Influence of Network Deployment on Resource Block Allocation

J.S. Gomez, A. Vergne, P. Martins, L. Decreusefond, and Wei Chen

***Abstract*—In this paper, the influence of network deployment on downlink dimensioning under an interference limited regime is studied. Theoretical and numerical results are derived for a network deployed as a Poisson point process. We show that slightly degraded propagation conditions has a beneficial effect on network dimensioning and performance. Spatial influence is also studied thanks to the β-Ginibre model. Simulations show that a regularly deployed network improves dramatically spectrum usage and network performances.**

***Index Terms*—β-Ginibre Point Process, Poisson Point Process, Network dimensioning.**

I. Introduction

SMARTPHONES and their ecosystem pushed user experience towards throughput intensive services. In this context, fourth generation wireless networks and 802.16e standard introduced OFDM on the radio link. The band is divided into orthogonal sub-carriers, which minimizes spectrum occupancy and fulfills capacity demand. In order to further increase spectrum capacity and mitigate interference, LTE and WiMax have introduced dynamic frequency and time access (OFDMA). Sub-carriers are aggregated in Resource Blocks (RB) of 1ms length and 180kHz wide. A typical LTE eNodeB can manage up to 100 RB per 10ms, for a total band of 20MHz. One can increase the number of RBs per eNodeB, however, provided some user demand and some loss requirement, is there a limit in the number of RBs one can allow? Inter-site interference also plays a great role since the number of resources block allocated to one user depends on the quality of the channel, which is strongly linked to the statistical properties of the eNodeB deployment.

Statistical properties of wireless networks have been widely explored in literature. Pioneer works of Baccelli [1] have provided a strong baseline for analyzing properties of networks deployed as a Poisson point process. However, such Poisson based models does not fit ideally real deployments, since there exists a strong probabilistic dependency between base station locations. Other point processes have been considered. Among them, the β-Ginibre point process, which properties has been studied in [2], successfully fitted the network deployment of four operators on Paris, France [3]. The β parameter gives insights on the regularity of the deployment: if β is near zero, the point process is considered as uniform and is tending to a Poisson point process. If β is near one, there is a strong dependency between the placement of the base stations. The point process is considered as regular and its statistical properties are the ones of a Ginibre point process. For the Poisson point process and the β-Ginibre point process, the SINR distribution has been studied respectively in [4] and [5]. This means that one can induce the quality of the radio signal on a network level, providing the type of point process used.

The objective of this paper is to study the influence of the network deployment on the dimensioning of the base stations. Especially, general downlink performances of a cell placed in the middle of a network deployed as a Poisson or β-Ginibre point process will be analyzed. Path-loss and fading are also considered in the channel model. As for frequency planning, the worst case scenario is considered since all base stations share the same band. The users are placed in the network following a Poisson point process. We evaluate the maximum number of RB that one base station can allocate, provided known user demand and outage probability. Since the exact maximum number of RB is computationally hard to obtain, we propose a pessimistic lower bound based on the concentration inequality, that fits both engineering requirements for computational complexity and robustness. We also evaluate the outage probability in function of the path-loss exponent, provided a fixed number of RB and user demand. Thanks to the results of Błaszczyszyn et al. [4] it is possible to provide an theoretical analysis for Poisson deployed networks. Simulation analysis will also explore the differences of performances induced by the type of deployment, as we consider several β-Ginibre deployed networks.

Resource allocation algorithms for OFDMA systems have been studied extensively. In [6], many optimization based algorithm are proposed for allocating and scheduling resources. Other works have included Markov chains [7], queuing theory or game theory to conceive original algorithms. In [8], the optimal resource allocation strategy is modeled as a Nash bargaining equilibrium. However, dimensioning and planning issues have been more recently investigated. In this paper, we adopt the same theoretical foundations as in [9]. However, its

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results are only valid in noise limited systems, since only one base station is considered. This paper extends and completes its results on a network level. It also gives insights on the link between network performances and deployment.

This paper is organized as follows. In the first section, the system model and theoretical hypothesis are given. A theoretical analysis is provided for Poissonnian deployed networks in Section II. In Section III, simulation analysis and performance analysis are provided for Poisson deployed and β-Ginibre deployed networks. In Section IV, conclusions are drawn.

II. System model

A. Channel model

The system is composed of a network of identical omni-directional OFDMA base stations. They are deployed in the plane according to a certain point process of a given intensity λ_n. The state of the network is considered at a given time slot. For each given time slot, active users are localized in the plane according to a Poisson point process with intensity λ_u. Each user asks for the same capacity C and each base station shares the same spectrum and without any cooperation scheme. Furthermore, each base station can allocate up to N_{avail} RBs per time slot. A user belongs to the base station under the condition that it provides him with the best SIR. Channel model includes path-loss and fading. Therefore, the power received by a mobile from a base station i is modeled by:

$$P(d_{ij}) = P_i G_{ij} d_{ij}^{-\gamma},$$

where P_i is the transmission power of the base station, d_{ij} the distance between the base station i and the mobile j and γ is the path-loss exponent. The channel fading G_{ij} is a random value following a exponential law of parameter one. Furthermore, the number of resource blocks allocated to each user, given by the following equation:

$$N_j = \min\left(\left\lceil \frac{C}{W_{rb}\log_2(1+\max_i(SIR_{i,j})}\right\rceil, l_m\right).$$

W_{rb} denotes the bandwidth of one resource block and $\lceil x \rceil$ is the ceil value of x. l_m is the maximum number of RBs that can be allocated to one user. The SIR between the ith base station and the jth user is given by:

$$SIR_{i,j} = \frac{P(d_{ij})}{\sum_{k\neq i} P(d_{kj})}.$$

The total number of RBs that are demanded in one cell is the random integer given by:

$$N_{tot} = \sum_j N_j$$

Quality of the network deployment is assessed thanks to the loss probability:

$$\mathbb{P}(N_{tot} > N_{avail}).$$

Three case scenarios are investigated:

- considering a given loss-probability, the influence of γ on $\mathbb{E}[N_{tot}]$ is studied,
- considering a given N_{avail}, the influence of γ on the loss-probability is studied,
- considering a given loss-probability, an approximation of N_{avail} thanks to the concentration inequality.

B. Influence of γ on dimensioning.

Theorem 1. In Poisson point process deployed network and for a provided loss probability ϵ, $\mathbb{E}[N_{tot}]$ decreases as γ increases.

Proof. For a given poissonian deployed network, the SIR at each point of the network is assumed to be independent to one another. This assumptions is reasonable since the homogeneous PPP is a stationary process. We model the users location and traffic demand by a marked PPP on $E = [|1,\dots,l_m|]\times\mathcal{C}$:

$$\lambda_u \mathrm{d}\nu(z) = \lambda_u\,(\alpha(\mathrm{l}) \otimes \mathrm{d}z)$$

$\alpha(l) = \alpha_l$ corresponds to the probability that a user demands l RBs and each α_l defined as follows:

$$\alpha_l = \begin{cases} \mathbb{P}(t_1 < SIR) & \text{if } l = 1, \\ \mathbb{P}(t_{l_m} \leq SIR) & \text{if } l = l_m, \\ \mathbb{P}(t_l \leq SIR < t_{l-1}) & \text{otherwise,} \end{cases}$$

where $t_l = 2^{C/lW_{rb}} - 1$.

The family $(\alpha_1,\dots,\alpha_{l_m})$ corresponds to the mean proportion of the plane where one can allocate receptively $1,\dots,l_m$ RBs to a user. Hence, for each cell in the network, the number of RBs demanded by the users N_{tot} follows a compound Poisson distribution:

$$N_{tot} = \sum_{l=1}^{l_m} l M_l.$$

Each M_l is an independent Poisson of parameter $m_l = \lambda_u \alpha_l$. The mean total number $\mathbb{E}[N_{tot}]$ of RBs allocated in the cell $\mathcal{C}$, for users distributed according to a PPP is the sum:

$$\mathbb{E}[N_{tot}] = \sum_{l=1}^{l_m} \int_{x\in\mathcal{C}} l\,\alpha_l \lambda_u \mathrm{d}x$$

According to Błaszczyszyn et al. [4], for all $t \geq 1$ and $\gamma > 2$ the SIR coverage probability for a poissonian network is expressed as:

$$\mathbb{P}(SIR \geq t) = \frac{\gamma \sin(2\pi/\gamma)}{2\pi} t^{-2/\gamma}$$

Therefore, it is proven in Appendix *citer* that the coefficient α_l becomes dominant for small values of l as γ increases. $\mathbb{E}[N_{tot}]$ thus decreases with γ.

□

C. Influence of γ on loss probability.

Theorem 2. In Poisson point process deployed network, for a fixed threshold N_{avail}, the loss probability decreases as γ increases.

Proof. As in [9], the distribution of lM_l s constructed by the rule:

$$p_l(\omega) = \begin{cases} 0 & \text{if } \omega \bmod l \neq 0, \\ \exp(-m_l)\, m_l^q/q! & \text{if } \exists\, q \in \mathbb{N}, \omega = ql. \end{cases}$$

As the random variable N_{tot} is a linear combination of independent random variable, its density can be recovered by convolution of the densities of lM_l. Since α_l for small values of l becomes more dominant, the probability of small values occurring increases. □

In interference limited networks, the path-loss γ has a beneficial impact on the overall quality of the signal since it limits mutual cell interference.

D. Approximation of N_{avail}.

Since the probability density of N_{tot} is known, numerical simulations are conducted to find N_{avail} such that:

$$\mathbb{P}(N_{tot} \geq N_{avail}) \leq \epsilon,$$

ϵ being the desired loss threshold. For engineering purposes, a lower robust bound is provided thanks to the concentration inequality.

Theorem 3. The lower bound is given by:

$$\mathbb{P}(N_{tot} \geq \mathbb{E}[N_{tot}] + a) \leq \exp\left(-\frac{\mathbb{E}[N_{tot}^2]}{l_m}\, \mathrm{g}\left(\frac{a\, l_m}{\mathbb{E}[N_{tot}^2]}\right)\right),$$

where $g(\theta) = (1+\theta)\ln(1+\theta) - \theta$ and the moments are given for any p natural integer:

$$\mathbb{E}[N_{tot}^p] = \sum_{l=1}^{l_m} \int_{x \in \mathcal{C}} l^p\, \alpha_l \lambda_u \mathrm{d}x.$$

Proof. Since the number of RBs l is a positive integer with maximum l_m, Theorem 4 of [9] is applied to derive the expression. □

The lower bound of N_{avail} is then given by:

$$N_{avail} = \mathbb{E}[N_{tot}] + a,$$

where a is the solution of:

$$\mathrm{g}\left(\frac{a\, l_m}{\mathbb{E}[N_{tot}^2]}\right) = -\frac{\ln(\epsilon)\, l_m}{\mathbb{E}[N_{tot}^2]}.$$

TABLE I: Simulation parameters

λ_n	3.0 per km square
λ_u	400 per unit square
l_m	10
C	1 Mb/s
W_{RB}	180 kHz
Path-loss exponent	from 2 to 5
Network radius	1,8 km

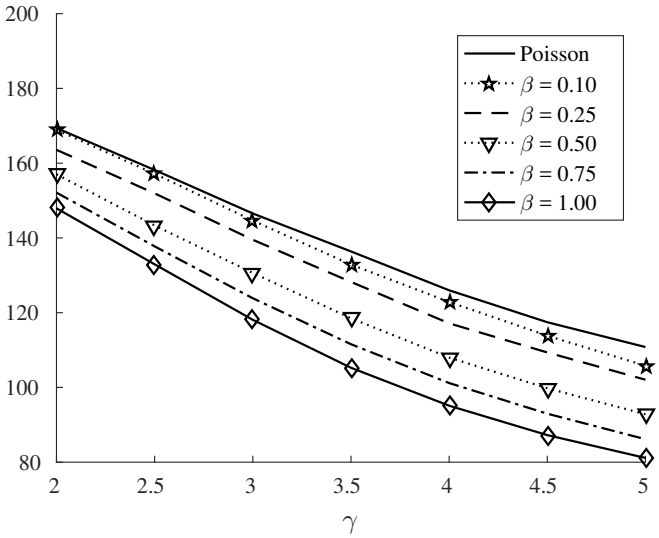


Fig. 1: $\mathbb{E}[N_{tot}]$ for a loss-probability of $\epsilon = 10^{-2}$ in function of γ

III. Simulations

In this section, the previous theoretical analysis are confirmed by numerical simulations. We also explore the influence of the regularity of the deployment on resource block allocation and loss probability in the network. Simulation parameters are summarized in Table I. The values λ_n and λ_u are chosen to match a busy urban environment. Users are assumed to be identical and ask for a download throughput of 1 Mb/s.

Figure 1 illustrates Theorem 1, where the mean number of RB needed decreases dramatically from 170 to 120 with γ. Such results are coherent with the fact that this study focuses on interference limited systems. Since radio propagation conditions becomes harsher with a high value of γ, interfering signals are therefore less strong compared to the main signal, granting more surface of the cell a higher SNR. Likewise, Figure 3 shows that for a fixed number of RBs per cell, the loss probability decreases with γ, as stated in Theorem 2. Figure 2 shows that the approximation given by the inequality concentration stated in Theorem 3 is as expected pessimistic and over estimate the exact result by 10%. The authors in [3] have shown that the β-Ginibre Point Process (β-GPP) provides a robust model for fitting real deployment. The PPP indeed fails to catch the underlying repulsiveness that exists between base station sites, since location of the points are considered to

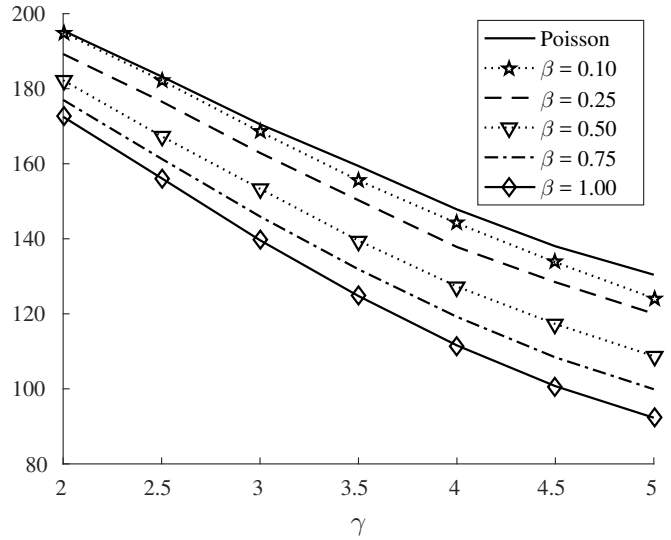


Fig. 2: $\mathbb{E}[N_{tot}]$ for a loss-probability of $\epsilon = 10^{-2}$ in function of γ

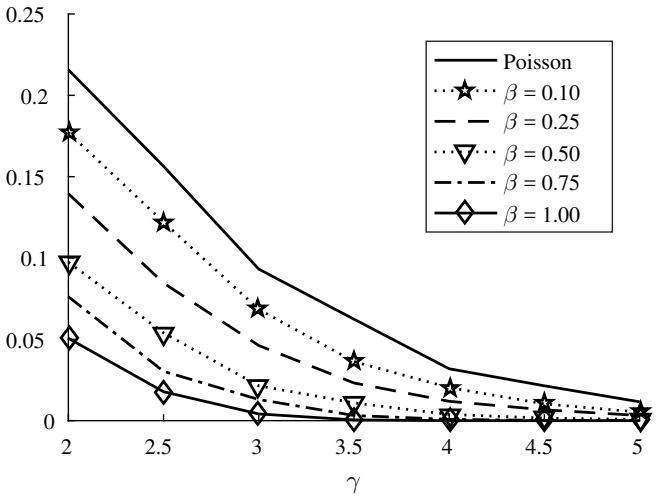


Fig. 3: Loss-probability for $N_{tot} = 100$ in function of γ

be independent with one another. Therefore, cluster of points might occur. A network deployed such as a PPP is qualified as uniform. On the contrary, a network where clusters occur with infinitesimal probability is qualified as regular. The β-GPP stands in between, thanks to β. When β goes to zero, the β-GPP converges to a PPP in law. However, the 1-GPP is the Ginibre Point Process. For each figure, the worst case scenario is held by the PPP whereas the best case scenario is given by the 1-GPP. The SIR is indeed improved in the network with β going to one as shown in [5]. Figure 1 and Figure 2 show that the improvement between PPP and 1-GPP varies between 12% and 26% depending on the value of γ. Likewise, the loss-probability is improved up to a factor of four between the PPP and 1-GPP.

IV. Conclusion

In this paper, the impact of the path-loss exponent on network dimensioning has been studied. Theoretical and numerical analysis demonstrated that in a PPP deployed network, a higher path-loss exponent has a beneficial effect on network performances under interference limited regime. Numerical analysis performed with β-GPP deployed networks has shown that spatial regularity has a dramatic effect on increasing network performances. Future works will aim at understanding the role of the noise in dimensioning cellular networks.

Appendix A

Influence of γ on the coefficients α_l.

According to the definition of the family $(\alpha_1, \ldots, \alpha_{l_m})$, α_1 and α_{l_m} are strictly monotonously increasing and decreasing respectively for $\gamma \geq 2$. In order to prove that the coefficients α_l for small l become preponderant in the weight sums, one must consider the pairwise difference:

$$\begin{aligned} d_l(\gamma) &= \alpha_{l+1} - \alpha_l, \\ &= \frac{\gamma \sin(2\pi/\gamma)}{2\pi} \left(2t_l^{-2/\gamma} - t_{l-1}^{-2/\gamma} - t_{l+1}^{-2/\gamma} \right). \end{aligned}$$

A necessary and sufficient condition for the pairwise differences to be positive is

$$q_l(\gamma) = \left(\frac{t_{l-1}}{t_l} \right)^{-\frac{2}{\gamma}} + \left(\frac{t_{l+1}}{t_l} \right)^{-\frac{2}{\gamma}} \leq 2.$$

A study of the variations of $q_l(\gamma)$ establishes that this function satisfies to the condition if and only if:

$$\frac{t_{l+2}\, t_l}{t_{l+1}^2} \geq 1,$$

The variation study of this ratio proves that this condition is always verified.

Therefore for γ large enough:

1) $\alpha_1 \geq \ldots \geq \alpha_{l_m}$,
2) $\lim_{\gamma \to \infty} \alpha_1 = 1$,
3) $\lim_{\gamma \to \infty} \alpha_l = 0$, with $l \neq 1$.

This proves that the coefficients α_l for small l become preponderant in the weight sums with γ increasing.

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