

Distributed Computation of the Čech Complex and Applications in Wireless Networks

Ngoc-Khuyen Le, Anais Vergne, Philippe Martins, Laurent Decreusefond

▶ **To cite this version:**

Ngoc-Khuyen Le, Anais Vergne, Philippe Martins, Laurent Decreusefond. Distributed Computation of the Čech Complex and Applications in Wireless Networks. [Research Report] Telecom Paristech. 2016. hal-01414603

HAL Id: hal-01414603
https://hal.science/hal-01414603

Submitted on 12 Dec 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Distributed Computation of the Čech Complex and Applications in Wireless Networks

Ngoc-Khuyen LE, Anaïs Vergne, Philippe Martins, Laurent Decreusefond
LTCI, CNRS, Télécom ParisTech, Université Paris-Saclay, 75013, Paris, France
Email: {ngoc-khuyen.le, anais.vergne, philippe.martins, laurent.decreusefond}@telecom-paristech.fr

***Abstract*—In this paper, we introduce a distributed algorithm to compute the Čech complex. This algorithm is aimed at solving coverage problems in self organized wireless networks. Two applications based on the distributed computation of the Čech complex are proposed. The first application detects coverage holes while the later one optimizes coverage of wireless networks.**

I. Introduction

Coverage is a key factor that determines the quality of service in wireless networks. In recent applications to solve coverage problems in wireless networks, simplicial complex is often utilized to represent the network topology. Unlike graph, which represents only the neighborhood of cells, simplicial complex presents the relationship of cells with a higher dimension.

The Čech complex is a simplicial complex that represents exactly the topology of the network [3]. The Čech complex is the right tool to describe and optimize the coverage for wireless networks. However, the computation of the Čech complex is complicated. The Čech complex represents a collection of cells by a simplex if they have a non-empty intersection. In [1], an algorithm to compute the Čech complex is introduced, but this algorithm is designed to be utilized in graphics science. Therefore, this algorithm only works with a collection of same sized cells. Although an algorithm to compute the Čech complex for a collection of differently sized cells is proposed in [6], this algorithm is still centralized.

In [7], authors introduce an application based on the Čech complex to optimize the coverage for wireless networks. However, as the computation of the Čech complex is still available in centralized way, this application is also centralized. The future wireless networks will be self organized and therefore require distributed computations. In [2], [4], [10], some algorithms to detect coverage holes in wireless networks are developed. Although these algorithms are available in both centralized and distributed way, they use the Rips complex instead of the Čech complex to represent the topology of the network. The Rips complex is an approximation of the Čech complex. The Rips complex represents a collection of cells by a simplex if every pair of cells in this collection are intersected. Therefore, the Rips complex may not capture exactly the topology of the network. So, some coverage holes may be undiscovered [9].

In this paper, we introduce the distributed computation of the Čech complex for a given collection of differently sized cells. This algorithm is aimed at solving coverage problems in wireless networks. We also propose two applications that are based on the distributed computation of the Čech complex. Given a wireless network, the first application detects coverage holes while the second one optimizes its coverage. The optimized coverage reduces as much as possible the intersection among cells. As a result, the waste power due to interference within intersected regions is avoided efficiently. Both of these applications are distributed.

This paper is organized as follows. Section II introduces a short background of simplicial homology and its applications in wireless networks. Section III describes all details about the distributed computation of the Čech complex. Section IV proposes a distributed coverage hole detection and a distributed coverage optimization for wireless networks. The last section concludes our paper.

II. Simplicial homology and applications

In this section, we introduce some notions of simplicial homology and its applications. For further details about the simplicial homology, see documents [8] and [5].

Given a set of vertices V, a k-simplex is an unordered subset $\{v_0, v_1, \ldots, v_k\}$, where $v_i \in V$ and $v_i \neq v_j$ for all $i \neq j$. The number k is its dimension. See Figure 1 for some instances. Any subset of vertices set of a simplex is a face of this simplex. An oriented simplex is an ordered type of

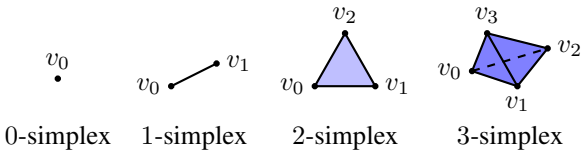


Fig. 1: An example of simplices.

simplex. The orientation of a simplex changes if two vertices of this simplex swap their position. The change of orientation is represented by a negative sign as:

$$[v_0, v_1, \ldots, v_i, v_j, \ldots, v_k] = -[v_0, v_1, \ldots, v_j, v_i, \ldots, v_k]$$

An abstract simplicial complex is a collection of simplices such that: every face of a simplex is also in the simplicial complex.

Let X be a simplicial complex. For each $k \geq 0$, we define a vector space $C_k(X)$ whose basis is a set of oriented k-simplices of X. If k is bigger than the highest dimension of X, let $C_k(X) = 0$. We define the boundary operator to be a linear map $\partial : C_k \to C_{k-1}$ as follows:

$$\partial[v_0, v_1, \ldots, v_k] = \sum_{i=0}^{k} (-1)^i [v_0, v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_k]$$

This formula suggests that the boundary of a k-simplex is the collection of its $(k-1)$-faces, as illustrated in Figure 2. For example, the boundary of a segment is its two endpoints. A filled triangle is bounded by its three segments. A tetrahedron has its boundary comprised of its four faces which are four triangles.

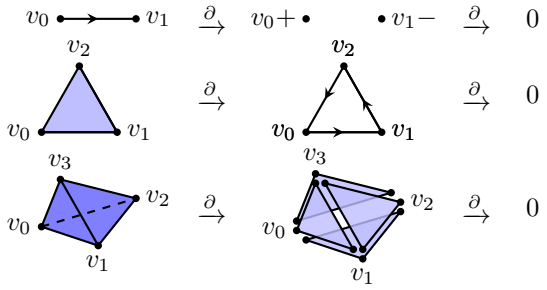


Fig. 2: Boundary operator.

The composition of boundary operators gives a chain of complexes:

$$\cdots \xrightarrow{\partial} C_{k+1} \xrightarrow{\partial} C_k \xrightarrow{\partial} C_{k-1} \cdots \xrightarrow{\partial} C_1 \xrightarrow{\partial} C_0 \xrightarrow{\partial} 0$$

Consider two subsets of $C_k(X)$: cycle-subset and boundary-subset, denoted as $Z_k(X)$ and $B_k(X)$ respectively. Let $\ker$ be the kernel and im be the image. By definition, we have:

$$Z_k(X) = \ker(\partial : C_k \to C_{k-1})$$
$$B_k(X) = \mathrm{im}(\partial : C_{k+1} \to C_k)$$

$Z_k(X)$ includes cycles which are not boundaries while $B_k(X)$ includes only boundaries. A k-cycle u is said to be homologous with a k-cycle v if their difference is a k-boundary: $[u] \equiv [v] \iff u - v \in B_k(X)$. A simple computation shows that $\partial \circ \partial = 0$. This result means that a boundary has no boundary. Thus, the k-homology of X is the quotient vector space:

$$H_k(X) = Z_k(X) \backslash B_k(X)$$

The dimension of $H_k(X)$ is called the k-th Betti number:

$$\beta_k = \dim H_k = \dim Z_k - \dim B_k \tag{1}$$

This number has an important meaning for coverage problems. The k-th Betti number counts the number of k-dimensional holes in a simplicial complex. For example, β_0 counts the connected components while β_1 counts the coverage holes, etc.

Definition 1 (Čech complex): Given (M, d) a metric space, ω a finite set of points in M and $\epsilon(\omega)$ a sequence of real positive numbers, the Čech complex with parameter $\epsilon(\omega)$ of ω, denoted $\check{\mathbf{C}}_{\epsilon(\omega)}(\omega)$ is the abstract simplicial complex whose k-simplices correspond to non-empty intersection of $(k+1)$ balls of radius $\epsilon(\omega)$ centered at the $(k+1)$ distinct points of ω.

If we choose $\epsilon(\omega)$ to be the cell's coverage range R, the Čech complex verifies the exact coverage of the system. In the Čech complex, each cell is represented by a vertex. A covered space between cells corresponds to a filled triangle, tetrahedron, etc. In contrast, a coverage hole between cells corresponds to an empty (or non-filled) triangle, rectangle, etc.

III. Distributed computation of Čech complex

A. System model

We consider a wireless network composed of N distinct cells. We assume that each cell uses isotropic propagation. The coverage of the i-th cell is modeled as:

$$c_i(v_i, r_i) = \{x \in \mathbb{R}^2 : \|x - v_i\| \leq r_i\},$$

where $\|.\|$ is the Euclidean distance, the vertex v_i represents the base station location and r_i is coverage radius of the i-th cell. Let $\mathcal{U}$ be the collection of cells, then $\mathcal{U} = \{c_i, i = 0, 1, \ldots, (N-1)\}$. The Čech complex of $\mathcal{U}$, $\check{\mathbf{C}}(\mathcal{U})$, is defined as the Čech complex of the wireless network. In the Čech complex, each vertex, i.e. a 0-simplex, v_i corresponds to the i-th cell c_i in the network. An edge, i.e. a 1-simplex, represents the connection, or the intersection, between two cells. Each k-simplex, where $k \geq 2$, represents the common intersection of the coverage of together $(k+1)$ corresponding cells of this simplex. For example, in the Figure 3, the 2-simplex $[v_2, v_3, v_6]$ means the overlap of coverage of cell c_2, cell c_3 and cell c_6. There is no coverage hole inside these cells. The higher dimensional simplex is, the higher overlap times is. The 3-simplex $[v_0, v_1, v_2, v_6]$ means the four corresponding cells: c_0, c_1, c_2 and c_6, together, have a common intersection. In contrast, a chain of 1-simplices indicates a coverage hole inside corresponding cells of the chain. For example, the chain $[v_3, v_4] + [v_4, v_5] + [v_5, v_6] + [v_6, v_3]$ shows a coverage hole inside four cells c_3, c_4, c_5 and c_6. To analyze the network topology, we use characteristics of the homology of the Čech complex.

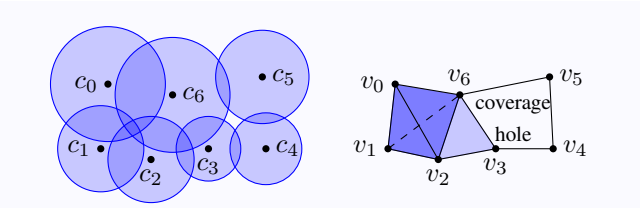


Fig. 3: Cells and their Čech representation.

As the definition of the Čech complex, each k-simplex represents the common intersection of together its $(k+1)$ corresponding cells. We denote $\mathbf{S}_k$ the collection of all k-simplices of the complex. Each vertex, that is a 0-simplex,

corresponds to a cell of the network. The collection of 0-simplices, $\mathbf{S}_0$, is then obviously the list of corresponding vertices of cells.

$$\mathbf{S_0} = \{v_i \mid i = 0, 1, \ldots, (N-1)\}.$$

Each 1-simplex is an edge that connects two overlapping cells. In other words, it is a pair of two neighbor cells. If two vertices belong to the vertices set of a k-simplex, where $k \geq 2$, then they form a 1-simplex. So, they are neighbors. Therefore, each cell needs to detect all its neighbors before computing all its simplices.

B. Neighbors detection

We assume that each cell c_i can communicate with other cells over radio within a distance $d_i = 2r_i$. We assume that there are enough frequency slots for cells to communicate over radio without collision. Every cell is also connected by a backhaul network. At the initial state, each cell broadcasts a ping message with its position and its radius over radio channel. If a cell receives a ping message, it verifies if the cell that sent this ping message is a neighbor. If they are neighbors, then the cell that received the ping message sends a relationship confirmation together with its position and its radius to the cell that sent the ping message by using the backhaul network. After receiving the confirmation, the cell that sent the ping message adds the cell that sent the confirmation into its collection of neighbors. We assume that all the cells can reply the confirmation within a period t_{ack}. After this period t_{ack}, every cell detects its collection of neighbors. We denote the collection of neighbors of the cell c_i as $\mathbf{N}_i$.

C. Distributed simplices computation

When the collection of neighbors is available, each cell computes its simplices by verifying the intersection among it and its neighbors. For the details about the intersection verification, see [6]. As each pair of neighbors forms a 1-simplex, the collection of 1-simplices of each cell is easily found. To find all k-simplices, where $k \geq 2$, each cell verifies if a group of it and its k neighbors has common intersection. If these cells have common intersection, then this group forms a k-simplex. However, neighborhood is a two-way relationship. Therefore, the verification of intersection could be duplicated by different cells that are neighbors. To avoid the redundant duplication, each cell verifies the intersection by following a right hand rule. This rule is that each cell verifies only with neighbors that are on its right hand side. If there is a neighbor which has the same horizontal coordinate, it verifies only this neighbor if it has higher vertical coordinate. If a simplex is found by a cell, this cell transmits this simplex to every cell that belongs to this simplex. As a result, every cell detects all its simplices. For example, in Figure 3, the cell c_2 verifies the intersection with only the cell c_3 and c_6. It detects the simplex $[v_2, v_3, v_6]$. It receives its other simplices from the neighbor c_0 and c_1. The Algorithm 1 reports the distributed computation of Čech complex for each cell. We denote the cell that is computing the simplices as c_i. The highest dimension of simplices that are considered is $\dim_{\text{max}}$. The output of this algorithm, the collection of simplices of the cell c_i is denoted as $\check{\mathbf{C}}_i$.

Algorithm 1 Distributed computation of Čech complex

Require: c_i the cell that is computing;
Ensure: $\check{\mathbf{C}}_i$ the collection of simplices of the cell c_i;
  broadcast {ping, v_i, r_i} over radio;
  $\mathbf{S}_0 = v_i$;
  while count time $< t_{\text{ack}}$ **do**
    if {ping, v_j, r_j} is received from c_j **then**
      if $d(v_i, v_j) < r_i + r_j$ **then**
        add v_j to $\mathbf{N}_i$;
        if $x_i < x_j$ **then**
          add v_j to $\mathbf{S}_0$;
          add $[v_i, v_j]$ to $\mathbf{S}_1$;
        end if
        if $x_i == x_j$ and $y_i < y_j$ **then**
          add v_j to $\mathbf{S}_0$;
          add $[v_i, v_j]$ to $\mathbf{S}_1$;
        end if
      end if
    end if
  end while
  for $k = 2$ **to** $\dim_{\text{max}}$ **do**
    for $l = 2$ **to** $C^k_{|\mathbf{S}_0|}$ **do**
      $\hat{s} = l$-th combination of k vertices in $\mathbf{S}_0$;
      $\hat{s} = \hat{s} \cup \{v_i\}$;
      $\hat{c} =$ corresponding cells of $\hat{s}$;
      if intersection of all cells in $\hat{c}$ is not empty **then**
        add $\hat{s}$ to $\mathbf{S}_k$;
      end if
    end for
  end for
  $\check{\mathbf{C}}_i = \{\mathbf{S}_0, \mathbf{S}_1, \ldots, \mathbf{S}_k\}$;
  send $\check{\mathbf{C}}_i$ to every corresponding cell of vertices in $\mathbf{S}_0$;
  $n_i = |\mathbf{N}_i|$;
  while $n_i > 0$ **do**
    if $\check{\mathbf{C}}_j$ is received from c_j in $\mathbf{N}_i/\mathbf{S}_0$ **then**
      $\check{\mathbf{C}}_i = \check{\mathbf{C}}_i \cup \check{\mathbf{C}}_j$;
      $n_i = n_i - 1$;
    end if
  end while
  return $\check{\mathbf{C}}_i$;

The global Čech complex that represents the topology of the whole network is sometimes needed. There should be a master cell that controls the topology of the network. This global Čech complex can be easily built by integrating the simplices computed from every cell. Each cell sends its computed simplices that contain only the vertices satisfied the right hand rule. One more time, this rule is useful, it avoids sending the duplicated simplices.

IV. Applications

A. Coverage hole detection

Coverage hole detection is one fundamental application in solving coverage problem. We introduce a distributed algorithm to detect all the edges that are on the cover (the boundary) of coverage holes. We assume that each cell has computed its simplices as in the Algorithm 1. If an edge (a 1-simplex) is not a cover edge of a coverage hole, then it musts connect with at least two neighbors that are on two different sides of this edge by filled triangles (a 2-simplex). Otherwise, the edge is a cover edge of a coverage hole. So, each cell cooperates with its neighbors to find all its cover edges. If a pair of neighbors can not find at least two cells that are their common neighbors and are on different sides of the edge formed by this pair, then the edge formed by this pair is a cover edge.

Algorithm 2 Distributed coverage hole detection

Require: $c_i(v_i, r_i)$ the cell that is on a cover edge;
Ensure: boundary cycle that contains v_i;
  $m_i = |\mathbf{S}_{1,i}|$;
  $n_i = |\mathbf{S}_{2,i}|$;
  $\mathbf{E} = \emptyset$;
  for $p = 1$ **to** m_i **do**
    $s_1 = p$-th 1-simplex in $\mathbf{S}_{1,i}$;
    $\mathbf{V} = \emptyset$;
    for **do**
      $s_2 = q$-th 2-simplex in $\mathbf{S}_{2,i}$;
      $v = s_2/s_1$;
      add v to $\mathbf{V}$;
    end for
    if there is no pair of vertices that are on different sides of s_1 **then**
      add s_1/v_i to $\mathbf{E}$;
    end if
  end for
  $h = \{$boundary detection, $v_i\}$;
  send h to every corresponding cell of vertices in $\mathbf{E}$;
  while count time $< t_0$ **do**
    if a boundary detection message h^* received **then**
      if v_i is in h^* **then**
        $l =$ vertices list in h;
        remove vertices in l that are above v_i;
        add l to the boundary cycle list $\mathbf{L}$;
      else
        add v_i to the end of h^*;
      end if
    end if
  end while
  return $\mathbf{L}$;

The cells that are on cover edge can cooperate to find the cycle that is boundary of a coverage hole. If a cell c_i is on a cover edge, it sends a boundary detection message $\{$boundary detection, $v_i\}$ to the other cell of this cover edge. If a cell, a receiver, received a boundary detection message from a sender, and it does not correspond to any vertex in the message, it adds its vertex to the end of this message, then it forwards the message to its neighbors that are on cover edges and are not the sender. If this receiver detects that its vertex is on the list of vertices in the message, then it remove all vertices that are above its vertex in the list. It announces the remained vertices as a boundary cycle that covers a coverage hole. We assume that all the message are sent and received in a period t_0. After this period, each cell detects all its boundary cycles. The Algorithm 2 describes the details of the coverage hole detection.

B. Coverage optimization

Consider a wireless network, we maximize its coverage and minimize its total transmission power at the same time. Firstly, we ensure a maximal coverage for the network. Each cell is turned on and is set to work with the highest transmission power. At this initial state, the network has the largest coverage. However, many cells are hardly overlapped. The overlap region between cells causes the waste of transmission power due to interference. We can optimize the transmission power by minimizing the overlap region. However, the global coverage of the network should be conserved. In other words, the two Betti numbers β_0 and β_1 of the Čech complex of the network should be unmodified. The optimization problem can be written as:

$$\begin{aligned} \min_{r} \quad & \sum_{i=0}^{N-1} r_i^{\gamma} \\ \text{s.t.} \quad & \beta_0 = \beta_0^* \\ & \beta_1 = \beta_1^* \\ & r = (r_0, r_1, \ldots, r_{N-1}), \end{aligned} \tag{2}$$

where β_0^* and β_1^* are the Betti numbers of the Čech complex of the network at the initial state where every cell is working with the maximal transmission power.

In this section, we introduce a distributed algorithm to optimize the coverage as well as to save energy for the network. This algorithm is applied for each cell in the network. We assume that all the fenced cells and boundary cells are already known. Only the cells that are not fenced or boundary cells can try to reduce the coverage radius.

At the first step, each cell needs to search for its neighbors as well as its simplices by following the Algorithm 1. Once the neighbors set is established, each cell now starts its reduction process. On each cell, there is a timer which counts down to zero. The timer is set to a uniform random value from 1 to $t_{\max}$, where $t_{\max}$ is the maximal value of the timer. When the timer of a cell is expired, this cell tries to do a reduction. If two cells that are neighbors try to reduce their radius at the same time, the coverage hole may not be detected due to the outdated information about the radius of each other. Therefore, before trying to reduce the radius, each cell sends a "pause" message to its neighbors. Then, the cell reduces its coverage radius and verifies the coverage. The radius reduction of one

cell only makes topology change in the local region that is comprised of this cell and its neighbors. If there is a new coverage hole, it must be inside this local region. This means that if there is no new coverage hole after the radius reduction, the Betti numbers β_0 and β_1 of the Čech complex of this local region are unchanged. The verification of the network coverage can be reduced to the coverage verification in only this local region as in the Algorithm 3.

Algorithm 3 Quick coverage verification after a radius reduction of one cell

Require: c^* the cell that changed its radius;
$\mathcal{N}$ = neighbors collection of the cell c^*;
$\check{\mathbf{C}}^*$ = the Čech complex of $\{c^*\} \cup \mathcal{N}$ before the radius reduction;

Ensure: **true** if and only if there is no new coverage hole.

compute Betti numbers β_0^* and β_1^* of $\check{\mathbf{C}}^*$;
$\check{\mathbf{C}}$ = the Čech complex of $\{c^*\} \cup \mathcal{N}$ after the radius reduction;
compute Betti numbers β_0 and β_1 of $\check{\mathbf{C}}$;
if $\beta_0 = \beta_0^*$ and $\beta_1 = \beta_1^*$ **then**
  verification = **true**;
else
  verification = **false**;
end if
return verification;

If no hole appears, the cell confirms the reduction and sends the new value of coverage radius to its neighbors. It also sends the "continue" message to its neighbors to tell them that they can continue. If a cell received a "pause" message, it pauses its process and waits until the message "continue" is received. Then, it continues its process normally. There is a special case where two neighbor cells whose timers expire at the same time send the "pause" message to each other simultaneously. One of these two cells receives a "pause" from another before it sends "continue" message to other cells. This cell cancels the current reduction step and sets a new value for its timer and waits to retry.

If a cell tries to reduce its coverage radius and makes a coverage hole, it reverses its coverage radius to the previous value and stops the reduction process. This cell is set to irreducible.

The distributed energy saving algorithm applied for each cell is described in the Algorithm 4.

Algorithm 4 Distributed downhill energy saving algorithm for each cell

Require: c a cell in the network;
Ensure: the optimal radius for c;
transmit the position and the coverage radius of c to other cells;
collect the information about the position and coverage of other cells;
while (1) **do**
  set timer = uniform$(0, t_{\max})$;
  wait until timer expires;
  $\mathcal{N}_c$ = the collection of neighbors of c;
  if a "pause" received **then**
    wait until "continue" received;
  end if
  $r_{\text{old}} = r_c$;
  $r_c = r_c - \Delta r_c$;
  if $r_c < r_{c,\min}$ **then**
    $r_c = 0$;
    verify the coverage;
    if no coverage hole appears **then**
      send "continue" to neighbors in $\mathcal{N}_c$;
      transmit the "turning off" to other cells;
      break;
    else
      $r_c = r_{\text{old}}$;
      send "continue" to neighbors in $\mathcal{N}_c$;
    end if
  else
    verify the coverage;
    if no coverage hole appears **then**
      send r_c to neighbors in $\mathcal{N}_c$;
      send "continue" to neighbors in $\mathcal{N}_c$;
    else
      $r_c = r_{\text{old}}$;
      send "continue" to neighbors in $\mathcal{N}_c$;
      break;
    end if
  end if
end while
return;

V. Conclusion

This paper introduces the distributed computation of the Čech complex aimed for wireless networks. Two distributed applications that are based on this computation of the Čech complex are also proposed. The first application detects the coverage hole and the second one optimizes the coverage for the network.

References

[1] Stefan Dantchev and Ioannis Ivrissimtzis. Efficient construction of the ech complex. *Computers & Graphics*, 36(6):708 – 713, 2012. 2011 Joint Symposium on Computational Aesthetics (CAe), Non-Photorealistic Animation and Rendering (NPAR), and Sketch-Based Interfaces and Modeling (SBIM).

[2] V. De Silva and R. Ghrist. Coordinate-free coverage in sensor networks with controlled boundaries via homology. *Int. J. Rob. Res.*, 25(12):1205–1222, December 2006.

[3] Vin de Silva, Robert Ghrist, and Abubakr Muhammad. Blind swarms for coverage in 2-D. In *Proceedings of Robotics: Science and Systems*, Cambridge, USA, June 2005.

[4] Robert Ghrist and Abubakr Muhammad. Coverage and hole-detection in sensor networks via homology. In *Proceedings of the 4th International Symposium on Information Processing in Sensor Networks*, IPSN '05, Piscataway, NJ, USA, 2005. IEEE Press.

[5] Allen Hatcher. *Algebraic topology*. Cambridge University Press, 2001.
[6] Ngoc-Khuyen Le, P. Martins, L. Decreusefond, and A. Vergne. Construction of the generalized cech complex. In *Vehicular Technology Conference (VTC Spring), 2015 IEEE 81st*, pages 1–5, May 2015.
[7] Ngoc-Khuyen Le, P. Martins, L. Decreusefond, and A. Vergne. Simplicial homology based energy saving algorithms for wireless networks. In *Communication Workshop (ICCW), 2015 IEEE International Conference on*, pages 166–172, June 2015.
[8] James Munkres. *Elements of algebraic topology*. Perseus Books, 1984.
[9] F. Yan, P. Martins, and L. Decreusefond. Accuracy of homology based approaches for coverage hole detection in wireless sensor networks. In *2012 IEEE International Conference on Communications (ICC)*, pages 497–502, June 2012.
[10] Feng Yan, P. Martins, and L. Decreusefond. Connectivity-based distributed coverage hole detection in wireless sensor networks. In *Global Telecommunications Conference (GLOBECOM 2011), 2011 IEEE*, pages 1–6, Dec 2011.