# Robust optimal sizing of a hybrid energy stand-alone system

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Alain Billionnet[(1)], Marie-Christine Costa[(2)], Pierre-Louis Poirion[(3)†]

[(1)] ENSIIE-CEDRIC, 1 Square de la Résistance F-91025 Évry Cedex

[(2)] ENSTA ParisTech, Université Paris-Saclay (and CEDRIC-CNAM) 828, Boulevard des Maréchaux F-91762 Palaiseau Cedex

[(3)] LIX, Ecole Polytechnique,A COMPLETER Palaiseau Cedex

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**Abstract**

This paper deals with the optimal design of a stand-alone hybrid system composed of wind turbines, solar photovoltaic panels and batteries. To compensate for a possible lack of energy from these sources, an auxiliary fuel generator guarantees to meet the demand in every case but its use induces important costs. We have chosen a two-stage robust approach to take account of the stochastic behavior of the solar and wind energy production and also of the demand. We seek to determine the optimal system, i.e. the one that generates a minimum total cost when the worst case scenario relating to this system occurs. We use a constraint generation algorithm where each sub-problem (the recourse problem) can be reformulated by a mixed-integer linear program and hence solved by a standard solver. We also propose a polynomial time dynamic programming algorithm for the recourse problem and show that, in some cases, this algorithm is much more efficient than mixed-integer linear programming. Finally, we report computational experiments on instances constructed from real data, that show the efficiency of the proposed approach and we study the addition of constraints linking the uncertainty in consecutive time periods.

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†Corresponding author: E-mail address: plpoirion@gmail.com

# 1 Introduction

The fast development of renewable energies brought new complex problems of combinatorial optimization in particular as regards the autonomous hybrid energy systems. These systems involve several energy sources as wind, biomass or sun and are not connected to the grid. They are particularly useful in islands as La Réunion [16] and in remote areas as there are in Canada [10]. Mathematical techniques have been used to optimize either the operation of the system [1, 9, 14, 20] or the design of the park which this paper is dealing with. The most recent references on this topic include [4], [5], [10], [15] and [17]. In [4], a methodology is introduced to perform the optimal sizing of an autonomous photovoltaic/wind system. In [5], the authors present a mathematical programming model to optimize the design of hybrid wind-PV systems that solves the location of the wind-PV generators and the design of the microgrids. In [10], integer linear programming is used to optimize the design of wind farm collection networks. In [15] the aim is to determine the types, numbers and placement of wind turbines to install, considering investment costs and power production criteria; a mixed-integer nonlinear model is proposed and tested. In [17], an integrated photovoltaic/wind system with battery storage is considered: a heuristic approach is proposed to find the sizes of the wind farm, photovoltaic array and battery.

In this paper, we study a stand-alone hybrid system composed of wind turbines, solar photovoltaic (or PV) panels and batteries. To compensate for a lack of energy from these sources, an auxiliary fuel generator guarantees to meet the demand in every case but its use induces important costs. We consider here that the type of wind turbines, PV panels, and batteries have been predetermined by the users according to the lands on which the park will be settled. The aim is to determine the optimal number of photovoltaic panels, wind turbines and elements of battery to install in order to serve a given demand while minimizing the total cost of investment and use.

Moreover, the stochastic behavior of the solar and wind energy production on the one hand, and the demand on the other hand, needs to search for a robust solution, i.e. a solution which is good enough whatever the scenario that occurs (see [7]). We assume that there is no known probability distributions of the data and following the approach proposed in [2] and [19], we consider that the uncertain data can vary between given bounds and that there are limits to the total variation of each kind of data. We propose a mixed-integer program in two stages to model the problem: investment decision variables also called here-and-now variables must be fixed in the first stage while operating recourse variables also called wait-and-see variables will be determined once the uncertainty has been revealed. Then we follow the approach proposed in [3] to solve the problem.

In Section 2 we describe the system operation and the notations. In Section 3 we give a mixed-integer program for the problem without uncertainty. In Section 4 we propose a model in two stages based on mixed-integer programming where the decision variables are integer and the recourse variables are real. In Section 5 we show that, in this case the recourse problem, i.e. the second stage problem, can be solved in polynomial time by using dynamic programming, contrary to the general case presented in [3]. In Section 6 we propose an exact approach based on constraints generation to solve the robust problem. In Section 7 we test our method on real instances obtained in [11]. In the last section, we take into account some dependencies between the uncertainty in consecutive time periods before concluding.

# 2 System operation

We have to design an energy system for a period spanning many years. The optimization model we propose focuses on one year which is decomposed in $T$ time periods of one hour, where a time period $t$ goes from time $t-1$ to time $t$. Considering that the main part of the data depends on the climate, this allows to take into account the variations of weather during the year. As explained before, the types of wind turbines and PV panels that will be installed are predetermined. They are defined by their expected nominal output power, respectively $E_t^w$ and $E_t^p$ (in Kw.h) for each time period $t$, which are functions of the characteristics of the equipments, the land where they will be installed and the mean meteorological data over the past few years for each time period. The costs of a wind turbine and a PV panel are denoted by $C^w$ and $C^p$ respectively: these costs include purchase and installation costs (reduced to one year according to the lifetime of equipments) and annual maintenance cost including the lease of land.

The purchase and maintenance of the diesel generator induce a fixed cost which is not involved in the optimization. However, its use induces a cost proportional to the energy it provides: let $C^g$ denote the unit cost (for 1 Kw.h).

The system is described in Figure 1. When weather conditions are favorable, the energy produced by wind turbines and PV panels is sufficient to serve the demand and the excess energy can be used to charge the battery. In case of unfavorable weather, the energy stored in the battery is used to serve the demand. It is only when battery is empty that the fuel generator is used: the cost-in-use of the generator is very high but it allows to meet the demand in any case.

The battery bank is composed of elements connected together. The cost of one element is denoted by $C^b$ and it has a minimal load denoted by $K_{min}$ and a (maximal) capacity denoted by $K_{max}$: w.l.o.g., in the following we consider

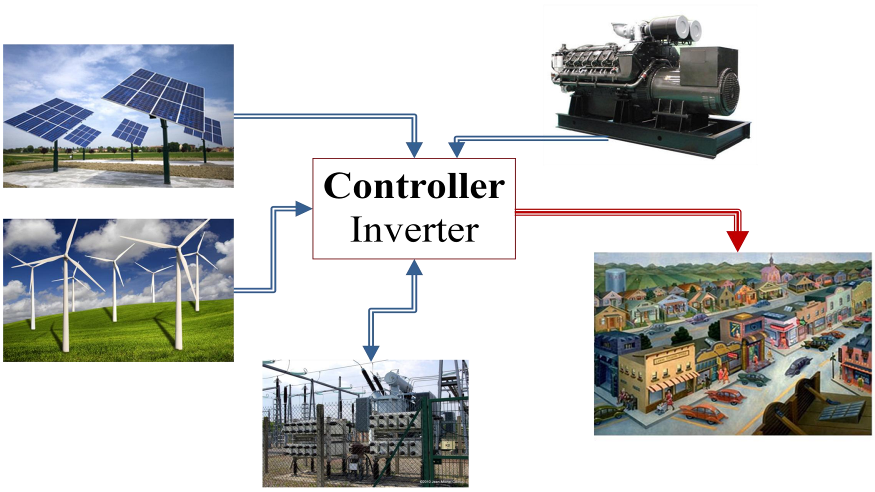


Figure 1: A stand-alone energy park

that, for each element, the minimal load is equal to 0 and the capacity is equal to $K = K_{max} - K_{min}$. We assume that the initial load of each element is equal to $0$. Each element has a maximum charge per hour, denoted by $E^{in}$, and a maximum discharge per hour, denoted by $E^{out}$, in Kw.h. The battery bank operating induces a loss of energy. The rate of return is denoted by $\gamma < 1$: for 1 Kw.h charged in the battery, only $\gamma$ Kw.h can be effectively used. The physical constraints impose that either all the elements are charging (or inactive) or all are discharging; notice that, since the rate of return is less than 1, it is easy to prove that this battery bank operating is optimal. Then, if there are $x^b$ elements in the battery bank, we can consider that there is only one battery with capacity equal to $x^b K$, with a maximum charge per hour equal to $x^b E^{in}$ and a maximum discharge per hour equal to $x^b E^{out}$. As in [14], we assume that the state of the battery does not change during a time period: either the battery is charging (or inactive) or it is discharging.

From the size of the lands, the characteristics of equipments and their placement on the land, the maximum number $N^w_{max}$ of wind turbines, $N^p_{max}$ of PV panels and $N^b_{max}$ of elements in the battery bank which can be installed have been calculated. The expected demand in energy has also been evaluated from the data of previous years for each time period and is denoted by $D_t$, $t = 1, ..., T$.

## 3 The problem without uncertainty

If there is no uncertainty, the objective is to determine the design of the park, that is the number of wind turbines, PV panels and elements in the battery bank, in order to meet the demand with a minimal global cost. We propose a mixed-integer model where variables $x^w, x^p$ and $x^b$ are respectively the (integral) number of wind turbines, PV panels and elements in the battery to install in the park, and for $t = 1, ..., T$, variable $e_t^g$ denotes the amount of energy produced by the generator during the time period $t$ (from $t-1$ to $t$), $e_t^{in}$ (resp. $e_t^{out}$) denotes the amount of energy being charged (resp. discharged) in the battery during the time period $t$ and for $t = 0, ..., T$, $e_t^b$ denote the load of the battery at time $t$.

The problem can be written as the following mixed-integer linear program:

$$
LP \left|
\begin{array}{lr}
\min\limits_{x,e} C^p x^p + C^w x^w + C^b x^b + C^g \sum\limits_{t=1}^{T} e_t^g & \\
E_t^p x^p + E_t^w x^w - e_t^{in} + \gamma e_t^{out} + e_t^g \geq D_t,\ t = 1, ..., T & (1) \\
e_t^{in} \leq x^b E^{in},\ t = 1, ..., T & (2) \\
e_t^{out} \leq x^b E^{out},\ t = 1, ..., T & (3) \\
e_t^b \leq x^b K,\ t = 1, ..., T & (4) \\
e_t^b = e_{t-1}^b + e_t^{in} - e_t^{out},\ t = 1, ..., T & (5) \\
x^p \leq N_{max}^p & (6) \\
x^w \leq N_{max}^w & (7) \\
x^b \leq N_{max}^b & (8) \\
x^w,\ x^p,\ x^b\ \in \mathbb{N}, & (9) \\
e_t^{in},\ e_t^{out},\ e_t^b,\ e_t^g \in \mathbb{R}^+,\ t = 1...T;\ e_0^b = 0 & (10)
\end{array}
\right.
$$

In the objective function, $C^p x^p + C^w x^w + C^b x^b$ corresponds to the investment cost as defined in Section 2, and $C^g \sum_{t=1}^{T} e_t^g$ represents the usage cost. For each time period, constraints (1) impose that the demand is met: the amount of energy obtained from wind and sun is equal to $E_t^p x^p + E_t^w x^w$; $-e_t^{in} + \gamma e_t^{out} + e_t^g$ is the energy supplied by the battery (see Remark 1) and the auxiliary generator. Constraints (2), (3) and (4) limit the charge, discharge and load of the battery. Constraints (5) gives the load of the battery at time $t$: it is equal to the load at time $t-1$ ($e_{t-1}^b$), plus the energy charged in the battery ($e_t^{in}$) minus the energy provided by the battery ($e_t^{out}$); as explained in Section 2, we suppose w.l.o.g. that $e_0^b = 0$. Constraints (6), (7) and (8) bounds the number of installed equipments. Finally, constraints (9) and (10) impose that all variables are positive, variables $x$ are integer and variables $e$ are real.

**Remark 1.** *There is an optimal solution of $LP$ such that $e_t^{in} e_t^{out} = 0$.*

*Proof.* Assume that, for some $t$, we have $e_t^{in} > 0$ and $e_t^{out} > 0$ in an optimal solution $S$ and consider $\hat{S}$ obtained form $S$ by modifying only two variables: $\hat{e}_t^{in} = e_t^{in} - e_t^{out}$ and $\hat{e}_t^{out} = 0$ if $e_t^{in} \geq e_t^{out}$, and $\hat{e}_t^{out} = e_t^{out} - e_t^{in}$ and $\hat{e}_t^{in} = 0$ if $e_t^{in} < e_t^{out}$. Constraints $(2)$, $(3)$ and $(5)$ are verified for $S$ and stay verified for $\hat{S}$ since both variables are decreased while $e_t^{in} - e_t^{out}$ remains constant. In addition, in constraints $(1)$, $-\hat{e}_t^{in} + \gamma \hat{e}_t^{out} = -e_t^{in} + e_t^{out}$ if $e_t^{in} \geq e_t^{out}$ and $-\hat{e}_t^{in} + \gamma \hat{e}_t^{out} = -\gamma e_t^{in} + \gamma e_t^{out}$ if $e_t^{in} < e_t^{out}$. In the two cases, $-\hat{e}_t^{in} + \gamma \hat{e}_t^{out} \geq -e_t^{in} + \gamma e_t^{out}$ and constraints $(1)$ are verified for $\hat{S}$. The value of the objective function and all the other variables are unchanged and $\hat{S}$ is an optimal solution of $LP$. So the model ensures that from any optimal solution we can deduce a solution such that the battery cannot be simultaneously in charge and discharge.

□

From any optimal solution of $LP$ we can deduce another optimal solution such that $e_t^{in} e_t^{out} = 0$ for all $t = 1, ..., T$. That is why these constraints, implying that the battery is either in charge or in discharge during a time period, may be omitted in $LP$.

In the following, we denote by $\mathcal{P}^x$ the polyhedron defined by constraints (6,7,8 9) (on $x$) and by $\mathcal{P}^e$ the polyhedron defined by constraints (2, 3, 4, 5, 10) (on $e$).

Here, the problem has only three integer variables and it can be easily solved with a mixed-integer linear programming software. More generally, the problem is $NP$-hard when there are $n$ sources of energy (see Annex 2) but it can be solved in polynomial time when $n$ is fixed [12].

# 4 A robust model

Now, let us consider that a part of the data are uncertain. For sake of clarity, we assume for the moment that there is uncertainty only on the demand which can vary in a given domain $\mathfrak{D}$. Our robustness objective is to find a feasible solution $(x, e)$ that minimizes the total cost involved by the worst possible scenario of $\mathfrak{D}$ in connection with $x$. We can state the robust problem as the following mathematical program:

$$
RP \left| \begin{array}{l} \min_{x \in \mathcal{P}^x} C^p x^p + C^w x^w + C^b x^b \\ \qquad + \max_{d \in \mathfrak{D}} \min_{e \in \mathcal{P}^e} C^g \sum_{t=1}^{T} e_t^g \\ \qquad\qquad E_t^p x^p + E_t^w x^w - e_t^{in} + \gamma e_t^{out} + e_t^g \geq d_t \ \ t = 1, ..., T \end{array} \right.
$$

Variables $x$ are the decision variables and variables $e$ are the recourse variables. For any feasible $x$, the following program $R(x)$ is called the "Recourse Program":

$$
R(x) \left| \begin{array}{l} \max_{d \in \mathfrak{D}} \min_{e \in \mathcal{P}^e} C^g \sum_{t=1}^{T} e_t^g \\ \qquad E_t^p x^p + E_t^w x^w - e_t^{in} + \gamma e_t^{out} + e_t^g \geq d_t \ \ t = 1, ..., T \end{array} \right.
$$

As in [19] or [3], we define the uncertainty set by:

$$
\mathfrak{D} = \{d \in \mathbb{R}_+^T : \ d_t = D_t + \delta_t \Delta_t, \ \sum_{t=1}^{T} \delta_t \leq \bar{\delta}, \ 0 \leq \delta_t \leq 1, \ \forall t = 1, ..., T\},
$$

where $D_t, \Delta_t$ and $\bar{\delta}$ are data, $\bar{\delta}$ being integer. Then, the demand $d_t$ will vary between $D_t$ and $D_t + \Delta_t$, for $t = 1, ..., T$. $\Delta_t$ is the maximum variation of $d_t$, $\delta_t$ represents the uncertainty on $d_t$, and $\bar{\delta}$ fixes a bound to the cumulated variations. Since we only consider the worst scenarios, $\delta_t$ vary between $0$ and $1$ (and not between $-1$ and $1$ as it could be expected). If $\bar{\delta} \geq T$, then whatever the values of $x$, the worst-case scenario will be $d_t = D_t + \Delta_t$ for all $t$; if $\bar{\delta} = 0$ then there is only one scenario: $d_t = D_t$ for all $t$; thus $\bar{\delta}$ will be chosen between $0$ and $T$: the choice of $\bar{\delta}$ is discussed in Section 7. This definition of the uncertainty implies that the total variation of the demand relative to its reference value $D$ is bounded.

We can generalize the model to the case where there is also uncertainty on the generation of solar and wind energies. Let $e_t^p$ (resp. $e_t^w$) be the uncertain energy produced by PV arrays (resp. wind turbines). Similarly to $\mathfrak{D}$, we define the uncertainty sets $\mathfrak{E}^p$ and $\mathfrak{E}^w$ associated to $E^p$ and $E^w$ as:

$$
\mathfrak{E}^p = \{e^p : \ e_t^p = E_t^p - \phi_t \Phi_t, \ \sum_{t=1}^{T} \phi_t \leq \bar{\phi}, \ 0 \leq \phi_t \leq 1, \ \forall t = 1, ..., T\}, and
$$

$$
\mathfrak{E}^w = \{e^w : \ e_t^w = E_t^w - \omega_t \Omega_t, \ \sum_{t=1}^{T} \omega_t \leq \bar{\omega}, \ 0 \leq \omega_t \leq 1, \ \forall t = 1, ..., T\}.
$$

As for the demand, since we only consider the worst scenarios, for given uncertainty budgets $\bar{\phi}$ and $\bar{\omega}$, we only consider cases where $e_t^p \le E_t^p$ and $e_t^w \le E_t^w$. From [3] the method proposed hereafter to solve the problem for an uncertain demand (right-hand side of contraints) can be extended for uncertain data on the left-hand side of the constraints. For sake of clarity, we present the method only for uncertain demands, but the tests in Section 7 are carried out for uncertainty on $e^p$ and $e^w$ as well.

For fixed values of $x$, the recourse problem $R(x)$ is generally a difficult problem [3]; nevertheless, we show in the next section that it can be solved in polynomial time in our case.

# 5 Solving the recourse problem

From [19], we know that there is an optimal solution of $R(x)$ such that $\delta_t \in \{0,1\}$ for all $t = 1, ..., T$ and we can rewrite $R(x)$ as follows:

$$R(x) \left| \begin{array}{l} \max\limits_{\substack{\sum_{t=1}^T \delta_t \le \bar{\delta} \\ \delta_t \in \{0,1\},\ t=1,...,T}} \quad \min\limits_{e \in \mathcal{P}^e} C^g \sum_{t=1}^T e_t^g \\ \qquad\qquad\qquad\qquad E_t^p x^p + E_t^w x^w - e_t^{in} + \gamma e_t^{out} + e_t^g \ge D_t + \delta_t \Delta_t \quad t = 1, ..., T \end{array} \right.$$

Clearly, for any $x$, the worst scenario is obtained with $\sum_{t=1}^T \delta_t = \bar{\delta}$ and then it will be determined by setting $\delta_t = 1$ for $\bar{\delta}$ periods and $\delta_t = 0$ for the others, or equivalently by setting $d_t = D_t + \Delta_t$ for $\bar{\delta}$ periods and $d_t = D_t$ for the others. The problem now is to select the $\bar{\delta}$ periods for which $\delta_t = 1$. We propose a polynomial dynamic programming approach to answer this question and solve the recourse problem.

We have to solve the recourse problem for given values of $x$, thus $E_t^p x^p + E_t^w x^w$ is fixed and we denote $\hat{D}_t = D_t - E_t^p x^p - E_t^w x^w$, $\hat{d}_t = d_t - E_t^p x^p - E_t^w x^w = \hat{D}_t + \delta_t \Delta_t$. In addition, since $x_b$ is fixed, we assumed w.l.o.g. that $x^b = 1$, or equivalently we set $K \leftarrow x_b K$, $E^{in} \leftarrow x_b E^{in}$ and $E^{out} \leftarrow x_b E^{out}$. Since the unit diesel cost does not depend on $t$, we can determine the battery load at time $t$ ($e_t^b$) and the amount of energy supplied by the generator during period $t$ ($e_t^g$), from the battery load at $t-1$ ($e_{t-1}^b$) and the amount of energy either in excess or in lack ($\hat{d}_t$). We have the following proposition :

**Proposition 1.** *For given values of $\hat{d}$ and $\beta = e^b_{t-1}$ ($0 \leq \beta \leq K$), for $t = 1, ..., T$, we have:*

$$\left.\begin{aligned} e^b_t &= f(\hat{d}_t, \beta) = \max\left(0, \beta - E^{out}, \beta - \frac{\hat{d}_t}{\gamma}\right) \\ e^g_t &= g(\hat{d}_t, \beta) = \hat{d}_t - \gamma \min\left(\beta, E^{out}, \frac{\hat{d}_t}{\gamma}\right) \end{aligned}\right\} \quad \text{if } \hat{d} \geq 0$$

$$\left.\begin{aligned} e^b_t &= f(\hat{d}_t, \beta) = \min\left(K, \beta + E^{in}, \beta - \hat{d}_t\right) \\ e^g_t &= g(\hat{d}_t, \beta) = 0 \end{aligned}\right\} \quad \text{if } \hat{d} < 0$$

*Proof.* During time period $t$,

- If $\hat{d}_t \geq 0$, then the energy produced by wind turbines and PV panels is not sufficient to serve the demand. The battery discharges a quantity of energy equal to $\min\left(\beta, E^{out}, \frac{\hat{d}_t}{\gamma}\right)$, and the generator must provide an amount of energy equal to $g(\hat{d}_t, \beta) = \hat{d}_t - \gamma \min\left(\beta, E^{out}, \frac{\hat{d}_t}{\gamma}\right) \geq 0$.

- If $\hat{d}_t < 0$, then the energy produced is greater than the demand and the system stores in the battery an amount of energy equal to $\min\left(K - \beta, E^{in}, -\hat{d}_t\right)$. The generator is not used and $g(\hat{d}_t, \beta) = 0$.

The battery load at time $t$, $e^b_t = f(\hat{d}_t, \beta)$, is immediately deduced from the amount of energy charged or discharged from the battery during time period $t$. □

The algorithm operates from $\tau = T$ to $\tau = 1$ and considers at each step a "truncated recourse problem" $R_x(\tau, \zeta, \beta)$ defined on the $(T - \tau + 1)$ last time periods. At each step, it looks for an optimal operating for the considered period, i.e. from time $\tau - 1$ to time $T$, in function of the two parameters $\beta$ and $\zeta$: $\beta$ is the battery load at time $\tau - 1$, i.e. at the beginning of period $\tau$, and $\zeta$ is the uncertainty budget for these periods, that is the number of time periods with $\delta_t = 1$ among the $(T - \tau + 1)$ time periods; then, $\overline{\delta}$ represents the global "uncertainty budget" and the only value of $\zeta$ to consider for $\tau = 1$ is $\zeta = \overline{\delta}$.

Then $R_x(\tau, \zeta, \beta)$ can be written as the following mathematical program where $\hat{d}_t = \hat{D}_t + \delta_t \Delta_t$:

$$R_x(\tau,\zeta,\beta)\left|\begin{array}{ll}\max\limits_{\substack{\sum_{t=\tau}^{T}\delta_t\le\zeta\\ \delta_t\in\{0,1\},\ t=\tau,...,T}} & \min\limits_{e} C^g\sum\limits_{t=\tau}^{T} e_t^g \\ & -e_t^{in}+\gamma e_t^{out}+e_t^g\ge\hat{D}_t+\delta_t\Delta_t,\ t=\tau,...,T \quad (1)\\ & e_t^{in}\le E^{in},\ t=\tau,...,T \quad (2)\\ & e_t^{out}\le E^{out},\ t=\tau,...,T \quad (3)\\ & e_t^{b}\le K,\ t=\tau,...,T \quad (4)\\ & e_t^{b}=e_{t-1}^{b}-e_t^{out}+e_t^{in},\ t=\tau,...,T \quad (5)\\ & e_{\tau-1}^{b}=\beta \quad (5')\\ & e_t^{in},\ e_t^{out},\ e_t^{b},\ e_t^{g}\in\mathbb{R}_+,\ t=\tau,...,T \quad (10)\end{array}\right.$$

Notice that $(\tau,\zeta,\beta)$ represents the "current state" of the decision process and that $R(x) = R_x(1,\bar{\delta},0)$. Let us denote $v(R_x(\tau,\zeta,\beta))$ by $v(\tau,\zeta,\beta)$. We obtain the following recurrence relation verified by $v(\tau,\zeta,\beta)$ for $0\le\beta\le K$ :

$v(\tau,0,\beta) = C^g g(\hat{D}_\tau,\beta) + v(\tau+1,0,f(\hat{D}_\tau,\beta))$, for $1\le\tau<T$,

$v(T,\zeta,\beta) = C^g g(\hat{D}_T+\zeta\Delta_T,\beta)$, for $0\le\zeta\le 1$,

$v(\tau,\zeta,\beta) = \max(C^g g(\hat{D}_\tau,\beta) + v(\tau+1,\zeta,f(\hat{D}_\tau,\beta)), C^g g(\hat{D}_\tau+\Delta_\tau,\beta)+$
$v(\tau+1,\zeta-1,f(\hat{D}_\tau+\Delta_\tau,\beta)))$, for $1\le\tau<T$ and $0<\zeta\le T-\tau+1$.

Now, let us study $v(\tau,\zeta,\beta)$. We are going to prove that there is a constant $B$ ($0\le B\le K$) such that $v(\tau,\zeta,\beta)$ is a function of $\beta$ linearly decreasing on $[0,B]$ and constant on $[B,K]$ (see Figure 2).

**Proposition 2.** *For any $\tau,\ 1\le\tau\le T$, any $\zeta,\ 0\le\zeta\le T-\tau+1$, and any $\beta\in[0,K]$,*

- *there are $B\ge 0$ and $C\ge 0$ s.t. $v(\tau,\zeta,\beta) = C-\gamma C^g\min(\beta,B)$ (11),*
- *$B=\frac{v(\tau,\zeta,0)-v(\tau,\zeta,K)}{\gamma C^g}$ and $C = v(\tau,\zeta,0)$ is a solution of (11).*

*Proof.* The complete proof is given in Annex 1. We just give here the main ideas it uses.

First, the proof is given for $\zeta = 0$: in this case, the problem reduces to a problem without uncertainty since $\delta_t = 0$ for all $t=\tau$ to $T$. The proof is by induction on $t$, from $T$ to $T-\tau+1$; first we prove the existence of $B$ and $C$ and then we compute them. When $\zeta = T-\tau+1$ we have $\delta_t = 1$ for all $t=\tau$ to $T$ and

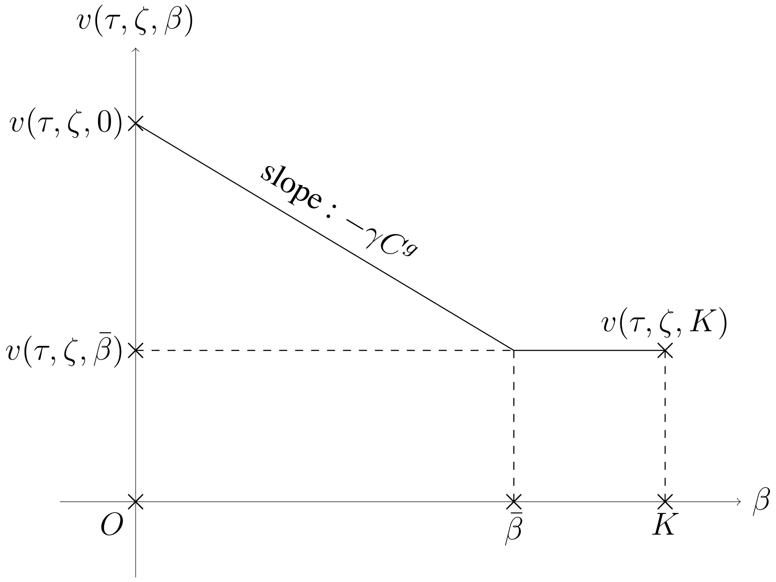


Figure 2: Variations of $v(\tau, \zeta, \beta)$ in function of $\beta$

the problem reduces to a problem without uncertainty too. The proof is similar. For values of $\zeta$ between 0 and $T - \tau + 1$, the proof is again by induction and it uses the results obtained for $\zeta = 0$ and $\zeta = T - \tau + 1$ before computing $B$ and $C$. □

Proposition 2 gives us an easy way to compute $v(\tau, \zeta, \beta)$. Indeed, for any $\beta$, the function $v(\tau, \zeta, \beta)$ is entirely determined by the two values $v(\tau, \zeta, 0)$ and $v(\tau, \zeta, K)$. We can therefore use two nested dynamic programming procedures computing $v(\tau, \zeta, 0)$ and $v(\tau, \zeta, K)$ for each $(\tau, \zeta)$, following the pattern shown on Figure 3.

Notice that since the computation of the initial states is $O(T^2)$, the complexity of the algorithm is $O(\bar{\delta}(T-\bar{\delta}+1)+T^2)$. Since $\bar{\delta} \leq T$, this algorithm is polynomial.

# 6 Solving the robust problem

The robust problem can be rewritten as:

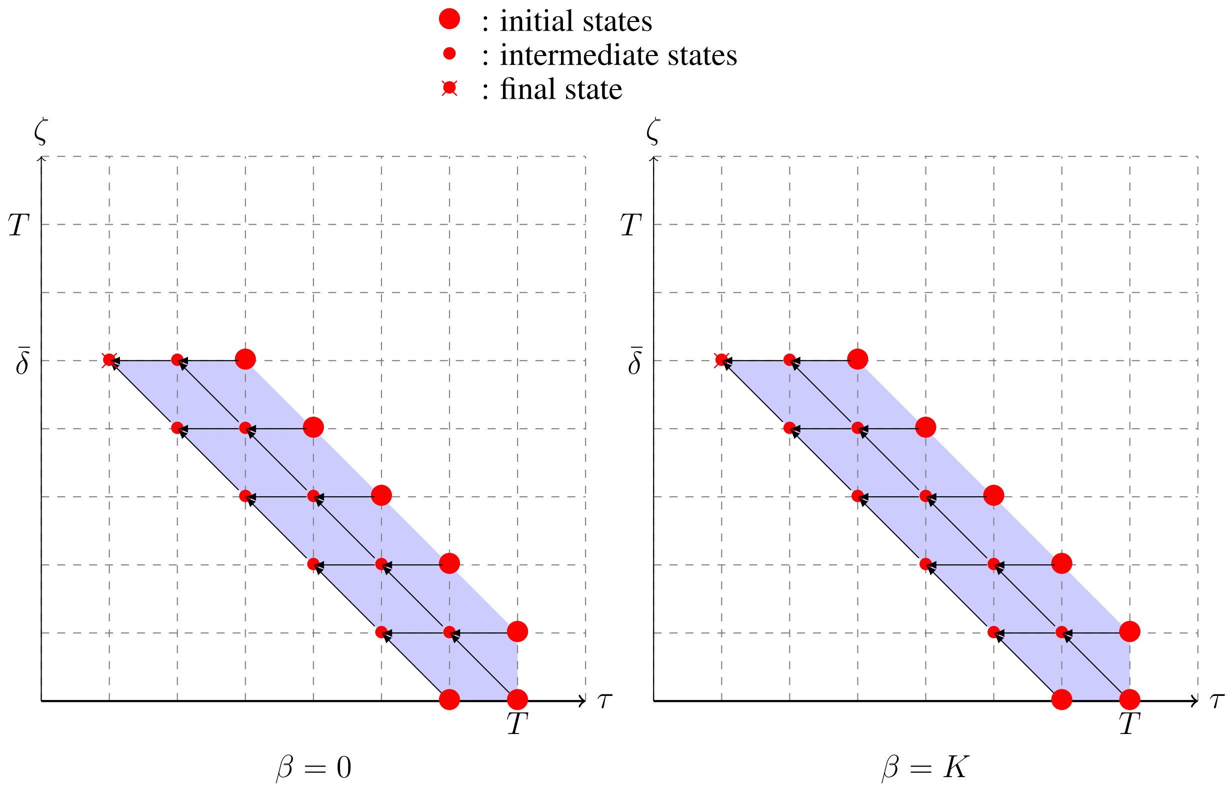


Figure 3: Pattern of calculation of $v(\tau, \zeta, \beta)$.

$$RP \left| \begin{array}{l} \min\limits_{x} C^p x^p + C^w x^w + C^b x^b + v(R(x)) \\ \qquad x^p \leq N^p_{max} \\ \qquad x^w \leq N^w_{max} \\ \qquad x^b \leq N^b_{max} \\ \qquad x^b,\ x^w,\ x^p \in \mathbb{N} \end{array} \right.$$

where $v(R(x))$ is the optimal value of $R(x)$ which can be obtained in polynomial time (see Section 5 ). Since in addition, $v(R(x))$ is a decreasing function of $x$, we could use a branch and bound algorithm to solve $RP$. Nevertheless, we prefer the approach proposed in [3] and [19] which proved efficient in practice. For any $x \in \mathcal{P}^x$ and $d \in \mathfrak{D}$, we define the following linear program:

$$
R(x,d)\left|\begin{array}{l} \min_{e\in\mathcal{P}^e} C^g \sum_{t=1}^{T} e_t^g \\ \quad - e_t^{in} + \gamma e_t^{out} + e_t^g \geq d_t - E_t^p x^p - E_t^w x^w,\ t=1,...,T\ (1) \end{array}\right.
$$

Let us associate the dual variables $\lambda$ to constraints $(1)$ and $\alpha, \beta, \mu, \pi$ respectively to constraints $(2)$, $(3)$, $(5)$ and $(4)$ of $\mathcal{P}^e$: we dualize $R(x,d)$. Then $R(x)$ becomes a $\max\max$ instead of a $\max\min$ problem and we can reformulate $R(x)$ to obtain a quadratic maximization program $DR(x)$ such that $v(DR(x)) = v(R(x))$:

$$
DR(x)\left|\begin{array}{lr} \max_{\alpha,\beta,\delta,\lambda,\mu,\pi} \sum_{t=1}^{T}[(D_t - E_t^p x^p - E_t^w x^w + \delta_t\Delta_t)\lambda_t \quad -x^b E^{in}\alpha_t - x^b E^{out}\beta_t - x^b K\pi_t] & \\ \quad \text{s.c. } \lambda_t \leq C^g, & t=1,...,T\ (13) \\ \quad - \lambda_t - \alpha_t + \mu_t \leq 0, & t=1,...,T\ (14) \\ \quad \gamma\lambda_t - \beta_t - \mu_t \leq 0, & t=1,...,T\ (15) \\ \quad \mu_{t+1} - \mu_t - \pi_t \leq 0, & t=1,...,T\ (16) \\ \quad \sum_{t=1}^{T} \delta_t \leq \bar{\delta} & (18) \\ \quad \delta_t \in \{0,1\}, & t=1,...,T\ (19) \\ \quad \alpha_t, \beta_t, \lambda_t, \mu_t, \pi_t \geq 0 & t=1,...,T\ (17) \end{array}\right.
$$

As in [3] we can linearize the quadratic terms in the objective function by substituting the variable $\nu_t$ to the product $\delta_t\lambda_t$. We obtain the following mixed-integer linear program

$$
LDR(x) \left| \begin{array}{ll}
\max_{\alpha,\beta,\delta,\lambda,\mu,\nu,\pi} \sum_{t=1}^{T} [(D_t - E_t^p x^p - E_t^w x^w)\lambda_t + \Delta_t \nu_t - x^b E^{in} \alpha_t - \quad x^b E^{out} \beta_t - x^b K \pi_t] & \\
\text{s.c. } \lambda_t \leq C^g, & t = 1, ..., T \ (13) \\
\quad - \lambda_t - \alpha_t + \mu_t \leq 0, & t = 1, ..., T \ (14) \\
\quad \gamma\lambda_t - \beta_t - \mu_t \leq 0, & t = 1, ..., T \ (15) \\
\quad \mu_{t+1} - \mu_t - \pi_t \leq 0, & t = 1, ..., T \ (16) \\
\quad \sum_{t=1}^{T} \delta_t \leq \bar{\delta} & (18) \\
\quad \delta_t \in \{0, 1\}, & t = 1, ..., T \ (19) \\
\quad \nu_t \leq C^g \delta_t, & t = 1, ..., T \ (20) \\
\quad \nu_t \leq \lambda_t, & t = 1, ..., T \ (21) \\
\quad \alpha, \beta, \lambda, \mu, \nu, \pi \geq 0 & (17)
\end{array} \right.
$$

Let $\mathcal{P}_Q$ be the polyhedron defined by the constraints $(13), ..., (21)$ of $LDR(x)$ where we replace $(19)$ by $0 \leq \delta_t \leq 1$, and let $(\mathcal{P}_Q)_I = conv(\mathcal{P}_Q \cap \{\delta \in \{0,1\}^T\})$, be the convex hull of the feasible solution of $LDR(x)$. Notice that this convex hull does not depend on $x$. $(\mathcal{P}_Q)_I$ being a polyhedron we can rewrite the robust problem $RP$ as the following linear program:

$$
PROB \left| \begin{array}{l}
\min_{x,z} C^p x^p + C^w x^w + C^b x^b + z \\
s.c. \ z \geq \sum_{t=1}^{T} [(D_t - E_t^p x^p - E_t^w x^w)\lambda_t^s + \Delta_t \nu_t^s \\
\qquad - x^b E^{in} \alpha_t^s - x^b E^{out} \beta_t^s - x^b K \pi_t^s], \ \ s = 1, ..., S. \\
\quad x^b \leq N_{max}^b, \\
\quad x^w \leq N_{max}^w, \\
\quad x^p \leq N_{max}^p, \\
\quad z \geq 0, \ x^b, \ x^w, \ x^p \in \mathbb{N}
\end{array} \right.
$$

where $S = |\mathcal{S}|$ and $\mathcal{S} = \{(\alpha^s, \beta^s, \lambda^s, \mu^s, \nu^s, \pi^s)_{1 \leq s \leq S}\}$ is the set of extreme points of $(\mathcal{P}_Q)_I$. However, due to the potentially tremendous number of constraints, we solve $PROB$ by a constraint generation algorithm as in [6]. Initially, we consider a subset $\mathcal{S}_0$ of $\mathcal{S}$; at a step $k$, we consider a subset $\mathcal{S}^k$ of $\mathcal{S}$ and we solve a relaxed program $PROB^k$ of $PROB$, called *master problem*, which consists in solving $PROB$ with the subset of constraints corresponding to $\mathcal{S}^k$ . The obtained solution is denoted by $(x^k, z^k)$.

Then we solve $LDR(x^k)$, called *slave problem*, to check if $(x^k, z^k)$ is optimal. If not, then a new constraint is added, i.e. an extreme point is added to $\mathcal{S}^k$ (See Algorithm 1).

On the basis that the number of extreme points of $(\mathcal{P}_Q)_I$ is finite, one can prove that this algorithm converges in a finite number of steps.

Two approaches can be applied to solve $LDR(x)$: either we directly use Cplex to solve this mixed-integer program or we use the approach presented in Section 5 to solve it in polynomial time. Indeed, by using dynamic programming we obtain in polynomial time the optimal values of $R(x)$, $e$ and $d$; let $d^*$ be the optimal associated value of $d$. So we have the optimal value of $R(x, d^*)$, which is, by strong duality, the optimal value of $DR(x, d^*)$ : $v(DR(x, d^*)) = v(DR(x)) = v(LDR(x))$. But we also need the values of the dual variables, $\alpha, ..., \pi$ : there are easy to obtain by applying the complementary slackness theorem to both dual programs $R(x, d^*)$ and $DR(x, d^*)$ which are in standard form. In our experiments, we directly get these values by Cplex.

---

**Algorithm 1** Constraint generation algorithm

---

1: We denote: $x = (x^p, x^w, x^b)$ and $Cx = c^p x^p + C^w x^w + C^b x^b$.
2: $(\alpha^0, \beta^0, \lambda^0, \mu^0, \nu^0, \pi^0) = 0$. Set $L \leftarrow -\infty, U \leftarrow +\infty, k \leftarrow 1$.
3: Solve the master problem :

$$PROB^k \left| \begin{array}{l} \min\limits_{x,z} Cx + z \\ s.c.\ z \geq \sum\limits_{t=1}^{T} [(D_t - E_t^p x^p - E_t^w x^w)\lambda_t^s + \Delta_t \nu_t^s \\ \quad - x^b E^{in} \alpha_t^s - x^b E^{out} \beta_t^s - x^b K \pi_t^s],\ \ 0 \leq s \leq k-1. \\ \quad x^b \leq N_{max}^b, \\ \quad x^p \leq N_{max}^p, \\ \quad x^w \leq N_{max}^w, \\ \quad z \geq 0,\ x^b,\ x^w,\ x^p \in \mathbb{N}. \end{array} \right.$$

Let $(x^k, z^k)$ be the obtained solution.
$L \leftarrow Cx^k + z^k$ .
4: Solve $LDR(x^k)$ either by Cplex or by dynamic programming. Let $(\delta^k, \lambda^k, \nu^k, \alpha^k, \beta^k, \pi^k)$ be the optimal solution.

$$U \leftarrow \min\{U, Cx^k + v(DR(x^k))\}.$$

**if** $U = L$, **then** return $(x^k, z^k)$ **else** go to 5.
5: Add the constraint

$$z \geq \sum_{t=1}^{T} [(D_t - E_t^p x^p - E_t^w x^w)\lambda_t^k + \Delta_t \nu_t^k - x^b E^{in} \alpha_t^k - x^b E^{out} \beta_t^k - x^b K \pi_t^k]$$

to the master problem $PROB^k$, $k \leftarrow k+1$ and go to 2.

---

# 7 Results

The proposed wind-PV model was applied for designing off-grid networks from three very different sets of real instances given for the state of Montana (USA) which has a continental climate, an island in the Philippines which has a tropical climate and Dunkerque (France) which has an oceanic climate.

The characteristics of the system components are the same for the three regions and come from the Homer website [11]. There are small wind turbines of type BWC XL.1 with a rated power of $1.24$ kW, PV panels of size $1$ kW and batteries Trojan L16P. The annual unit costs $C^p, C^w$ and $C^b$ are computed in the following way: $\frac{\text{purchase cost}}{\text{lifetime}}$ + annual O&R cost (Operations and Regulatory); $C^g$ is the cost for producing 1 Kw.h by the generator. The number $T$ of time periods is equal to $8760$ which is the number of hours in one year. Table 1 gives the main parameter values. For the three regions, the demand values $D_t$, and the solar and wind production values $E_t^p$ and $E_t^w$, $t = 1, ..., T = 8760$, are the average values for each hour (or time period) over several passed years. For the Montana and the Philippines, the data come from the Homer website [11]. The average values for Dunkerque were calculated from climatic data provided by Meteo France [13] and demand data provided by RTE-France [18]. We do not recall here the $8760$ hourly mean values of these parameters. Finally, we bounded the number of PV panels and wind turbines to $120$ and batteries to $700$.

| $C^p$ | $C^w$ | $C^b$ | $C^g$ | $K$ | $\gamma$ | $E^{in}$ | $E^{out}$ | $N^p_{max}, N^w_{max}$ | $N^b_{max}$ |
|---|---|---|---|---|---|---|---|---|---|
| 280$ | 295$ | 26$ | 3.9$ | 2.16 | 0.85 | 0.11 | 2.16 | 120 | 700 |

Table 1: Main data values.

We made several kinds of tests in order to verify the efficiency and the accuracy of our approach. First we study the influence of the demand uncertainty on the design of the park, as well for the parameter fixing the level of uncertainty ($\bar{\delta}$) as for the parameter limiting the demand deviation from the mean demand value ($\Delta_t$). Then we compare the dynamic programming approach to the approach based on the use of CPLEX from the computational point of view. Finally, we give some results where the uncertainty concerns both the demand and production by wind turbines and photovoltaic panels. Additional tests are presented in Section 8 to take into account some dependencies between demands in consecutive time periods.

## 7.1 Results in function of the uncertainty level

In this section we give for the three regions the results for different values of $\bar{\delta}$ (see Section 4). For each $\bar{\delta}$ we give the optimal number of wind turbines, PV panels and elements in the battery. We also give the optimal value of the robust problem, that is the minimal cost including the investment cost for the park and the fuel consuming.

Table 2 gives the results for Montana. In the first part, we assume that the maximum deviation from the mean value $D_t$ of the demand at each time period is $\Delta_t = 10\% D_t$, $t = 1, ..., 8760$. In the second part, we assume that $\Delta_t = 30\% D_t$ which corresponds to the actual deviation for the Montana.

| | $\bar{\delta}$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0 | 100 | 200 | 300 | 500 | 700 | 900 | 8760 |
| $\Delta_t = 10\% D_t$ | | | | | | | | |
| $x^p$ | 45 | 45 | 46 | 45 | 51 | 50 | 50 | 50 |
| $x^w$ | 64 | 65 | 66 | 66 | 66 | 70 | 71 | 71 |
| $x^b$ | 467 | 473 | 469 | 482 | 490 | 514 | 504 | 504 |
| Cost (in $) | 49874 | 51554 | 52757 | 53255 | 54525 | 54874 | 54874 | 54884 |
| $\Delta_t = 30\% D_t$ | | | | | | | | |
| $x^p$ | 45 | 45 | 46 | 47 | 55 | 56 | 59 | 59 |
| $x^w$ | 64 | 66 | 69 | 68 | 72 | 84 | 84 | 83 |
| $x^b$ | 467 | 489 | 504 | 524 | 567 | 601 | 597 | 608 |
| Cost (in $) | 49874 | 54787 | 57966 | 60392 | 63893 | 64908 | 64857 | 64857 |

Table 2: Montana. Park design and total associated robust cost as a function of the global level of uncertainty $\bar{\delta}$ and the maximal variation on the demand $\Delta_t$.

As expected, the optimal value of the robust problem increases as a function of $\bar{\delta}$ corresponding to an increase of the demand but it reaches its maximum value for a certain threshold (here 900). Indeed, the optimal value of the robust problem cannot decrease when the global uncertainty level increases since any solution for $\bar{\delta} = \hat{\delta}$ is admissible for $\bar{\delta} < \hat{\delta}$. Furthermore, once the park is sufficient to cover the demand for some critical set of periods, it is also sufficient to cover any other periods. Therefore the cost remains constant as soon as $\bar{\delta}$ is large enough to cover the critical periods. In addition we note that there is no significant differences between the two cases $\Delta_t = 10\% D_t$ and $\Delta_t = 30\% D_t$: evidently the cost and the number of elements in the park are slightly larger in the second case since the demand can be larger but there is also a stabilization of their values for a threshold (around 850).

In Tables 3 and 4 we give the results for Dunkerque and the Philippines, respectively. We give the results only for $\Delta_t = 10\%D_t$ because the results obtained for $\Delta_t = 30\%D_t$ are very similar. We see that the stabilization is obtained for a lower threshold ($\bar{\delta} = 160$) for Dunkerque. On the contrary, for the Philippines we must compute the park until $\bar{\delta} = 6000$ to obtain a stabilization but the increasing is very slow from $\bar{\delta} = 2000$ to $6000$. The differences between the fuel costs and the thresholds of stabilization in the two regions are probably due to the fact that the climate is much more irregular in the Philippines than in Dunkerque.

| | $\bar{\delta}$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0 | 40 | 60 | 100 | 120 | 140 | 160 | 8760 |
| $x^p$ | 9 | 7 | 6 | 6 | 5 | 7 | 7 | 7 |
| $x^w$ | 17 | 19 | 20 | 20 | 21 | 24 | 24 | 24 |
| $x^b$ | 299 | 299 | 305 | 311 | 324 | 377 | 384 | 384 |
| Cost (in $) | 15773 | 17611 | 18461 | 19636 | 20157 | 20216 | 20220 | 20220 |

Table 3: Dunkerque. Results as a function of $\bar{\delta}$.

| | $\bar{\delta}$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0 | 500 | 1000 | 1300 | 1500 | 2000 | 6000 | 8760 |
| $x^p$ | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| $x^w$ | 60 | 61 | 62 | 64 | 64 | 66 | 67 | 67 |
| $x^b$ | 99 | 95 | 98 | 100 | 101 | 103 | 105 | 105 |
| Cost (in $) | 31631 | 32906 | 33765 | 34049 | 34175 | 34353 | 34650 | 34650 |

Table 4: Philippines. Results as a function of $\bar{\delta}$.

## 7.2 Dynamic programming versus integer linear programming

In Table 5, we compare the CPU times required by Algorithm 1 when the recourse problem ($LDR(x)$ at Step 4 of Algorithm 1) is solved either by using the polynomial time dynamic programming algorithm of Section 5 or by using CPLEX.

| | $\bar{\delta}$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Montana | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 850 | 8760 |
| D.P. CPU(s) | 82 | 90 | 90 | 93 | 93 | 97 | 97 | 95 | 94 | 67 |
| Cplex CPU(s) | 46 | 88 | 88 | 103 | 2845 | 7422 | 4358 | 375 | 204 | 22 |
| | $\bar{\delta}$ | | | | | | | | | |
| Philippines | 0 | 100 | 500 | 1000 | 2000 | 2500 | 3000 | 3500 | 4000 | 8760 |
| D. P. CPU(s) | 75 | 91 | 90 | 95 | 112 | 115 | 124 | 120 | 118 | 85 |
| Cplex CPU(s) | 74 | 87 | 117 | 199 | 720 | 1103 | 1620 | 1792 | 900 | 123 |

Table 5: Montana and Philippines. Comparison of the two approaches from the CPU time point of view (CPU times in seconds).

We notice that for intermediate values of $\bar{\delta}$ the dynamic programming approach is much faster than the approach using Cplex. In particular, for values of $\bar{\delta}$ between $400$ and $600$ for the Montana, the Cplex approach can take several hours to solve the problem while the dynamic programming approach only needs a few minutes. Indeed, there are about 40000 variables including 8000 integer variables, and 50000 constraints. Moreover, solving $LDR(x)$ is difficult because there is an important gap between the values of its continuous relaxation and its optimal integer solution: this gap is in part due to the linearization of the quadratic terms. Furthermore, with Cplex, the CPU time increases until $\bar{\delta} = 900$ and then decreases until $\bar{\delta} = 8760$ while it is almost constant for dynamic programming. We verified that the non efficiency of CPLEX in median cases is due to the very big number of nodes explored in the branch and bound. Indeed, median values correspond to difficult instances and extremal values correspond to cases where there is little uncertainty because the actual demands are all close either to the mean value or to the largest value.

## 7.3 Uncertainty in demand and energy production

Now we consider that the demand as well as the energy production are uncertain and we bound the uncertainty level respectively to $\bar{\delta}$ for the demand, $\bar{\omega}$ for the wind turbine energy production, and $\bar{\phi}$ for the photo-voltaic production. From the results presented in [3] we know that the our approach can be used to solve the problem when $\bar{\omega}$ and $\bar{\phi}$ correspond to the maximum number of periods where the unit productions $E_t^w$ and $E_t^p$ can reach their minimal values. But the size of the problems to solve is much more larger since we have to introduce three set of 0-1 variables, $z_t$ for the demand (as before), $z_t^\omega$ (for wind turbine production) and $z_t^\phi$ (for photo-voltaic production), $t = 1, ..., T$: the number of variables and linearization constraints is very large. Moreover, the dynamic programming approach becomes intractable to solve the recourse problem for large values of $T$, although the algorithm remains polynomial. Nevertheless, we could test the constraint generation algorithm for small values of $(\bar{\delta}, \bar{\phi}, \bar{\omega})$ and some significant results are presented in Table 6. Solving the most difficult instances ((300,100,100), (300,300,300) or (500,500,500)) requires more than two hours and it was not possible to solve bigger instances.

| | $x^p$ | $x^w$ | $x^b$ | Cost |
|---|---|---|---|---|
| $\bar{\delta} = \bar{\phi} = 0$ | 64 | 45 | 467 | 49874 |
| $\bar{\delta} = 0, \bar{\phi} = 0, \bar{\omega} = 500$ | 66 | 48 | 465 | 51348 |
| $\bar{\delta} = 8760, \bar{\phi} = 0, \bar{\omega} = 0$ | 71 | 50 | 504 | 54884 |
| $\bar{\delta} = 0, \bar{\phi} = 8760, \bar{\omega} = 0$ | 71 | 48 | 456 | 51852 |
| $\bar{\delta} = 0, \bar{\phi} = 0, \bar{\omega} = 8760$ | 64 | 51 | 469 | 51438 |
| $\bar{\delta} = \bar{\phi} = \bar{\omega} = 8760$ | 78 | 54 | 523 | 58672 |
| $\bar{\delta} = 100, \bar{\phi} = 100, \bar{\omega} = 100$ | 67 | 47 | 468 | 53743 |
| $\bar{\delta} = 300, \bar{\phi} = 100, \bar{\omega} = 100$ | 66 | 48 | 499 | 55566 |
| $\bar{\delta} = 100, \bar{\phi} = 300, \bar{\omega} = 100$ | 70 | 46 | 489 | 54334 |
| $\bar{\delta} = 100, \bar{\phi} = 100, \bar{\omega} = 300$ | 69 | 39 | 522 | 54297 |
| $\bar{\delta} = 300, \bar{\phi} = 300, \bar{\omega} = 300$ | 70 | 49 | 504 | 56809 |
| $\bar{\delta} = 500, \bar{\phi} = 500, \bar{\omega} = 500$ | 74 | 49 | 500 | 58183 |

Table 6: Montana ($\Delta_t = 10\% D_t$). Design and cost of the park when considering energy and demand uncertainties .

We do not present all the results but our tests show that, as previously the optimal value of the robust problem increases as a function of $\bar{\delta}, \bar{\phi}$ and $\bar{\omega}$, until a certain threshold. Furthermore, we notice that the combined influence of uncertainties in the energy generated by wind turbines and PV-panels induces a larger cost augmentation than the sum of the ones induced by considering separate uncertainties in wind and solar energy generation. Indeed, the production of the park must cover the demand for the $\bar{\delta}$ critical periods of demand as for the $\bar{\phi}$ and $\bar{\omega}$ critical periods of production.

# 8 Other definitions of demand uncertainty

In this section we only consider demand uncertainty and the uncertainty corresponding to a worst scenario is defined by the values taken by $\delta_t, t = 1, ..., T$ in the recourse problem. Our tests were made on real world data sets which show some natural dependency between demands $D_t$ in consecutive time periods; but in the definition of uncertainty given in Section 4 we assume that the variations $\delta_t$ on these demands are independent. Doing so, we probably oversize the park and obtain an upper bound of the worst cost: indeed, introducing new constraints linking $\delta_t$ values will lead up to reduce the uncertainty domain and so to reduce the cost.

Several possible models allow to take into account some dependencies between variables $\delta_t$. For instance, we could consider additional constraints as $(1-\rho)\delta_{t-1} \le \delta_t \le \min((1+\rho)\delta_{t-1}, 1)$, $\forall t = 2, ..., T$, $0 < \rho < 1$, where $\rho$ is a fixed parameter which induces a small variation of uncertainty between consecutive time periods. But our approach is not compatible with such constraints because they imply non integer values of $\delta_t$ in the optimal solutions of the recourse problem. Then $R(x)$ cannot be easily solved: the dynamic programming approach is no more valid, and the dual $DR(x)$ is a non convex quadratic program which cannot be linearized. So, since there is no specific real world justifications for such constraints, we try other ways to track this aspect.

## 8.1 Sliding time window constraints for uncertainty

A first idea to avoid erratic behaviour of the data variation is to forbid a sawtooth oscillation of the $\delta_t$ values, for instance sequels as 1 0 1 0 1 0 1... . For this purpose, we add a new constraint limiting the total number of gaps 0 to 1 (or 1 to 0) between $\delta_{t-1}$ and $\delta_t$: $\sum_{t=2}^{T} |\delta_t - \delta_{t-1}| \le H$ where $H < \bar{\delta}$ is a fixed integer parameter measuring the "oscillation level". The new definition of $\mathfrak{D}$ is:

$$\mathfrak{D} = \{d \in \mathbb{R}_+^T : d_t = D_t + \delta_t \Delta_t, \delta_t \in \{0,1\} \forall t, \sum_{t=1}^{T} \delta_t \le \bar{\delta}, \sum_{t=2}^{T} |\delta_t - \delta_{t-1}| \le H\}.$$

To introduce the constraint $\sum_{t=2}^{T} |\delta_t - \delta_{t-1}| \le H$ in the recourse problem $R(x)$, we need to linearize it. We add new variables $\hat{\delta}_t \ge 0; t = 2, ..., T$ and the constraint becomes:

$$\begin{aligned} &\hat{\delta}_t \geq \delta_t - \delta_{t-1} \quad \forall t = 2, ..., T \quad (a) \\ &\hat{\delta}_t \geq \delta_{t-1} - \delta_t \quad \forall t = 2, ..., T \quad (b) \\ &\sum_{t=2}^{T} \hat{\delta}_t \leq H \quad (c) \end{aligned}$$

The recourse problem with these new constraints can still be solved in polynomial time by dynamic programming: we add a new parameter $\eta = 1, ..., H$ and consider a new recurrence formulation of the recourse problem $R_x(\tau, \zeta, \beta, \eta)$, where $\eta$ measures the oscillation level used in the time periods $\tau$ to $T$. The presentation of the algorithm would be tedious and is not given here. Moreover, the computing time increases greatly and is no more competitive with a linear programming approach using CPLEX, where the new constraints (*a*), (*b*) and (*c*) are added in $R(x)$.

On another hand, too long sequels of time periods with $\delta_t = 1$ are unlikely and lead also up to oversize the park; to avoid such scenarios, we can add the constraints:

$$\textstyle\sum_{t=t_0}^{t_0+T^s} \delta_t \leq \bar{T}, \ \forall t_0 = 1 \text{ to } T - T^s \quad \text{(d)}$$

where $\bar{T}$ and $T^s$ are fixed parameters which limit to $\bar{T}$ the number of consecutive periods with $\delta_t = 1$ in any set of $T^s$ consecutive periods.

## 8.2 Results

First we tested the complemented model obtained by adding constraints $(a, b, c)$ to the definition of $\mathfrak{D}$ and using CPLEX, on the data provided for Montana tested in Section 7. Some of the results are given in Table 7.

| | time (s) | $x^p$ | $x^w$ | $x^b$ | Cost |
|---|---|---|---|---|---|
| $\bar{\delta} = 200,\ H = 10$ | 3540 | 46 | 64 | 470 | 52088 |
| $\bar{\delta} = 200\ H = 50$ | 900 | 47 | 64 | 478 | 52620 |
| $\bar{\delta} = 200\ H = 100$ | 600 | 46 | 65 | 478 | 52735 |
| $\bar{\delta} = 300\ H = 10$ | 5998 | 46 | 64 | 474 | 52568 |
| $\bar{\delta} = 300\ H = 50$ | 2460 | 47 | 64 | 481 | 53066 |
| $\bar{\delta} = 300\ H = 100$ | 2160 | 46 | 65 | 485 | 53205 |

Table 7: Montana ($\Delta_t = 10\% D_t$). Effect of a bound on the oscillation level.

The costs are somewhat lower than the previous ones but there is no significant differences. In fact, for the main part of the results obtained in Section 7, the worst scenarios defined by $\delta_t$ were obtained by sequels of 1 followed by sequels of 0, which can be explained as follows: firstly, a worst scenario corresponds to $\delta_t = 1$ when the demand at $t$ is high (and the production is low), and an analysis of the data show that there are generally several consecutive such time periods; secondly, the recourse cost increases when the battery is empty and this occurs after several periods of high demands. Moreover, the computing time is much greater with these new constraints. In fact, when $H$ increases, the problem is less constrained, which can explained that the computing time decreases while the cost increases.

We also tested the model with additional constraints ($a, b, c, d$) but in this case the computing time becomes very high (several hours) and there is no significant differences in the obtained solutions.

# 9 Conclusion

We studied a robust two-stage mixed integer optimization problem associated to the design of an hybrid energy system. This problem is characterized by the basic assumptions made in [3] or [6], i.e. a linear model of the problem without uncertainty, a smooth definition of uncertainty with no given distribution or mean values of the data, an objective which is to minimize the cost of a worst scenario, mixed integer first stage variables and continuous second stage variables. We proved that in this special case Dynamic Programming can be used to solve the recourse problem. Our results could be applied to other problems of the same type as inventory management problems.

The method allows to take into account sliding time window constraints between the variables defining the uncertainty. However, for our problem the dynamic programming approach fails to take into account these new constraints because of the great number of time periods (8760 ). Nevertheless, it could probably be used for smaller problems.

The next objective for solving robust two-stage mixed integer linear optimization problems will be to build an efficient method for integer recourse variables: that is a real challenge for operational research specialists.

# Annex 1. Proof of Proposition 2

First, we give the proof for $\zeta = 0$:

**Proposition 3.** *If $\zeta = 0$ then for for any $\tau,\ 1 \leq \tau \leq T$ and any $\beta \in [0, K]$,*

- *there are $B \geq 0$ and $C \geq 0$ s.t. $v(\tau, \zeta, \beta) = C - \gamma C^g \min(\beta, B)$ (11),*
- *$B = \frac{v(\tau,\zeta,0)-v(\tau,\zeta,K)}{\gamma C^g}$ and $C = v(\tau, \zeta, 0)$ is a solution of (11).*

*Proof.* The problem reduces to a problem without uncertainty: if $\zeta = 0$ then $\delta_t = 0$ for all $t = \tau, ..., T$. We first prove that for each $\tau \in \{1, ..., T\}$ there are two constants $B$ and $C$ such that: $v(\tau, 0, \beta) = C - \gamma C^g \min(\beta, B)$ (12), then we verify the expression of $B$ and $C$. From the recurrence relation (12); we have, for $\tau < T$: $v(\tau, 0, \beta) = C^g g(\hat{D}_\tau, \beta) + v(\tau + 1, 0, f(\hat{D}_\tau, \beta))$. We then proceed by induction from $T$ to 1.

- Step 1: $\tau = T$, $v(T, 0, \beta) = C^g g(\hat{D}_T, \beta)$.
  If $\hat{D}_T < 0$, then $v(T, 0, \beta) = 0$ ($B_1 = 0, C_1 = 0$).
  Else $v(T, 0, \beta) = C^g \hat{D}_T - \gamma C^g \min\left(\beta, E^{out}, \frac{\hat{D}_T}{\gamma}\right)$ ($B_1' = \min\left(E^{out}, \frac{\hat{D}_T}{\gamma}\right)$, $C_1' = C^g \hat{D}_T$. )
- Step 2: $\tau = \bar{\tau} + 1$.
  Assume there are $B_2 \geq 0$ and $C_2 \geq 0$ such that

$$v(\bar{\tau} + 1, 0, \beta) = C_2 - \gamma C^g \min(\beta, B_2).$$

- Step 3: $\tau = \bar{\tau}$,

$$v(\bar{\tau}, 0, \beta) = C^g g(\hat{D}_{\bar{\tau}}, \beta) + C_2 - \gamma C^g \min(f(\hat{D}_{\bar{\tau}}, \beta), B_2).$$

  - If $\hat{D}_{\bar{\tau}} \geq 0$, we have:

$$\begin{aligned} v(\bar{\tau}, 0, \beta) &= C^g \hat{D}_{\bar{\tau}} - \gamma C^g \min\left(\beta, E^{out}, \hat{D}_{\bar{\tau}}/\gamma\right) \\ &\quad + C_2 - \gamma C^g \min\left(\max\left(\beta - \hat{D}_{\bar{\tau}}/\gamma, \beta - E^{out}, 0\right), B_2\right) \\ &= (C^g \hat{D}_{\bar{\tau}} + C_2) \\ &\quad - \gamma C^g \left(\min\left(\beta, E^{out}, \hat{D}_{\bar{\tau}}/\gamma\right) + \min\left(\max\left(\beta - \hat{D}_{\bar{\tau}}/\gamma, \beta - E^{out}, 0\right), B_2\right)\right) \end{aligned}$$

    By considering the three cases $\beta \leq \min\left(E^{out}, \frac{\hat{D}_{\bar{\tau}}}{\gamma}\right)$, $\min\left(E^{out}, \frac{\hat{D}_{\bar{\tau}}}{\gamma}\right) < \beta \leq B_2 + \min\left(E^{out}, \frac{\hat{D}_{\bar{\tau}}}{\gamma}\right)$, and $B_2 + \min\left(E^{out}, \frac{\hat{D}_{\bar{\tau}}}{\gamma}\right) < \beta$, it is easy to verify that:

$$\begin{aligned} v(\bar{\tau}, 0, \beta) &= (C^g \hat{D}_{\bar{\tau}} + C_2) - \gamma C^g \min\left(\beta, B_2 + \min\left(E^{out}, \hat{D}_{\bar{\tau}}/\gamma\right)\right) \\ &= C_3 - \gamma C^g \min(\beta, B_3) \end{aligned}$$

with $B_3 = \min\left(B_2 + \min\left(E^{out}, \frac{\hat{D}_{\bar{\tau}}}{\gamma}\right), K\right)$ (we can choose $B_3 \leq K$ since $\beta \leq K$) and $C_3 = (C^g \hat{D}_{\bar{\tau}} + C_2)$.

– If $\hat{D}_{\bar{\tau}} < 0$, then:

$$\begin{aligned} v(\bar{\tau}, 0, \beta) &= 0 + C_2 - \gamma C^g \min\left(\min(\beta - \hat{D}_{\bar{\tau}}, \beta + E^{in}, K), B_2\right) \\ &= C_2 - \gamma C^g \min(\beta - \hat{D}_{\bar{\tau}}, \beta + E^{in}, K, B_2) \\ &= C_2 - \gamma C^g \min(\beta + \min(-\hat{D}_{\bar{\tau}}, E^{in}), \min(K, B_2)) \\ &= (C_2 - \gamma C^g \min(-\hat{D}_{\bar{\tau}}, E^{in})) - \gamma C^g(\beta, \min(K, B_2) - \min(-\hat{D}_{\bar{\tau}}, E^{in})) \\ &= C'_3 - \gamma C^g \min(\beta, B'_3) \end{aligned}$$

$$\text{with} \left\{ \begin{array}{l} B'_3 = \min(K, B_2) - \min(-\hat{D}_{\bar{\tau}}, E^{in}) \text{ and } C'_3 = C_2 - \gamma C^g \min(-\hat{D}_{\bar{\tau}}, E^{in}) \\ \text{if } \min(K, B_2) \geq \min(-\hat{D}_{\bar{\tau}}, E^{in}) \\ B'_3 = 0 \text{ and } C'_3 = C_2 - \gamma C^g \min(-\hat{D}_{\bar{\tau}}, E^{in}) - \gamma C^g(\min(K, B_2) - \min(-\hat{D}_{\bar{\tau}}, E \\ \text{if } \min(K, B_2) < \min(-\hat{D}_{\bar{\tau}}, E^{in}) \end{array} \right.$$

Now, let us verify the expression of $B$ and $C$. Taking $\beta = 0$ in (2) gives $C = v(\bar{\tau}, 0, 0)$. Furthermore, $v(\bar{\tau}, 0, B) = v(\bar{\tau}, 0, 0) - \gamma C^g B = v(\bar{\tau}, 0, K)$, and we have

$$B = \frac{v(\bar{\tau}, 0, 0) - v(\bar{\tau}, 0, K)}{\gamma C^g}.$$

The proof if $\zeta = T - \tau + 1$ is similar and is not given here. □

The proof for $\zeta = T - \tau + 1$ is very similar and is not given here. Just notice that $\delta_t = 1$ for all $t$. We now prove the proposition for any value of $(\zeta)$.

**Proposition 3.** *For any $\tau$, $1 \leq \tau \leq T$, any $\zeta, 0 \leq \zeta \leq T - \tau + 1$, and any $\beta \in [0, K]$,*

- *there are $B \geq 0$ and $C \geq 0$ s.t. $v(\tau, \zeta, \beta) = C - \gamma C^g \min(\beta, B)$ (11),*
- *$B = \frac{v(\tau,\zeta,0) - v(\tau,\zeta,K)}{\gamma C^g}$ and $C = v(\tau, \zeta, 0)$ is a solution of (11).*

*Proof.* We prove the proposition by recurrence on $(\tau, \zeta)$.

- Step 1: from Proposition 3, The proposition is true for any $\tau$ if $\zeta = 0$ or $\zeta = T - \tau + 1$ (i.e. for $(\tau, \zeta) \in \{(T, 0), (T-1, 0), ..., (1, 0), (T, 1), (T-1, 2), ..., (T-i, i+1), ..., (T-\bar{\delta}-1, \bar{\delta})\}$)

- Step 2: assume it is true for $(\tau+1, \bar{\zeta}-1)$ and for $(\tau+1, \bar{\zeta})$, we have

$$\forall \beta \in [0, K], v(\tau+1, \bar{\zeta}-1, \beta) = C_2 - \gamma C^g \min(\beta, B_2),$$
$$v(\tau+1, \bar{\zeta}, \beta) = C_2' - \gamma C^g \min(\beta, B_2'),$$

- Step 3: from the recurrence relation (1) we have:

$$\begin{aligned} v(\tau, \bar{\zeta}, \beta) = \max(&C^g g(\hat{D}_\tau, \beta) + v(\tau+1, \bar{\zeta}, f(\hat{D}_\tau, \beta)), \\ &C^g g(\hat{D}_\tau + \Delta_\tau, \beta) + v(\tau+1, \bar{\zeta}-1, f(\hat{D}_\tau + \Delta_\tau, \beta))). \end{aligned}$$

Let $Q = C^g g(\hat{D}_\tau, \beta) + v(\tau+1, \bar{\zeta}, f(\hat{D}_\tau, \beta))$.

  - If $\hat{D}_\tau \geq 0$:

$$\begin{aligned} Q &= C^g \hat{D}_\tau - \gamma C^g \min\left(\beta, E^{out}, \hat{D}_\tau/\gamma\right) \\ &\quad + C_2' - \gamma C^g \min\left(\max\left(\beta - \hat{D}_\tau/\gamma, \beta - E^{out}, 0\right), B_2'\right) \\ &= (C^g \hat{D}_\tau + C_2') \\ &\quad - \gamma C^g \left[\min\left(\beta, E^{out}, \hat{D}_\tau/\gamma\right) + \min\left(\max\left(\beta - \hat{D}_\tau/\gamma, \beta - E^{out}, 0\right), B_2'\right)\right] \end{aligned}$$

    As in the proof of Proposition 3, we get

$$\begin{aligned} Q &= (C^g \hat{D}_\tau + C_2') - \gamma C^g \min\left(\beta, B_2' + \min\left(E^{out}, \hat{D}_\tau/\gamma\right)\right) \\ &= C_3' - \gamma C^g \min(\beta, B_3') \end{aligned}$$

    with $B_3' = \min\left(B_2' + \min\left(E^{out}, \frac{\hat{D}_\tau}{\gamma}\right), K\right)$ and $C_3' = (C^g \hat{D}_\tau + C_2')$.

  - If $\hat{D}_\tau < 0$, then:

$$\begin{aligned} Q &= 0 + C_2' - \gamma C^g \min\left(\min(\beta - \hat{D}_\tau, \beta + E^{in}, K), B_2'\right) \\ &= C_2' - \gamma C^g \min(\beta - \hat{D}_\tau, \beta + E^{in}, K, B_2') \\ &= C_2 - \gamma C^g \min(\beta + \min(-\hat{D}_\tau, E^{in}), \min(K, B_2)) \\ &= C_3' - \gamma C^g \min(\beta, B_3') \end{aligned}$$

$$\text{with} \begin{cases} B_3' = \min(K, B_2') - \min(-\hat{D}_\tau, E^{in}) \text{ and } C_3' = C_2' - \gamma C^g \min(-\hat{D}_\tau, E^{in}) \\ \text{if } \min(K, B_2') \geq \min(-\hat{D}_\tau, E^{in}) \\ B_3' = 0 \text{ and } C_3' = C_2' - \gamma C^g \min(-\hat{D}_\tau, E^{in}) - \gamma C^g(\min(K, B_2') - \min(-\hat{D}_\tau, E \\ \text{if } \min(K, B_2') < \min(-\hat{D}_\tau, E^{in}) \end{cases}$$

Let $R = C^g g(\hat{D}_\tau + \Delta_\tau, \beta) + v(\tau + 1, \bar{\zeta} - 1, f(\hat{D}_\tau + \Delta_\tau, \beta))$. We prove similarly that there exists $C_3" \geq 0$, and $B_3" \geq 0$, such that

$$R = C_3" - \gamma C^g \min(\beta, B_3").$$

Therefore,

$$v(\tau, \bar{\zeta}, \beta) = \max(C_3' - \gamma C^g \min(\beta, B_3'), C_3" - \gamma C^g \min(\beta, B_3")).$$

By considering the four cases: $C_3' < C"_3$ and $B_3' < B"_3$; $C_3' < C"_3$ and $B_3' \geq B"_3$; $C_3' \geq C"_3$ and $B_3' < B"_3$; $C_3' \geq C"_3$ and $B_3' \geq B"_3$, it is easy to verify that there is $B_3,\ B_3 \in [B_3', B"_3]$, such that:

$$v(\tau, \bar{\zeta}, \beta) = C_3 - \gamma C^g \min(\beta, B_3),$$

with $C_3 = \max(C_3', C"_3)$. we conclude the recurrence referring to the scheme shown on figure 2. Indeed, by proposition 3, the proposition is true for $(\tau, \zeta) \in \{(T, 0), (T, 1)\}$, and it is easy to verify that if the proposition is true for $(\tau, \zeta) \in \{(\bar{\tau}+1, 0), (\bar{\tau}+1, 1), ..., (\bar{\tau}+1, T-(\bar{\tau}+1)+1)\}$, then it is true for $(\tau, \zeta) \in \{(\bar{\tau}, 0), (\bar{\tau}, 1), ..., (\bar{\tau}, T-\bar{\tau})\}$ and for $(\tau, \zeta) = (\bar{\tau}, T-\bar{\tau}+1)$, by proposition 3.

□

# Annex 2. Complexity of the general problem without uncertainty

Let us consider a general model where there are $n$ sources of renewable energy. We will call $LP_{gen}$ the resulting model. In this case, one unit of source $i$ costs $C^i$ and produces a quantity $E_t^i$ of energy during period $t$, $t = 1, ..., T$. The objective function becomes $\sum_{i=1}^n C^i x^i + C^b x^b + C^g \sum_{t=1}^T e_t^g$. Constraint (1) becomes $\sum_{i=1}^n E_t^i x^i - e_t^{in} + \gamma e_t^{out} + e_t^g \geq D_t, t = 1...T$ and constraints (6)-(7) are replaced by $x^i \leq N_{max}^i, i = 1, ..., n$. Constraints (2)-(5), (8) and (10) are retained.

We are going to show that this generalized problem is NP-hard. Thus, there is no polynomial time algorithm solving $LP_{gen}$ unless $P = NP$.

**Proposition 4.** *$LP_{gen}$ belongs to the class of NP-hard problems.*

*Proof.* We will show that the bounded knapsack problem $BKP$ reduces to $LP_{gen}$. Consider the decision problem $DBKP$ associated with $BKP$:

$$DBKP \begin{cases} \textit{Data: } n, a_1, a_2, ..., a_{n+1}, c_1, c_2, ..c_{n+1}, u_1, u_2, ..., u_{n+1}, c, V \text{ in } \mathbb{N} \\ \textit{Question: } \text{Is there a vector } (y_1, ..., y_{n+1}) \in \mathbb{N}^{n+1} \text{ such that} \\ \sum_{i=1}^{n+1} a^i y^i \geq V,\ \sum_{i=1}^{n+1} c^i y^i \leq c \text{ and } y_i \leq u_i,\ i = 1, ..., n+1. \end{cases}$$

Now we define the decision problem $DLP_{gen}$ associated with $LP_{gen}$:

$$DLP_{gen} \begin{cases} \textit{Data: } n, T, C, C^b, C^g, E^{in}, E^{out}, e_0^b, \gamma, K, N_{max}^b, D_t, C^i, E_t^i, N_{max}^i, \\ t = 1, ..., T, i = 1, ..., n. \\ \textit{Question: } \text{Is there a vector } (x^1, ..., x^n) \in \mathbb{N}^n, \text{ an integer } x^b, \text{ and} \\ \text{nonnegative reals } e_t^{in}, e_t^{out}, e_t^b, e_t^g, t = 1, ..., T, \text{ satisfying the} \\ \text{constraints of } LP_{gen} \text{ and such that the value of the objective} \\ \text{function is less than or equal to } C? \end{cases}$$

From an instance of $DBKP$, let us construct the following instance of $DLP_{gen}$: $T = 1$; there are $n$ sources of energy with $C^i = c_i$, $E_1^i = a_i$ and $N_{max}^i = u_i$ for $i = 1, ..., n$; $C^b = c_{n+1}$, $K = a_{n+1}$, $N_{max}^b = u_{n+1}$, $E^{in} = E^{out} = e_0^b = a_{n+1}$, $\gamma = 1$; $C^g = 2c$; $D_1 = V$; $C = c$.

Notice that we can set $e_1^{in} = 0$ and $e_1^{out} = a_{n+1}$, which results in $e_1^b = 0$ from constraint (5). Then constraints (2), (3), (4) and (5) are verified and the remaining problem is to determine if there are $(x^1, ..., x^n) \in \mathbb{N}^n$, $x^b \in \mathbb{N}$, $e_1^g \in \mathbb{R}^+$ such that the following set of constraints is satisfied :

$$\left| \begin{array}{l} \sum_{i=1}^{n} c_i x^i + c_{n+1} x^b + 2c e_1^g \leq c \quad (**) \\ \sum_{i=1}^{n} a_i x^i + a_{n+1} x^b + e_1^g \geq V \quad (*) \\ x^i \leq u_i,\ i = 1, ..., n \\ x^b \leq u_{n+1} \\ x^b, x^i (i = 1, ..., n) \in \mathbb{N}, e_1^g \in \mathbb{R}^+ . \end{array} \right.$$

Let $y_1, y_2, ..., y_{n+1}$ be a solution of $DBKP$. It is clear that $x^i = y_i$ for $i = 1, ..., n$, $x^b = y_{n+1}$ and $e_1^g = 0$ is a solution of $DLP_{gen}$.

Conversely, let $((x^i)_{1 \leq i \leq n}, x^b, e_1^g)$ be a solution of $DLP_{gen}$, i.e. of the above set of constraints. According to constraint (*), $\sum_{i=1}^n a_i x^i + a_{n+1} x^b \geq V - e_1^g$ and, according to constraint (**), $e_1^g \leq 1/2$; then we have $\sum_{i=1}^n a_i x^i + a_{n+1} x^b \geq V - 1/2$. Since the quantities $\sum_{i=1}^n a_i x^i + a_{n+1} x^b$ and $V$ are integers, we finally obtain:

$\sum_{i=1}^{n} a_i x^i + a_{n+1} x^b \geq V$. Thus, the vector $y$ defined by $y_i = x^i$ for $i = 1, ..., n$, $y_{n+1} = x^b$ is a solution of $DBKP$, and the two problems are equivalent.

Since $DBKP$ is NP-complete [8], we have proved that the decision problem $DLP_{gen}$ associated with the generalized problem $LP_{gen}$ is also NP-complete.

□


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