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Toward variational data assimilation for coupled models: first experiments on a diffusion problem

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Context

Ocean-atmosphere coupled models have a key role in weather forecast nowadays.
• The coupling methods may severely modify the model solution. An exact solution of the coupling problem can be obtained using a Global-in-Time Schwarz method (Lemarié et al. 2014).
• The initialisation of coupled models also has a major impact on the forecast solution (Mutihardjo et al. 2015).
• Few coupled DA methods started to be developed (Smith et al. 2015, Laloyaux et al. 2015…)

Our approach

• The dynamical equations of our system are coupled using an iterative Schwarz domain decomposition method (Gander 2008).
• We are using variational DA techniques, which require minimization iterations and we are looking to benefit of the minimization iterations to converge toward the exact solution of the coupling problem: the minimisations iterations substitute the Schwarz iterations.
• Three general variational DA algorithms, are presented here and applied to a simple coupled system (Pelaez et al. 2016).

1. Model problem and coupling strategy

Let us define two models on each space-time domain \( (x, t) \to \{ d = 1 \} \), with a common interface \( \Gamma = \{ x = 0 \} \).

Problem: How to strongly couple the two models at their interface \( \Gamma \) ?

→ We propose to use a global-in-time Schwarz algorithm (Gander 2008).

For a given initial condition \( u_0 \in L^2(\Omega_0) \) and first-guess \( \tilde{u}(i, 0) \), the coupling algorithm reads

\[
L_{\Gamma_{d2}} \tilde{u}_{d2} = f_{d2} \quad \text{on} \quad \Gamma_{d2} \cap \Gamma_{d1} \quad \text{and} \quad L_{\Gamma_{d1}} \tilde{u}_{d1} = f_{d1} \quad \text{on} \quad \Gamma_{d1} \cap \Gamma_{d2}
\]

\[
J_{\Gamma_{d2}} \tilde{u}_{d2} \quad \text{and} \quad L_{\Gamma_{d1}} \tilde{u}_{d1}
\]

\( \mathcal{J} \) and \( \mathcal{I} \) are the interface operators, \( k \) is the iteration number, \( \Gamma_{d2} = \{ x = 0 \} \), and \( \tilde{u}_d \) is in \( L^2(\Omega_2) \), \( L^2(\Omega_1) \), and \( L^2(\Omega) \).

At convergence, this algorithm provides a mathematically strongly coupled solution which satisfies

\[
J_{\Gamma_{d2}} \tilde{u}_{d2} = J_{\Gamma_{d1}} \tilde{u}_{d1}
\]

\( \tilde{u} \) converges speed of the method greatly depends on the choice for \( \mathcal{J} \) and \( \mathcal{I} \) operators, and the choice of the first-guess.

2. Classic data assimilation

Let us introduce the classic cost function for variational data assimilation in the uncoupled case, for a domain \( \Omega_d \)

\[
u_a = u_a(x, t) - u(x, t)
\]

where \( u_a(x, t) \) is the usual Euclidian inner product on a spatial domain \( \Sigma \).

3. Toward a coupled variational data assimilation

If the DA process is done separately on each subdomain, the initial condition \( u_0 = \{ u_{d1}^0, u_{d2}^0 \} \) obtained on \( \Gamma \) does not satisfy the interface conditions. The interface imbalance in the initial condition can severely damage the forecast skills of coupled models (Mutihardjo et al. 2015).

Objective: properly take into account the coupling in the assimilation process

Full Iterative Method (FIM)

• \( u_0 = u(a, t) \), \( a \in \Omega \)
• We iterate the models till convergence of the Schwarz algorithm \( (k_{\text{max}} \) iterations) \n• The first-guess \( \tilde{u}_d \) in (1) is updated after each minimization iteration

\[
J_{\text{FIM}}(k_d) = J_{\text{FIM}}(k_{d-1}) + \int \left( \nabla - H(\nabla u)^{-1} - \Gamma u - H(\nabla u)^{-1} \right) \nabla \left( u - u_0 \right)
\]

where \( \nabla x u \) is \( \{ u_{d1}^0, u_{d2}^0 \} \) and \( \Gamma = \{ x = 0 \} \).

Truncated Iterative Method (TIM)

• \( u_0 = u(a, t) \), \( a \in \Omega \)
• The Schwarz iterations are truncated at \( k_{\text{max}} \) iterations
• Extended cost function (must fit the interface conditions) (Guigas and Marnier 2007)

\[
J_{\text{TIM}}(k_d) = J_{\text{TIM}}(k_{d-1}) + \int \left( \nabla - H(\nabla u)^{-1} - \Gamma u - H(\nabla u)^{-1} \right) \nabla \left( u - u_0 \right)
\]

Coupled Assimilation Method with Uncoupled models (CAMU)

• \( u_0 = \{ u_{d1}^0, u_{d2}^0 \} \)
• We suppress the coupling between both models
• The cost function for the CAMU is

\[
J_{\text{CAMU}}(k_d) = J_{\text{FIM}}(k_d) + 0.01 \int \left( \nabla - H(\nabla u)^{-1} - \Gamma u - H(\nabla u)^{-1} \right) \nabla \left( u - u_0 \right)
\]

The originality of these algorithms is the use of a Schwarz algorithm to couple our models jointly to the DA process with an extended cost function.

4. Application to a 1D diffusion problem

Previous algorithms are applied on a 1D linear diffusion problem.

\[
\frac{\partial u}{\partial t} = \alpha G \nabla^2 u
\]

\( \alpha G \) and \( \alpha F \) are the diffusion coefficients in each subdomain

\( a = \{ 1, 2 \} \) and \( \mathcal{I} = \{ 1, 2 \} \) the interface operators on \( \Gamma \) (Dirichlet-Neumann)

\( \mathcal{J} \) and \( \mathcal{I} \) the analytical solution

Single column observation experiment:

• Observations are available in \( \Gamma \) (\( \Omega \)) at the end of the time-window (i.e. \( t = T \))

• We define the interface imbalance indicator, equal to \( J_{\text{FIM}} \) with \( \nu_{a1} = 0.01 \) and \( \nu_{a2} = 0 \)

References


Table 1: Overview of the properties of the coupled variational DA methods described

In the framework of an iterative coupling, we set up few data assimilation algorithms.

• Adding a physical constraint on the interface conditions in the cost function can have a beneficial effect on the performance of the method and allow to save coupling iterations.

• An approach which only requires the adjoint of each individual model but not the adjoint of the coupling showed promising results

• The methods are very sensitive to the parameters choices

• We only test the algorithms on a simple linear problem

Perspectives

• Algorithm convergence and conditioning problem when \( J_\mathcal{D} \) is part of the cost function will be studied.

• Since the objective is to apply such methods to ocean-atmosphere coupled models, increasingly complex models including physical parameterisations for subscales and non-linearities will be considered.