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A NUMERICAL EFFICIENT APPROACH FOR COUPLED AEROTHERMAL SIMULATIONS IN FLUID-STRUCTURE SYSTEMS

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ABSTRACT

Conjugate heat transfer procedures are commonly used in many scientific and aerospace problems in which accurate heat transfer predictions are needed. But these procedures may suffer from stability and performance limitations. This paper presents a numerical approach for steady conjugate heat transfer problems based on a stability analysis in a canonical coupling prototype. The main characteristics of the Dirichlet-Robin transmission procedure are presented and analyzed. Two fundamental parameters are introduced, a “numerical” Biot number controlling the stability process and an optimal coefficient that ensures unconditional stability and a high numerical efficiency of the coupled aerothermal simulations. An algorithm that chooses the locally most efficient numerical approach is proposed. An industrial test case then illustrates the excellent behavior of the interface treatments. Finally it is shown, on the basis of the numerical Biot number, that the choice of a relevant transmission condition is critically important in achieving an even faster convergence.

1. INTRODUCTION

Optimizing heat transfer between fluids and solids is a crucial issue in a wide range of engineering applications, especially in the aerospace industry in which heat transfer enhancement is considered as a key parameter for developing more efficient and effective thermal devices. For instance, the tendency toward higher turbine inlet temperatures is continuing and it continues to raise considerable problems for the design of turbine cooling circuits.

In many situations it is not adequate to consider heat transport in the fluid only, with some idealized boundary conditions at the fluid-solid interface. Instead, the fully coupled problem must be considered, including heat conduction in the solid body. Conjugate heat transfer (CHT) analysis addresses such coupled problems as it is able to study processes which involve variations of temperature within the solids and the fluid simultaneously, due to their mutual thermal interaction.

In partitioned procedures, systems are spatially decomposed and the solution is separately advanced in time over each partition [1,2]. This strategy allows the direct use of designed solvers for different fields but may suffer from stability issues and performance limitations.

As has often been stressed, the nature of instabilities derived from a 1D model can give insight into the potential instabilities in 2D/3D flows computations. As a result, the behavior of interface conditions in CHT is often studied using a normal mode analysis, for instance by Roe et al. [3], Henshaw & Chand [4], Kazemi-Kamyab et al. [5], Joshi et al. [6], even if other means of investigation exist [7]. This is because one may reasonably assume that the modes that may be unstable are those whose variation is in the direction normal to the coupled interface.

Recently, it was shown, using a 1D thermal problem, that a numerical transition can be identified [8]. This transition was derived from a normal mode stability analysis, and results in an unconditionally stable coupling procedure that provides high convergence rates. The goal of this paper is to present the implementation of stable and efficient numerical schemes at the fluid-structure interface for aerothermal simulations on the basis of this new concept.

This paper is composed of two parts. In the first part, the theoretical study is presented. The numerical interface treatment is briefly described for a Dirichlet-Robin transmission procedure. The pros and cons of this specific scheme are then presented along with the
appropriate numerical treatments that optimize it. In order to further enhance the numerical efficiency of the coupled approach, we put forward another complementary numerical alternative based on the physical properties of the local coupled problem. These general numerical concepts are then illustrated in the second part of the paper via a CHT 3D industrial test case with emphasis put on the stability and convergence properties of the coupling schemes.

2. COUPLING MODEL

2.1. The partitioned CHT procedure

The partitioned CHT strategy presented hereafter is motivated by the desire to obtain a fast and stable steady solution of the fluid-structure thermal problem. It takes advantage of the significant discrepancy that exists between the characteristic time scales of the phenomena, namely a fast transient process in the fluid and a very slow one in the structure. Therefore, since the physical time-dependent response is not sought, the steady fluid-structure problem is solved by iteratively coupling a transient fluid solver to a steady state solid solver to achieve temperature and flux convergence at the interface. This implies that only the final fluid-solid steady state result is relevant.

Partitioned techniques are very popular as they allow the direct use of a specialized solver for each subdomain, offering significant benefits in terms of efficiency and code reuse.

The basic conventional serial staggered (CSS) algorithm was used to sequentially execute the CFD and the conduction solvers. The generic cycle is described in Fig. 1 where \( \Omega_f \) is the fluid sub-domain and \( \Omega_s \) is the solid sub-domain.

![Figure 1. CSS Algorithm](image)

The interval \([t^n, t^{n+1}]\) corresponds to the current coupling step at which the two solvers exchange their interface values. This procedure is repeated until a global steady-state solution is obtained.

2.2. Interface coupling prototype

The transient energy equation in the fluid domain is discretized using a finite volume method (FVM) while the steady state heat conduction equation within the solid is solved using a finite element method (FEM).

An implicit backward Euler method is employed for the time derivative in the fluid and the heat diffusion terms in both domains are computed using a second order central difference formulation (see details in [8]).

If the convection term is assumed to be negligible in the first fluid cell, the CHT problem may be modelled by a simple one-dimensional thermally coupled problem [8]. The equations in both domains are then iteratively solved using the CSS algorithm, with interface data being exchanged at each coupling iteration.

A general Robin transmission condition, considered on the solid side, leads to the equation:

\[
\hat{Q}_r + \alpha_f \hat{T}_r = Q_f + \alpha_f T_f
\]

where \( Q \) is the heat flux, \( T \) the temperature and \( \alpha_i \) is a coupling coefficient, with (^) denoting the sought values. The same type of interface condition can be defined on the fluid side:

\[
\hat{Q}_f + \alpha_s \hat{T}_f = Q_s + \alpha_s T_s
\]

Applying the right hand side of Eq.(2) as a boundary condition on the fluid domain, may technically be challenging in most research and commercial CFD packages. To overcome this hurdle, the convective boundary condition used in the CFD solver may be expressed in terms of a reference temperature \( T_f \) and a convective heat transfer coefficient \( h_{ext} \) (W.m\(^{-2}\).K\(^{-1}\)).

This is obtained easily from Eq. (2):

\[
\hat{Q}_f = h_{ext}(T_{ref} - \hat{T}_f)
\]

with

\[
\begin{align*}
    h_{ext} &= \alpha_s \\
    T_{ref} &= \frac{\hat{Q}_f}{\alpha_s} + T_s
\end{align*}
\]
2.3. Stability analysis

The Godunov & Ryabenkii theory [9,10] is used to analyze the stability of the coupled fluid-solid procedure because it includes the effects of boundary conditions on the numerical stability of the problem.

The first step consists in introducing the normal mode solution in the discrete model equations, composed of the transient energy equation in the fluid and the steady heat conduction in the solid with Robin conditions on both sides of the interface. The second step consists in considering the eigen-solutions of the discrete problem. After elementary transformations (detailed in [8,11]), the temporal amplification factor $g(z)$ of the coupled discrete thermal problem can be expressed as follows

$$g(z) = \frac{(\alpha_f + \alpha_s) K_f}{(K_f + \alpha_f)(\alpha_f + \alpha_s) K_f} \kappa_f + \frac{(K_f - \alpha_f)(K_s - \alpha_s)}{(K_f + \alpha_f)(\alpha_f + \alpha_s)}$$

where $K_f$ and $K_s$ denote the fluid and solid thermal conductances, respectively defined by

$$K_f = \frac{k_f}{\Delta x_f} = \frac{\text{fluid conductivity}}{\text{grid size}}$$

The same applies to the conductance of the solid domain, but this time it worth noting that as only a steady state is sought at every coupling step, the characteristic size to be considered is the width of the solid domain $\Lambda$.

$$K_s = \frac{k_s}{\Lambda_s} = \frac{\text{solid conductivity}}{\text{solid size}}$$

The temporal amplification factor $g(z)$ depends upon the complex function $\kappa_f = \kappa_f(z,D_f)$. It can be shown that this function, solution of a quadratic equation obtained from the interior scheme in the fluid domain, introduces the mesh Fourier number $D_f$ defined by

$$D_f = a_f \Delta t / \Delta x_f^2$$

where $a_f$ is the fluid thermal diffusivity ($m^2/s$), $\Delta t$ is the time step. The mesh Fourier number introduced in Eq. (8) characterizes heat conduction in the layer mesh of the transient domain. The stability of the coupled 1D CHT model is thus characterised by the modulus of its temporal amplification factor $g(z)$.

3. EFFICIENT CHT TREATMENTS

3.1. Optimal coefficient

A general Robin-Robin interface condition simultaneously involves two coefficients. It is thus useful to initially limit the interfacial scheme to a Robin condition on the solid side and a Dirichlet condition on the fluid side (prescribed temperature). This is the most commonly used configuration in literature, because it is simple to implement and can deal with most situations likely to arise. But in order to go further, we will not limit ourselves to this scheme, as we shall see later.

The function $\max |g(z)|$ is plotted in Figure 2 in terms of $\alpha_f \geq 0$, $\alpha_s = \infty$ for two different Fourier numbers $D_f$. We can observe that this function is defined and continuous and that each curve is composed of two half-lines with a singular point similar to a cusp at the intersection. At this point, $\max |g(z)|$ attains its minimum value.

![Figure 2. $\max |g(z)|$ for 2 different Fourier numbers](image)

$D_f = 10$ - stable $\forall \alpha_f \geq \alpha_f^{\text{opt}}$

$D_f = 1000$ - unconditionally stable

In other words, against all expectations, the existence of a transition value for $\alpha_f$ is highlighted. At this transition value, the shape of the curve changes, resulting in the lowest amplification factor. This value, denoted $\alpha_f^{(\text{opt})}$, is given by

$$\alpha_f^{(\text{opt})} = \frac{K_f}{2(1 - D_f)}$$

where $D_f$ is a dimensionless Fourier number defined by
\[
\overline{D}_f = \frac{D_f}{1+D_f + \sqrt{1+2D_f}} \tag{10}
\]

\(D_f\) describes the interval \([0, +\infty]\) as \(\overline{D}_f\) ranges over \([0, 1]\). It is a rapidly increasing function as can be seen in Tab. 1.

<table>
<thead>
<tr>
<th>(D_f)</th>
<th>(1)</th>
<th>(2)</th>
<th>(4.2)</th>
<th>(10)</th>
<th>(100)</th>
<th>(179)</th>
<th>(1000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\overline{D}_f)</td>
<td>0.27</td>
<td>0.38</td>
<td>0.50</td>
<td>0.64</td>
<td>0.87</td>
<td>0.90</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 1. Normalized Fourier number

As shown in Fig. 2, two diametrically opposite behaviors can be identified:

1. \(\alpha_f < \alpha_f^{(\text{opt})}\): the amplification factor is a decreasing function of \(\alpha_f\). It is a branch of hyperbola that depends directly upon the Fourier number \(D_f\). The greater \(D_f\), the smaller the amplification factor.

2. \(\alpha_f > \alpha_f^{(\text{opt})}\): the amplification factor is an increasing function of \(\alpha_f\). It is also a branch of hyperbola that tends to one as \(\alpha_f\) tends to infinity.

At the intersection \(\alpha_f = \alpha_f^{(\text{opt})}\), the two branches of the amplification factor join up. At this remarkable value, the amplification factor attains its absolute minimum.

3.2. Numerical Biot number and stability

The expression of the mathematical expression of the optimal coefficient was provided recently at ONERA [8]. But it is now fundamental to identify all the stability zones involved in the Dirichlet-Robin transmission procedure. To the best of our knowledge, this has never been presented before.

The stability condition \(g(z, \alpha_f, \alpha_s = \infty) < 1\) applied to Eq.(5) leads to the following lower bound stability

\[
\alpha_f^{\text{min}} = \frac{K_f}{2}(1-\overline{D}_f)-\frac{K_s}{2} \tag{11}
\]

which can also take the following form

\[
\alpha_f^{\text{min}} = \frac{K_f}{2}\left[B(1-\overline{D}_f)-1\right] \tag{12}
\]

In Eq. (12), we have introduced the dimensionless number

\[
Bi^{(\Delta)} = \frac{K_f}{K_s} = \text{thermal conductance of the 1st fluid cell} / \text{thermal conductance of the solid domain} \tag{13}
\]

\(Bi^{(\Delta)}\) may be regarded as a non-conventional local Biot number involved in a transient CFD calculation. Let us recall that the conventional Biot number needs a heat transfer coefficient that can be defined only at steady state. The new dimensionless number introduced in Eq. (13), is defined at any coupling iteration of a CHT computation and results from a balance between the unsteady fluid and steady solid domain properties.

From Eq. (12), it can be seen that two zones can be considered:

\[
Bi^{(\Delta)}(1-\overline{D}_f) \leq 1 \tag{14}
\]

Then, the CHT procedure is stable for any positive value of \(\alpha_f\), and especially for \(\alpha_f = 0\). In other words, a Dirichlet-Neumann transmission condition can be used, with no relaxation, without affecting the stability of the coupled problem. In practice however, a Dirichlet-Robin transmission is preferred in such cases, in order to avoid an ill posed thermal problem on the solid side.

<table>
<thead>
<tr>
<th>(\alpha_f)</th>
<th>(0)</th>
<th>(\alpha_f^{\text{min}})</th>
<th>(\alpha_f^{(\text{opt})})</th>
<th>(\infty)</th>
<th>condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>g</td>
<td>)</td>
<td>\text{UNSTABLE}</td>
<td>1</td>
<td>(g^{(\text{opt})})</td>
</tr>
<tr>
<td>(</td>
<td>g</td>
<td>)</td>
<td>(g^0)</td>
<td>(g^{(\text{opt})})</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2. Stability zones
The second zone is

\[ \text{Bi}^{(\Delta)}(1 - D_f) \geq 1 \quad (15) \]

Then, the CHT procedure is inherently prone to instability and a relaxation coupling coefficient must be used such that \( \alpha_f > \alpha_{f}^{\text{min}} \). This lower stability limit may be very large. For example in Fig. 2, \( \alpha_{f}^{\text{min}} = 10^4 \) for \( D_f = 10 \).

The main results of this stability analysis are summarized in Tab. 2.

As we can see, the unconditional stability of the coupling procedure can always be ensured by increasing \( D_f \) (and hence \( D_f \)) such that Condition (14) holds. When this criterion is not satisfied, a certain amount of relaxation is needed and the optimal amount of relaxation is provided by the choice of the optimal coefficient (Eq. (9)).

Fig. 2 illustrates, for two different Fourier numbers (\( D_f =10 \) and \( D_f =10^3 \)), the general trends just discussed. At low or moderate values of \( \alpha_f \), the amplification factor is partly outside the stability domain (\( D_f =10 \) and first row in Tab. 2) and totally inside the stability domain for \( D_f =10^3 \) (second row in Tab. 2). From Fig. 2, the fundamental role of \( \alpha_f^{\text{opt}} \) that controls and "manages" the behavior of the curve is highlighted.

3.3. Practical consequences

The theoretical results proposed in this paper can have the widest practicable effect in reducing the CPU time and in ensuring unconditional stability. Indeed, substituting Eq. (9) back into Eq. (5), we obtain the following amplification factor

\[ g(z, \alpha_f^{\text{opt}}) = \frac{\alpha_f^{\text{opt}}}{K_s + \alpha_f^{\text{opt}}} < 1 \quad (16) \]

As a result, the optimal coefficient always provides a stable procedure and the lowest amplification factor.

But special attention has to be given to the conditions (14) and (15). If condition (14) holds, this means that the "dynamic" thermal resistance of the fluid domain at the shared interface is greater than the resistance offered by the whole solid domain. A Dirichlet-Robin condition is therefore appropriate.

If condition (15) holds, and at extreme cases where \( \text{Bi}^{(\Delta)}(1 - D_f) \gg 1 \) or \( \alpha_f^{\text{opt}} \gg K_s \), imply that the solid thermal gradients are not negligible or that the thermal fluid conductance is large compared to that of the solid. Here, a Dirichlet condition imposed on the fluid does not provide the most efficient solution, even though the optimal coefficient is able to stabilize the procedure. A coupling procedure that is more suited for the physics and the numerical setup at hand would be to impose a Neumann condition on the fluid side.

3.4. Conventional and numerical Biot

In this paper a "numerical" local Biot number has been introduced and particular emphasis has been placed on its key role in the stability process. This new local Biot number takes into account the thermal and dynamic response of the boundary layer and directly participates in the stability of the coupled process. It is worth noting that diffusion, on both sides of the interface, dominates and guides the coupling process, as long as transients effects prevail.

On a different level, the conventional Biot Number is a criterion which gives a direct indication of the relative importance of conduction and convection. It measures how is the resistance to heat flow within the solid relative to the resistance presented by the convection processes at the surface. As a result, this number can be a key parameter, at steady state and only at steady state, to determine the stability of the fluid-solid equilibrium. But this parameter, not defined during the transients, cannot be used to set up a numerical CHT procedure, as long as a transient fluid state is involved in this procedure.

3.5. Efficient strategy at the interface

Realistic and accurate thermal conjugate heat transfer solutions require high-resolution CFD meshes. Consequently the placement of the first fluid grid point should fall in the near-wall boundary-layer region, in order to ensure \( y^+ \) values close to unity. This is mandatory when low Reynolds RANS turbulence models are used, due to the sensitivity of their thermal wall fluxes and temperatures to the boundary layer resolution. In such cases, it is safe to neglect the effects of convection on the first fluid mesh element, which allows us to consider it as a small perturbation, that does not affect the nature of the diffusion equation.

Another important point concerns the canonical prototype proposed in this paper. The relevance of this predictive model problem, particularly the so-called "optimal coefficient" has been confirmed in many CHT problems, but it is clear that this model does not contain
velocity component normal to the wall and thus, this approach is valid if the Péclet number which measures the ratio of convection to diffusion is small. In other words, a good representation of the flow structure is here necessary. This also implies that the grid must be refined enough so that convection may thus be considered as a small perturbation.

The two preceding considerations imply that the ratio $\text{Bi}^{(\Lambda)}$ is likely to be very high as it directly depends on the first fluid cell size. This is becoming a tendency, with the ever growing computational resources dedicated to CFD computations.

It is therefore essential to develop an efficient strategy for coupled CHT calculations that are shifting towards very fine grids at the interface. In such configurations, the widely used Dirichlet condition on the fluid side, quickly show its limitations. Of course, the dynamic effect of the fluid domain, represented by $(1 - D_f)$, is capable of stabilizing any CHT calculation, but the numerical performance of such coupling will be far from ideal, as it does not take into account the physics on either side of the interface.

Table 3. Adaptive coupling strategy

<table>
<thead>
<tr>
<th>Condition</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Bi}^{(\Lambda)}(1 - D_f) \leq 1$</td>
<td>$\alpha_f = \alpha_f^{(\text{opt})}$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_f$ = $\alpha_f^{(\text{opt})}$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_f$ = $\alpha_f^{(\text{opt})}$</td>
</tr>
<tr>
<td>$\text{Bi}^{(\Lambda)}(1 - D_f) &gt; 1$</td>
<td>$\alpha_f = \alpha_f^{(\text{opt})}$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_f$ = $\alpha_f^{(\text{opt})}$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_f = \alpha_f^{(\text{opt})}$</td>
</tr>
</tbody>
</table>

Figure 3 shows the streamlines and the temperature contours of the flow in a central cross section. The complex geometrical setup of the problem generates

4. TEST CASE

4.1. First CHT tests

The formulation of the optimal Robin coefficient was published relatively recently [8], and consequently very few CHT computations that take advantage of it are found in literature. However, some interesting results have been achieved.

For instance, in a steady CHT computation of an effusion cooling system [12], the CPU time necessary to converge was divided by a factor of ten as compared to a conventional method.

In another work devoted to test systematically the values of various coupling coefficients, it has been shown that the optimal coefficient can be applied in an efficient manner as a tool for predicting and obtaining excellent stability properties [13]. Indeed, the CHT results obtained by employing the optimal coefficient have always proved to be stable and oscillation-free. This would suggest that the one-dimensional normal mode analysis can provide relevant coefficients directly applicable to industrial 3D CHT problems.

We now present results to illustrate the importance of the interface treatment in CHT problems, with emphasis put on the optimal treatment and the strategy proposed in this paper. The details about this computation can be found in [14].

4.2. 3D industrial test case

The considered 3D industrial setup represents a cell of a reheat oven and puts into play various interface configurations, ranging from metallic parts to thick insulating components, simultaneously interacting with the fluid flow. This provides a realistic setup with a large number of heterocline coupled interfaces. The heterogeneous properties of this set-up are ideal to study the nature of the coupling procedures. Radiative heat transfer is not considered in this work, because it is outside the scope of the current work.

The walls of the cell are made out of, thick, high temperature insulating material. Molybdenum electrical heaters are positioned above and below the steel element. Turbulent air, flows from the inlet and interacts with the walls of the chamber and its metallic elements before exiting through the outlet. The upper wall has an imposed external temperature, while the external faces of the lower and side walls are assigned a convection boundary condition.
several recirculation zones in the fluid domain, some of which are illustrated in this Figure.

![Temperature contours & flow streamlines](image)

Figure 3. Temperature contours & flow streamlines

The near-wall fluid mesh density that was used in the CHT calculations is refined enough \( (y^+ \approx 1) \) to correctly capture the flow boundary layer. Four coupling coefficients have been used:

1. \( \alpha_f = \alpha_0 = H_{\text{stand}} = K_f > \alpha_f^{\text{opt}} \) (see [15])
2. \( \alpha_f = \alpha_1 > \alpha_f^{\text{opt}} \)
3. \( \alpha_f = \alpha_2 = \alpha_f^{\text{opt}} \)
4. \( \alpha_f = \alpha_3 < \alpha_f^{\text{opt}} \)

As might be anticipated, the convergence history largely depends on the value of the coupling coefficient as shown in Fig. 4. For low values of the coefficient \( \alpha_f = \alpha_1 \), regardless of the fluid Fourier number employed, the CHT computations are unstable. In other words, oscillations that grow without bound are obtained.

![Convergence vs coupling iterations (D-R)](image)

Figure 4. Convergence vs coupling iterations (D-R)

This tendency can be observed in Fig. 4 at the beginning of the computation before divergence. Larger values produce a stable computation with a low or very low convergence rate (for instance at \( \alpha_f = \alpha_0 \)). This latter coefficient provides an over-stable solution. When the coefficient is equal \( \alpha_f = \alpha_2 = \alpha_f^{\text{opt}} \) to the optimal value, the convergence is, not surprisingly, always monotonic (for all the Fourier numbers considered) and provides the best numerical behavior.

These convergence behaviors were predictable from the model problem. As expected, \( \alpha_f^{\text{opt}} \) appears to be the optimal choice, just at the intersection of the Neumann and the Dirichlet branches [13]. This coefficient provides a monotonic, stable and fast convergence of the coupled simulations.

### 5. Optimized Transmission Scheme

The preceding results clearly show that for a given transmission condition scheme (in this case Dirichlet-Robin), it is possible to optimize the numerical procedure in terms of stability and convergence. It is already a significant achievement. However, as stated before and explicitly described in Tab. 2, it is useful to know if the algorithm proposed in Tab. 3 can be used. In other words, is the Dirichlet-Robin condition the most appropriate scheme? Of course, we must recognize that the Dirichlet-Robin condition and the resulting optimal coefficient (Eq. (9)) always gives a monotonic and stable behavior. But can we proceed a step further?

Indeed, the computational domain in the CHT case contains may disparate zones and it is instructive to consider the values of the coupling coefficients computed by the model problem. In some of these zones, the coupling coefficient needed for stability is clearly very high. This means that stability is achieved at the expense of computational efficiency (CPU time) and obviously, a Dirichlet condition imposed on the fluid side is not the best choice. The problem arises when the numerical Biot number \( Bi^{(A)} \) is too high. In this case, a natural physics-based approach is to impose a Neumann condition on the fluid side, as identified in the algorithm of Tab. 3. The first tests with such an approach show that convergence occurs 3.2 times faster than the standard coupling scheme.

In the preceding test case, the ratio between the thermal conductances is approximately

\[
Bi^{(A)} = \frac{K_f}{K_s} = 1.25 \times 10^2 \quad (17)
\]

This high value indicates that a Dirichlet condition may be not appropriate. Of course, it can always be
stabilized, as shown theoretically and in the numerical results. But the price to pay can be high. A large value of the coupling coefficient indicates an over-relaxation, even if this relaxation is "optimal"!

It was therefore of great importance to use the strategy defined in Section 3.5. Fig. 5 shows the residual of the Neumann-Dirichlet (N-D) procedure as well the convergence history of the optimal Dirichlet-Robin condition. These CHT computations highlight the importance of a relevant transmission procedure. Let us recall that the N-D procedure works with no relaxation parameters and shows the fastest convergence properties. Clearly, the strategy proposed in this paper (Tab. 3) seems to be the most physics-based approach, while always being immune to oscillations.

6. CONCLUSION

We have presented the stability analysis of the CHT problem, with emphasis put on the Dirichlet-Robin procedure. It has been shown that the Fourier and the numerical Biot numbers were the key parameters predicting the numerical behavior of the coupling process. They are used to obtain the optimal Dirichlet-Robin procedure that can be regarded as an adaptive procedure to always obtain the fastest rate of convergence and the best stability properties.

This is certainly a first essential step and further improvement of the CHT computations were proven to be attainable. The local Biot number introduced in this paper can also be regarded as a good criterion to choose a physics-based approach. With this insight, the most appropriate interface transmission condition can be implemented as opposed to being limited to a single procedure. As a result, a strategy based on two complementary interface schemes has been presented.

This strategy has been applied to a 3D CHT test case exhibiting disparate coupling surfaces making it an excellent candidate to testing the new coupling strategy. This numerical example has clearly illustrated that a single interfacial approach can provide effective and practical solutions directly applicable to aerothermal coupled computations. But a higher numerical efficiency and substantial computing time savings could be attained when using the proposed adaptive algorithm.

The proposed strategy, involving two complementary interface treatments, should be extended to cover the most general coupling approach, based on the Robin-Robin transmission procedure, in order to cope with the increasing complexity of CHT simulations.

7. REFERENCES


