gravity. He aimed to extend some results obtained by Levi-Civita and his students to more general cases. In particular, in the second part of his dissertation, he intended to focus on the following "unsolved" problem Levi-Civita suggested to him – to determine the two-dimensional motion of a fluid in vacumm, submitted to gravity, variable, and reaching stationary state in some time (Mazet, 1929; 3-4). However, as Mazet confessed, such a problem presents mathematical difficulties he was not able to overcome. As he explained in his PhD thesis, some discussions with Brillouin, Vergne and Villat convinced him to simplify the original problem and to introduce a further hypothesis, by considering a horizontal circular nozzle opening progressively in time. In his work, Mazet applied Levi-Civita's results on two-dimensional jets, and used his "fruitful" method of conformal transformation as developed by Cisotti in his treatise (1921-1922).

As it used to be in France at that time, Mazet had to print his PhD thesis before defending it. Nevertheless, the impression was generally very expensive. Young PhD students tried to publish their works on scientific journals that, however, did not meet all demands and hardly accepted very long memoirs for publication. A good solution was to publish in the *Rendiconti del Circolo matematico di Palermo*, an international and well-reputed journal. By following, once more, the suggestion of Vessiot and Villat, Mazet asked Levi-Civita for publishing his memoir in the *Rendiconti* of Palermo. Thanks to the intercession of Levi-Civita, his request was accepted and his work was actually published in volume 53 of the journal.[50]

Villat et de M. Thiry. Il vient, d'autre part de rédiger leçons d'aérodynamique que M. Painlevé a professé cette année à la Sorbonne." The letter is in Fondo Levi-Civita.

[49] In his letter to Levi-Civita (dated August 10, 1926), Mazet wrote : "Je vais soumettre l'ensemble de mon travail à M. Villat pour savoir si ce que j'ai fait est suffisant pour ma thèse. L'international Education Board m'a avisé récemment qu'une bourse Rockefeller m'était accordée. J'aurai donc le plaisir de retourner à Rome au mois de novembre". The letter is in Fondo Levi-Civita. For more details on French students at the University of Rome see (Mazliak 2015)

[50] Mazet wrote to Levi-Civita on December 6th, 1927: "… Je viens recevoir le permis d'imprimer [ma thèse] et je pourrai soutenir ma thèse dès que l'impression sera terminée. Comme celle-ci entraîne toujours à des gros

Finally, Mazet defended his thesis in 1929 – Painlevé, Vessiot, and Villat (*rapporteur*) belonged to his PhD committee. Anyway, as already remarked, Mazet worked on a subject proposed by Levi-Civita and then developed by his students, especially by Cisotti. Mazet went on researching on hydrodynamics, in particular on fluid friction, during his scientific career. He soon involved himself in the creation of new institutions related to practical mechanics and applied hydrodynamics. Mazet spent the most part of his career at the University of Lille, where he became assistant professor in 1932 and professor of mechanics in 1936; the same year he was appointed "director of studies" at the *Institut Industriel du Nord*. Very involved in local scientific life, in 1934 Mazet founded a *Laboratoire de Mécanique Expérimentale* related to the University of Lille, the first in France, aiming at practising students in experimental mechanics and making them able to put concrete problems into equations. This Laboratory was largely appreciated[51] and had vocation to become a great institution of applied mechanics involving local industries.[52] Since 1947 Mazet was director of the ONERA (*Office National d'Etudes et de Recherches Aérospatiales*) of Lille, succeeding to Joseph Kampé de Fériet (1893-1982).

Levi-Civita was then attentive to applications and was proud to show that his theoretical results were in agreement with practical problems of hydraulics. We mentioned the case of Mazet, who was forced to renounce to the original and more general subject of his thesis, proposed by Levi-Civita, and finally reduced it to a case easier to solve mathematically. It was Villat himself who suggested the new version of the problem to him. Villat, Levi-Civita and the other *mathematicians* working on hydrodynamics in Italy and France shared a similar approach to hydrodynamics – to simplify the physical problem in order to find solutions without renouncing to mathematical rigour.

5. Concluding remarks

frais, M. Vessiot et M. Villat m'ont suggéré de m'adresser à une publication scientifique susceptible d'accueillir mon manuscrit. […] J'ai pensé que je serais peut-être plus favorisé en m'adressant à un recueil italien tel que les Rendiconti del Circolo di Palermo qui ont déjà publié plusieurs thèse françaises ". The letter is in Fondo Levi-Civita.

[51] For instance Paul Montel, in a letter addressed to the Dean of the Faculty of Sciences of Lille University (dated November 8, 1935), put in evidence the importance of Mazet's Laboratoire de Mécanique Expérimentale in France. In particular he wrote: "Enfin j'ai proposé aussi pour une promotion Mazet. […] Il a realise ici une organisation unique en France de travaux pratiques de Mécanique Rationnelle qui fait d'ailleurs l'objet d'un ouvrage en cours de publication." The letter is in Dossier Mazet, ASA, Université de Lille 1.

[52] See the "Rapport de la commission chargée d'examiner les conséquences pour la Faculté de la nomination à Paris de M. Fleury", in Dossier Mazet, ASA, Université de Lille 1.

In this section we focus on some crucial aspects, which explain why Levi-Civita became a reference point in the field of hydrodynamics for French students and colleagues. Finally, we put in evidence the historiographical elements emerged by our analysis.

5.1 Levi-Civita as a *Master*

We have shown that Levi-Civita's ideas influenced Villat's thesis and the work of many French scholars. He was a Master for young French students, eager to improve their mathematical education in Rome. We add that Levi-Civita played an important role for the publication strategy of French students. In fact, several PhD students published (in French) their thesis in *Rendiconti* of the *Circolo Matematico di Palermo* also thanks to his intercession with the *Circolo*. In addition, during their stay in Italy and even later, young French mathematicians regularly published brief notes in *Rendiconti* of the Academy of the Lincei, often communicated by Levi-Civita.

Moreover, our transnational approach allows us to observe that ENS students could count not only on their supervisor (*patron*) – currently Villat for hydrodynamics – but also on other scholars (internal or external to ENS) with whom they regularly discussed or exchanged letters, as in the case of Levi-Civita. Let us give some examples. Jacotin-Dubreil's supervisor was Villat; nevertheless, she decided by herself the subject of her thesis by reading Levi-Civita's paper on wave theory. Levi-Civita suggested the subject of the thesis to Mazet, though Villat was his supervisor. Mazet mentioned Volterra, Levi-Civita, but also Villat, Painlevé, and Vessiot as supervisors. Moreover, he referred to his essential discussions with Vergne and Brillouin, though none of them belonged to his PhD scientific committee (*jury*). Mazet, and other PhD-students of ENS, accomplished their work by attending lectures, discussing and corresponding with many researchers.

Finally, we point out that Levi-Civita's activity as a Master is not limited to an institutional engagement. In fact, he not only supervised several PhD theses or Rockefeller research, but he also exchanged with students and colleagues, who had never studied with him and to whom he gave his help. For instance, we mention the case of Dimitri Pavlovitch Riabouchinski (1882-1962), a Russian student of Jukowski, who had founded in 1904 in Kotchino (Russia) an aerodynamics laboratory of which he was director. In 1919 he moved to Paris where he met Painlevé, Villat, and other experts of aerodynamics. Riabouchinski asked Levi-Civita to support him in order to obtain a Rockefeller fellowship. Also thanks to Levi-Civita's help, he got the grant and went to Oxford with Augustus Edward Hough Love (1863-

1940).[53] The following year, he asked Levi-Civita for another testimonial in order to obtain the renewal of his Rockefeller fellowship.[54]

5.2 Levi-Civita's activity in France

Levi-Civita strongly participated in scientific activities in France – he published French translations of his treatises on hydrodynamics, accepted invitations to publish papers in French journals or books, and gave lectures at the Institute of fluid mechanics of Villat and in other French institutions. Corresponding member of the Academy of Sciences of Paris since 1911, Levi-Civita published in *Comptes Rendus* of the French Academy, but also in the *Journal des Mathématiques pures et appliquées*, accepting the invitations of its director Villat. For instance, the paper on liquid jets, which summarized a lecture held at the Institute of fluid mechanics, was published in *Journal des Mathématiques* in 1932.

Just after his appointment as director of the Institute of Fluid Mechanics on November 22nd, 1930, Villat asked Levi-Civita to give lectures on "a topic of your choice concerning your most recent research" (Nastasi, Tazzioli, 2003; 393). In the following years, Villat would renew his request. In Villat's Institute several students and colleagues could meet the Italian Master and discuss with him. Levi-Civita was pleased to accept Villat's invitations, although in the thirties the fascist regime required a special authorization from the government upon submission of documentation. A reference to this fact is for example in an undated letter – but very probably drafted about 1930 – where Levi-Civita asked Villat for a "diplomatic invitation" addressed to the Italian Ministry of Foreign Affairs. However, Levi-Civita added, permission was usually granted even to those who are not "in the good graces of the regime" (in Nastasi, Tazzioli, 2003; 394); the reference to him is implicit.

Villat got Levi-Civita's support for all his most important enterprises. In particular, Villat asked Levi-Civita for his collaboration in his *Mémorial des Sciences mathématiques*. On June 5th, 1925, he wrote to Levi-Civita: "Thank you very much for your kind letter and your cordial wishes you sent me for the Mémorial des Sciences Mathématiques. I am particularly pleased

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[53] On March 6th, 1926 Riabouchinski wrote to Levi-Civita: "Permettez moi de vous entretenir d'une question personnelle. Le Prof. Paul Painlevé a bien voulu me présenter comme candidat à une bourse de l'International Education Board fondé par Rockefeller. Cette bourse me permettrait de continuer pendant un an mes recherches théoriques et expérimentales et, peut-être, d'obtenir ensuite un Laboratoire. La proposition de M. Painlevé est secondé par le Prof. Love de Oxford." The letters by Riabouchinski to Levi-Civita are contained in Fondo Levi-Civita.

[54] Riabouchinski wrote to Levi-Civita on August 10th, 1927: "Ne m'en voulez pas trop si je prends la liberté de vous prier aussi à me faire l'honneur de me donner encore une fois l'appui de votre grand nom, dont le poids, comme j'ai eu encore maintes fois m'en convaincre en Angleterre, est très grand, en insistant une fois de plus que vous considérez si je puis être utile comme directeur d'un Laboratoire d'Aérodynamique ou comme research-professor."

with your high approval, which evidence to me our current initiative responded well to a need, and will render service to mathematical public. Your authoritative opinion reinforces my conviction." (Nastasi, Tazzioli, 2003; 392) In 1934 Levi-Civita accepts to write a volume of the *Mémorial* on the relativistic two-body problem, after preparing the result "in the paper devoted to Brillouin" and detailing it in "following lectures". For the moment, he preferred not to precise when the volume should be ready.[55] His volume, actually, will be published posthumously, in 1950 (Levi-Civita, 1950), because of criticism moved by American scholars – especially by Howard Percy Robertson (1903-1961) – against crucial points of his theory.[56] The above-mentioned letter refers to the book for Brillouin's Jubilee to which Levi-Civita contributed with a paper on the relativistic two-body problem (Levi-Civita, 1935). Grateful, Villat wrote him on January 31st, 1935: "Your manuscript will be one of the jewels of the book to honour Mr. Marcel Brillouin, and I read it with intense pleasure." (Nastasi, Tazzioli, 2003; 403)

5.3 Hydrodynamics in the inter-war period in France

The historiography often claimed the importance of pure mathematics (especially analysis) over applied mathematics during the inter-war period. As Gispert and Leloup argue, oral and written testimonies on mathematics in the period between the two world wars concern almost exclusively members of the Bourbakist movement.[57] That can explain, at least partially, the historiographical distortion of French mathematics. In fact, such an image should be nuanced; several mathematicians – among whom some specialists of pure mathematics such as Maurice Fréchet – began to work on the theory of probability from the early years of the 20th century.[58] As we have seen, in the twenties and thirties, hydrodynamics also seems a very dynamic research field thanks to the work by M. Brillouin, Villat, Painlevé and their students.

The strong scientific and institutional interests of Paul Painlevé in aviation certainly contributed to the development of hydrodynamic studies in France. In 1895 Painlevé gave a course on friction, a topic related to aerodynamics (Painlevé, 1895), and, moreover, was actively involved in the first research on mechanical flight and on the propulsion of aeroplanes.[59] A professor at the Sorbonne, Painlevé played an important role in supporting mathematicians, such as Villat, leading major research centres of hydrodynamics and

[55] See the letter by Levi-Civita to Villat on November 23, 1934, in (Nastasi, Tazzioli, 2003; 403).
[56] For details about this history see (Nastasi, Tazzioli, 2006 ; 207-210).
[57] For bibliography on Bourbakist historiography see (Leloup, 2009; 21), (Gispert, Leloup, 2009; 41).
[58] See for instance (Mazliak, 2010).
[59] See the biography of Paul Painlevé (Anizan, 2012). For Painlevé's engagement in aviation see (Fontanon, 2005).

aerodynamics, and in giving impetus to mathematical research on such topics. Villat, who was Painlevé's successor in the chair of fluid mechanics, supervised 21 theses on fluid mechanics defended at the Sorbonne between 1914 and 1945,[60] and was one of the most influential scholars in France in the inter-war period.

5.4 Mathematical methods in hydrodynamics

The fact that mathematical methods are the only prerequisite for concrete applications underlies the contribution of Levi-Civita and Villat to hydrodynamics. We argue that the work of Levi-Civita's school was relevant for the development of mathematical hydrodynamics in France, i.e. hydrodynamics based on mathematical principles.

Concerning the judgement on "the Italians" expressed by Brillouin and Boussinesq and discussed in section 3, we think that their criticism should be downsized. In fact, both appreciated Italian contribution to hydrodynamics, as evidenced by the fact that in 1909 and 1910 Brillouin focused his courses at the *Collège de France* on the memoirs by Levi-Civita, Picciati and Boggio, while Boussinesq recommended Villat to read carefully the results on fluid dynamics published by Levi-Civita and Almansi.

Brillouin and Boussinesq feared that a too theoretical point of view could undermine the discipline and warned Villat of that danger. On September 9th, 1919, when Villat was appointed professor at the University of Strasburg, Brillouin congratulated Villat, who had been able "to forge […] the mathematical instrument needed to approach the physical problem of hydrodynamics little by little."[61] Villat's courses at the *École Superieure d'Aéronautique* had the same purpose, i.e. to strengthen the theory in view of applications, and similarly the works of Villat's students, as in the case of Thiry, Mazet, Jacotin-Dubreil.

In his course on fluid mechanics held at the same school of aeronautics (Villat, 1930), Villat focused more on theorems than on applications – he dealt with Levi-Civita's method of conformal representation for wake theory, the theorem of Kutta-Jukowski and its generalizations, and the theory of Prandtl. This course is the same as the one held at the Sorbonne. By using the words of the reviewer, Adolphe Buhl: "Nothing is more natural […] that Mr. Villat do not speak at the *École d'Aéronautique* a language essentially different from that spoken at the Sorbonne" And he added that "the author certainly wanted to put [in his

[60] See (Gispert, Leloup, 2009; 92).

[61] "chercher […] à forger l'instrument mathématique nécessaire pour approcher peu à peu le problème physique de l'hydrodynamique".

lectures] only things that are aesthetically pleasing and relatively elementary" (Buhl, 1930; 361).

The second edition of Villat's treatise appeared in 1938. Buhl's criticism is even harsher: "it must be remembered that the author, who has an eminently elegant spirit, has always presented his topics in aesthetic forms, which sometimes expressed regret concerning their non complete agreement with experimental verifications" (Buhl, 1938; 227). Buhl concludes his review by mentioning *inter alia* the physical *École de Toulouse*, headed by Charles Camichel, whose experiences have made great contributions to hydrodynamics; then, he hopes that is the path that we will follow in future.

Similarly to Villat, in his lectures at the *École Supérieur d'Aéronautique* held in the years 1925-27, Painlevé underlines that "we tried to push as far as possible the theoretical solution of this great problem." (Painlevé, 1930; 2) Apparently, theoretical hydrodynamics represents for Painlevé the solid foundation for any actual applications; that is why he mainly deals with the "rigorous" analysis of the motion of a non-deformable body in a perfect fluid.

Therefore, the polemics between mathematical hydrodynamics and experimental hydrodynamics does not involve "the Italians" on one side and French scholars on the other; on the contrary, Levi-Civita, Villat, Painlevé and their students agree on the essential points. The controversy seems, rather, much more extensive. On the one hand, there were hydraulic engineers and experimental physicists who were proponents of a highly experimental hydrodynamics (in Villat's words, "rudimentary" hydrodynamics), and, on the other hand, the supporters of a "mathematical" or "rational" hydrodynamics" founded on rigorous mathematical bases but not always in agreement with experimental data.[62] In 1929 Paolo Straneo, an Italian mathematician interested in aerodynamics, wrote that "the flight by aircraft has lead several specialists of fluid dynamics to revise their discipline in order to better understand the intolerable gaps between reality [and theory] [...] Therefore, many older theories accepted should be amended." (Straneo, 1929; 298) We point out that Levi-Civita did not actively work in a laboratory, but he was proud to show that his theoretical results are in agreement with experimental data found in some hydraulic laboratories (see section 4). Villat did not mention any physical experiments explicitly – even when he dealt with "applied hydrodynamics", he did not go out of the field of mathematical physics.

[62] On the difficulty between mathematical hydrodynamics and practical problems of fluid mechanics see (Darrigol, 2008).

We conclude by quoting a letter by Theodore von Kármán (1881-1963) to Jerome Clarke Hunsaker (1886-1984),[63] an organiser of the International Congress of Applied Mechanics in Cambridge (Mass.) in 1938: "Concerning Levi-Civita's recommendation of French representatives of Applied Mechanics, I agree that Villat and Pérès are excellent men; however, Villat is far beyond the line of what we would call the frontier of useful or applied mechanics. I really believe that the man we could use best for a general lecture is Kampé de Fériet, director of the Institute for fluid mechanics, in Lille. In the last years, he published two reviews on recent progress concerning waves and turbulence. Both reports were excellent and just on the limit between the practical and theoretical viewpoint as we like it. Besides that, he follows the experimental research, whereas Villat, in spite of the fact that he is director of an experimental institution, has no idea of experimental questions".

This letter shows once more the sharp contrast between Villat's theoretical research and his institutional position as director of the Institute of Fluid Mechanics; a contradiction that did not escape his physicist and engineer colleagues even in France. Finally, Villat's name does not appear in the *Proceedings* of the Fifth International Congress of Applied Mechanics, while Pérès gave one of the three general lectures on "the methods of analogies in applied mechanics" (Pérès, 1939). In the same Congress, Kampé de Fériet lectured on recent research about turbulence (Kampé de Fériet, 1939). Furthermore, except for Jules Drach (1871-1949) and Henri Beghin (1876-1969) who were academic-mathematicians, the other French lecturers at the Cambridge Congress were "engineers" working in technical laboratories and teaching in various Engineering Schools.[64] In the forties, various research institutions in aerodynamics appeared even in France, where mathematicians, physicists and engineers worked together.[65]

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[63] The letter dated 2 March 1937 is contained in the von Karman Papers, box 47, folder 3, California Institute of Technology. I thank Giovanni Battimelli for giving me the content of this letter.

[64] We mention for instance: Georges Darrieus (1888-1979), an electric engineer working in the Compagnie Électro-Mécanique ; Henri de Leiris (1903-1991), a General of the Marine Engineering; Albert Métral (1902-1962), polytechnicien, both teacher and engineer working in private industries; Louis Bergeron (1876-1948) a hydraulic engineer and professor at École Centrale, École Supérieure d'Électricité, and École de Physique et Chimie.

[65] See (Weber, 2008).

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