Toward variational data assimilation for coupled models: first experiments on a diffusion problem
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3. Toward a coupled variational data assimilation

If the DA process is done separately on each subdomain, the initial condition $x_0 = \{x^{(1)}_0, x^{(2)}_0\}_\Omega$ obtained on $\Omega$ does not satisfy the interface conditions. The interface imbalance in the initial condition can severely damage the forecast skills of coupled models (Mauliand et al. 2015).

Objective: properly take into account the coupling in the assimilation process

Full Iterative Method (FIM)
- $x_0 = x^{(i)}_0, i \in \Omega$
- We iterate the models till convergence of the Schwarz algorithm ($\alpha^{(k)}$ iterations)
- The first guess $x^{(1)}_0$ in (1) is updated after each minimization iteration

\[ J_{\text{FIM}}(x_k) = J^{(1)}(x_0) + \int_\Omega \left( y - \mathcal{H}(x^{(1)}_k) \right)^2 d\Omega \]  

Truncated Iterative Method (TIM)
- $x_0 = \{x^{(1)}_0, x^{(2)}_0\}_\Omega$
- The Schwarz iterations are truncated at $k_{\text{max}}$ iterations
- Extended cost function (mismatch in the interface conditions) (Gagdol and Monnier 2007)

\[ J_{\text{TIM}}(x_k) = J^{(1)}(x_0) + \int_\Omega \left( y - \mathcal{H}(x^{(1)}_k) \right)^2 d\Omega + J^{\text{int}}(x_k) \]  

Coupled Assimilation Method with Uncoupled models (CAMU)
- $x_0 = \{x^{(1)}_0, x^{(2)}_0\}_\Omega$ with $x^{(1)}_0 = \{x^{(1)}_0\}_\Omega$
- We suppress the coupling between both models

The cost function for the CAMU is

\[ J_{\text{CAMU}}(x_k) = \frac{1}{2} \left( J^{(1)}(x_k) + J^{(2)}(x_k) \right) + J^{\text{int}}(x_k) \]  

The originality of these algorithms is the use of a Schwarz algorithm to couple our models jointly to the DA process with an extended cost function.

4. Application to a 1D diffusion problem

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4. Application to a 1D diffusion problem

Previous algorithms are applied on a 1D linear diffusion problem. We consider:
- $L_j = \alpha_j \frac{\partial^2}{\partial x^2} + \beta_j \frac{\partial}{\partial x} + \gamma_j$ for $j \in \{1, 2\}$
- $f_j$ is the diffusion coefficients in each subdomain
- $F_j = \{f_j, \beta_j, \gamma_j\}$ and $I_j$ is the interface operators on $\Gamma$ (Dirichlet-Neumann)
- $x_j(t), f = \{a + \cos(\omega t)\}$ on $\Omega_j \times T_p$, the analytical solution

Single column observation experiment:
- Observations are available in $(\Omega_j \setminus I_j)$ at the end of the time-window (i.e. at $t = T$)
- We define the interface imbalance indicator, equal to $J^{\text{int}}$ with $\alpha^{(1)} = 0.01$ and $\alpha^{(2)} = 0.001$

Acknowledgments

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