Toward variational data assimilation for coupled models: first experiments on a diffusion problem
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Context
Ocean-atmosphere coupled models have a key role in weather forecast nowadays.

The coupling methods may severely modify the model solution. An exact solution of the coupling problem can be obtained using a Global-in-Time Schwarz method (Lemarié et al. 2014).

The initialisation of coupled models also has a major impact on the forecast solution (Mullholland et al. 2015).

Few coupled DA methods started to be developed (Smith et al., 2015; Lataux et al., 2015...), for coupled systems, and showed promising results.

Our approach
The dynamical equations of our system are coupled using an iterative Schwarz domain decomposition method (Gander, 2008).

We are using variational DA techniques, which require minimization iterations and we are looking to take benefit of the minimization iterations to converge toward the exact solution of the coupling problem: the minimizations iterations substitute the Schwarz iterations.

Three general variational DA algorithms, are presented here and applied to a simple case: the analytical solution of the problem is known.

1. Model problem and coupling strategy

Let us define two models on each space-time domain \( \Omega_a \times [0,T] \) \((a=1,2)\), with a common interface \( \Gamma = \{ x = 0 \} \).

Problem: How to strongly couple the two models at their interface \( \Gamma \) ?

We propose to use a global-in-time Schwarz algorithm (Gander, 2008).

For a given initial condition \( u_0 \in L^2(\Omega_a \times T) \) and first-guess \( \varphi_0(x,t) \), the coupling algorithm reads

\[
\begin{align*}
L_{a2}u_2 + J_{a1}(u_2) & = f_2 & \text{on } & \Omega_2 \times T \\
L_{12}u_1 + J_{21}(u_1) & = f_2 & \text{on } & \Omega_1 \times T \\
L_{11}u_1 + J_{12}(u_1) & = f_1 & \text{on } & \Gamma \\
J_{21}(u_2) & = J_{12}(u_1) & \text{on } & \Gamma
\end{align*}
\]

\( J_{a1}(u_a) \) and \( J_{a2}(u_a) \) are the interface operators, \( k \) is the iteration number, \( T_{a2} = \{ (x,t) \in \Omega_a \times T \mid x \in \Gamma \} \)

At convergence, this algorithm provides a mathematically strongly coupled solution which satisfies \( J_{21}(u_2) = J_{12}(u_1) \) on \( \Gamma \).

The convergence speed of the method greatly depends on the choice for \( J_{a1} \), \( J_{a2} \) operators, and the choice of the first-guess.

2. Classic data assimilation

Let us introduce the classic cost function for variational data assimilation in the uncoupled case, for a domain \( \Omega \)

\[
\mathcal{J} = \int_{\Omega} \left( \frac{1}{2} (u_0 - u(x,t))^2 + \int_0^T \left( \frac{1}{2} (u_a - u(x,t))^2 + R \left( J_{a1}(u_a) - J_a(u_a) \right)^2 \right) dt \right) dx
\]

where \( J_a \) is the usual Euclidean inner product on a spatial domain \( \Omega \).

3. Toward a coupled variational data assimilation

If the DA process is done separately on each subdomain, the initial condition \( u_0 \) obtained on \( \Omega \) does not satisfy the interface conditions. The interface imbalance in the initial condition can severely damage the forecast skills of coupled models (Mullholland et al. 2015).

Objective: properly take into account the coupling in the assimilation process.

Full Iterative Method (FIM)

\( \mathcal{J}_a = \int_{\Omega_a} \left( \frac{1}{2} (u_a - u(x,t))^2 + R \left( J_{a1}(u_a) - J_a(u_a) \right)^2 \right) dx \)

The first-guess \( u_1 \) in (1) is updated after each minimization iteration

\[
J_{FIM}(u_1) = J_a(u_1) + \int_0^T \left( y - H (x) \dot{y} - H (x(t)) \right)^2 dt
\]

Truncated Iterative Method (TIM)

\( \mathcal{J}_a = \int_{\Omega_a} \left( \frac{1}{2} (u_a - u(x,t))^2 + R \left( J_{a1}(u_a) - J_a(u_a) \right)^2 \right) dx \)

The Schwarz iterations are truncated at \( k_{max} \) iterations.

Extended cost function (missfit in the interface conditions) (Gajda and Markov 2007)

\[
J_{TIM}(u_1) = J_a(u_1) + \int_0^T \left( y - H (x) \dot{y} - H (x(t)) \right)^2 dt + \int_0^T \left( y(t) - H(x(t)) \right)^2 dt
\]

Coupled Assimilation Method with Uncoupled models (CAMU)

\( \mathcal{J}_a = \int_{\Omega_a} \left( \frac{1}{2} (u_a - u(x,t))^2 + R \left( J_{a1}(u_a) - J_a(u_a) \right)^2 \right) dx \)

We suppress the coupling between both models

The cost function for the CAMU is

\[
\mathcal{J}_{CAMU}(u_1) = \int_0^T \left( J_{FIM}(u_1) + J_{FIM}(u_2) \right) dt
\]

The originality of these algorithms is the use of a Schwarz algorithm to couple our models jointly to the DA process with an extended cost function.

4. Application to a 1D diffusion problem

Previous algorithms are applied on a 1D linear diffusion problem. We consider:

\[
\begin{align*}
L_1 & = \partial_t u_1 - \nabla \cdot (K_1 \nabla u_1) \\
L_2 & = \partial_t u_2 - \nabla \cdot (K_2 \nabla u_2) \\
J_{12} & = \int_{\Gamma} \left( \frac{1}{2} (u_1 - u_2)^2 + R \left( J_{12}(u_1) - J_{12}(u_2) \right)^2 \right) dx \int_0^T \left( y - H (x) \dot{y} - H (x(t)) \right)^2 dt
\end{align*}
\]

Single column observation experiment:

Observations are available in \( \{ x, t \} \) at the end of the time-window (i.e. \( t = T \)).

We define the interface imbalance indicator, equal to \( J_{12}(x,T) \) with \( u_1 = 0.01 \) and \( u_2 = 0.02 \).

Table 1: Overview of the properties of the coupled variational DA methods described

5. Single column observation experiment results

Table

6. Conclusions and perspectives

In the framework of an iterative coupling, we set up few data assimilation algorithms.

- Adding a physical constraint on the interface conditions in the cost function can have a beneficial effect on the performance of the method and allow to save coupling iterations.
- An approach which only requires the adjoint of each individual model but not the adjoint of the coupling showed promising results.
- The methods are very sensitive to the parameters choices.
- We only test the algorithms on a simple linear problem.

Perspectives

- Algorithm convergence and conditioning problem when \( J_{21} \) is part of the cost function will be studied.
- Since the objective is to apply such methods to ocean-atmosphere coupled models, increasingly complex models including physical parameterisations for sub-scales, and non-linearities will be considered.

References


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