Toward variational data assimilation for coupled models: first experiments on a diffusion problem
Rémi Pellerej, Arthur Vidard, Florian Lemarié

To cite this version:
Rémi Pellerej, Arthur Vidard, Florian Lemarié. Toward variational data assimilation for coupled models: first experiments on a diffusion problem. ISDA 2016, Jul 2016, Reading, United Kingdom. 2016. <hal-01412165>

HAL Id: hal-01412165
https://hal.archives-ouvertes.fr/hal-01412165
Submitted on 8 Dec 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Toward variational data assimilation for coupled models: first experiments on a diffusion problem

Rémi Pellerej, Arthur Vidard and Florian Lemarié

remi.pellerej@inria.fr

Inria, Univ. Grenoble-Alpes, CNRS, Laboratoire Jean Kuntzmann, F-38000 Grenoble

1. Model problem and coupling strategy

Let us define two models on each space-time domain \( \Omega = [0, 1] \) with a common interface \( \Gamma = \{ x = 0 \} \).

Problem: How to strongly couple the two models at the interface \( \Gamma \)?

We propose to use a global-in-time Schwarz algorithm (Gander [2006]).

For a given initial condition \( u_0 \in H^1(\Omega) \), the coupling algorithm reads

\[
\begin{align*}
F_{\alpha}(u_0) = f_1 & \quad \text{on } \Omega \setminus \{ x = 0 \} \\
G_{\alpha}(u_0) = 0 & \quad \text{on } \Gamma \\
\end{align*}
\]

The convergence speed of the method greatly depends on the choice for \( F_{\alpha} \) and \( G_{\alpha} \), and the choice of the first-guess

2. Classic data assimilation

Let us introduce the classic cost function for variational data assimilation in the uncoupled case, for a domain \( \Omega \):

\[
\begin{align*}
J_{\alpha}(u_0) & = \frac{1}{2} \int_{\Omega} (u_0 - \hat{u}(x))^2 \, dx \\
& = \frac{1}{2} \int_{\Omega} (u_0 - \hat{u}(x))^2 \, dx + \frac{\alpha}{2} \int_{\Gamma} (u_0 - \hat{u}(x))^2 \, dx
\end{align*}
\]

where \( \frac{1}{2} \) is the usual Euclidean inner product on a spatial domain \( \Sigma \).

3. Toward a coupled variational data assimilation

If the DA process is done separately on each subdomain, the initial condition \( u_0 = (u_0^1, u_0^2)^T \) obtained on \( \Gamma \) does not satisfy the interface conditions. The interface imbalance in the initial condition can severely damage the forecast skills of coupled models (Muñoz et al. [2015]).

Objective: properly take into account the coupling in the assimilation process

Full Iterative Method (FIM):

- Initial guess \( u_0 \) in \( \Omega \)
- We iterate the models till convergence of the Schwarz algorithm (\( k_{\text{max}} \) iterations)
- The first guess \( u_0 \) in \( \Omega \) is updated after each minimization iteration

\[
J_{\alpha}(u_0) = J_{\alpha}(k_{\text{max}}) + \frac{1}{2} \int_{\Gamma} (u_0 - \hat{u}(x))^2 \, dx + \frac{\alpha}{2} \int_{\Gamma} (u_0 - \hat{u}(x))^2 \, dx
\]

Truncated Iterative Method (TIM):

- The Schwarz iterations are truncated at \( k_{\text{max}} \) iterations
- Extended cost function (motivated by the interface conditions) (Gajjola and Mariner [2007])

\[
J_{\alpha}(u_0) = J_{\alpha}(k_{\text{max}}) + \frac{1}{2} \int_{\Gamma} (u_0 - \hat{u}(x))^2 \, dx + \frac{\alpha}{2} \int_{\Gamma} (u_0 - \hat{u}(x))^2 \, dx
\]

Coupled Assimilation Method with Uncoupled models (CAMU):

- We suppress the coupling between both models
- The cost function for the CAMU is

\[
J_{\alpha,G}(u_0) = \frac{1}{2} \int_{\Omega} (u_0 - \hat{u}(x))^2 \, dx + \frac{\alpha}{2} \int_{\Gamma} (u_0 - \hat{u}(x))^2 \, dx
\]

The originality of this algorithm is the use of a Schwarz algorithm to couple our models jointly to the DA process with an extended cost function.

4. Application to a 1D diffusion problem

Previous algorithms are applied on a 1D linear diffusion problem. We consider:

\[
\begin{align*}
\mathcal{L} u_1 &= \partial_t u_1 + \partial_x u_1 \\
\mathcal{L} u_2 &= \partial_t u_2 + \partial_x u_2
\end{align*}
\]

and the diffusion coefficients in each subdomain

\[
\mathcal{L} u_i = \partial_t u_i + \partial_x u_i
\]

The Schwarz iterations are truncated at \( k_{\text{max}} \) iterations

- We define the interface imbalance indicator, equal to \( J_T \) with \( u_T = 0.01 \) and \( \alpha_T = 40 \)

5. Single column observation experiment results

Table 1: Overview of the properties of the coupled variational DA methods described

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Control vector</th>
<th># of coupling iterations</th>
<th>Extended cost function</th>
<th>Adapt of the coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIM</td>
<td>( u_0 )</td>
<td>k_{\text{max}}</td>
<td>no</td>
<td>strong</td>
</tr>
<tr>
<td>TIM</td>
<td>( u_0 )</td>
<td>k_{\text{max}}</td>
<td>yes</td>
<td>strong</td>
</tr>
<tr>
<td>CAMU</td>
<td>( u_0 )</td>
<td>k_{\text{max}}</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

References


