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A DATA STRUCTURE FOR PROGRESSIVE VISUALISATION AND EDITION OF VECTORIAL GEOSPATIAL DATA

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ABSTRACT:

Virtual cities are an ever more growing research topic, with a great variety of applications for urban planning, routing, simulation or cultural heritage (Biljecki et al., 2015). Technical challenges still remain, especially regarding the high amount of data in a virtual city model. To allow the best user experience, this data needs to be streamed. While there exist efficient methods for the progressive streaming of raster data, few satisfying solutions exist for vectorial data. Most of these solutions require a long preprocessing stage, where the data is organised, simplified or generalised and often duplicated. The result is generally a new dataset suitable only for visualisation that needs to be recomputed whenever the original data is modified.

An alternative to the costly generalisation of the geometry is the transmission of a subset of the data. In a lot of use cases, only a handful of features are of interest to the user. Transmitting and displaying only the most relevant data allows the user to rapidly access a view containing most of the information he needs at a low cost.

When transmitted from the server to the client, the city model’s geometries are usually not sent individually, but packed together in a tile. With the surge in mobile devices, there is a need to control the amount of resources allocated to an application, and therefore the quantity of data within each tile. Since city models do not have a constant density, vectorial data has to be organised in hierarchical structures in order to have tiles of roughly the same size.

Being able to carry out geometrical operations or to edit the data opens up the use cases of virtual cities. For this purpose, having a single model for both the visualisation and the storage of the data seems convenient. Furthermore, a unified model removes the need to duplicate the data (base model and visualisation model). With datasets covering larger and larger areas, exceeding the city scale, as well as existing in multiple temporal versions (Chaturvedi et al., 2015a), storage may well become an issue in the future.

The contributions of our paper are as follows:

- an efficient hierarchical structure for vectorial data allowing:
  - progressive loading of data based on a weight function
  - dynamic addition and removal of data
  - compatible with rising exchange formats such as 3D-tiles (3D Tiles - Specification for streaming massive heterogeneous 3D geospatial datasets, 2016)

- an architecture for visualising data from a geospatial database featuring:
  - fast deployment
  - original storage format, suitable for 3D analysis

The paper is organised as follows: we will review the state of the art in the fields of web 3D and virtual cities, then we will describe our data model and, in the next section, the detailed algorithms for creating and updating our data structure as well as ways of accessing the data. We will then present some results of our implementation of the model, both client-side with visual results of the progressive streaming and server-side with performance measurements. Finally, we will conclude and present leads for future works.

1. INTRODUCTION

Progressive streaming of 3D geometries is a problem that has been addressed by numerous papers in the last 20 years (Hoppe, 1996). It is especially relevant nowadays, with web applications becoming so common and embedding 3D content thanks to WebGL (Evans et al., 2014). The geospatial field is especially concerned by these works since it deals with a huge quantity of data. A lot of the research has already been done for terrain streaming (Lerbour, 2009), but streaming buildings and other city objects (roads, bridges, tunnels, etc.) is a much more recent topic.

The existing approaches for streaming 3D vectorial data can be classified in two main categories: generalisation on the one hand and progressive transmission on the other.

Generalisation is the process of simplifying a geometry or a group of geometries into a less complex and lighter geometry. It is a
method for generating levels of details where each level is an independent geometry: transitioning from one level of detail to the other requires the loading of a whole new object. (Mao, 2011) presents a variety of methods for creating simplified representations of city models. (He et al., 2012) and (Guercke et al., 2011) generate multiple LoDs for building groups by generalising the footprint of the buildings.

Since its levels of details are independent, generalisation can produce better quality approximations than other methods that use additive refinement as level of detail. Such methods fall within the progressive transmission class. Additive refinement is a desirable feature for web applications since data transmission is a frequent bottleneck. Progressive data structure may still have a cost, but it is usually far lower than that induced by generalisation methods. (Ponchio and Dellepiane, 2015) presents a method for streaming high-quality meshes. Portions of the meshes are refined depending on the camera position.

Web city viewers often use very simple progressive loading methods for buildings. A 2D grid manages the loading of data in (Chaturvedi et al., 2015), (Gesquiere and Manin, 2012) or (Gaillard et al., 2015). While its simplicity can be attractive for web applications, the visual result is unsatisfactory: data is loaded non-uniformly, in chunks. Areas of interest may be loaded last since only geometric metrics are used to define which grid tiles are loaded: only the camera position counts, not the nature of the data.

Hierarchical data structures are popular in the geospatial field, since they are well suited for data of varying scale. (Suárez et al., 2015) presents a globe application, where the use of a quadtree enables constant resource usage. However, this only works for regularly meshed tiles, where each tile has the same number of triangles. Furthermore, vectorial data stored in quadtrees often overstep the boundary of the tiles, making them somewhat complex to manage. In geospatial databases, r-trees (Guttman, 1984) are often used to index the data. While it offers quick access to the features and solves the problem of overstepping features, it is not suitable for progressive streaming: all the features are stored at the same depth, in the leaves of the tree (there is no hierarchy of features), and features of the same leaf tile are always close to one another, meaning that a progressive loading based on this structure will be non-uniform and chunky, similarly to a grid. (Christen, 2016) uses a bounding volume hierarchy to organise 3D buildings, but doesn’t provide any insight on how the BVH should be built. In the ray tracing field, common methods to build bounding volume hierarchies rely on heuristics such as SAH (Wald, 2007). However, constructing BVHs in this manner does not allow the user to control the depth in which the objects are in the BVH.

Most of recent approaches are based on a strong relation between the client and its dedicated server. For several years, the Open Geospatial Consortium has been proposing standards to facilitate communication between different servers and clients. Recent works (3D Portrayal Interoperability Experiment, 2016) tend to propose a 3D portrayal specification, that may be followed in an implementation of this paper’s method, in order to provide a non-intrusive solution server-side.

In this paper, we present a progressive transmission method based on a data structure which allows a uniform loading of features over the whole extent of a scene, that takes into account not only the spatial position of the features, but also their semantic information. This produces a partial representation of a city composed of the most relevant features without the need to generate a new dataset or to download redundant data.

3. DATA MODEL

In this section, we will present our solution for storing and editing a geospatial dataset. A description of the data structure is presented in figure 1. This dataset \( S \) is defined as a collection of features \( F \). A feature \( F_i \) is a 3D geometry to which is assigned a number of semantic attributes \( A_i \). For instance, a feature could be a building with attributes such as height, energy consumption, etc. A feature could also be a bridge with its name and date of construction as attributes.

![Figure 1: The bounding box hierarchy (left) and an exploded view of it (right).](image)

Our goal is to retrieve data in chunks of similar sizes. Additionally, we wish to retrieve the most important data first. Since a city has variable density, we either need an irregular organisation of the data or a hierarchical data structure. In our case, hierarchical data structures are preferable since they allow the ranking of data. Therefore, we store our data in a Bounding Box Hierarchy (BBH): a tree where each node has a bounding box which strictly encloses all the underlying features (see figure 2). This is an advantage compared to the usual structures used for tiling cities (quadtreers or regular grids), where some features may overstep their tile’s boundaries and causes imprecisions in the loading of its features. In this paper, we use bounding boxes since our data has almost no verticality. In some cases, Bounding Volume Hierarchies (BVH) might be a better choice. The presented methods work with both BBH and BVH with virtually no change.

![Figure 2: A 2D representation of the bounding box hierarchy structure.](image)
deeper than those of sparse parts. This is not a problem: our main goal is to have tiles which have a similar weights and progressive loading, it is therefore natural that we have theses disparities.

In this paper, a tile is the spatial aspect of a node, i.e. the bounding box and the features contained in a node.

There are various ways of limiting the number of features in a node. In the examples and algorithms of this paper, we define a maximum number of features per node, but it is also possible to set a maximum number of points or triangles per node. This is especially useful for low-end clients that need to control the amount of data they download or render. Using these more complex metrics (where the limiting property varies from feature to feature) raises the issue of packing the features into nodes in an optimal manner. This is a classic knapsack problem. We use a very simple approach, where as soon as a feature overfills a node, it is considered full. This solution offers satisfying results, further optimisation is left for future works.

Bounding box hierarchies (or bounding volume hierarchies) are a fairly common data model. Some exchange formats, most notably AGI’s 3D-tiles (3D Tiles - Specification for streaming massive heterogeneous 3D geospatial datasets, 2016), already support these kind of structures, assuring compatibility with standard-compliant web clients. The specificity of our method, namely the way we order the features inside the hierarchy, takes place on the server and is completely transparent for the client.

4. DATA PROCESS

In this section, we will describe methods for initialising, editing and accessing the data of the BBH. An overview of the processes the BBH undertakes is shown in figure 3. Detailed algorithms are available in the appendix.

4.1 Classification

The classification of the features in the bounding box hierarchy is based on the weight associated to each feature. This weight is determined by a function $f$ based on the feature’s geometry $G_i$ and semantic attributes $A_i$: $W(F_i) = f(G_i, A_i)$.

The choice of weight function depends on the target application. The most important features for the application should get the highest weight. For example, in a simple city visualisation application, the biggest buildings should be displayed first, and thus have a higher weight. If the application is focused on tourism, monuments should be favoured by the function. In section 4.4 we will present ways of creating and combining weight functions.

In the examples throughout this paper, for the sake of simplicity, we will use a purely geometric weight function based on the ground surface of a feature: $W(F_i) = ground\_surface(G_i)$.
Add the first features to the node (3). Create the bounding based on these features (4). Divide the quadtree tile (5) and assign the remaining features to each of its children based on the position of their centroids (6). Repeat the process for each child, until no features are left (7).

Recursively adjust the bounding box of the nodes such as the parent node’s bounding box encloses its features and its children’s bounding box (8). The resulting structure is shown in (9).

4.2 Adding and Removing Features

Adding or removing data from the structure requires careful reorganisation to maintain the integrity of the database: the ordering of the features in accordance with their weight must be preserved, the maximum number of features per node must not be exceeded and the bounding boxes must be adjusted.

4.2.1 Addition The insertion of a feature (figure 5) requires finding the first node which fulfils the two following conditions:

- the node is not full or the weight of the new feature is between the weight of the lowest and highest weight features of the node;
- the new feature’s centroid is inside the quadtree tile associated to the node.

The quadtree tiles that contain the new feature’s centroid are traversed from top to bottom until a node that validates the first conditions is found.

The new feature is added to the node. If the node’s number of features exceeds \( n \), the lowest weight feature of the node is removed and added to the child node. The process is repeated until the addition of a feature does not cause a node to be overfull.

Once the hierarchy has been reorganised, the bounding boxes of the altered nodes are recomputed.

4.2.2 Removal Removing a feature (figure 6) from a node requires rebalancing the hierarchy. The removed feature of a node is replaced by the highest weight feature of its children. The child from which the feature was taken undergoes the same process, until the leaves of the tree are reached.

The bounding boxes are then resized for each altered node.

4.3 Accessing the Data

We recommend storing the features in a spatial database on the server. The features are indexed by the identifier of the tile to which they belong. This allows the server to select very quickly the features that a client needs, as the client sends requests for specific tiles.

The client accesses the data by querying the tiles that are inside its field of view and match its desired precision. For this task, the client must know in advance the bounding box and the level of detail of each tile. We call refinement scheme the way each tile is subdivided into its child tiles, that is the number of child tiles and their bounding boxes.

There are two main ways of managing the transmission of the refinement scheme. The refinement scheme of the whole structure can be sent on initialisation of the application, solving the issue once and for all. This has the advantage of allowing the client to query multiple levels of tiles if needed, but can slow the initialisation of an application if the dataset is large and has a lot of tiles. The alternative is to transmit the scheme progressively, along the features. When answering a tile query, the server sends both the features and the tile’s refinement scheme.

4.4 Creating a weight function

In this section, we will define guidelines for creating and combining weight functions. A weight function assigns to a feature a weight, represented as a floating point number between 0 and 1. The higher the weight, the more importance is attached to the feature.

\[ F \rightarrow W(F), \mathbb{R} \rightarrow [0..1] \]

Surface Weight A classification based on the 2D footprint of a building allows the largest buildings to be displayed first. It is a pertinent choice for visualisation applications since these buildings are big enough to be seen from afar and can be used as landmarks for orientation purposes.

The surface weight function is defined as a function which returns 0 for a null surface and 1 for the surface of the largest building of...
the dataset. Cities often have a few buildings that are very large in comparison to the others. To avoid the weight values of most of the buildings to be clumped up near 0 as a result, we recommend using a function that returns 0.5 for a building that has the average size. Here is such a function, with $S$ the surface, $\text{avg}$ the average surface of features in the data set and $\text{max}$ the maximum surface:

$$\text{surface}_\text{weight}(F) = \begin{cases} \frac{S}{2\text{avg}} & \text{if } S < \text{avg} \\ 1 - \frac{S - \text{max}}{2\text{avg} - \text{max}} & \text{if } S \geq \text{avg} \end{cases}$$

**Attribute Weight** Semantic attributes can be used as a classification method. As they vary wildly in nature, it is hard to define a precise guideline for these weight functions.

**Combining Weight Functions** We can combine weight functions to create new, more complex functions. To each function $W_i$ is assigned an importance factor $w_i$. The weighted arithmetic mean of these functions form the new weight function.

$$W(F) = \frac{\sum_{i=0}^{n} w_i W_i(F)}{\sum_{i=0}^{n} w_i}$$

For example, let’s combine into a function $W_C$ the surface weight function $W_1$ described above and an attribute weight function $W_2$ that returns 1 for red features, 0 for blue features. For $w_1 = w_2 = 1$, $W_C$ will be a function that first prioritises the red features, then order features according to their ground surface. For $w_1 = 1$ and $w_2 = 0.5$, $W_C$ will be a function that guarantees that the red features will be displayed first only if their are larger than average. Figure 7 illustrates this example.

![Figure 7: Different weight functions. Highest weight feature is on the left, lowest weight feature is on the right. (1) Surface weight. (2) Combined surface and attribute weight. $w_1 = w_2 = 1$. (3) Combined surface and attribute weight. $w_1 = 1$, $w_2 = 0.5$.](Image)

5. **RESULTS**

5.1 **Implementation and performance**

All the following results were obtained on an Intel®Core™i5-4590 CPU @ 3.3GHz x 4, Nvidia GeForce GTX 970.

We implemented the algorithms for creating and manipulating our data structure in Python, using psycopg2 (psycopg2 - Python-PostgreSQL Database Adapter, 2016) to interact with a postgres database (PostgreSQL - The world’s most advanced open source database., 2016). We use the postGIS extension to handle geospatial data (PostGIS - Spatial and Geographic Objects for PostgreSQL, 2016).

We tested our algorithms on a dataset of close to 30,000 features of the city of Lyon and a second dataset of 80,000 features of the city Montréal. Results for Montréal are written in parentheses. The initial state of the database for our study was a postGIS database filled with the aforementioned data indexed by gid.

We chose a weight function that returns the area of the footprint of the building and a maximum of 50 features per tile. It took 6.1 (11.4) seconds to create our data structure. 1.3 (1.9) seconds were spent fetching the data from the database and computing the weight of each feature, 4.6 (8.9) seconds were spent updating the database with a new tile index. The remaining 0.2 (0.6) second was used to organise the data inside our structure.

Adding and removing features from the database and updating the data structure is also fast. For the same database, which had 1165 tiles and up to 5 depth levels, each of these operations took barely more than 10 milliseconds (see table 1).

<table>
<thead>
<tr>
<th></th>
<th>mean (ms)</th>
<th>standard deviation (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lyon</td>
<td></td>
<td></td>
</tr>
<tr>
<td>add</td>
<td>11.5</td>
<td>4.1</td>
</tr>
<tr>
<td>remove</td>
<td>10.4</td>
<td>3.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Montréal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>add</td>
<td>17.8</td>
<td>5.5</td>
</tr>
<tr>
<td>remove</td>
<td>16.1</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Table 1: Add and remove operations performance.

5.2 **Test client**

We use an Apache2 server combined with Python scripts to manage the server-side processes.

Client-side, we visualise the data using Cesium (Cesium - WebGL Virtual Globe and Map Engine, 2016). Our Javascript implementation features the progressive loading of buildings as intended by our data structure.

![Figure 8: Screenshot of our web application based on Cesium. Buildings in red belong to level 0 tiles, those in orange to level 1 tiles, those in yellow to level 2 tiles and those in green to level 3 tiles.](Image)

Figure 8 shows the result of the classification of the dataset’s features in the web application. We can clearly see that the buildings with the biggest 2D footprints are assigned to the highest level of the hierarchy and are therefore displayed first. Figure 9 is an example of this behaviour: the greater the distance to the camera, the fewer tiles are loaded; only the buildings with the highest weight remain.
We limited the number of triangles loaded in the scene and compared the result for different loading methods. Figure 10 shows a comparison between our method, with a weight function based solely on the ground surface of the buildings, and a grid of 500m x 500m tiles.

We implemented different weight functions to observe the effect changing these functions has on the displayed scene:

\[
W_1(F) = \text{surface weight}(F) \\
W_2(F) = \frac{\text{surface weight}(F) + \text{is noteworthy}(F)}{2} \\
\text{is noteworthy}(F) = \begin{cases} 
1 & \text{if } F \text{ has the noteworthy attribute} \\
0 & \text{otherwise}
\end{cases} \\
W_3(F) = \frac{\text{surface weight}(F) + \text{distance weight}(F)}{2} \\
\text{distance weight}(F) = \max(0, 1 - \frac{\text{distance to Rhône}}{500})
\]

In figure 11, we see that more noteworthy buildings are loaded while using \(W_2\) instead of \(W_1\). Figure 12 shows similar results, far more buildings are loaded next to the Rhône river when \(W_3\) is used.

5.3 Limitations

Changing the weight function requires the reprocessing of the full structure. It is however possible to have multiple indexes to provide different classifications from which to choose.

A drawback of this data structure is that not all the loaded data will be in the camera’s field of view. Features are stored in each level of the hierarchy and the topmost tiles can be quite large. Therefore, if the camera does not cover the full extent of a tile, which is likely for the tiles that are high in the hierarchy, it is likely that some features will be loaded despite not being visible. A way to mitigate this problem is to have multiple level-0 tiles by cutting the original extent into a grid, as suggested in section 4.1.

6. CONCLUSION AND FUTURE WORKS

We have presented a novel data structure for the storage of vectorial data. Its hierarchical nature allows the data to be easily streamed and is therefore suitable for web applications. Moreover, features are ordered according to a weight function, making the first features to be loaded the most relevant ones. This property also means that if only a limited number of features can be loaded, due to limited processing power or storage, these features will be the most relevant ones for the current view.

We have also presented algorithms to edit the structure, removing the need to rebuild the structure when changes are made. Adding
or removing features is fast, a few milliseconds, compared to the
time it takes to create the hierarchy, which takes seconds.

We achieve better visual results using our bounding box hierarchy
construction method than using traditional data structures such
as grids. Furthermore, thanks to the weight function, we allow
semantic data to have an impact on visualisation. This can be
used to create specific views for use cases by giving higher weight
to features that have semantic attributes that are relevant to the use
case.

Future works include adding support for geometric level of de-
tail. Deeper levels of the hierarchy could be loaded with a lower
geometric level of detail while top levels would be loaded at full
detail.

Another improvement would be the possibility to combine differ-
ent weight functions to generate a new structure on the fly, based
on the classification of each function. This would allow the user
to customise his view without needing to recompute the whole
hierarchy.

A future improvement could be the use of new metrics to de-
cide which tile needs to be refined. Traditionally, only geometric
metrics like screen space error are used to choose which tiles of a
hierarchical data structure have to be displayed. Choosing to load
a tile not only for its position relative to the camera but also for
the quantity of information it brings to the scene (see figure 13)
would allow semantic information to play a bigger role in city
viewing applications, and hopefully lead to a smoother experi-
ence for the user.

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Figure 12: Different weight functions and 500,000 triangles limit. The top image uses $W_1$ and the bottom one $W_3$.


A. FULL ALGORITHMS

Notations:

- $N$ a node of the tree;
- $B(A)$ the bounding box of the node or the feature A;
- $C(N)$ the list of the child nodes of the node N;
- $P(N)$ the parent node of the node N;
- $F(N)$ the list of the features of a node;
- $mF(N)$ the lowest weight feature of a node;
- $MF(N)$ the highest weight feature of a node;
- $QB(N)$ the bounding box of the quadtree’s tile associated to the node N;
- $F$ a feature;
- $W(F)$ the weight associated to the feature F;
- $n$ the maximum number of features that a node can contain.

A.1 Classification

1: for $F$ in features do
2: \hspace{1cm} $W(F) = f(F)$ \hspace{1cm} \Comment Compute all weights
3: end for
4: \Comment Order features by descending weights
5: $F = \text{ASSIGN(features, root)}$
6: \Comment Function
7: function Assign(features, N)\hspace{1cm}$\textbf{count} \leftarrow 0$
8: for $F$ in features do
9: \hspace{1cm} if $F \in QB(N)$ then
10: \hspace{2cm} if $\text{count} < n$ then \Comment Add feature if node not full
11: \hspace{3cm} $F(N) \leftarrow F(N) \cup F$
12: \hspace{3cm} $B(N) \leftarrow B(N) \cup B(F)$
13: \hspace{2cm} $\text{count} \leftarrow \text{count} + 1$
14: \hspace{1cm} end if
15: \hspace{1cm} end if
16: else \Comment Stop process when node full
17: \hspace{2cm} break
18: end if
19: end for
20: \Comment Subdivide node if features remain
21: if $\text{features} \neq \emptyset$ then \Comment Subdivide node if features remain
22: \hspace{1cm} $F = \text{DIVIDE(features, N)}$
23: for $Ni$ in $C(N)$ do \Comment Adjust bounding box
24: \hspace{1cm} $B(N) \leftarrow B(N) \cup B(Ni)$
25: end for
26: end if
27: end function
28: \Comment Function
29: function Divide(features, N)
30: Create children nodes for N
31: for $Ni$ in $C(N)$ do \Comment Assign each feature to the right child
32: \hspace{1cm} $Fi \leftarrow \emptyset$
33: for $F$ in features do
34: \hspace{2cm} if $F \in QB(Ni)$ then
35: \hspace{3cm} $Fi \leftarrow Fi \cup F$
36: \hspace{3cm} features $\leftarrow$ features $\setminus F$
37: \hspace{2cm} end if
38: end for
39: Assign($Fi, Ni$)
40: end for
41: end function

A.2 Addition

1: function Add($F$, N)
2: \hspace{1cm} if $|F(N)| < n$ then \Comment Replace lowest weight feature
3: \hspace{2cm} $F(N) \leftarrow F(N) \setminus F$
4: \hspace{2cm} $F(N) \leftarrow F(N) \cup F$
5: \hspace{2cm} $F \leftarrow mF(N)$
6: \hspace{1cm} end if
7: if $C(N) = \emptyset$ then \Comment Create children nodes for N
8: \hspace{2cm} Create children nodes for N
9: end if
10: for $Ni$ in $C(N)$ do \Comment Add feature to the right child
11: \hspace{2cm} if $F \in QB(Ni)$ then
12: \hspace{3cm} Add($F$, $Ni$)
13: \hspace{3cm} break;
14: \hspace{2cm} end if
15: \hspace{2cm} end if
16: \hspace{2cm} end for
17: \hspace{1cm} end if
18: \hspace{1cm} end for
19: Update bounding box
20: end function

A.3 Removal

1: function Remove($F$, N)
2: if $F \subset F(N)$ then \Comment Get highest weight children feature
3: \hspace{2cm} $F(N) \leftarrow F(N) \setminus F$
4: \hspace{2cm} $F \leftarrow \text{maxF} \leftarrow MF(Ni)$
5: \hspace{2cm} end if
6: if $C(N) \neq \emptyset$ then \Comment Remove N
7: \hspace{2cm} end for
8: \hspace{2cm} end if
9: \hspace{2cm} end for
10: \hspace{2cm} end if
11: \hspace{2cm} end for
12: if $F(N) = \emptyset$ then
13: \hspace{2cm} Remove N
14: \hspace{2cm} end if
15: \hspace{2cm} end if
16: \hspace{2cm} end if
17: \hspace{2cm} end if
18: else
19: for $Ni$ in $C(N)$ do \Comment Remove($F$, $Ni$)
20: \hspace{2cm} if $F \in QB(Ni)$ then
21: \hspace{3cm} Remove($F$, $Ni$)
22: \hspace{3cm} break;
23: \hspace{2cm} end if
24: \hspace{2cm} end for
25: \hspace{2cm} end if
26: \hspace{2cm} Update bounding box
27: end function