Function Call Re-Vectorization
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To cite this version:
Rubens Moreira, Sylvain Collange, Fernando Pereira. Function Call Re-Vectorization. ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP), Feb 2017, Austin, Texas, United States. <hal-01410186>

HAL Id: hal-01410186
https://hal.archives-ouvertes.fr/hal-01410186
Submitted on 8 Dec 2016

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Abstract
Programming languages such as C for CUDA, OpenCL or ISPC have contributed to increase the programmability of SIMD accelerators and graphics processing units. However, these languages still lack the flexibility offered by low-level SIMD programming on explicit vectors. To close this expressiveness gap while preserving performance, this paper introduces the notion of Call Re-Vectorization (CREV). CREV allows changing the dimension of vectorization during the execution of a kernel, exposing it as a nested parallel kernel call. CREV affords programmability close to dynamic parallelism, a feature that allows the invocation of kernels from inside kernels, but at much lower cost. In this paper, we present a formal semantics of CREV, and an implementation of it on the ISPC compiler. We have used CREV to implement some classic algorithms, including string matching, depth first search and Bellman-Ford, with minimum effort. These algorithms, once compiled by ISPC to Intel-based vector instructions, are as fast as state-of-the-art implementations, yet much simpler. Thus, CREV gives developers the elegance of dynamic programming, and the performance of explicit SIMD programming.

Categories and Subject Descriptors D - Software [D.3 Programming Languages]: D.3.4 Processors - Compilers

General Terms Languages, Performance

Keywords SIMD, SIMT, Function, Programmability

1. Introduction
New hardware asks for new programming idioms. As an example, the appearance of general purpose Graphics Processing Units (GPUs) has led to a revolution in programming languages [12, 23], which has culminated in the materialization of languages such as C for CUDA [29], OpenCL [22], ISPC [25] and PyCuda [18]. These Multi-Threaded (MT) languages let programmers express computations as single kernels executed by many threads. They target architectures that combine SIMD and multi-threaded execution, like GPUs and multi-core CPUs with vector instructions.

However, such high-level abstractions come at a cost in flexibility and composability. Most programming languages that target hardware accelerators pack threads into SIMD vectors or GPU warps for the whole duration of a kernel call. They also suspend and resume individual threads to simulate thread-dependent control flow. These constraints affect device-side library functions, which cannot assume any particular organization of parallelism nor thread activity. For instance, invoking full SIMD functions within divergent regions might lead to incorrect behavior. Consequently, library functions often contain two versions of each routine, and dynamically dispatch the proper version depending on whether threads diverge at the call site or not.

On GPUs, developers circumvent the restrictions listed above in two ways: via warp-synchronous programming, or via dynamic parallelism. In the first case, programmers use the knowledge that threads are grouped in warps to achieve thread communication without synchronization or memory sharing. Yet, warp-synchronous programming is not easily composable with classic multi-thread programming. Programmers must ensure that every thread in a warp participates in each collective operation; e.g., the CUDA shfl function has undefined behavior when reading data from an inactive thread. However, multi-thread programming puts thread divergences out of the hands of programmers. Consequently, such warp-synchronous functions may not be called from multi-thread code that may have divergent control flow.

CUDA's dynamic parallelism (or OpenCL's device-side enqueue) lets threads already in flight create new groups of threads [36]. This feature gives developers the opportunity to implement strikingly elegant algorithms [21]. However, this construct is too heavyweight for our simpler purpose of re-activating threads within a warp. For instance, invoking new threads from within a thread in CUDA involves the global scheduling of a new grid of threads [16], a very expensive event. In short, currently, either we have the programmability and elegance of the multi-threaded model, or the efficiency of warp-synchronous programming, but not both.

The goal of this paper is to allow the composability of SIMD and MT through a programming construct syntactically similar to dynamic parallelism. To this end, we intro-

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1 We shall call groups of threads that execute in lock-step a warp.
2 Ex.: see Trove at https://github.com/bryancatanzaro/trove.
duce the notion of Call Re-Vectorization (CREV). CREV is a programming idiom that modifies function calls. Functions marked with the crev tag, henceforth called r-functions, are executed by all the threads in a SIMD unit. This implies a context switch: to run an r-function, the runtime must change the state of all the threads, including those inactive due to divergent control flows. Upon completion, workers return to their previous state, in the same way that a function call is handled. Thus, we achieve a new level of recursion, in which threads can spawn new threads in a stack-based fashion. However, contrary to traditional dynamic parallelism, the call stack is handled. Thus, we achieve a new level of recursion, in the same way that a function call returns.

To validate our ideas, we have implemented them in ISPC [3, 25]. ISPC is a programming language, plus its compiler. This compiler produces industrial quality code for SIMD units such as Intel Streaming SIMD Extensions (SSE), Intel Advanced Vector Extensions (AVX) including AVX-512 for Xeon Phi accelerators [31], or ARM NEON. We chose to implement CREV in ISPC because this framework supports the notion of unmasked or everywhere blocks [25]: the ability to activate – in a new context – threads that are idle due to divergences. This feature is a requirement of CREV. We have used this new ISPC’s extension to implement several classic algorithms using CREV. We show that these implementations are as efficient as warp-synchronous versions of them, and as clear and elegant as if they had been implemented using dynamic parallelism.

Summary of our Contributions. The key contribution of this paper is the notion of Call Re-Vectorization, which comes out of the observation that it is possible to capitalize on divergent threads to help speedup the work of active threads. We explain the concept of CREV through examples, a formal semantics and an industrial quality implementation:

- **Examples**: Section 2 shows examples of algorithms that benefit from our notion of Call Re-Vectorization. Further examples are discussed in Section 4.
- **Semantics**: Section 3.2 formalizes the semantics of μ-SIMD, a low-level instruction set sufficient to implement CREV. We have written a Prolog interpreter to validate that semantics. This interpreter made it easy to prototype different implementations of CREV, until we had a design that we could graft into a state-of-the-art compiler.
- **Translation**: Section 3.3 describes the translation of the high-level “crev” keyword into the low-level representation. Core properties of the final, low-level code, as produced by the translator, are listed in Section 3.4.
- **Evaluation**: Section 4 provides an empirical evaluation of our implementation. To perform this evaluation, we have implemented some algorithms, which are faster and cleaner than their original versions without CREV.

2. Overview

The goal of this section is to explain Warp-Synchronous Programming, Dynamic Parallelism (DP), and our notion of Call Re-Vectorization (CREV). To this end, we shall use Algorithm 1 as an example. This program receives a book \( b_i \), plus a pattern \( p \). It then copies out all the lines \( l \in b_i \) that match \( p \). Pattern matching is performed by \( \text{memcpy} \), and memory copying is done by \( \text{memcpy} \). The book is represented as a matrix of characters; thus, each of its lines, and also the pattern \( p \), is a vector of up to \( N \) characters. Algorithm 1 runs in parallel: \( t_{id} \) is a thread identifier. Hence, each thread is in charge of matching a line \( l \) in \( b_i \) against \( p \). In case the match is positive, this thread must copy \( l \) to an output matrix \( b_o \). For clarity, we assume a single warp in this example, although the techniques we described can be applied independently to multiple warps. The number of threads that run simultaneously in Algorithm 1 is \( W \), the warp width.

Algorithm 1: SIMD Book Filter

1. \( W \leftarrow \text{warp size}; \; t_{id} \leftarrow \text{thread index}; \)
2. \( \text{Function bFilter(mtx } b_i, \text{ mtx } b_o, \text{ vec } p, \text{ int } N) \)
3. \( \text{for } k \leftarrow t_{id} \text{ to num_lines}(b_i) \text{ - 1 step } W \text{ do} \)
4. \( l \leftarrow b_i[k]; \)
5. \( \text{if memcpy}(l, p, N) == 0 \text{ then} \)
6. \( \text{memcpy}(l, b_o[k], N); \)

A naive multi-thread implementation of \( \text{memcpy} \) iterates sequentially over the arrays within each thread. This implementation is highly inefficient due to branch and memory divergence. Branch divergence occurs if the number of iterations \( N \) differs across threads. Threads with few iteration would finish the loop earlier and wait for threads with more iterations in order to restore convergence at the end of the loop. Memory divergence also happens as threads within a warp access data in unrelated locations. Such accesses, referred to as uncoalesced in the CUDA literature or as gather/scatter on SIMD platforms, are bandwidth-inefficient compared to accesses to consecutive elements.

2.1 Warp Synchronous Programming

It is possible to write function \( \text{memcpy} \) in a way that distributes operations on contiguous elements across consecutive threads. Algorithm 2 does it. Function \( \text{memcpy}_{\text{shfl}} \) is aware of the SIMD nature of a warp. Variables are stored as vectors, having each position belonging to a specific thread. Instruction \( \text{shfl}(v, i) \) allows thread \( t_{id} \) to read the value stored in variable \( v \), but in the register space of thread \( i \). This implementation give us an efficient way to copy data between arrays, as copies are distributed evenly between threads, removing most of the branch divergence. Memory divergence is also eliminated as threads of a warp access consecutive elements at each iteration of the loop on line 7.
Nevertheless, this function has an important limitation: it requires all threads in the warp to be active. It cannot safely be called from a point that has potential branch divergence. Indeed, the loop on line 7 would skip elements if some threads were inactive. To support calls to `memcpy_shfl` within divergent regions, we need a way to re-activate threads and put them to work on the copy loop. In addition, the warp-synchronous programming construct is more complex and error-prone than the naive implementation.

```
Algorithm 2: Warp synchronous memcpy
1 W ← warp size; tid ← thread index;
2 Function memcpy_shfl(vec s, vec d, int N)
3     for j ← 0 to W - 1 do
4         d[m] ← shfl(d[j]);
5         s[m] ← shfl(s[j]);
6         N[m] ← shfl(N[j]);
7     for i ← tid to N[m] - 1 step W do
8         d[i] ← s[i];
```

2.2 Dynamic Parallelism in CUDA

In NVIDIA’s CUDA and OpenCL 2.0, dynamic parallelism (DP) is the ability to invoke a new kernel \( K_2 \) from within a kernel \( K_1 \) [32]. In this case, programmers may request a large number of threads, i.e., multiple new warps in multiple thread blocks. As the inner \( K_2 \) is a new kernel, all its threads are active upon entry, regardless of branch divergence in \( K_1 \). Algorithm 3 shows an implementation of `memcpy` that we could invoke from Algorithm 1 using dynamic parallelism. This algorithm splits, among all the threads in a warp, the work of copying vector \( s \) to vector \( d \). Its main advantage is simplicity; its disadvantage is efficiency.

```
Algorithm 3: Implementation of memcpy that could be invoked dynamically from Algorithm 1.
1 W ← warp size; tid ← thread index;
2 Function memcpy_dp(vec s, vec d, int N)
3     for k ← tid to N - 1 step W do
4         d[k] ← s[k];
```

Wang et al. demonstrate that the overhead of a new kernel launch can be as high as one millisecond [32]. The new kernel must be scheduled and wait until there is resources available for its execution. Then, the requested number of warps and memory blocks must be allocated before execution starts. For large workloads, the overhead of launching a nested kernel is paid off by the massive data parallelism available in the GPU [8]. However, for small tasks, this extra cost might degrade performance.

2.3 Call Re-Vectorization

Introducing an inner dimension of parallelism is desirable to implement irregular algorithms such as graph traversal and recursive sorting. Unfortunately, current abstractions based on warp-synchronous programming or Dynamic Parallelism either compromise efficiency or programmability. To solve this conundrum, we introduce Call Re-Vectorization (CREV), a new programming idiom. Synthetically, CREV is akin to CUDA’s dynamic parallelism. Semantically, it avoids the cost of scheduling new kernels.

```
Function bFilter(mtx b, mtx b0, vec p, int N)
3     for k ← tid to num_lines(b) - 1 step W do
4         l ← b0[k];
5         if memcmp(l, p, N) ≠ 0 then
6             crev memcpy_crevel(l, b0[k], N);
7 Function memcpy_crevel(vec s, vec d, int N)
8     for k ← tid to N - 1 step W do
9         d[k] ← s[k];
```

3. Semantics of CREV

This section presents the semantics of Call Re-Vectorization. First, in Section 3.1, we state informally key features of CREV. In Section 3.2, we introduce \( \mu \)-Simd, a low-level programming language with a set of primitives that lets us
implement CREV. In Section 3.3, we show how to implement the crev high-level construct using the building blocks available in \(\mu\)-SIMD. Finally, in Section 3.4, we use our semantics to state some properties of CREV. Before we dive in these details, Example 3.1 arms the reader with some intuition on how CREV works.

**Example 3.1** A function is called with the crev prefix to indicate that every thread, whether enabled or disabled, should execute the function. Every thread should execute the \(r\)-function multiple times if multiple enabled threads in the warp call it. For instance, if the warp size is 32 and 7\( \begin{aligned} \text{memcpy} \end{aligned} \) the \(r\)-function multiple times if multiple enabled threads in the warp are enabled when the program flow hits line 6 in Algorithm 4, all 32 threads execute \texttt{memcpy\_crev} 7 times. In each case, the 32 threads temporarily take on the local state of the active thread that they are helping. Once done, these workers all get their local state restored.

### 3.1 The Cornerstones of CREV

CREV is defined as follows: for each active thread that reaches a call tagged with crev, we execute the target function once, forwarding global parameters (scalars) and extracting private ones per active thread (vectors). This principle of Call Re-Vectorization lays on three pillars: \textit{thread re-activation, SIMD function call} and \textit{data distribution}.

**Thread Re-Activation.** CREV does not lead to the creation of new threads. A function invoked via a CREV call is executed by every thread that is part of a warp, regardless of its threads’ state. As mentioned in Section 2, a thread might be inactive due to divergences. However, dormant threads are re-activated to perform work. The former state of the thread is saved into the context stack, used for divergence management. On software context stack implementations such as used on AMD GPUs and Intel AVX-512 platforms, this operation is performed entirely in software. For platforms with hardware context stack implementations, like NVIDIA GPUs, it will require a new machine instruction.

**SIMD Function Call.** Multi-thread and SIMD languages have different definitions of function calls. In MT, a function call is only performed by active threads. Only register lanes that correspond to active threads are saved. The other threads are guaranteed to stay inactive during execution of the function and need no context save. Although each thread conceptually has its own private call stack, the call stacks of a warp are typically synchronized for performance reasons and to allow the sharing of a single scalar stack pointer for a warp. Implementations of SIMD languages, on the other hand, save whole vector registers on function calls, keeping one stack pointer per warp. Unlike regular MT functions, \(r\)-functions follow an SIMD application binary interface. This ensures that all registers in-use are saved before being overwritten inside the function, including lanes of threads that were inactive. Because no threads are created, context switch is similar to the cost of invoking a new function.

```plaintext
branch if zero
unconditional branch
branch if thread previously active
write to shared memory
read from shared memory
binary operations
copy
shuffle data between lanes
synchronization barrier
halt the machine
begin everywhere block
done everywhere block
```

**Labels** (\(L\)) := \(l \in \mathbb{N}\)
**Constants** (\(C\)) := \(c \in \mathbb{N}\)
**Variables** (\(V\)) := \(T_{id} \cup \{v_1, v_2, \ldots\}\)

**Figure 1.** \(\mu\)-SIMD instruction set. Operands (\(o\)) can be either variables or integer constants.

| Labels | := l ∈ \mathbb{N} |
| Constants | := c ∈ \mathbb{N} |
| Variables | := \(T_{id} \cup \{v_1, v_2, \ldots\}\) |
| Instructions | := \(\text{Figure 1}\) |

- **Active Threads** \(\Theta \subset \mathbb{N}\)
- **Local Memory** \(\sigma \subset V \rightarrow \mathbb{Z}\)
- **Local Memory Bank** \(\beta \subset T_{id} \rightarrow \sigma\)
- **Shared Memory** \(\Sigma \subset \mathbb{N} \rightarrow \mathbb{Z}\)
- **Synch Stack** \(\Pi \subset (L \times \Theta \times L \times \Theta \times \Pi)\)
- **Context Stack** \(\Lambda \subset (\Theta \times \Pi \times \Lambda)\)
- **Program** \(P \subset L \rightarrow I\)
- **Program Counter** \(pc \in \mathbb{N}\)

**Figure 2.** The state of \(\mu\)-SIMD machine is a seetuple \(M(\Theta, \beta, \Sigma, \Pi, \Lambda, P, pc)\). \(\Theta\) is the set of active threads. A thread \(t \in \Theta\) has a local memory \(\sigma\), accessible through a memory bank \(\beta\). Threads communicate through shared memory \(\Sigma\). The stack \(\Pi\) tracks control flow divergences. A key component of Call Re-Vectorization is the thread stack \(\Lambda\). The program counter, \(pc\), keeps track of the next instruction \(t \in P\) to be executed. The program \(P\) is a linear sequence of instructions. Although it never changes, we include it as state for convenience.

**Data Distribution.** Each formerly active warp thread is serialized to have a full warp operate on its data. As in the warp-synchronous \texttt{memcpy} example (Algorithm 2), this requires extracting and broadcasting each thread’s register lane. Data distribution will be later detailed in Algorithm 5.

### 3.2 Low-Level Semantics

We formalize the notion of Call Re-Vectorization on top of a core language, \(\mu\)-SIMD. This language provides the low-level constructs necessary to implement \(r\)-functions. Most of the syntax of \(\mu\)-SIMD comes from Sampaio et al. [28], who, in turn, have reused ideas from Bougé et al. [2] and Farrell et al. [10]. A \(\mu\)-SIMD program is a sequence of instructions indexed by a \(pc\). Figure 1 shows \(\mu\)-SIMD’s syntax.
\[ \text{split}(\Theta, \beta, v) = (\Theta_0, \Theta_n) \text{ where} \]
\[ \Theta_0 = \{ t \mid t \in \Theta \text{ and } \beta[t] = \sigma_t \text{ and } \sigma_t[v] = 0 \} \]
\[ \Theta_n = \{ t \mid t \in \Theta \text{ and } \beta[t] = \sigma_t \text{ and } \sigma_t[v] \neq 0 \} \]
\[ \text{push}(\pi, \Theta_n, pc, l) = [(pc, [], l, \Theta_n)] \]
\[ \text{push}(pc', [], l', \Theta'_n) : \Pi, \Theta_n, pc, l) = \Pi' \text{ if } pc \neq pc' \]

where \( \Pi' = (pc, [], l, \Theta_n) : (pc', [], l', \Theta'_n) : \Pi \)

**Figure 3.** Auxiliary functions used to define \( \mu\)-SIMD. split is a filter, dividing threads into two divergent sets (\( \Theta_0 \) and \( \Theta_n \)). Auxiliary function push updates the synchronization stack \( \Pi \) due to control flow divergences.

**Operational Semantics.** The state \( M \) of a program is a tuple \( (\Theta, \beta, \Sigma, \Pi, \Lambda, P, pc) \), as described in Figure 2. Threads are uniquely identified by a natural \( t_{id} \), having a local memory \( \beta[t_{id}] \), and sharing a global memory \( \Sigma \). Memory is vectorized, thus, a local address \( v \) denotes a vector of variables \( v \in \beta[t_{id}] \); hence, each thread sees its private version of \( v \).

To formalize the semantics of \( \mu\)-SIMD, we use the auxiliary functions shown in Figure 3. The semantics of \( \mu\)-SIMD is given by Figures 4 and 5. The former shows the behavior of instructions that change the program’s control flow; the latter shows the behavior of logic and arithmetic instructions. The result of executing a control flow instruction is a tuple \( (\Theta, \beta, \Sigma) \). The interface between Figure 4 and Figure 5 is performed by Rules IT and TL. The result of executing an arithmetic or logic instruction is a pair \( (\beta, \Sigma) \), i.e., they only update the program memory.

**The semantics of control flow divergences.** To simulate the effect of divergences, \( \mu\)-SIMD has a stack \( \Pi \). Each element in \( \Pi \) is a tuple \( (l_{id}, \Theta_{done}, l_{next}, \Theta_{todo}) \), which indicates the point where divergent threads must re-converge. A new tuple is pushed onto \( \Pi \) due to a conditional branch, located at \( l_{id} \), that has caused a divergence, as described by Rules BT, BF, and BD, in Figure 4. \( \Theta_{done} \) is the set of threads that have reached the synchronization point, \( \Theta_{todo} \) is the set of threads waiting to execute. These threads, once active, will resume execution at label \( l_{next} \). The stack is popped by instructions sync, whose behavior is given by Rules SS and SP.

**The Thread Stack.** To implement CREV, we have added a thread stack \( \Lambda \) to \( \mu\)-SIMD. This stack is fundamental to the implementation of everywhere blocks. \( \Lambda \) holds pairs \( (\Theta, \Pi) \). Figure 4 shows that instructions everywhere (Rule Eb) push elements onto \( \Lambda \), and instructions end_everywhere (Rule Ee) pop it. The first element in this tuple is the set of threads active immediately before the execution of an everywhere block. The second element is the divergence stack, also in the state before the execution of the last everywhere block traversed by the program flow. In Rule Eb
\( \Sigma(v) = c \)
\( \Sigma \vdash v = c \)

(MM) \( \Sigma(v) = c \) \( \Sigma \vdash v = c \)

(TL) \( (t, \beta, \Sigma, \Theta_{mask}, t) \rightarrow (\sigma_1, \Sigma') \)
\( (\Theta, \beta [\beta[t] \rightarrow \sigma_1], \Sigma', \Theta_{mask}, t) \rightarrow (\beta'', \Sigma'') \)

(MT) \( t, \beta \vdash t_{id} = t \)

(BP) \( t, \beta \vdash v_2 = c_2 \)
\( t, \beta \vdash v_3 = c_3 \)
\( \beta[t] = \sigma_t \)
\( c_1 = c_2 \oplus c_3 \)

(MV) \( \beta[t] = \sigma_t \)
\( \sigma_t(v) = c \)

(SI) \( t, \beta \vdash v_1 = c_1 \)
\( t, \beta \vdash v_{lane} = c_{lane} \)
\( \beta[t] = \sigma_t \)
\( c_{lane} \in \Theta_{mask} \)
\( \sigma_{lane} = \sigma_{lane}(v_1) = c_2 \)

(Ct) \( t, \beta \vdash v_x = c_x \)
\( (t, \beta, \Sigma, \sigma, v = \downarrow v_x) \rightarrow (\sigma_1 \backslash [v \mapsto c], \Sigma) \)

(AS) \( t, \beta \vdash v' = c \)
\( (t, \beta, \Sigma, \sigma, v = v') \rightarrow (\sigma_1 \backslash [v \mapsto c], \Sigma) \)

(LD) \( t, \beta \vdash v_x = c_x \)
\( (t, \beta, \Sigma, \sigma, v = \downarrow v_x) \rightarrow (\sigma_1 \backslash [\sigma \mapsto c], \Sigma) \)


<table>
<thead>
<tr>
<th>Instructions</th>
<th>Address</th>
<th>Shared Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>v0 = {tid }</td>
<td>0 1 2 3</td>
<td></td>
</tr>
<tr>
<td>v1 = {v0 == 0 }</td>
<td>0 1 1 0</td>
<td></td>
</tr>
<tr>
<td>bz v1, Done</td>
<td>4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>v2 = 4 * {tid + 1 }</td>
<td>2 1 3 4</td>
<td></td>
</tr>
<tr>
<td>everywhere</td>
<td>8 9 10 11</td>
<td></td>
</tr>
<tr>
<td>v8 = 0</td>
<td>1 5 6 1</td>
<td></td>
</tr>
<tr>
<td>Loop</td>
<td>12 13 14 15</td>
<td></td>
</tr>
<tr>
<td>jmp, mask v8, Call</td>
<td>2 3 1 7</td>
<td></td>
</tr>
<tr>
<td>v3 = shfl(v2, v8)</td>
<td>16 17 18 19</td>
<td></td>
</tr>
<tr>
<td>v4 = v3 + tid</td>
<td>3 4 0</td>
<td></td>
</tr>
<tr>
<td>v5 = i v4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>v6 = v5 + 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \uparrow v_4 = v_6 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Next</td>
<td>v8 = v8 + 1</td>
<td></td>
</tr>
<tr>
<td>v7 = {v8 == 4 }</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bz v7, Loop</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \uparrow v_8 = v_7 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( {v8 } )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>end_everywhere</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( {tid } = 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Done, sync</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Private Memory</th>
<th>Address</th>
<th>Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>v0</td>
<td>v1</td>
<td>v2</td>
</tr>
<tr>
<td>Contents</td>
<td>( \ast )</td>
<td>( \ast )</td>
</tr>
<tr>
<td>v4</td>
<td>v5</td>
<td>v6</td>
</tr>
<tr>
<td>Contents</td>
<td>( \ast )</td>
<td>( \ast )</td>
</tr>
<tr>
<td>v8</td>
<td>Contents</td>
<td>( \ast )</td>
</tr>
</tbody>
</table>

**Figure 5.** Semantics of arithmetic, logic and data-related instructions. Rule TL loops over every thread \( t \in \Theta \), and for each one of them, executes instruction \( \iota \). No assumption can be made on the order in which instructions run.

**Figure 6.** Program written in \( \mu \)-SIMD, plus its initial state.

Different thread contexts; hence, programs might contain an arbitrary number of nested **everywhere** blocks. After executing the instruction **end_everywhere**, threads previously inactive will go back into sleeping mode. In other words, after **end_everywhere**, the pair \((\Theta, \Pi)\) at the top of \( \Lambda \) is popped, and the diverging configuration \( \Pi \) becomes part of the current state of threads. If necessary to check if a thread is active due to an **everywhere** block, then \( \mu \)-SIMD provides a conditional **jmp_mask**. The statement **jmp_mask**(tid, l) will divert execution to \( l \) if \( t_{id} \) is active in the mask at \( \Lambda \)’s top.

**Example 3.2** Figure 6 shows a program written in \( \mu \)-SIMD. We assume \( |\Theta_{all}| = 4 \). This program increments a 4 \( \times \) 4 matrix; however, line \( i \) is incremented only if \( \Sigma ith = 0 \). The figure shows the initial state of the shared (\( \Sigma \)) and local memory of each thread (\( \sigma \)). The initial state of the variables in the local memory is immaterial for this example. Figure 7 shows a trace of the execution of the program, given its initial state. Only threads \( t_{id} = 0 \) and \( t_{id} = 3 \) will enter the **everywhere** section, because \( \Sigma[0] = \Sigma[3] = 0 \). Nevertheless, all the four threads will execute the commands within that block. Instruction \( v3 = shfl(v2, v8) \) lets each thread read into \( v3 \) the value of \( v2 \) seen by thread \( v8 \).

**3.3 High-Level Semantics**

The \( \mu \)-SIMD assembly gives us the primitive building blocks to implement CREV in higher-level languages. As a proof of concept, we have implemented CREV onto ISPC, using instructions of ISPC that are equivalent to those seen in \( \mu \)-SIMD. By focusing on an abstract notation, \( \mu \)-SIMD, instead of on a concrete language, such as ISPC, we claim generality: CREV can be implemented in any environment that supports our notions of **everywhere** and shuffle. In this section we show how to implement the **crev** modifier, which marks a function call as an r-function. For simplicity, our high-level language provides only syntax to declare and invoke functions. A function declaration consists of a name \( f \), plus a list of formal parameters, e.g. \( f(T p_1, \ldots, T p_n) \). We let \( T \) denote a type modifier, which can be either **uniform** or **varying**. We have borrowed this notation from ISPC. Other programming languages have different ways to express these modifiers. For instance, in CUDA we have **shared** and **global** allocation filling the role of ISPC’s uniform variables.

Figure 8 shows the code that we produce for a r-function call \( f(a_1, \ldots, a_n) \), where each \( a_i \), \( 1 \leq i \leq n \) is an actual argument of \( f \). Such an r-function call will trigger up to \( |\Theta| \) executions of \( f \), one for each active thread \( t \in \Theta \). The test
in lines 3 or 8 in Figure 8 are used to single out the function invocation performed by each thread. A different call will happen due to each handle label. In another dimension of parallelism, each function call will be executed by \( \Theta_{all} \) threads, due to the everywhere block at lines 10 and 13. Thus, we might have up to \( \Theta_{all}^2 \) computations.

**Figure 7.** Execution trace of the program in Figure 6. Column Var shows contents of last variable assigned. T indicates branch taken; F indicates otherwise. The symbol • marks inactive threads. For the syntax of instructions, we refer the reader to Fig. 1; for their semantics, Figs. 4 and 5.

**Figure 8.** Low-level code produced to call r-function \( f \).

Algorithm 5: Data distribution

1. Function declaration: \( f(p_1, \ldots, p_n) \);
2. Function call: \( f(t_1, a_1, \ldots, t_n, a_n, t) \);
3. Function extract(\( t^n, p^n, a^n, i \))

for \( k = 1 \ldots n \) do

if \( t_k \) is uniform then

\( p_k = a_k \);

if \( t_k \) is varying then

\( shfl(a_k, i) \);

Example 3.3 The program in Figure 9 shows three functions called via \( \text{crev} \). We are assuming an architecture with four SIMD lanes, i.e., \( \Theta_{all} = \{ T_0, T_1, T_2, T_3 \} \). When \( \text{foo} \) is invoked, the value of a, main’s local variable, is broadcasted to foo’s formal parameter. Thus, \( T_0 \) sees \( \text{foo}(0) \), \( T_1 \) sees \( \text{foo}(1) \), etc. When \( T_0 \) calls \( \text{bar} \) from \( \text{foo} \), the same behavior is observed. However, when \( T_1 \) calls \( \text{baz} \) from \( \text{bar} \), all the four threads activated in this context see \( \text{baz}(2) \), because \( \text{baz} \) receives an uniform argument. The fact that \( \text{baz} \)’s
local variable d is marked as varying is immaterial in this example, as this variable is initialized with uniform values.

### 3.4 Properties of CREV

The semantics of CREV, given by μ-SIMD’s primitive building blocks, and the translator seen in Section 3.3, lets us establish a few properties that are true about this programming abstraction. In this section we go over a few of these properties. They are valid under the assumption that programs are well-formed. We define well-formed programs below:

**Definition 3.4 (Well-Formed Program)** A μ-SIMD program is well-formed if any occurrence of an everywhere instruction at label $l_1$ is matched by an occurrence of an end_everywhere instruction at label $l_2$, and these two labels are control equivalent.

Definition 3.4 borrows the concept of control equivalence from Ferrante et al. [11]. Two points, $l_1$ and $l_2$, in a program’s control flow graph are said to be control equivalent if $l_1$ dominates $l_2$, and $l_2$ post-dominates $l_1$. We say that $l_1$ dominates $l_2$ if, and only if, any path from the root of the CFG to $l_2$ must cross $l_1$. Dually, $l_2$ post-dominates $l_1$ if, and only if, any path from $l_1$ to the end of the CFG must cross $l_2$. Our translator produces well-formed programs, as long as the program flow cannot leave a function through points other than its return address.

**Theorem 3.5 (Well-Formed Translation)** The translator of Figure 8 produces well-formed programs.

**Proof:** This result follows trivially from the fact that an everywhere block surrounds only Algorithm 5 and the r-function. Well-formedness holds as long as none of these routines let the program flow escape the enclosing end_everywhere instruction. This implies that the r-function cannot throw exceptions, for instance.

**Composability.** CREV allows the nesting of everywhere blocks. Composition happens due to nested function calls. The thread stack $\Lambda$ ensures that the last invoked r-function will be the first to remove pending computation. In what follows, we visit three consequences of this property.

**Composition is multiplicative.** An crev call will put all the warp threads in active mode. By coupling this observation with composability, we have that, in the absence of divergences, a sequence of $n$ nested crev calls will create $|\Theta_{alt}|^n$ tasks. Notice that CREV produces new tasks, but not new threads: we still have only $|\Theta_{alt}|$ threads to solve these tasks.

**Commutativity.** The translator of Figure 8 calls an r-function in a lexicographic order defined by thread identifiers. However, μ-SIMD’s primitives do not impose any order on the threads pushed onto $\Lambda$. Therefore, the multiple SIMD calls of an r-function can be handled in any order.

**Synchronization parity.** There is no distinction between the top level of parallelism and the inner level of parallelism with regards to the synchronization primitive. In other words, divergences are handled transparently by the synchronization stack $\Pi$, and, from a synchronization standpoint, it is not possible to tell if execution exists within the context of an r-function or not. To ensure this property, μ-SIMD’s everywhere instruction pushes onto $\Lambda$, together with the set of active threads, the divergent state $\Pi$.

**The interplay between CREV and nested function calls.** The implementation of CREV does not interfere with the implementation of function calls. Programming languages that support recursion use a structure known as activation stack to manage function calls. Entries in the activation stack are called activation records, and they store functions’ local variables, return address, arguments, etc. Upon invocation, the activation record of a function is pushed onto the activation stack. For each thread pushed onto the thread stack there will exist one activation record on the activation stack. The multiplicative nature of CREV also implies on a multiplication of activation records. Therefore, $n$ nested r-calls will generate $|\Theta_{alt}|^n$ activation records; however, the maximum depth of the activation stack is still $n + 1$: activation records owned by different threads will not exist simultaneously.

**Example 3.6** Figure 10 reuse the program from Example 3.3 to illustrate these points. Again, we assume $|\Theta_{alt}| = 4$. Thus, three non-divergent nested r-calls will create $4 \times 4 \times 4 \times 4 = 256$ tasks. At any time, the thread stack will contain at most $4 + 4 + 4 + 4 = 16$ tasks waiting for execution. The activation stack will contain, at any given point, at most $4$ activation records, corresponding to the activation of functions main, foo, bar and baz.

### 4. Evaluation

To evaluate the ideas presented in this paper, we have implemented CREV on ISPC. We use this implementation to demonstrate that CREV allies the efficiency of warp synchronous programming with the clarity and elegance of dy-
namic parallelism; hence, avoiding the complexity of the former, and the heavy scheduling cost of the latter.

**Experimental Setup.** Because CREV is a novel concept within ISPC, this compilation framework does not provide benchmarks that use it. Thus, we have re-implemented seven classic algorithms using the new keyword `crev`. Our seven benchmarks are: (bk) Book Filter (Algorithm 1); (sm) String Matching; (bf) Bellman-Ford [1]; (df) Depth-First Search; (le) Leader Election; (qs) quick-sort; and (ms) merge-sort.

**Runtime Environment.** We have implemented CREV onto ISPC v 1.9.1, and have used it to target a 6-core 2.00 GHz Intel Xeon E5-2620 CPU with 8-wide AVX vector units, running Linux Ubuntu 12.04 3.2.0. This running environment gives us warps with eight threads, e.g., $|\Theta_{all}| = 8$.

**The Competing Approaches.** We compare four different ways to implement our benchmarks.

- **Seq:** serial implementation of each algorithm, as defined in Cormen’s book [5]. String-matching was implemented after the Knuth–Morris–Pratt (KMP) [19] algorithm.
- **Par:** warp synchronous implementation using constructs available in the ISPC language, but without CREV.
- **Launch:** dynamic parallelism, implemented via the `launch` keyword, which starts a PThread per function call.
- **CREV:** the implementation of the algorithms using the ideas introduced in this paper.

The Seq version of each benchmark is implemented in C++, and is compiled with clang version 3.7.1, with the optimization flags `-O2/-std=c++11`. Benchmarks in the other three categories (Par, Launch and CREV) are compiled with ISPC. The Launch and CREV implementations look the same, except for the keyword that precedes function calls: `launch` in the first case, and `crev` in the second. The implementations in the Par group are different: they do not contain function calls within divergent regions, to ensure correctness.

Implementations are not available for all the programs. Even though it is trivial to implement quick-sort or merge-sort with dynamic parallelism or with CREV, the craft of a warp synchronous implementation of them is not obvious; hence, we omit them. Additionally, we omit a sequential implementation of book filter, because this problem does not have a canonical, textbook-like solution. Notice, however, that this algorithm reuses two r-functions: `string match (sm)`, and `memcpy` (Algorithm 2). Thus, we show results for sequential string matching, but not for sequential book filter.

**How to read our results.** Results are measured in millions of execution cycles, as reported by ISPC’s testing environment. Numbers are the average of five, out of six samples. We have removed the first, to avoid cold-start discrepancies. The reader must bear four observations in mind, when analyzing our results: (i) speedups of CREV over pure ISPC (Par) are due to the better load distribution that CREV accomplishes by transporting work to inactive threads; (ii) the large slowdowns observed with Launch are due to the heavy cost of scheduling millions of new Posix threads to perform small chunks of work. (iii) CREV’s slowdowns are due to the boilerplate code necessary to serialize threads, before invoking r-functions; (iv) we are comparing against an industrial-strength compiler; hence, speedups tend to be modest.

### 4.1 Results and Discussion

Figure 11 summarizes our results. The table shows runtime, measured as number of execution cycles (in millions), for all the implementations that we have of the seven different benchmarks. These numbers are produced with the largest inputs that we have for each benchmark:

- **bk:** 10K strings of 0s and 1s, each with 20K bits, and random target pattern with 16 bits.
- **sm:** 256MB taken from books available in the Project Gutenberg, and a target pattern with 16 characters.
- **bf:** random Erdős-Rényi [9] graph with 2,048 nodes and 80% edge probability.
- **df, le:** 8-ary complete tree, depth 5 (root + five full levels).
- **qs, ms:** 16K random integers in the range $[0, 100000]$.

Figure 11 shows that ISPC, with or without CREV, is competitive against mainstream compilers: Seq is the best C implementation that we could produce of each algorithm; furthermore, these programs are compiled with clang -O2 (which was faster than clang -O3 in this experiment). Nevertheless, CREV and Par outperform these implementations in several cases, although they are not embarrassingly parallel. The table also reveals that Launch is sometimes orders of magnitude slower than the other approaches, being the fastest in only one case: bk. In bf and df, the excessive number of threads created by Launch forced earlier termination.

CREV usually outperforms Par by a small margin. However, the main benefit of the former over the latter is not performance. Rather, it is readability. The Launch and the CREV versions of each benchmark are exactly the same, except that whereas in one case we prepend the call of functions with the launch keyword, in the other we use `crev`.

On the other hand, the Par implementations are very different, because they cannot call functions within potentially divergent regions. To circumvent this restriction, the five Par benchmarks forgo recursion. As an example, the ISPC ver-

<table>
<thead>
<tr>
<th>Pg</th>
<th>Seq</th>
<th>Par</th>
<th>Launch</th>
<th>CREV</th>
</tr>
</thead>
<tbody>
<tr>
<td>bk</td>
<td>×</td>
<td>8,530.99</td>
<td>7,857.98</td>
<td>7,405.17</td>
</tr>
<tr>
<td>sm</td>
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<td>3,576.14</td>
<td>393,166.27</td>
<td>2,737.94</td>
</tr>
<tr>
<td>bf</td>
<td>141,088.73</td>
<td>493,619.69</td>
<td>•</td>
<td>529,856.06</td>
</tr>
<tr>
<td>df</td>
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<td>3,786.26</td>
<td>•</td>
<td>3,790.44</td>
</tr>
<tr>
<td>le</td>
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<td>3,983.09</td>
<td>5,272.92</td>
<td>3,984.79</td>
</tr>
<tr>
<td>qs</td>
<td>2.87</td>
<td>×</td>
<td>204.28</td>
<td>2.87</td>
</tr>
<tr>
<td>ms</td>
<td>7.30</td>
<td>×</td>
<td>104.98</td>
<td>4.11</td>
</tr>
</tbody>
</table>
sion of depth-first search uses a stack of tasks, instead of performing recursive functions to traverse the graph.

Algorithm 6: Pattern matching: CREV vs. Naïve

1: \( P \leftarrow \text{pattern}; T \leftarrow \text{target text}; \)
2: \( W \leftarrow \text{warp size}; t_{id} \leftarrow \text{thread index}; \)
3: \text{Function \texttt{memcpy}(Offset \( k \))}
4: \( m \leftarrow \text{True}; \)
5: for \( i \leftarrow t_{id} \text{ to } |P| \) do
6: \( \quad \text{if } P[i] \neq T[i + k] \text{ then } m \leftarrow \text{False}; \)
7: \( \quad \text{if all } (m = \text{True}) \text{ then } \text{Found}(k); \)
8: \text{Function \texttt{StringMatch}}
9: for \( i \leftarrow t_{id} \text{ to } (|T| - |P|) \) step \( W \) do
10: \( \quad \text{if } P[0] = T[i] \text{ then } \text{crev \texttt{memcpy}(i)}; \)
11: \text{Function \texttt{ParStringMatch}}
12: for \( i \leftarrow t_{id} \text{ to } (|T| - |P|) \) step \( W \) do
13: \( j \leftarrow 0; k \leftarrow i; \)
14: \( \quad \text{while } j < |P| \text{ and } P[j] = T[k] \text{ do} \)
15: \( \quad \quad j \leftarrow j + 1; k \leftarrow k + 1; \)
16: \( \quad \text{if } j = |P| \text{ then } \text{Found}(k); \)

As another example, Algorithm 6 shows the CREV-based implementation of string matching. This is a warp-synchronous implementation of parallel matching: each thread \( t_{id} \) tries to match \( P \) at positions \( T[t_{id} + n \times W] \), where \( n \leq |T| \), and \( W \) is the warp size. Thus, in the best scenario, runtime is divided by \( W \). This implementation is irregular: divergences might happen at lines 6 and 10. Each call to \texttt{memcpy} will commence a CREV sequence of computations. Figure 12 compares our implementation, seen in Algorithm 6 (\texttt{StringMatch}) against the equivalent Par version, which does not perform any function call.

The Impact of Data on Runtime. The numbers seen in Figure 11 are input dependent. However, the overall conclusions remain the same, once we feed our benchmarks with inputs of different sizes. Figure 12 compares the runtime different implementations of string-matching and leader election. In string-matching, we omitted the Launch-based version, as it was too slow. Notice that KMP has lower asymptotic complexity than Algorithm 6. It runs in \( O(|T| + |P|) \), whereas Algorithm 6 runs in \( O(|T| \times |P|/|W|) \). For this experiment, we searched for prefixes of the pattern “She had been watching him the la”, of sizes 4, 8, ... , 28, 32 in Jane Austen’s book Pride and Prejudice, taken from Project Gutenberg. CREV is always faster than \texttt{ParStringMatch}, and runs faster than KMP in more than half the cases. CREV beats plain parallelism because it distributes function \texttt{memcpy} among the eight available vector lanes. On the other hand, \texttt{ParStringMatch} has a potentially long divergent block in line 14. In our best result, observed for patterns of size eight, CREV runs in 44% of the time taken by \texttt{ParStringMatch}, and in 40% of the time taken by KMP.

Figure 12 (Bottom) shows a comparison between the four implementations of leader-election. Dynamic parallelism is still the slowest, overall; however, we notice that it tends to catch up with the other approaches for very large inputs, in this benchmark. This behavior is expected: if the amount of parallel work is large, then the greater flexibility and independence of pthreads start paying for the cost of creating them. Nevertheless, depending on how the algorithm is implemented, dynamic parallelism might lead to the creation of a very large number of threads, as the size of the input grows. Due to this observation, we have not been able to run the Launch versions of \texttt{df} and \texttt{bf} for the largest available inputs. On the other hand, in the case of leader-election (\texttt{le}), parallelism is coarser: each thread receives the task of finding strong components in a graph. Thus, even though the number of threads grows as the size of the input increases, this growth is less accentuated than in \texttt{df} and \texttt{bf}.

5. Related Work

GPUs’ increasing programmability and decreasing costs have made them very popular for the development of general purpose high performance applications [23]. This popularity has attracted the interest of programming language researchers. Control flow divergences have been a particularly important source of attention. Therefore, the compiler-

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4 Function \texttt{memcpy} is also used at line 5 of Algorithm 1
5 \text{https://www.gutenberg.org/ebooks/42671}
related literature contains a vast body of work describing analyses [6, 27, 28, 30] and optimizations [6, 7, 35, 37] that reduce the effects of divergences in GPGPU code. CREV is not a competitor of these analyses and optimizations. On the contrary, Call Re-Vectorization complements such techniques, giving programmers a tool that lets them deal with divergences at the software level. In the rest of this section we touch work that is more closely related to ours.

**Everywhere Blocks.** The problem of expressing nested SIMD loops in multi-thread style is not new. Some data-parallel programming languages for SIMD computers in the 80’s and 90’s allow to re-enable temporarily dormant threads. The C* language [26], the Maspar Programming Language [20] and the POMPC language [15] incorporate a control flow construct named either everywhere or all to this end. We have re-used these instructions to implement CREV. However, these are low-level primitives: they are not programmer-friendly, nor have any interface with function calls. Using everywhere directly is difficult, as this abstraction has no knowledge nor control over the state of dormant threads. CREV, on the contrary, is as easy to use as dynamic parallelism. It manages register saves and restores automatically, relieving the programmer from this task.

**Warp-level convergence guarantees.** Previous work enforces guarantees on where threads converge after control divergences to make warp-synchronous programming safer. For instance, Pharr et al. have proposed the maximal convergence guarantee [25], and Gaster has proposed a divergence-aware execution model for OpenCL [13]. CREV goes further by actually enforcing convergence at arbitrary program points, allowing warp-synchronous functions to be called from divergent sections. To the best of our knowledge, this is the first attempt to provide developers with such possibility.

**Grid-level Dynamic Parallelism.** Much effort has been spent to reduce the overhead of dynamic parallelism. Alternatives to CUDA Dynamic Parallelism such as DTBL [33]. Free Launch [4] and LaPerm [34] reduce sub-kernel launch overhead or improve cache locality. By relying on global schedulers, they allow load-balancing between GPU stream multiprocessors. We are not competing with these efforts, because CREV is not an alternative to dynamic parallelism. CREV is a static code transformation with no dynamic scheduling; hence, it does not create extra parallelism. In other words, we move work to threads that are already in flight, instead of spawning new threads. The main benefit of CREV, when compared to these previous work comes in terms of programmability and efficiency: by supporting composability of multi-thread and SIMD code, we give developers the chance to benefit from efficient warp-synchronous idioms without neither having to deal with primitives like shuffle, vote and population count, nor having to worry about saving the context of threads.

**Thread-level divergence aware optimizations.** Compilers may reorder computations across loop iterations within each thread to mitigate branch divergence [6, 14, 17, 24]. However, each thread performs the same set of tasks as in the original version, so divergences induced by load balance between threads of a warp remains an issue. CREV is a way to deal with irregular programs whose performance divergences hurt. However, CREV is not an optimization implemented by the compiler: programmers must adapt algorithms to use this construct. CREV deals well with divergences because it lets developers balance workload between threads in flight. In other words, it changes the loop structure by distributing iterations across different threads.

## 6. Final Thoughts

Primitives such as warp vote and shuffle have given experts the possibility of writing efficient SIMD code, by programming from the point of view of one warp. This coding style has been used in CUB and many other CUDA libraries. However, warp-synchronous code does not play well with branch divergence. Most warp-synchronous algorithms require all threads in a warp to be active. This is a problem for the common usage scenario of a simple MT-style CUDA kernel that calls warp-synchronous library functions. It is our vision that the application developer writing the kernel should not be concerned with the internal implementation of library functions, and should be able to call any function inside divergent program regions. To meet the demands of this vision, this paper has introduced the notion of Call Re-Vectorization (CREV). We have described the building blocks necessary to implement CREV. Looking towards compatibility with future hardware, we have proposed low-level primitives with well-defined semantics and a high-level interface, the crev idiom, that makes programmer intent explicit. Thus, our notion of CREV does not rely implicitly on current hardware behavior, which might eventually change. We have implemented CREV into ISPC, using Intel instructions, and have shown how to code irregular algorithms in this environment. Our implementations are not only clearer than non-CREV based approaches, but also more efficient, as they balance work among inactive warp threads.

## Acknowledgement

This project is supported by the Inria-Fapemig Associate Team Prospiel and by CNPq. We thank the members of the Dragon Nest group and the PPoPP referees for their insightful comments and suggestions. Example 3.1 has been suggested by one of the referees. Finally, we thank Albert Cohen for shepherding this paper. Our implementation is publicly available at [http://cuda.dcc.ufmg.br/swan/](http://cuda.dcc.ufmg.br/swan/).

## References


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See [https://nvlabs.github.io/cub/](https://nvlabs.github.io/cub/)


